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David Malament

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Causal Theories of Time and the Conventionality of Simultaneity

DAVID MALAMENT

THE UNIVERSITY OF CHICAGO

1. Adolf Grünbaum maintains that, within the framework of special relativity, the relation of *simultaneity relative to an inertial observer* is conventional rather than factual in character. His argument turns on two assertions:

(1) The relation is not uniquely definable in terms of the relation of causal connectibility.
(2) Temporal relations are non-conventional if and only if they are so definable.

The second assertion constitutes a version of the "causal theory of time".

So far as I know, criticism of Grünbaum's argument has always focused on (2). Michael Friedman ([1]), for example, sees no reason why we must adopt a causal theory of time or, for that matter, any other reductionist analysis of temporal relations. Even if (1) is true, he argues, it does not follow that there is no fact to the matter whether two events are simultaneous relative to a particular inertial observer.

I am entirely sympathetic with Friedman's scepticism concerning (2). But even while avoiding debate over conventionalism and causal theories of time, one has grounds for rejecting Grünbaum's argument. On what seems to me a natural reading, *assertion (1) is false*. In a straight forward sense, the relative simultaneity relation of special relativity *is* uniquely definable from the causal connectibility relation. It is rather ironic. To the extent that one is committed to the "if" clause of (2) one

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should be committed to the *non-conventionality* of the relative simultaneity relation.

Making precise and proving this claim of unique definability is the object of this note. Technically speaking, the result established is a trivial consequence of basic facts about the geometric structure of Minkowski spacetime first noted by A. A. Robb.[1] Unfortunately, most philosophers have couched their discussions of the relative simultaneity relation in terms of ϵ-values and coordinate transformations, rather than invariant four-dimensional structure. In doing so they buried Robb's insights.

2. Two point events are said to be *causally connectible* if and only if it is possible for a photon or particle with non-zero rest mass to travel between them (in either direction). One standardly characterizes the causal connectibility relation in terms of a mathematical model. Suppose $p = (p_0, p_1, p_2, p_3)$ and $q = (q_0, q_1, q_2, q_3)$ are points in R^4. The Minkowski inner product on R^4 is defined by

$$(p, q) = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3 .$$

The inner product induces a norm on R^4 *defined by* $|p| = (p, p)$ and a symmetric two-place relation κ defined by $p \kappa q \equiv |p - q| \geqslant 0$. It is a basic assumption of the theory of special relativity that the class of all point events under the relation of causal connectibility is isomorphic to (R^4, κ).[2] To simplify notation, it is convenient to suppress the distinction between the two structures and refer directly to κ as the causal connectibility relation. This will be done in what follows.

Our problem, then, is that of determining what would-be "simultaneity relations" are definable in terms of κ. The problem could be posed in terms of other "causal relations". One standardly considers the relations of *timelike* and *lightlike relatedness*

$$p \tau q \equiv |p - q| > 0$$
$$p \lambda q \equiv |p - q| = 0$$

too. But the three relations κ, τ, and λ are explicitly, first order definable in terms of one another.[3] So it makes no difference which one works with.

