TIME, LENGTH, AND MASS CHANGES IN A GRAVITATIONAL FIELD

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Seen from afar, time dilates, rulers contract, and masses increase in the induced field inside either a hollow accelerating cylinder or the equivalent GM/r^2 field. Frame dragging causes these effects to be due to velocity when viewed close at hand.

In a recent letter Brans [1] takes issue with Einstein's statement [2] that general relativity predicts a mass increase by $1+\bar{\sigma}$ near ponderable bodies, and is therefore in harmony with Mach's Principle. He argues that Einstein was thinking of a locally detectable effect. According to Brans, $\bar{\sigma}$ cannot be detected locally (and experiments agree) [3], but he leaves the impression that it might be detectable by non-local means. The following simple thought experiments in three dimensions (based on induced g fields) show that in a GM/r^2 field (for $GM/rc^2 \ll 1$) local observers (by local measurements) see no time dilation, no length contraction, and, indeed, no increase in inertial mass. Distant observers, however (using telescopic measuring devices) see time dilation, radial length contraction, and an increase in inertial mass that is anisotropic.

It was recognized long ago that accelerating masses should produce a g field by induction in a manner analogous to producing an E field by induction [4]. An argument by Good [5] is very convincing and bears repeating in part. In fig. 1, a long line of charges is set in motion with constant acceleration at $t = 0$. At a time $t = r/c$ later, a test charge at P sees that all the charges from A to B have moved to the right, but since information has not arrived from the remainder, they must appear where they were at $t = 0$. This is approximately the same thing as adding $+q$ at B and $-q$ at A, and Good shows that the dipole field at P is just the familiar induced field $E = -\mathrm{d}A/\mathrm{d}t$. Hence the arrangement in fig. 1 is basically a simple transformer. Then, by substituting mass points for charges, Good shows that an induced field of opposite sign must appear at P.

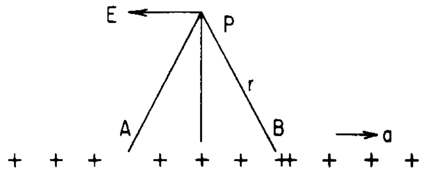

Fig. 1. Long row of accelerating charges.

Such being the case, fig. 1 (with mass points for charges) describes a simple gravitational transformer. If the speed, v_g, of gravitational effects is c, g will be closely analogous to E. If $v_g \neq c$, but is finite, the closeness becomes more remote, but a finite induced g field will still exist.

Let us replace the charges in fig. 1 by a large hollow cylinder of charges (several kilometers long) accelerating axially at a rate a along the z axis. This cylinder looks like the primary of a coaxial high frequency transformer, where the static E and B fields are zero inside, and $E_z = -\mathrm{d}A/\mathrm{d}t$ is uniform (except near the ends). If we replace the charges by mass points we can conceptually think of accelerating this large hollow mass-point cylinder (with no change in mass point spacing, and with no local perturbing masses) by simply attaching long massless strings to each mass point. By analogy with the electrical case, a gravitational vector potential A_g will exist in the z direction, and we will have an induced field $g_z = \mathrm{d}A_g/\mathrm{d}t$ that is uniform for many kilometers throughout the interior, but falls off as $1/(x+y)^{1/2}$ just outside.

In this uniform induced field, scattered mass points should fall as a rigid body. Hence we can assume (and justify shortly) the existence of a large volume spatially uniform accelerating intertial frame. For a *first* thought experiment we will need observers in this accelerating frame. Before the onset of acceleration at $t = 0$, they will assemble (along the z direction) two very long rows of touching rulers (each cemented to a clock) as shown in fig. 2a. At $t = 0$ both rows are stationary and all clocks read zero.

