THE JOURNAL OF PHILOSOPHY

VOLUME LXX, NO. 21, DECEMBER 6, 1973

GEOMETRODYNAMICS AND ONTOLOGY *

FOR nearly two decades before 1972, Professor John Wheeler pursued a research program in physics that was predicated on a monistic ontology which W. K. Clifford had envisioned in 1870 and which Wheeler[1] epitomized in the following words: "There is nothing in the world except empty curved space. Matter, charge, electromagnetism, and other fields are only manifestations of the bending of space. *Physics is geometry*" (225). In an address to a 1960 Philosophy Congress,[2] he began with a qualitative synopsis of the protean role of curvature in endowing the one presumed ultimate substance, empty curved space, with a sufficient plurality of attributes to account for the observed diversity of the world. He said:

> . . . Is space-time only an arena within which fields and particles move about as "physical" and "foreign" entities? Or is the four-dimensional continuum all there is? Is curved empty geometry a kind of magic building material out of which everything in the physical world is made: (1) slow curvature in one region of space describes a gravitational field; (2) a rippled geometry with a different type of

* This essay grew out of an Introduction which I presented as chairman of the symposium on J. C. Graves's *The Conceptual Foundations of Contemporary Relativity Theory,* held in December, 1972, at the Boston meetings of the American Philosophical Association.

I owe a very substantial debt to the physicist John Stachel, with whom I discussed the original, oral version of this paper in detail and who was also generous with his time in providing guidance on matters of content and literature pertinent to the present published version. I also had the benefit of valuable reactions from my Pittsburgh colleagues Allen I. Janis and John R. Porter to mathematical questions relating to the geodesic method. Warm thanks are due to these three friends for their kind help and to the National Science Foundation for the support of research.

1 *Geometrodynamics* (New York: Academic Press, 1962).

2 "Curved Empty Space-Time as the Building Material of the Physical World," in E. Nagel, P. Suppes and A. Tarski, eds., *Logic, Methodology and Philosophy of Science* (Stanford: University Press, 1962).

> curvature somewhere else describes an electromagnetic field; (3) a knotted-up region of high curvature describes a concentration of charge and mass-energy that moves like a particle? Are fields and particles foreign entities immersed *in* geometry, or are they nothing *but* geometry?
>
> It would be difficult to name any issue more central to the plan of physics than this: whether space-time is only an arena, or whether it is everything (361).

For nineteen years, Wheeler and his co-workers, such as Charles Misner, developed some of the detailed physics of Clifford's ontology of curved empty space-time, as an outgrowth of general relativity, under the name 'geometrodynamics' ('GMD'). In Wheeler's parlance, "a geometrodynamical universe" is "a world whose properties are described by geometry, and a geometry whose curvature changes with time—a *dynamical geometry*" (*loc. cit.*). But in a lecture at a 1972 conference, Wheeler disavowed his erstwhile long quest for a reduction of all of physics to space-time geometry.[3] In a brief notice of that conference the pertinent part of this lecture was summarized as follows: "He [Wheeler] also developed the theme that the structure of space-time could only be understood in terms of the structure of elementary particles rather than the converse statement which he has advocated for many years." [4]

To emphasize his new conception of the fundamental and indispensable ontological role played by entities or processes *other than* space-time, Wheeler repeatedly spoke of PRE-geometry. As I understood him, he sought to emphasize in this way that he now regards space-time not as the basic stuff of a monistic ontology but rather as an *abstraction* from the events in which quantum processes are implicated. That is to say, according to Wheeler's new conception of PRE-geometry, space-time is an abstraction from the constitution of physical events ONTOLOGICALLY no less than epistemologically! In Wheeler's *erstwhile* view of space-time as the only autonomous substance, space-time is absolute in the older familiar sense of being *empty*. In the perspective of Wheeler's new program

[3] Conference on Gravitation and Quantization, held in October and November, 1972, at the Boston University Institute of Relativity Studies, directed by John Stachel. I am grateful to Professor Stachel for having given me the opportunity to attend this conference.

This disavowal can now be seen to have been heralded by Wheeler's January, 1971, Foreword to J. C. Graves, *The Conceptual Foundations of Contemporary Relativity Theory* (Cambridge, Mass.: MIT Press, 1971), p. viii.

[4] *Nature*, CCXL (Dec. 15, 1972): 382. Wheeler's own published repudiation of GMD has just become available in C. W. Misner, K. S. Thorne, and Wheeler, *Gravitation* (San Francisco: Freeman, 1972), § 44.4 pp. 1203–1208.

of a PRE-geometric ontology, his earlier all-out geometric absolutism appears as a kind of ontological chauvinism. Wheeler now wishes to supplant that absolutism by a neo-Leibnizian view which is RELATIONAL in the sense that it considers that space-time structure is only one aspect of a quantum universe whose ontological furniture cannot be constituted out of space-time. I believe that one reason for Wheeler's explicit mention of Leibniz was to emphasize the relational character of his notion of PRE-geometry.

