Teaching gravitational waves: a lesson in heuristic (mis)understanding of how an interferometer detects gravitational waves

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LIGO-G060362-00-Z

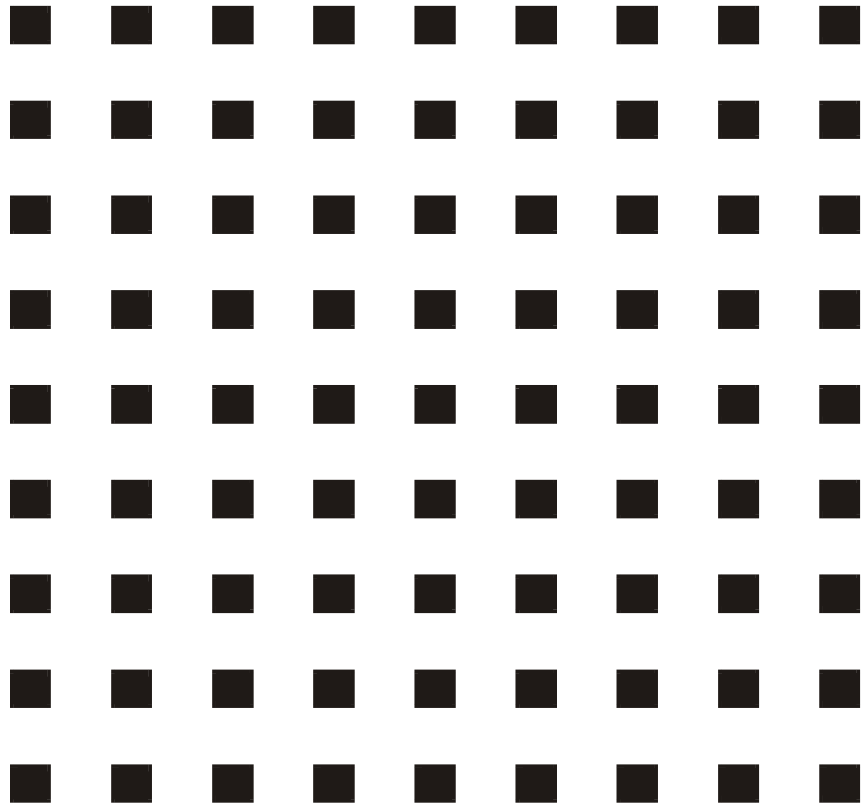
Outline

- The heuristic (and mostly right) picture
- A correct calculation, right in all cases
- What does the calculation mean?
  - Do test masses move in response to a gravitational wave?
  - Do light waves stretch in response to a gravitational wave?
  - If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?
- How to avoid confusion (in this case)
- A lesson about heuristics in general relativity?