At the onset of acceleration, row B will be restrained to remain at rest. Everything else, however, will fall freely along $-z$, so, from a distance we will see row A and the observers in the uniform inertial frame (hereinafter called frame A) start accelerating at a rate $a' \ll a$. Furthermore, we will see the rulers at the far ends of row A (kilometers apart) start accelerating in the uniform field at exactly the same rate. Although the local observers can see the stars (through the sieve-like walls of the cylinder) accelerating in the opposite direction, their local acceleration is not detectable by local means because every atom in the g_z field accelerates in exactly the same way, and the atom-to-atom spacing remains unchanged in the local falling inertial frame. Hence the rulers remain *touching*, and if (from afar) we should see contraction of the rulers in row A, it must be by contraction of the row as a *whole*. But such contraction, if it should exist, would force different accelerations at the two ends of row A (kilometers apart), and this *cannot* happen if g_z is uniform throughout. Hence (from afar) we see the rulers in row A accelerate *with the inertial axes* as a rigid body, and, by contrast with special relativity, where contraction is due to motion *with respect* to the axes, we see no contraction of A's rulers even though, with constant acceleration in the g_z field, they can attain high velocities as seen by us at our distant location.

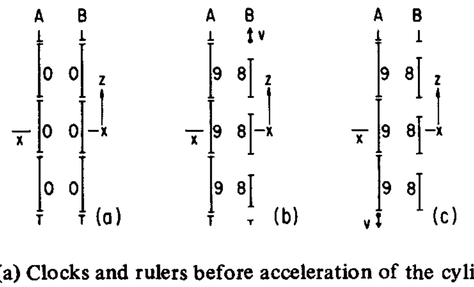

Fig. 2. (a) Clocks and rulers before acceleration of the cylinder (cylinder not shown). (b) View of B rulers, held fixed with respect to the distant stars (as seen by observers in frame A). (c) Our view (from afar) of A's uncontracted rulers accelerating past slow clocks and contracted rulers at rest in row B.

In row B, however, the rulers *will* contract, because the observers in row A have been instructed to exert forces that will keep the *lower* end of each B ruler *at rest* with respect to *us* at our distant location near the fixed stars. Hence the frame A observers will see each B ruler speed up axially and shrink (not along x or y) but along z by $1/\gamma = (1 - v_z^2/c^2)^{1/2}$, and, because each ruler must cover the same distance in the same time (as measured by the A rulers and synchronized clocks), the A observers will see *spaces* appear, as shown in fig. 2b. The A observers also see B's clocks losing time by the factor $1/\gamma$. At some later time, when B's rulers have shrunk to 3/4 of their initial value, the A observers will see B's clocks ticking at 3/4 of their initial rate, and, starting from a reading of zero, as shown in fig. 2a, the A observers might (depending upon how the acceleration was programmed) see an integrated value of 8 arbitrary time units on B's clocks while the clocks in their frame (row A) all read 9, as shown in fig. 2b. A picosecond laser could easily photograph a few of the contracted rulers and paired clock readings. This flash technique, described by McGill [6], avoids Terrell's rotational effects [7]. The picture will look like fig. 2b, and shows that whenever a B clock reads 8, it must be alongside an A clock reading 9.

Observers on the rulers in row B however, see a very different picture. Since they are the ones that feel the forces that accelerate them with respect to the falling inertial frame A, they see row A contract *as a whole* with *no* spaces between A's rulers. This view (of no use to us) is included for completeness.

From afar (through a telescope) we see the picture in fig. 2c. We will see it in a slightly different location from the true location because every ray from the two rows, while emerging from the cylinder, will fall with the inertial axes and will be bent through the same angle. This does not matter. B's rulers will appear *at rest* with respect to us; so, as a function of time, while row A accelerates along $-z$ as a rigid body, we will see B's rulers shrink and spaces open up. At a later instant (since there is no preferred B clock) we see *all* of them reading 8, and, as seen in fig. 2b, they *must* be alongside A clocks reading 9 at this instant, as shown in fig. 2c. Hence we see the same paired readings as the A observers, and this means that (in addition to seeing no contraction for A's rulers) we see no slowing down of A's falling clocks. The ratio of space length to ruler length must be the same for all observers, so, from fig.

