Electromagnetism and gravitation

(electromagnetic fields induced by rotation/ionic crystals/electrets)

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ABSTRACT   Generation of electric fields in rapidly rotating insulators is discussed and calculated. An interesting effect is expected in TlCl. A possible appearance of magnetic fields near rapidly rotating gravitating bodies is proposed. The simple suggestion made here would lead to magnetic fields of negligible magnitude.

In the last decades of his life, Einstein attempted to construct a unified field theory. Having reduced gravitation to the principles of geometry, he hoped for a similar system that would include gravitation and electromagnetism. Today, after being puzzled by nuclear forces and after having discovered scores of "elementary particles," we know that in a unified field theory much more needs to be unified. It seems that in physics we did not run fast enough to remain in the same place. This paper is an attempt to discuss some possible relations between gravitation and electromagnetism.

We shall make use of the equivalence principle and replace gravity by acceleration. First, the straightforward problem will be discussed of how an acceleration, conveniently available in rotating bodies, will give rise to electromagnetism. The well-known effects due to the orientation of magnets (i.e., orientation of spins) by the rotation will not be reviewed. We shall concentrate on effects due to the acceleration of charged particles.

In the second part of this paper, magnetism that might arise in the vacuum induced by the rotation of a gravitating body (pulsar or black hole) will be considered.

Polarization induced by acceleration

Electric potentials due to centrifugal forces do occur in rotating metals. The idea that the free electrons are crowded toward bigger radii is naive. The main effect of the centrifugal forces is exercised on the positive ions. The density will be lower near the axis of rotation than at bigger radii. The electrons will neutralize the positive ions. However, when the degeneracy of the electron gas is taken into account the electrons are squeezed out of the denser regions. Thus, a small positive charge will appear near the surface of the rotating body (1).

A much bigger effect can be expected in ionic crystals in which the positive and negative ions, having different masses, will be subject to different centrifugal forces. The polarization of a rotating needle made of an alkali halide will actually be quite similar to the polarization caused by an electric field. The difference is that an electric field acts both on the electrons and on the ions, whereas the centrifugal force acts practically only on the heavy ions. This obvious effect seems to have received little attention.

Let us call the relative displacement of the positive and negative ions x, the masses of the two ions M_+ and M_-, the distance from the axis of rotation r, and the number of revo-

lutions per second ω; then the displacement is given by:*

$$qx = \frac{1}{2}(M_+ - M_-)r\omega^2 \qquad [1]$$

where q is the constant of proportionality in the restoring forces that keep the ions in their equilibrium positions.

The polarization P_{rot} due to the rotation will be

$$P_{rot} = nxe_n = (ne_n/2q)(M_+ - M_-)r\omega^2 \qquad [2]$$

where n is the number of ion pairs per cm^3 and e_n is the effective charge of the ions (that is, the dipole moment due to the relative displacement x divided by that displacement). This charge e_n is dependent on the shape of the crystal (2). The subscript n indicates that the crystal we use has the shape of a needle. The polarization P_{rot} can be measured, and this will give an experimental determination of the quantity e_n/q.

Another application of the same principle is to replace in a complex solid one isotope by another one with the mass difference ΔM. The resulting difference of polarizations caused by the rotation ΔP_{rot} will be rather involved since a force on one atomic species will produce displacement in other atoms. The measurement of ΔP_{rot}, together with other properties of the solid, can contribute to the information about effective charges that are analogous to e_n.

A particularly interesting application would be to rotate an electret at the temperature near the transition point. If the crystal has no ordered permanent dipole moment but we are near the phase transition where such a dipole is established, the rotation will produce big effects which go to infinity as the transition temperature is approached. These measurements may turn out to be relatively easy. Furthermore, they may give interesting insight into the nature of the transition. More explicitly, the application to an electret near its transition point will indicate the relative contributions that the various atoms make in the fluctuations from a state with no dipole to a state with a permanent dipole. Such fluctuations may be enhanced by an electric field or by the forces due to rotation. It remains true that the average polarization induced by the rotation will give information concerning the contributions of various atoms. The measurement of polarizations will lead to quantitative structural information only in combination with data on the vibrational spectrum and the position of atoms before and after the phase transition.

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* The right-hand side of [1] is the part of the centrifugal force that acts in opposite directions on the positive and negative ions. The portion $1/2(M_+ + M_-)r\omega^2$ that acts on them in the same direction gives rise to compression and makes a negligible contribution in ionic crystals to the polarization. In an element that is an insulator, for instance, sulfur, one would expect only a polarization due to the variation of compression along the needle. This case should be investigated for the sake of comparison. It might yield some unexpected results.

Table 1. Table of data substituted in Eq. 12.