A heuristic picture

Mostly right …

A set of
freely-falling test masses



A gravitational wave meets some test masses

- Transverse

  No effect along direction of propagation

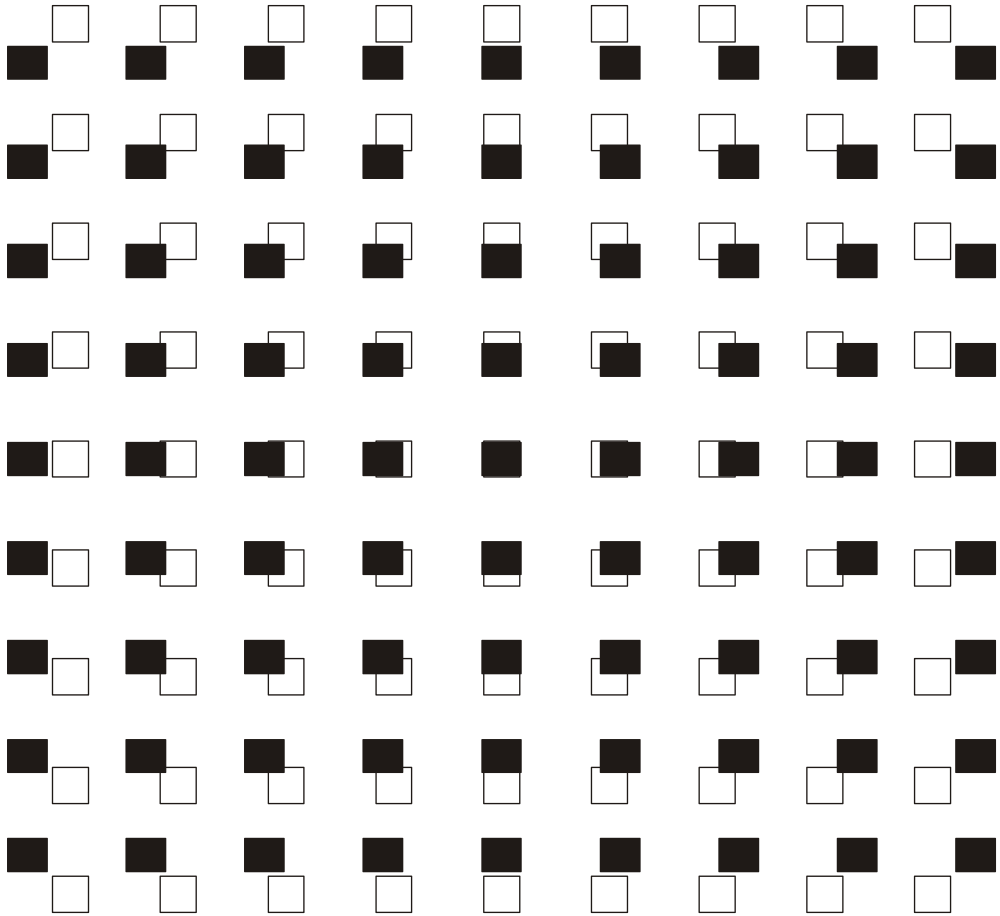
- Quadrupolar

  Opposite effects along *x* and *y* directions

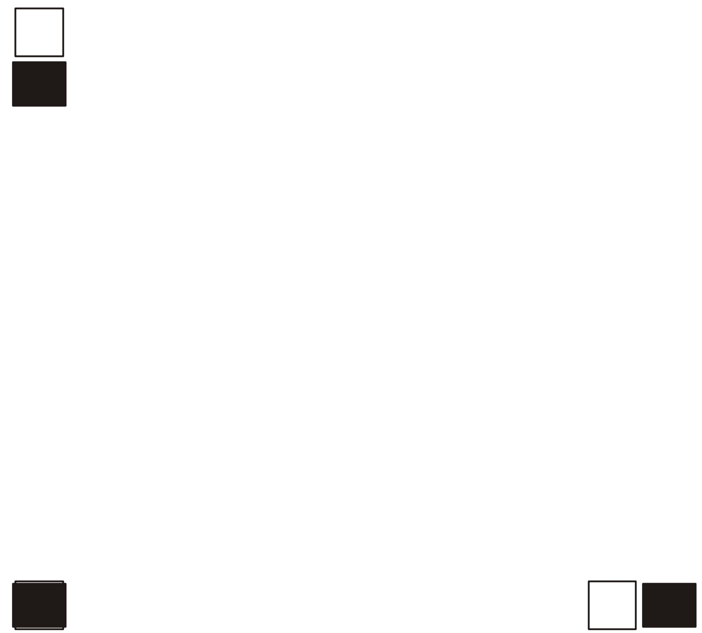
- Strain

  Larger effect on longer separations

$$h \equiv 2\frac{\Delta L}{L}$$



