A computer program to visualize gravitational lenses

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Gravitational lenses presently play an important role in astrophysics. By means of these lenses the parameters of the deflector such as its mass, ellipticity, etc., and Hubble's constant can be determined. Using C, XFORMS, MESA, and IMLIB, a computer program to visualize this lens effect has been developed. This program has been applied to generate sequences of images of a source object and its corresponding images. It has also been used to visually test different models of gravitational lenses. © *2001 American Association of Physics Teachers.*
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I. INTRODUCTION

Computer visualization is nowadays an important field of scientific research, because it often allows a better understanding of natural phenomena. Gravitational lenses (hereafter GLs), predicted by Einstein's general relativity theory, deflect the light rays coming from a distant object. GLs allow the formation of multiple images of the same source object. Moreover, this gravitational effect opens the possibility of determining the mass of the deflector and the computation of the age of the universe (Hubble's constant), as well as other lens parameters and cosmological parameters.

An interactive computer program to visualize this gravitational effect has been developed. The program was written in the C programing language and uses the XFORMS (tool kit), MESA (graphic library), and IMLIB (image library). The program runs on systems with Linux or Unix. An SGI version of this program is also available.

In Sec. II the gravitational lens effect is briefly reviewed. The visualization program is presented in Sec. III. In Sec. IV some applications are shown: image sequences and easy visual modeling. Improved versions of this program can be produced and the author wishes to invite the interested reader to participate in this process. The author is preparing a website for downloading this program: `http://lia.efis.ucr.ac.cr/~frutos/`

II. THE GRAVITATIONAL LENS THEORY

A. The gravitational lens equation

Due to the curvature of space–time, light rays coming from a distant source object (quasar, star, or galaxy) are deflected when passing close to a lens or deflector (star, galaxy, or cluster of galaxies). For weak gravitational fields the *Post–Newtonian* approximation is applicable. Under this approximation the deflection angle or Einstein's angle does not depend on the direction of propagation and the trajectories are approximated by straight lines. This gravitational light deflection is depicted in Fig. 1. From this figure an equation—the *GL equation* or the *ray tracing equation*[1]—which is obeyed by light rays passing near a lens object can be deduced:

$$\mathbf{y} = \mathbf{x} - \boldsymbol{\alpha}(\mathbf{x}), \tag{1}$$

where

$$\boldsymbol{\alpha}(\mathbf{x}) = \frac{1}{\pi} \int \kappa(\mathbf{x}') \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} \, d^2\mathbf{x}', \tag{2}$$

$$\mathbf{y} = \frac{\boldsymbol{\eta}}{\eta_0} = \frac{D_s \boldsymbol{\beta}}{\eta_0}, \quad \mathbf{x} = \frac{\boldsymbol{\xi}}{\xi_0} = \frac{D_l \boldsymbol{\theta}}{\xi_0}, \quad \eta_0 = \frac{D_s}{D_l} \xi_0, \tag{3}$$

and ξ_0 is the length scale on the lens surface. The angle $\boldsymbol{\alpha}(\mathbf{x})$ is called Einstein's angle. The surface density of the lens is given by

$$\kappa(\mathbf{x}) = \frac{\Sigma(\boldsymbol{\xi})}{\Sigma_{\text{cr}}}, \tag{4}$$

where Σ_{cr} represents the *critical density*:

$$\Sigma_{\text{cr}} = \frac{c^2 D_s}{4\pi D_l D_{ls}}. \tag{5}$$

A generalization of Eq. (1) which considers the perturbation of a *macrolens* (a galaxy or cluster of galaxies) is given by

$$\mathbf{y} = \mathcal{M} \cdot \mathbf{x} - \boldsymbol{\alpha}(\mathbf{x}), \tag{6}$$

where the matrix $\mathcal{M}$ is given by the expression

$$\mathcal{M} = \begin{pmatrix} 1 - \kappa - \gamma\cos\phi & -\gamma\sin\phi \\ -\gamma\sin\phi & 1 - \kappa + \gamma\cos\phi \end{pmatrix}.$$

The components of the matrix depend upon the parameters κ, γ, and ϕ which are, respectively, the dimensionless constant surface mass density, the dimensionless shear of the macrolens, and the shear angle. The microlens (a star in a galaxy or a galaxy in a cluster of galaxies) is represented by means of the Einstein's angle $\boldsymbol{\alpha}(\mathbf{x})$.