Suppose that O is a time-like line representing an inertial

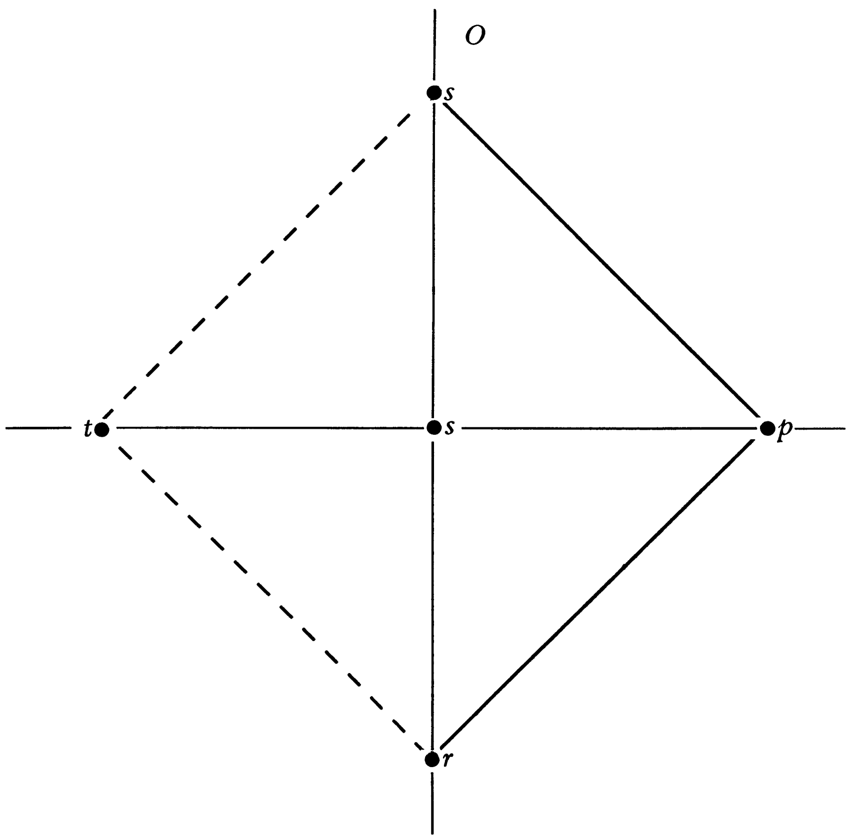

Figure 1'

observer. Let Sim_O be the standard ($\epsilon = \frac{1}{2}$) relation of simultaneity relative to O. Our first claim is that Sim_O is explicitly, first order definable from κ and the relation of membership in O. (One cannot expect to define simultaneity *relative to O* without making reference to O.) It is convenient to state this claim in the following form.

Proposition 1 Suppose Orth is the four-place relation defined by $\mathrm{Orth}\,(p, q, r, s) \equiv (p\text{-}q, r\text{-}s) = 0$. Then:

(i) $\mathrm{Sim}_O\,(p, q)$ iff $(\exists r)(\exists s)(r \neq s\ \&\ r \in O\ \&\ s \in O\ \&\ \mathrm{Orth}\,(p, q, r, s))$
(ii) Orth is explicitly, first order definable in terms of κ.

Discussions of the relative simultaneity relation conducted solely in terms of ϵ-values tend to obscure the fact that the relation, conceived geometrically, is nothing but orthogonality relative to O. This is what is asserted in (i). To see the connection between ϵ-values and orthogonality it suffices to perform a simple calculation. (An elaboration of the computation would constitute a proof of (i).) Suppose that r, q, s are distinct points on O satisfying: (a) $q = r + \epsilon(s - r)$ where $0 < \epsilon < 1$. Suppose further that p is a point not on O satisfying: (b) $r \lambda p$ and $p \lambda s$. Then it is easy to check that:

$$(*) \qquad (q\text{-}p, s\text{-}r) = 0 \text{ iff } \epsilon = \tfrac{1}{2}.$$

From (b) we have:

$$\begin{aligned} 0 &= (s\text{-}p, s\text{-}p) = (s\text{-}r\text{+}r\text{-}p, s\text{-}r\text{+}r\text{-}p) \\ &= (s\text{-}r, s\text{-}r) + 2(s\text{-}r, r\text{-}p). \end{aligned}$$

From this and (a) we have:

$$\begin{aligned} (q\text{-}p, s\text{-}r) &= (r\text{+}\epsilon(s\text{-}r)\text{-}p, s\text{-}r) = (r\text{-}p, s\text{-}r) + \epsilon(s\text{-}r, s\text{-}r) \\ &= (\epsilon\text{-}\tfrac{1}{2})(s\text{-}r, s\text{-}r). \end{aligned}$$

Hence (*) follows from the fact that $(s\text{-}r, s\text{-}r) > 0$.