2c, we see B's rulers contract and B's clocks slow down by the factor $1/\gamma = (1 - v_z^2/c^2)^{1/2}$. The frame A observers attribute the contraction and time dilation *in motion* to the velocity, v_z, of *row* B with respect to their *inertial axes,* but we see the contraction and time dilation *at rest* as the result of the velocity, $-v_z$ of their *inertial axes* with respect to *row B*. Furthermore, all of these directly visible events can be described by any coordinate system, and therefore are completely *coordinate independent.*

In fig. 2c we see the inertial axes falling past B's rulers, but we also realize there are forces holding B's rulers at rest in the induced field, and since this field is the cause of the accelerating axes, we can (alternatively) invoke this field as the cause of B's contracted rulers and slow running clocks. This immediately makes us think of equivalence arguments. Just as a charge cannot locally distinguish between induced and static E fields, a material particle should not be able to locally distinguish between induced and static g fields. So let us set the stage for a *second* thought experiment. We will assemble (in free space) a long row of identical touching rulers (each cemented to a clock), and then, as we hold the *lower ends fixed* with respect to the distant stars, another collaborator will superimpose a $1/r^2$ field by placing a mass M at one end where $r = 0$. Then, using a telescope with a very restricted field of view, we will follow a particle for a limited time and distance as it falls past these rulers. At the same time, with another telescope, we will follow (inside the accelerating mass-point cylinder) another particle as it falls past the *stationary* rulers and clocks in row B that are shrinking and slowing down (respectively) as a function of *time*. By means of mirrors, the two very limited fields of view, showing both particles (each with a ruler and clock instantaneously alongside) will be displayed side by side, as shown in fig. 3. If we program the cylinder's acceleration such that we see both falling particles cover exactly the same distance at exactly the same rate, then we know the induced and real fields (at the *locations* shown in the two fields of view) must have the same value. Inside the cylinder, *as the fall velocity increases,* we see the particle falling past stationary rulers that get shorter and shorter, and clocks that run slower and slower, so we know that in the static field, *as the fall velocity increases,* we must see the other particle falling past stationary rulers that get shorter and shorter and clocks that run slower and slower.

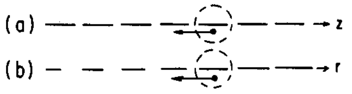

Fig. 3. (a) Particle falling past stationary rulers in row B that all contract at the same rate with time. (b) Particle falling past stationary rulers that contract with decreasing r in the field of M. Everything looks identical in the two fields of view as the telescopes follow the two particles.

Furthermore the contraction and slowing of time in the GM/r^2 field must correspond exactly to what we see in the induced field, otherwise it becomes possible, by viewing *local* behavior, to distinguish between induced and static fields.

The equivalence just described holds over a limited range of r and t, but we can conceptually extend this range and imagine free-fall from $r = \infty$ in the field of M (corresponding to zero induced field inside the cylinder). As the particle picks up speed, the contraction inside the cylinder (for $v_z/c \ll 1$), is given by $1/\gamma = (1 - v_z^2/2c^2)$. For the same fall velocity from ∞ in the real GM/r^2 field, we have $\frac{1}{2}mv^2 = GMm/r$, so the factor $1/\gamma$ is equivalent to $(1 - GM/rc^2)$. This means that, (with a powerful telescope) we could actually *see* clocks and *radial* rulers, *at rest* in a weak GM/r^2 field, slow down and contract (respectively) by the factor $(1 - GM/rc^2)$ by comparison with clocks and rulers at infinity. Azimuthal rulers, of course, would be unaffected. (Previous thought experiments that show contractions in a g field without invoking induction have been severely criticized [8].)