B. Gravitational lens models

To simulate Einstein's angle there are several models for the lens mass distribution. The gravitational lens models are divided as follows:

- parametric models
- nonparametric models.

At the present time the program has only parametric models.[1–3] The nonparametric models have recently been used to model gravitational lenses.[4–6] For weak lensing the *Kaiser–Squires* method can be applied to model lenses.[7–10] In the program these techniques have not been implemented. The program includes the following parametric models:

- Chang–Refsdal
- double plane lens
- transparent sphere
- singular isothermal sphere
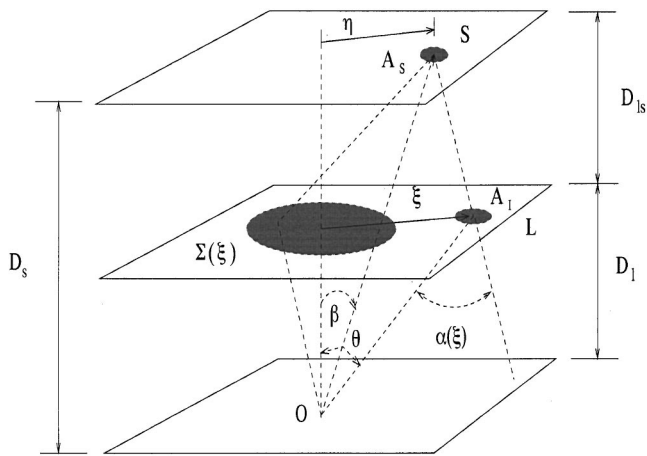- nonsingular isothermal sphere

Fig. 1. The post–Newtonian approximation of the light deflection.

- elliptical
- King
- truncated King
- Hubble
- de Vaucouleur
- spiral
- multipole lens
- rotation lens
- uniform ring.

The interested reader is referred to the literature for a discussion of these models.[1–3]

III. THE VISUALIZATION PROGRAM

A. Description of the program

The program was written in C and the MESA GRAPHIC LIBRARIES (free version of the Open GL) have been used. These graphic subroutines are available for Unix or Linux systems. The XFORMS LIBRARY was used to design the control panel program[11] (see Fig. 2). The IMAGE LIBRARY permits one to load an image file on the program. These libraries can be found at the following addresses:
`http://bragg.phys.uwm.edu/xform`
`http://www.mesa3d.org/`
`ftp://ftp.enlightenment.org/pub/`
`enlightenment/imlib/`
The program creates a window: the *Control Panel* (see Fig. 2). The user can control all items on it by clicking. A second window, the *Image Window*, appears when the Image Window button is clicked on this panel (see Fig. 3). The Help button on the panel gives the user a concise program guide. A version of this program which employs the SGI *Graphic Libraries*[12] is also available.

B. The control panel

All the objects are adjusted interactively with the mouse. This control panel interface has the following items:

- Source Menu (Filled Circle, Colored Rings, and Image File)
- Model Menu (the aforementioned models)
- Source Positioner
- three counters, two for source positioning (*X, Y*) and one for adjusting the pixel resolution (*N*)
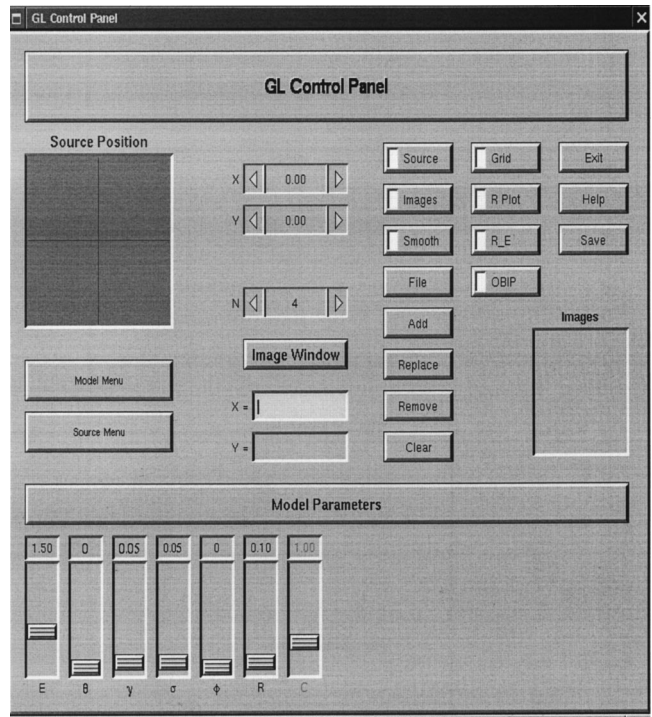

Fig. 2. Control panel.

- two inputs for positioning the observed lens images (*X, Y*)
- buttons (Model Parameters, Image Window, Source, Images, Smooth, Grid, *R* Plot, *R_E*, File, Add, Replace, Remove, Clear, OBIP, Save, Help, Exit),
- Sliders (if one chooses a model, then the corresponding sliders appear)
- a browser for showing the observed lens images

For a detailed discussion of these items, see the program manual (to get the program manual send an e-mail to the author or click the Help button on the control panel).
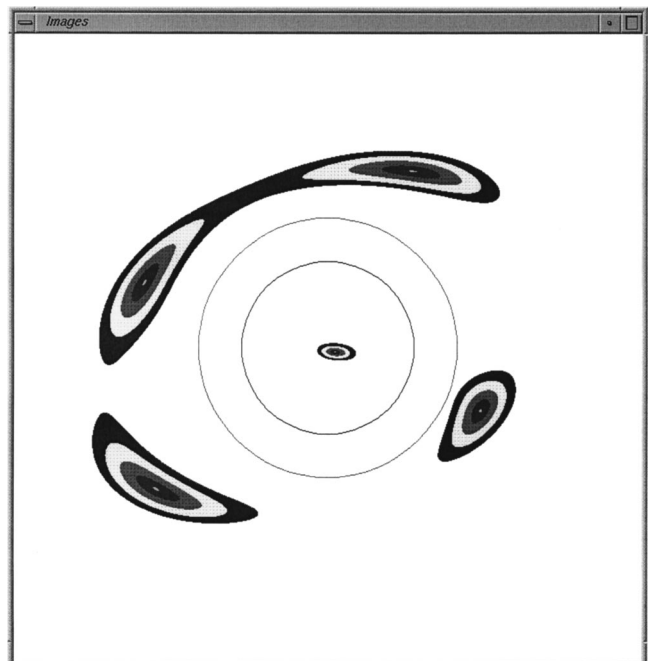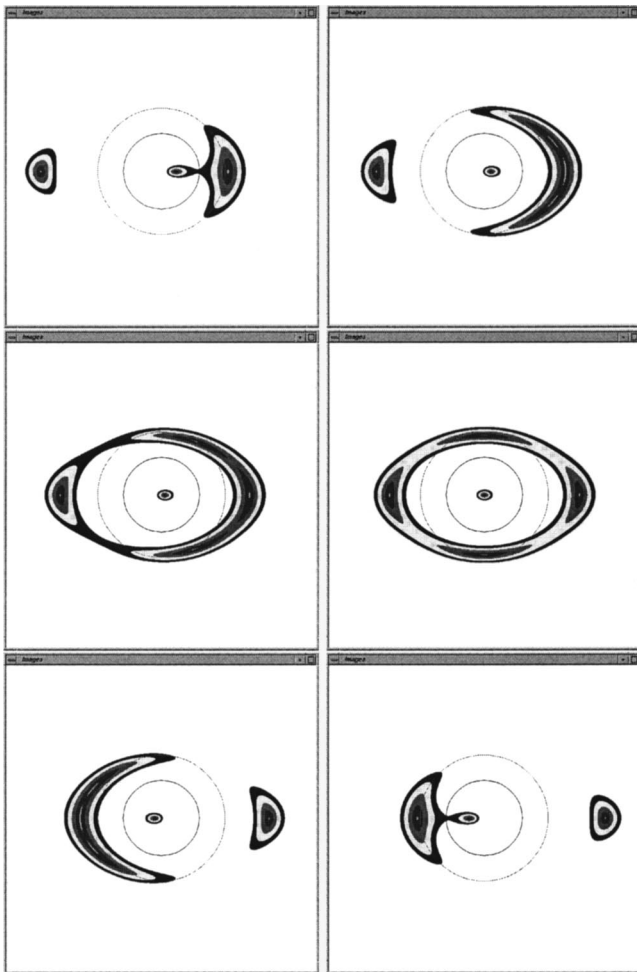