Part (ii) of Proposition 1 is (essentially) due to Robb.[4] Details of the construction can be found in Robb's book and in John Winnie's exposition ([4]). At least in the very special case where p, q, r, s are as in the figure, $\mathrm{Orth}\,(p, q, r, s)$ can be expressed in the following simple form:

$$\begin{aligned} \mathrm{Orth}\,(p, q, r, s) \equiv\ & (\exists t)\,(t \lambda s\ \&\ t \lambda r\ \& \\ & \sim(\exists u)\,(u \lambda t\ \&\ u \lambda q\ \&\ u \lambda p)). \end{aligned}$$

The imbedded condition '$\sim(\exists u)(u\lambda t \;\&\; u\lambda q \;\&\; u\lambda p)$' asserts that the light cones of the three points p, q, t do not have a common point of intersection. For triples of points no two of which are causally connectible, this condition is equivalent to collinearity.

3. It remains to show that the relation Sim_O is the *only* would-be relative simultaneity relation which is definable from κ and O. To be sure, there are other two-place relations which are definable from κ and O. But all these are ruled out if minimal, seemingly innocuous conditions are imposed.

Suppose S is a candidate for the relation of "simultaneity relative to O". Whatever else is the case, one may argue, S should at least be an equivalence relation. It should also not be vacuous. It should render at least *some* point on O "simultaneous" with *some* point not on O. And it should do so without rendering absolutely *all* points "simultaneous". Our claim is that Sim_O is the only relation which satisfies these conditions while being definable from κ and O (in any sense of "definable" no matter how weak).

To make the claim precise we need a few definitions. A bijective map f: $R^4 \to R^4$ is a *causal automorphism* iff for all points p and q in R^4, $p\kappa q$ iff $f(p)\kappa f(q)$. If in addition f satisfies $p\in O$ iff $f(p)\in O$, then f is an *O causal automorphism*. If an n-place relation is definable from κ and O, in any sense of "definable" no matter how weak, then it will certainly be preserved under all O causal automorphisms. So it is useful to work with a notion of implicit definability. We say that an n-place relation R is *implicitly definable from κ* (respectively *implicitly definable from κ and O*) iff for all causal automorphisms (respectively all O causal automorphisms) f and all points $p_1, \ldots, p_n$:

$$R(p_1, \ldots, p_n) \text{ iff } R(f(p_1), \ldots, f(p_n)).$$

Proposition 2 Suppose S is a two-place relation on R^4 where

(i) S is (even just) implicitly definable from κ and O;
(ii) S is an equivalence relation;
(iii) S is non-trivial in the sense that there exist points $p\in O$ and $q\notin O$ such that $S(p, q)$.

Then S is either Sim_O or the universal relation (which holds of all points).

To prove the proposition one need only keep in mind what the class of O causal automorphisms looks like. It includes all rotations, translations, and scalar expansions which map O onto itself. It also includes all reflections of R^4 with respect to hypersurfaces orthogonal to O which map O onto itself.

Proof By (iii) there exist points $p \in O$ and $q \notin O$ such that $S(p, q)$. There are two cases to consider: either $\mathrm{Sim}_O(p, q)$ holds or it does not. Suppose Q is the set of points r such that there exists an O causal automorphism f with $f(p) = p$ and $f(q) = r$. If $\mathrm{Sim}_O(p, q)$ holds then Q is just the hypersurface orthogonal to O containing p. If $\mathrm{Sim}_O(p, q)$ does not hold then Q is a "double cone" with vertex p which (in general) is distinct from the null cone with vertex p. Since S is preserved by all O causal automorphisms, we have (in both cases) that $S(p, r)$ for all r in Q.

Suppose first that $\mathrm{Sim}_O(p, q)$ does *not* obtain. We show that $S(p, v)$ for *all* v in R^4. It will follow that S must be the universal relation. Let v first be an arbitrary point on O. There is an O causal automorphism such that $f(p) = v$. Function f maps Q onto a double cone $f[Q]$ with vertex $f(p)$ which is congruent to Q. Since f preserves S it follows that $S(f(p), r)$ for all $r \in f[Q]$. Now Q and $f[Q]$ must intersect in some point w. So we have $S(p, w)$ and $S(f(p), w)$. Hence $S(f(p), p)$; i.e. $S(v, p)$. Now let v be any point not on O. There is an O causal automorphism f where $f(q) = v$. Since f preserves S we have $S(f(p), f(q))$; i.e. $S(f(p), v)$. But $f(p) \in O$ and so (invoking the first half of this argument) $S(p, f(p))$. Hence $S(p, v)$.