Crystal	k^*	k_0	ω_{tr}[†] $(\times 10^{-13})$	ρ[‡]
NaCl	5.90	2.385[§]	3.62	2.165
KCl	4.75	2.22[¶]	2.97	1.984
KBr	4.90	2.46[§]	2.23	2.75
KI	5.6	2.81	1.98	3.13
AgCl	11.2	4.29	2.31	5.56
AgBr	12.2	5.08	1.67	6.473
TlCl	31.9	5.05[¶]	2.06	7.004

* Squares of refractive indices (3).
† Calculated from values of λ (4).
‡ Ref. 3.
§ Ref. 5.
¶ Ref. 6.

Cubic lattices of the type A⁺B⁻

We shall now return to the case of the alkali halides and similar ionic lattices. Here the magnitude of the effect can be calculated from known data. This case, therefore, yields no new information but may serve as a test.

The force constant q of Eq. 1 is simply related to the frequency of the residual rays ω_{tr}, the frequency of a vibration whose wavelength is long compared to the lattice distance (but short compared to the dimensions of the crystal) and whose propagation vector is perpendicular to the displacement of the ions.

According to Lyddane et al. (formula 5' of ref. 2),

$$\frac{M_+ M_-}{M_+ + M_-} \omega_{tr}^2 = \frac{4\pi n e_n^2}{k - k_0} \qquad [3]$$

where k is the dielectric constant of the crystal and k_0 is the smaller value that k assumes if the ions are not permitted to move. This latter value, k_0, may be obtained by extrapolating the square of the refractive index from the optical region to long wavelength, with exclusion of the infrared spectrum.

In the same paper the formula

$$q = \omega_{tr}^2(M_+ M_-/M_+ + M_-) \qquad [4]$$

is implied.

Since Eq. 3 is not derived in the quoted paper but is stated as a generalization of other results. its derivation will be indicated here. We consider a needle of the alkali halide crystal in an external electric field E_0 parallel to the needle. The dipole $e_n x$ will be induced per ion pair. Equating the force on this ion pair with the restoring force qx, one has

$$e_n E_0 = qx. \qquad [5]$$

One should note that the force constant q depends on the shape of the crystal and the orientation of the displacement within the crystal. (It may have been more consistent to use the notation q_n.)

The polarization of the crystal is connected with the electric field by

$$4\pi P = (k - 1)E_0. \qquad [6]$$

This polarization is composed of the ionic and electronic contributions. The electronic contribution is defined by

$$4\pi P_{el} = (k_0 - 1)E_0 \qquad [7]$$

while the ionic contribution is the sum of the dipole moments due to the ionic displacements

$$4\pi P_{ion} = 4\pi n e_n x. \qquad [8]$$

Table 2. Values for $4\pi P_{rot}$ electric field at tip of rotating needle for $r\omega^2 = 10^8$ cm·sec⁻² in units of mV/cm

Crystal	$4\pi P_{rot}$
NaCl	−1.77
KCl	0.39
KBr	−4.48
KI	−9.87
AgCl	16.71
AgBr	6.51
TlCl	70.25

From [5], [6], [7], [8], and $P = P_{ion} + P_{el}$

$$q = 4\pi n e_n^2/(k - k_0) \qquad [9]$$

follows.

In a transverse vibration the electric field is parallel to the nodal planes. Let us assume that our needle is thick compared to the wavelength and the nodal planes are parallel to the needle. Then the electric field is continuous on the faces of the needle and, disregarding end-effects near the point of the needle, all formulas given above apply. Eq. 4 holds for the force constant for a needle (with displacement along the needle) as well as for the force constant of a transverse vibration. Eq. 3 follows from [4] and [9].

From [3] and [4], we obtain

$$\frac{e_n}{q} = \left[\frac{(k - k_0)(M_+ + M_-)}{4\pi n M_+ M_- \omega_{tr}^2}\right]^{1/2}. \qquad [10]$$

Substituting this into [2] we get the electric field $4\pi P_{rot}$ which should be observed at the tip of the needle rotated around an axis perpendicular to its long dimension

$$4\pi P_{rot} = (M_+ - M_-)\frac{r\omega^2}{\omega_{tr}}\left[\frac{\pi(k - k_0)\,n(M_+ + M_-)}{M_+ M_-}\right]^{1/2}. \qquad [11]$$

This formula may be simplified by introducing the density $\rho = n(M_+ + M_-)$ of the ionic crystal

$$4\pi P_{rot} = (M_+ - M_-)\frac{r\omega^2}{\omega_{tr}}\left[\frac{\pi\rho(k - k_0)}{M_+ M_-}\right]^{1/2}. \qquad [12]$$

(Incidentally, this polarization gives rise to a volume-density of charge within the needle of the value $(M_+ - M_-)\omega^2/4\omega_{tr}[\rho(k - k_0)/\pi M_+ M_-]^{1/2}$ with a compensating surface density of charge at the needle. From this the potential in the neighborhood of the needle can be derived.)