We also have mass changes to consider. The transverse mass of a high-speed moving body is usually found in a thought experiment where a transverse action–reaction impulse is applied, at the right instant, between the moving mass and an identical mass at rest. Transverse velocities, $v_\perp$, are the same in both frames (as measured by local clocks and rulers), but due to time dilation we see $v'_\perp$ as $v_\perp/\gamma$. Conservation of momentum must hold for everything we see in our frame. Hence we see $m'_\perp = m$.

In a *third* thought experiment we will use a variation of this method to find the mass, at rest, of a central B ruler in fig. 2c. A collaborator at a great distance along x, will give an identical ruler a velocity $-v_x$ toward ruler B. If the velocity change due to the static field of the accelerating cylinder is subtracted out, $-v_x$ should not change (as measured by local clocks and rulers). But we (at a great distance along y) see time slow down by $1/\gamma$ as the ruler enters the cylinder (through a mass-point-free-slot) and approaches ruler B. An instant before impact the force holding ruler B at rest will be released, so, at impact (using our own clocks, and the distance scales in our telescope eyepiece) we will see ruler B acquire a velocity $-v_x/\gamma$. We know that momentum conservation must hold for everything we measure in our frame. Hence we see a transverse mass $m'_x = \gamma m$. The mass experiment, with an initial ruler velocity v_z or $-v_z$ (starting from a great distance along $-z$ or $+z$) will (after subtracting out changes due to the static field and the accelerating axes) will give ruler B a velocity $\pm v_z/\gamma^2$, because we see stationary lengths (e.g. B's rulers) contracted along z by $1/\gamma$. Hence the longitudinal mass of a stationary ruler in row B must be given by $m'_z = \gamma^2 m$. (By contrast, the longitudinal mass of a moving body in special relativity is $m'_{\parallel} = \gamma^3 m$ [9]. By the equivalence set up in our second thought experiment we must see these same transverse and longitudinal masses for objects *at rest* in the field of a mass M, except that γ and γ^2 are replaced by $(1 + GM/rc^2)$ and $(1 + 2GM/rc^2)$, respectively. In summary, *as seen from afar*, the clock rate in a GM/rc^2 field will be $R' = R(1 - GM/rc^2)$, the length of a radial ruler will be $L' = L(1 - GM/rc^2)$, and transverse and longitudinal masses will be $m'_{\perp} = m(1 + GM/rc^2)$ and $m'_{\parallel} = m(1 + 2GM/rc^2)$, respectively. As seen by local observers (using local clocks and rulers) none of these effects are observable.

The induced field and the acceleration of frame A cannot be maintained forever, and this leads to our *final* thought experiment. Eventually $g_z \to 0$; and with respect to us, the velocity of frame A (and row A) levels off at a constant value $-v_z$. Observers in frame A, in turn, see row B coasting at constant v_z relative to their axes; so they see B's clocks running slowly, and B's rulers contracted by a factor $1/\gamma$ that no longer changes with time. From afar, we see A's uncontracted rulers moving with velocity $-v_z$ past B's slow stationary clocks, and contracted rulers (and spaces), The induced and static fields are both zero, so it appears that the history of immersion in a field is needed to produce time dilation and length contraction, but the field is not needed to maintain such a state. The state is maintained by the velocity of the inertial frame past B's clocks and rulers. (In a future communication this behavior will be related to the region inside a mass shell.) Strange things happen if we decide to travel to row B and see all this close at hand. The static field along x should not affect lengths along z, so we might think that with no induced field along z, there should be no forces exerted along z, and no z contraction. Hence we should be able to approach a ruler in row B and find it shortened with respect to ours, even though it was originally identical. This, of course, does not make sense. Hence we must conclude that as we approach the stationary rulers in row B, our inertial axes will be dragged into the frame of row A, and consequently we will see the contraction of B's rulers as due to relative velocity. This frame dragging effect, produced by a linearly *coasting* mass distribution, is not emphasized in the usual interpretations of Mach's Principle [10].

Many lengthy and fruitful discussions with Professor Alfredo Banos and recent discussions with Professor Kimball Milton have helped immensely in formulating the thought experiments described in this paper.

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