Next suppose that $\mathrm{Sim}_O(p, q)$ *does* obtain. It follows that $\mathrm{Sim}_O \subseteq S$. For suppose $\mathrm{Sim}_O(r, s)$. Then there exists an O causal automorphism f (it is just a translation) such that $f(r)$ and $f(s)$ belong to Q. Hence $S(p, f(r))$ and $S(p, f(s))$. Therefore $S(f(r), f(s))$ and $S(r, s)$. Now if $S \not\subseteq \mathrm{Sim}_O$ then there must exist points r and s such that $S(r, s)$ but not $\mathrm{Sim}_O(r, s)$. Furthermore we may assume that exactly one of the points r and s lies on O. (If r and s are both on O then we can work with the points s and $f(q)$ where f is an O causal automorphism such that $f(p) = r$. If neither r nor s is on O then we can work with s and $f(p)$ where f is an O causal automorphism such that $f(q) = r$.) But now the argument from the previous paragraph is applicable (with r and s playing the roles of p and q). It follows that S must be the universal relation. Q.E.D.

Propositions 1 and 2 together make precise the sense in which the relative simultaneity relation of special relativity *is* uniquely definable from the causal connectibility relation. In contrast, of course, no "absolute" simultaneity relation is definable from that relation. This fact is made precise in the following simple proposition.

Proposition 3 Suppose S is a two-place relation of R^4 where

(i) S is (even just) implicitly definable from κ;
(ii) S is an equivalence relation;
(iii) S is non-trivial in the sense that there exist distinct points p and q such that $S(p, q)$.

Then S is the universal relation (which holds of all points).

Proof By (iii) there exist distinct points p and q such that $S(p, q)$. We may as well suppose that p and q are spacelike related. (If p and q are timelike or null related than a parallel argument will apply.)

Now all translations, scalar expansions, and Lorentz transformations (i.e. linear bijections which preserve the norm $|\ |$) are causal automorphisms of R^4. So given any pair of spacelike related points $\{r,s\}$, there is a causal automorphism f where $f(p) = r$ and $f(q) = s$. By (i) it follows that $S(r, s)$. Thus $S(r, s)$ holds for *all* spacelike related points r and s.

Finally let u and v be any points whatsoever. There must exist a point t which is spacelike related to both u and v. Hence $S(u, t)$ and $S(v, t)$ and, therefore, $S(u, v)$. Thus S must be the universal relation. Q.E.D.

Proposition 3 pinpoints the difference between the situation in special relativity and in classical physics.

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[3] A. A. Robb, *A Theory of Time and Space* (Cambridge: Cambridge University Press, 1914).
[4] J. Winnie, "The Causal Theory of Minkowski Spacetime," forthcoming in *Minnesota Studies in the Philosophy of Science*.

NOTES

[1] Cf. [3]. Discussions of Robb's work can be found in [2] and [4].

[2] Notice that in the formulation of this assumption no reference is made to clocks, rigid rods, or the velocity of light as measured in different directions by inertial observers. Furthermore, at least in some idealized sense, one can justify the adoption of the assumption without reference to any of these things. One can do so by means of a representation theorem. In a suitable second order formal language containing a single two-place predicate symbol (for κ) one can formulate a finite set of axioms every model of which is isomorphic to (R^4, κ). One can think of these axioms as "laws of causal connectibility" and then their adoption commits one to the basic assumption. This representation theorem is proven, more or less, in Robb's book.

[3] For example, $p\lambda q$ is equivalent to:

$$p\kappa q \ \& \ (p = q \ \vee \ (\exists r)(r \neq p \ \& \ r \neq q \ \& \ (s)(s\kappa p \ \& \ s\kappa q \rightarrow s\kappa r))).$$

So λ is explicitly first order definable in terms of κ.

[4] The qualification is necessary since Robb used an asymmetic relation 'after' in his construction rather than one of our symmetric relations. Also he did not worry about the niceties of first order (as against higher order) definitions. This qualification also applies to the attribution made above in Note 2.