As examples we shall consider the alkali halides NaCl, KCl, KBr, and KI and also the analogous compounds AgCl, AgBr, and TlCl. The data that were used for k, k_0, ω_{tr}, and ρ are given in Table 1 (3–6). Values for the electric field $4\pi P_{rot}$ near the tip of the needle are listed in Table 2 for $r\omega^2 = 10^8$ cm·sec⁻² in units of mV/cm. It seems quite possible to measure these surface fields $E = 4\pi P_{rot}$ because values of $r\omega^2 > 10^9$ cm· sec⁻² can be easily obtained in the centrifuge. The samples in question will probably not stand these accelerations. Nevertheless, the effects probably can be observed or even measured. The actual limit of observation is about 10^{-3} mV/cm.

One should note the high value for $4\pi P_{rot}$ in the case of TlCl. (Incidentally, the measurements for k, as given by different authors, differ by as much as 50% for this compound.) The corresponding values for e_n exceed by a great amount the charge of an electron. The picture of the ion carrying a given charge is no longer a good approximation. The vibration corresponding to ω_{tr} may induce a gradual change in the nature

of the bond. Indeed, the salt may be in the neighborhood of a phase change. For TlCl, the sign given in Table 2 (which is based on the assumption that Tl carries the positive charge) need not be correct. To carry out the experiment on a rotating needle may be, therefore, particularly interesting in this case.

Magnetic fields near massive rotating stars

The general question of an interaction between gravitation and electromagnetism is, of course, far more basic. Indeed, if the vacuum is a potential source of electrons, positrons, and other particles, then one might imagine that in a vacuum a rotating gravitational field may give rise to electromagnetic forces, just as such forces do arise in rotating crystals. Without attempting to construct a theory one may try to find a formula that has the correct dimensions and the right symmetry properties.

Near a rapidly rotating gravitational object one may, for instance, assume that a magnetic field will appear given by[†]

$$H = (e/c^3)[\vec{g} \times \vec{\omega}] \qquad [13]$$

with e equal to the charge of the electrons, $\vec{g}$ the local gravitational acceleration, and $\vec{\omega}$ the angular velocity of the rotating body. The expression $[\vec{g} \times \vec{\omega}]$ is the vector product of $\vec{g}$ and $\vec{\omega}$.

The dimensions of [13] are easily checked. Time reversal inverts the sign of ω but leaves g unchanged. Thus, H changes its sign under this operation, which is the correct behavior. Inversion (replacement of all space coordinates by their negative values) changes g but not ω. Thus, H would change its sign under inversion, which is incorrect. One should remember, however, that inversion should be accompanied by the interchange of particles and antiparticles. This inverts the sign of e and leads to the correct result that H will not change under inversion combined with charge conjugation. It should be remembered that we are looking for an effect in vacuum. Therefore, *a priori*, electrons and positrons must have an equal role. Beyond these arguments, which show consistency, I can find no physical reason that fields as given in [13] will really exist. It should be noted that this field, even if real, is exceedingly small.

On the surface of a rotating mass [13] will give, of course, a higher value of H than at greater distances. Since, on the surface, the centrifugal acceleration should be less than g, we find

$$\omega < (g/r)^{1/2}. \qquad [14]$$

Therefore, the absolute value of H will be limited by

$$|H| < \frac{e}{r^2}\left[\frac{(gr)^{1/2}}{c}\right]^3 \approx \frac{e}{r^2}\left(\frac{v_{esc}}{c}\right)^3. \qquad [15]$$

Here v_{esc} is the escape velocity on the surface of the rotating object. The magnetic field will be, therefore, numerically less than an electric field due to a single electron located at the center of the rotating body. The factor $(v_{esc}/c)^3$ can become unity only on the surface of a black hole.

It seems that the elementary conditions to satisfy symmetry relations and to give the correct dimensional behavior suggest an extremely weak coupling between gravitation and magnetism. It cannot, of course, be excluded that a big numerical factor should appear in an equation analogous to [13]. But the factor would have to be truly enormous to make the coupling significant.

Conclusion

The first part of this paper deals with a practical matter that does not appear to be novel; the second part deals with an approach that may be novel but does not seem practical. I am reminded of the professor who told his student: "Your thesis contains material which is new and material which is correct. Unfortunately, what is new is not correct and what is correct is not new." Even so, I hope that some may find one or the other part of this discussion interesting.

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[†] It would be equivalent to assume a vector potential equal to $e\phi\vec{\omega}/c^3$ where ϕ is the gravitational potential.