Fig. 3. The image window.

Fig. 4. An elliptical lens.

C. The image window

On this window the events (images, ray plot, etc.) appear. All variations of the parameters on the control panel are shown immediately on this window. To show the images of a chosen gravitational lens model, the *ray tracing method* has been used. The caustic is represented by means of the *ray plot method*.[1]

IV. APPLICATIONS

In this section two of the many applications of the program are discussed:

- *image sequences*
- *easy visual modeling*.

An image sequence for an elliptical model and a visual modeling of the gravitational lens 2237+0305 will be shown.

A. Image sequences

An image sequence is easy to generate with this program. The structure of the images for different positions of the source can be investigated by means of such an image sequence.

An elliptical lens. A sequence of images is shown in Fig. 4. The sequence begins on the top left-hand panel of the sequence. The source, concentric rings, moves from left to right. The source is not shown, only the images. The best
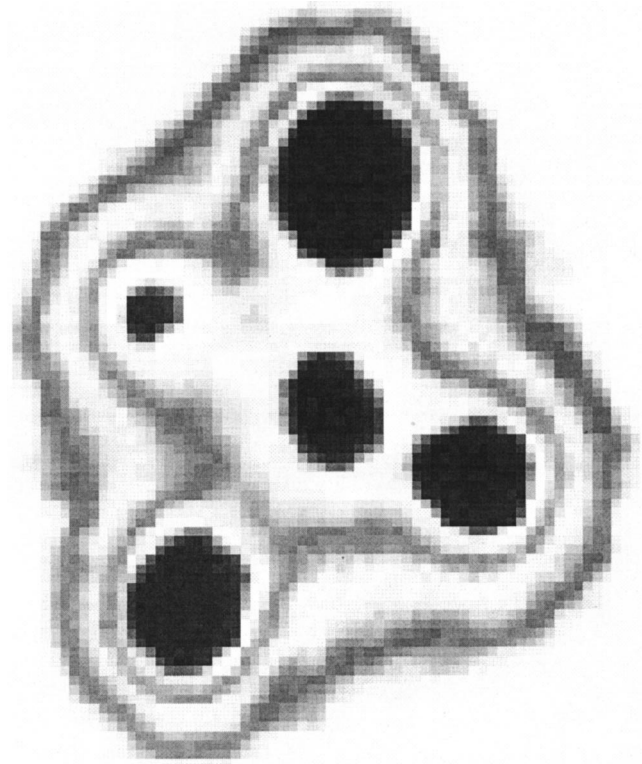

Fig. 5. Gravitational lens 2237+0305.

possible resolution given by the control panel was used. In this sequence the shear γ and the constant surface mass density σ are nonzero and the shear angle $\phi=0$. There are two circles: the Einstein ring or lens scale and the core radius. The formation of a small and a large arc can be seen. In the middle of the sequence one can recognize the deformation of the Einstein ring in an ellipse. Due to the core radius an image appears in the center, which fuses with an arc, in the case that the source moves away.