In the minds of great scientists like Wheeler, there often is a subtle interplay between empirical and conceptual, or philosophical, promptings for abandoning, no less than for espousing, a major theory with its research program. It seems to me that this state of affairs need not at all betoken a gratuitous and pernicious apriorism: at least generally, there is no sharp divide between legitimate empirical and conceptual or philosophical reasons for the rejection (acceptance) of a major theory. Thus, I believe that predominantly philosophical considerations are likewise germane to the appraisal of certain facets of Wheeler's erstwhile purely geometrical ontology vis-à-vis the rival ontology adumbrated in his more recent notion of PRE-geometry. Hereafter, when I speak of "geometrodynamics" or use its acronym 'GMD', I shall disregard Wheeler's basic change of view and use this term to refer to his earlier relativistic theory of *empty* space, rather than to the ontologically more noncommittal standard version of general relativity. Even though it was *not,* of course, relativistic, Clifford's so-called "space-theory of matter" shared the essential ontological assumptions of Wheeler's GMD. Hence I shall also occasionally denote Clifford's theory-sketch by 'GMD'.

Partly in response to a 1972 GMD Symposium [5] which focused on John C. Graves's *The Conceptual Foundations of Contemporary Relativity Theory* (*op. cit.*), the present essay aims to provide an analysis and appraisal of some of the major facets of the ontology of GMD as developed by Clifford and Wheeler.

But we shall first be concerned in the next section with foundational aspects of the Riemannian metric of space-time that are central to Einstein's *standard* 1915/6 general relativity theory. This prior consideration of classical general relativity, which is important in its own right, will furnish a basis for the ontological articulation and assessment of the GMD thesis that the metric geometry of the space-time manifold in which we live is the physical world's only autonomous substance.

[5] This JOURNAL, LXIX, 19 (Oct. 26, 1972): 621–649.

I. THE PHILOSOPHICAL STATUS OF THE METRIC OF SPACE-TIME IN THE GENERAL THEORY OF RELATIVITY

Three major alternative methods for physically grounding and theoretically circumscribing the metric of space-time up to a constant scale factor merit consideration, although none of them is wholly unproblematic. We shall consider two of them rather briefly before giving our main attention to a third, which has been adduced as foundationally most germane to the ontological vision of GMD. Let us begin with the most traditional of the three alternative methods.

(i) *The Method of Rods-and-Clocks.* Both in his 1917 Prussian Academy Lecture "Geometry and Experience," and in his "Autobiographical Notes," [6] Einstein appealed to special rods and clocks in order to give a provisional explication of the physical significance of the space-time metric. He regarded this theoretical foundation for the metric as provisional ontologically, because he noted (59) that macroscopic rods and clocks are "objects consisting of moving atomic configurations" and in that sense not, as it were, "theoretically self-sufficient entities." In this same vein, others have gone on to point out that there is the following kind of serious question of principle concerning the ability of solid rods to fulfill the role of a metric standard in the general theory of relativity (hereafter "GTR"): If such a rod is allowed to fall freely in a permanent gravitational field such as the earth's, its end-points will be driven either closer together or farther apart by that field. Thus, *as judged by the space-time metric prescribed by the GTR,* the length of the rod will change, although the amount of that change will depend on the constitution of the rod. Making relativistic corrections for such deviations from rigidity may not only be very difficult but also involve much of the *whole* GTR on pain of vicious logical circularity.

But, as I have emphasized elsewhere,[7] the *ontology* of the GTR-metric that I espouse in section II below *allows* fully that the following statement by Hilary Putnam [8] be true: "the metric is implicitly specified by the whole system of physical and geometrical laws and 'correspondence rules' " (206).

[6] In P. A. Schilpp, ed., *Albert Einstein, Philosopher-Scientist* (New York: Tudor, 1949), pp. 59–60.

[7] *Geometry and Chronometry in Philosophical Perspective* (Minneapolis: University of Minnesota Press, 1968), p. 367.

[8] "An Examination of Grünbaum's Philosophy of Geometry," in B. Baumrin, ed., *Philosophy of Science, The Delaware Seminar,* II (New York: Interscience, 1963), pp. 205–255.

Having seen some of the difficulties of reliance on solid rods even infinitesimally, one might follow J. L. Synge[9] and use another method which is *chronometric*.

(ii) *Synge's Chronometric Method.* Synge writes (in *The Special Theory*):

> To measure time one must use a *clock*, a mechanism of some sort in which a certain process is repeated over and over again under the same conditions, as far as possible. The mechanism may be a pendulum, a balance wheel with a spring, an electric circuit, or some other oscillating system, and out of these one passes by idealisation to the concept of a *standard clock*. . . .
>
> Let us however make the concept of a standard clock more definite by thinking of it as an atom of some specified element emitting a certain specified spectral line, the "ticks" of the clock corresponding to the emission of the crests of successive waves of radiation from the atom. By associating numbers 1, 2, 3, . . . with these ticks, we have a scale for assigning times to events occurring in the history of the atom, or (since that would give us a very small unit of time) we may more conveniently associate with the ticks the numbers $a, 2a, 3a, \ldots,$ where a is some chosen small number, to be used universally for all clocks.
>
> To base the measurement of time, as above, on a standard atomic frequency is very like the plan of basing the measurement of length on a standard wave length. But in relativity the concept of length is not an easy one, and it seems best to start with an atomic clock as the basic concept and introduce the idea of wave length at a later stage (14/5).