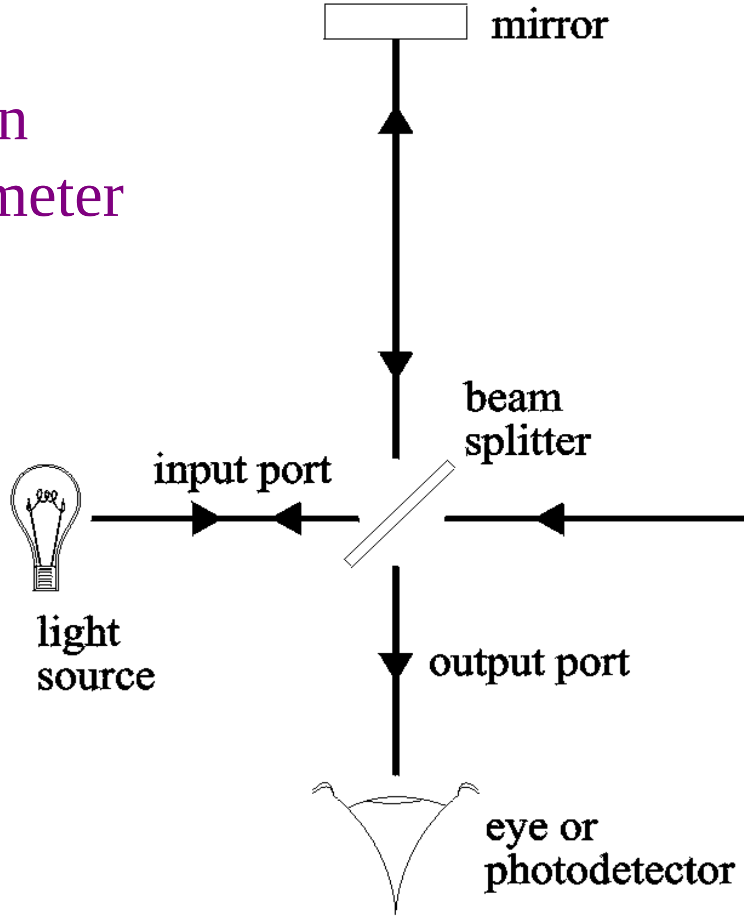
The interferometer detection strategy



Since the wave amplitude is a *strain*, this argues for test masses as far apart as practicable. We use masses hung as pendulums, kilometers apart.

Sensing relative motions of distant free masses

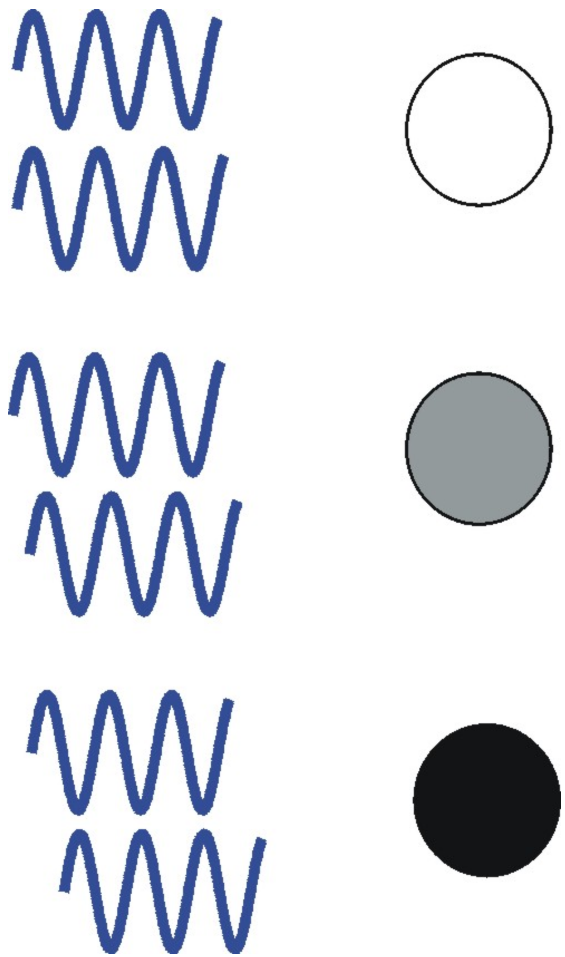
Michelson interferometer



A transducer from length difference to brightness

Wave from x arm.