B. Easy visual modeling

Here an easy visual modeling of the gravitational lens system 2237+0305 is shown. The model images are considered well fit when they overlap most of the area of the observed images (represented through concentric circles), as visually estimated. At the present time a fitting subroutine is not implemented. When such a subroutine is included, this program would become an excellent tool to model gravitational lenses.

Visual model for the gravitational lens 2237+0305. The system 2237+0305 (see Fig. 5) was discovered in 1985.[13] Due to its unusual shape, this lens is called the *Einstein cross*. Attempts to model this lens have been carried out.[14–20]

The data used in Tables I and II were taken from Kent and Falco, and Schneider *et al.*[15,16] The cosmological distances of Table II were calculated using the equations from Schneider *et al.*[15] ($H_0=100$ km s^{-1} Mpc^{-1} and $q_0=1/2$).

Since the positions of the observed images (Table I) are too small to be shown in the image window, a scale factor of

Table I. Image positions for the lens 2237+0305.

| | Positions | |
Object	X (″)	Y (″)
2237+0305 A	0.08±0.01	−0.94±0.01
2237+0305 B	−0.60±0.02	0.74±0.01
2237+0305 C	0.70±0.01	0.26±0.01
2237+0305 D	−0.77±0.02	−0.41±0.02
Galaxy	0.00±0.02	0.00±0.02

200 was chosen (the scale factor can be chosen by the user). Then all positions are multiplied by this scale. The positions are also rotated around the X axis.

The lens galaxy is elliptical, so it is reasonable to choose an elliptical model in the program, although one can also try other models.[15–18] In Schneider *et al.* an N point mass lens ($N=6724$) was used.[15] An *elliptical King* model and an *elliptical de Vaucouleur* model were used in Kent and Falco.[16] In Rix *et al.* the *de Vaucouleur* model was used.[17] A general SIS model with shear was used in Wambsganss and Paczynski.[18]

The major axis of the lens has a particular orientation; therefore the coordinate system is rotated around an angle θ, so that the elliptical potential has the same orientation. The angle θ is taken as a parameter. The constant surface mass density and the shear are zero. The visual fitting is shown in Fig. 6. In Table III the model parameters of the elliptical model are shown. These were found through variation of sliders and source positioners.

The data for the lens scale (*Einstein radius*), the core scale, and the source position must be divided by 200. Thus one obtains $\alpha_E=0.''90$, $\theta_c=0.''35$, and $(X,Y)=(0.''14, 0.''06)$. The Einstein radius and core radius are determined by means of the equations $\mathcal{R}_E=\alpha_E D_l$ and $r_c=\theta_c D_l$ ($\mathcal{R}_E=0.48\,\text{kpc}$ and $r_c=0.19\,\text{kpc}$).

From the following equation the lens mass can be found:

$$\mathcal{R}_E^2=\frac{4GM}{c^2}\frac{D_l D_{ls}}{D_s}.$$

The velocity dispersion is determined through an equation from Blandford and Kochanek.[21,22]

The major axis angle can be calculated from $\theta'=90°+\theta$ ($\theta=-24°$).

In Table IV all values for the different models are shown: the elliptical King model (model A),[16] the de Vaucouleur model (model B, the source position is referred to the image

Table II. Parameters of the lens galaxy and the source.