Wave from y arm.



Light exiting from beam splitter.

As relative arm lengths change, interference causes change in brightness at output.

A correct calculation

Can be generalized to be right in all cases …

- Arms of arbitrary length
- Waves of arbitrary waveform (frequency)
- Arrival from arbitrary angle

Distance measurement in relativity…

… is done most straightforwardly by measuring the light travel time along a round-trip path from one point to another.

- Because the speed of light is the same for all observers.

Examples:

- light clock
- Einstein’s train *gedanken* experiment

The *space-time interval* in special relativity

Special relativity says that the interval

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

between two events is *invariant* (and thus worth paying attention to.)

In shorthand, we write it as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$
with the Minkowski *metric* given as

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Generalize a little

General relativity says almost the same thing, except the metric can be different.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The trick is to find a metric $g_{\mu\nu}$ that describes a particular physical situation.

The metric carries the information on the space-time curvature that, in GR, embodies gravitational effects.

Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

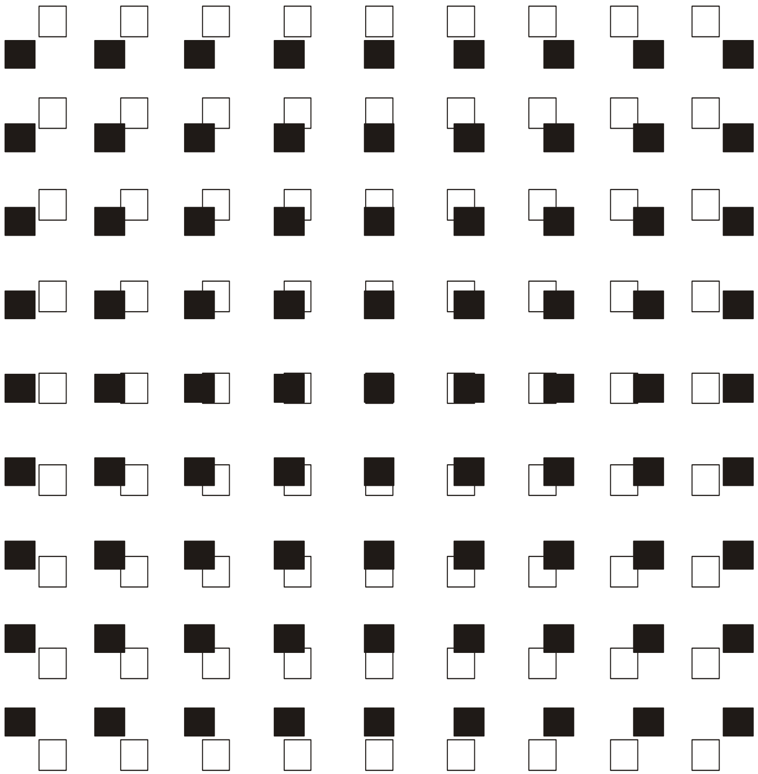
with a wave propagating in the *z*-direction described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Two parameters = two polarizations