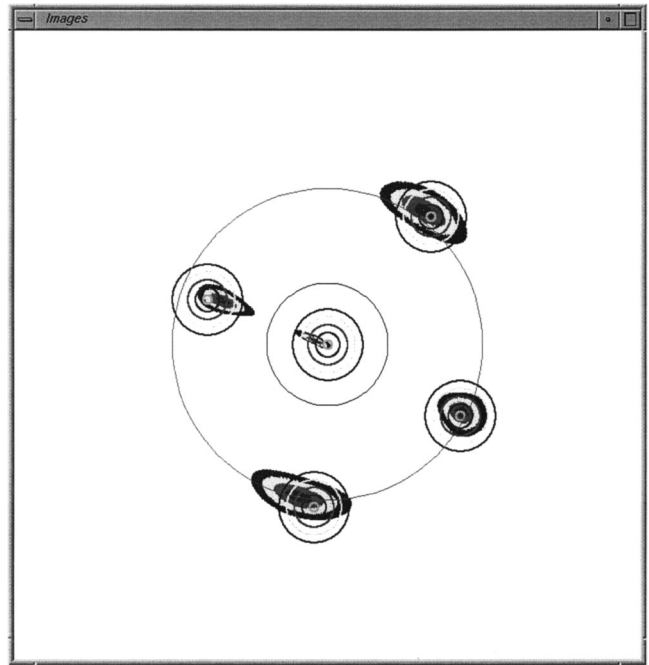| Parameter	Value
z_l	0.0394
Distance D_l	110 Mpc
Ellipticity ϵ	0.57
Angle of the mayor axis	77°
Angle of the bulk	38°
Velocity dispersion σ_v	215 km/s
z_s	1.695
Distance D_s	867 Mpc
Distance D_{ls}	825 Mpc

Fig. 6. Visual fitting of the gravitational lens 2237+0305.

A),[17] the generalized SIS model (model C),[18] and the elliptical model. The models A, B, and C were fitted. Our elliptical model is not reliable, because the program does not possess a fitting subroutine, but one can estimate and compare with other models.

V. CONCLUSIONS AND FUTURE WORK

From both the didactical and scientific points of view a program to visualize gravitational lenses is useful. This versatile program works quickly and interactively with the mouse.

With this computer program the user has a tool to visualize and to visually model gravitational lenses. The applications of the program we have shown in this paper are:

- sequences of images
- easy visual modeling.

The user can produce sequences of images for a chosen gravitational lens model. Through the variation of model parameters, he or she can investigate the structure of the images. The user can also attempt to visually model observed

Table III. Model parameter of the elliptical model and source position.

Parameter	Value
E: lens scale	180
θ: rotation angle	336°
γ: shear	0
σ: constant surface mass density	0
ϕ: shear angle	0°
R: source radius	50
C: core scale	70
ϵ: ellipticity	0.54
α: softness	0.05
κ: constant central mass density	1.00
(X, Y): source position	(28.0, 12.0)

Table IV. Model parameter for the system 2237+0305.

Parameter	Model			
	A	B	C	Elliptical
Einstein ring	0″.90	0″.90	0″.874	0″.90
Shear	⋯	⋯	0.02	⋯
Angle of the mayor axis °	66.8	68	66.84	66
Core radius kpc	0.10	⋯	⋯	0.19
Ellipticity ϵ	0.42	0.3	⋯	0.54
Softness α	⋯	⋯	⋯	0.05
Central surface mass density κ	⋯	⋯	⋯	1.00
Velocity dispersion km/s	170	209	⋯	109
Lens mass $10^{10} \mathcal{M}_\odot$	1.2	1.08	1.49	1.15
Source position				
X	−0″.07	0″.159	⋯	0″.14
Y	−0″.02	0″.877	⋯	0″.06

gravitational lenses. The observed position can be used as input data and the model parameters can be easily varied in order to approximate the observed images. So the user can quickly obtain model parameter estimations. Some observed lenses have already been modeled and the user can compare those results with the output from a chosen model of the control panel.

Future Work. The program can be improved by the inclusion of some additional subroutines:

• contour subroutine for the isochrones (time delay)
• light curves subroutine (dependence of brightness with time)
• subroutine for computing the image magnification
• subroutine to calculate critical curves and caustics
• fitting subroutine
• root finder subroutine
• subroutine to load images of observed gravitational lenses
• subroutine with more complex (elliptical) models
• subroutine for superposition of models in different lens planes
• subroutine with cosmic string lens models
• subroutine for nonparametric reconstruction
• Kaiser–Squires subroutine.

The author is working on implementing some of the above-mentioned improvements.

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