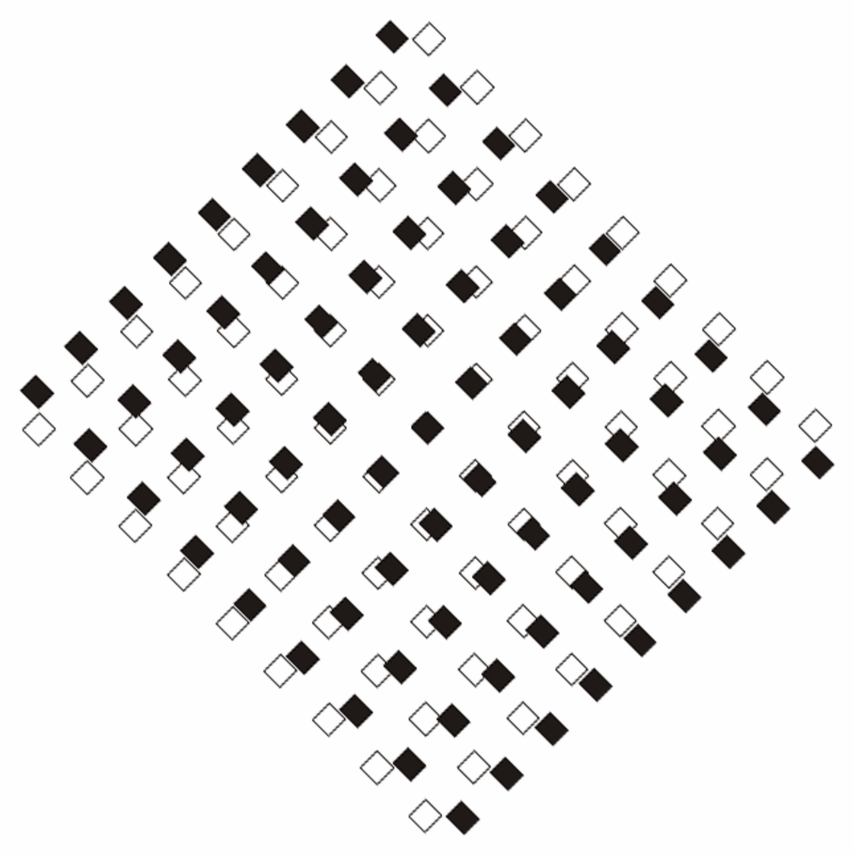
Plus polarization

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Cross polarization

$$\hat{h}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Solving for variation in light travel time: start with *x* arm

$$ds^2 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2 = 0$$

- First, assume *h(t)* is constant during light's travel through interferometer.
- Rearrange, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11}\right) dx$$

and integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$

Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

"Transverse-traceless (TT) gauge"

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

y-arm ($h_{22} = - h_{11} = -h$):

$$\Delta t_y = -hL / c$$

Difference between x and y round-trip times:

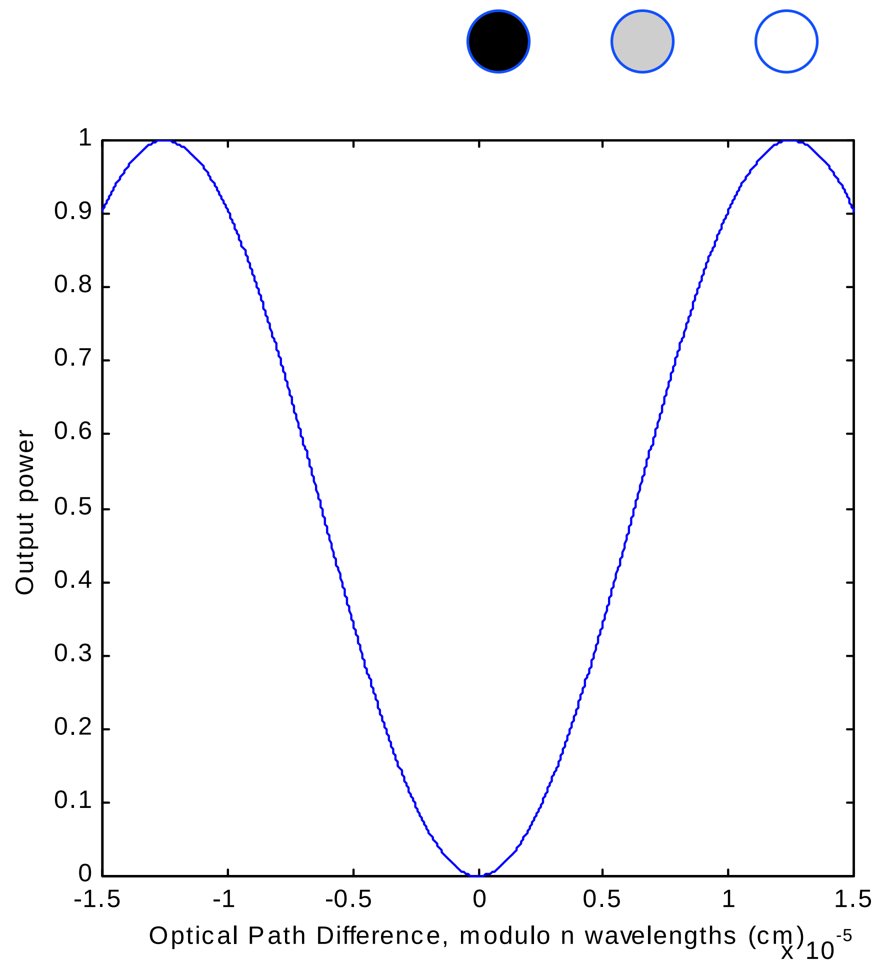
$$\Delta \tau = 2hL / c$$

Expressed as a phase difference

It is useful to express this as a <u>phase</u> difference, dividing time difference by radian period of light in the interferometer:

$$\Delta\phi = h\,\tau_{roundtrip}\,\frac{2\pi c}{\lambda}$$

Interferometer output vs. phase difference



What does the calculation mean?

- Do test masses move in response to a gravitational wave?
- Do light waves stretch in response to a gravitational wave?
- If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?

Do test masses move in response to a gravitational wave?

Yes and no.

No, if you describe physics in the coordinate system that we used to do the calculation. That used the *transverse traceless gauge*, and in that description freely-falling masses define the coordinates. Only a non-gravitational force can cause a mass to move, *i.e.* to change its coordinates.

Yes, if you describe physics the way we normally do in the laboratory, by defining coordinates with marks on a rigid rod.

This description doesn't work well when distances are long …

Do light waves stretch in response to a gravitational wave?

Yes and no, for the same reasons as for the previous question.

In the ordinary physical description, we'd say that the light waves stretch by the same fractional amount *h* as the masses move apart.

This is the same kind of effect as is responsible for the cosmological redshift in the expanding universe.

If light waves are stretched by gravitational waves, …

… how can we use light as a ruler to detect gravitational waves?

After all, we are using a “rubber ruler” that participates in the same distortions as the system whose distortions we are trying to measure?

As the masses move apart or together, light waves stretch or compress.

How, indeed, can the phase of the light waves register the effect of a gravitational wave?

The answer

The answer has to do with the fact that the light waves aren't a static ruler, but are traveling through the arms.

It is true, absolutely, that the *instantaneous* response of the light in an interferometer to a gravitational wave is in fact null.

But, the light travels through arms that are lengthened or shortened. Over the time it takes light to travel through an interferometer arm, the response builds up to the "naïve" amount.

This is what the exactly correct calculation says, too.

A confession

- It took me years to understand this.
- Rai Weiss taught me how to do the correct calculation, and I believed its results.
- But linking it to other heuristic ways of thinking was hard, and I carried some wrong pictures for a long time.
- For instance, I thought that light waves weren’t stretched by gravitational waves, because if they were then I “knew” that then the interferometer wouldn’t respond to a gravitational wave.
- The textbooks that I consulted weren’t much help on heuristics.

What might have helped me

No one explained to me clearly that the transverse traceless gauge corresponded to the coordinates marked by freely-falling masses.

- Although Rai Weiss knew this and used it in calculations.
- A clear statement of this point is often lacking in textbooks.

No one linked the physics of this situation to that of the cosmological redshift.

- There, too, there has also been a lot of confusing language, but people know how to get the right answer.

What did help me was having to answer questions from “cranks”. I finally decided that I needed a clear statement at the heuristic level.

Prospects for doing better in the future

- Relativity started as a pretty abstract subject, with a huge amount of mathematical baggage.
  - Eddington's remark that only a dozen people in the world could understand it.
- This, even though the whole point is to make statements that are true in any coordinate system.
  - And, Einstein proposed three experimental tests.
- As a new golden age of experimental relativity develops, clear interpretations are required. We always develop these as they are needed. Let's just keep remembering that heuristics are important.