Synge goes on to claim (15/6) that (a) For two neighboring events on the world line of a standard (atomic) clock whose coordinates are x^i and $x^i + dx^i$, respectively, the invariant infinitesimal *space-time* separation ds is equal to the *time* interval between them as measured by that clock, and (b) The chronometrically ascertained values ds for such pairs of events are equal to those of some function $f(x^i, dx^i)$ which is positive homogeneous of first degree in the coordinate differentials dx^i. This means that if the differentials dx^i are each multiplied by the same positive factor k, ds will be multiplied by that factor, so that

$$f(x^i, k \cdot dx^i) = k \cdot f(x^i, dx^i) \quad \text{for } k > 0$$

[9] *Relativity: The Special Theory* (Amsterdam: North-Holland, 1956), ch. I, §§ 9, 13, 14; *Relativity: The General Theory* (Amsterdam: North-Holland, 1960), ch. III, § 4.

An invariant line element $ds = f(x^i, dx^i)$ that has this mathematical property is said to be the metric of a *Finsler space* (19).

Synge does not explain on what grounds it is assumed that the *ds* values that are ascertained chronometrically as specified are those of a Finslerian metric, but merely points out (19) that a Riemannian metric is only a special case of a Finslerian metric. For example, in a Finslerian space-time, the separation *ds* might have the *non*-Riemannian form $ds = (g_{mnrs}\, dx^m\, dx^n\, dx^r\, dx^s)^{1/4}$. Furthermore, the reason given by Synge (19) for now choosing the Riemannian species $ds = |g_{mn}\, dx^m\, dx^n|^{1/2}$ of Finslerian line element is that this choice is dictated by the objective of formulating Einstein's theory. For the latter purpose, Synge selects in particular the kind of indefinite Riemannian metric that is of signature + 2.[10]

Note that Synge renounced the foundational use of the solid rods of method (i) above, in his chronometric ontology of the GTR space-time metric. By so doing, Synge's method forfeited such justification as the presumably Pythagorean infinitesimal metric behavior of rigid rods can furnish for adopting the Riemannian species of space-time metric within the Finslerian genus. Also, despite the essential immunity of the rate of atomic clocks to outside perturbations, it is not entirely clear that the observational use of atomic clocks—say, by tuning the vibrations of caesium atoms to synchronism with an oscillating electric circuit—can altogether avoid being epistemically parasitic on *non*relativistic theory. Thus, it is not fully clear whether Synge's chronometric method is any more free from such possible parasitism than method (i) when the latter rods-and-clocks method attempts to correct solid rods for gravitational distortions.

So far, Synge's prescriptions for the chronometric determination of *ds* were confined to event pairs whose separations were time-like. But for any given event *P* of a relativistic space-time, there will be other events *Q* whose separation from *P* is space-like, and still other events *R* which can belong to the career of a free photon whose world-line also crosses *P*. Therefore, Synge proceeds to point out very interestingly (23) that, if we are given the coordinate differences dx^i between *P* and any other nearby event as needed, then his chronometric method permits the *indirect* determination of the *ds* values even for event pairs (*P, Q*) whose separation is *space*-like! For, given the values of dx^i corresponding to ten events that lie, respectively, on as many different world lines of atomic clocks

[10] For a definition of the "signature" of a quadratic differential form, see, for example, E. C. Weatherburn, *Riemannian Geometry and the Tensor Calculus* (New York: Cambridge, 1957), pp. 11 and 14.

that cross P, the chronometric determination of the ten respective separations of these events from P enables us to use ten corresponding equations $ds^2 = |g_{mn}\ dx^m\ dx^n|$ to *calculate* the generally ten independent values g_{mn} at P. Although these g_{mn} values were obtained solely from determinations pertaining to the *time*-like world-lines of ten atomic clocks, they qualify as the metric coefficients at P for *any kind* of infinitesimal space-time interval of which P is an end-point. Hence their substitution in the equation for the Riemannian metric of space-time now permits us to calculate, in turn, the *space*-like separation of a nearby event Q from P for which the coordinate differentials dx^i are known.

In Synge's construction, it can then be postulated that, for the now known values of g_{mn} at P, the equation for the Riemannian metric will yield the null value $ds = 0$ for the separation between P and nearby events R which can be linked to P by the world-lines of free photons and whose coordinate differences dx^i from P are known (23). Given these results, Synge (ch. I, §14) is able to furnish a second kind of ideal experiment for the measurement of a *space*-like separation, which is more direct than the first kind presented above and uses as apparatus only a standard clock and photons. In summary, he writes:

> . . . we shall physicise the geometrical element ds of separation between two adjacent events, P and Q.
>
> First we find out by trial whether it is possible to make a material particle contain these two events in its history. If it is possible, then ds is measured by the difference in the readings at P and Q of a standard clock carried by the particle which includes them in its history.
>
> If it is impossible for a material particle to include both the events in its history, we know that PQ either lies on the null cone or is spacelike. We test whether it lies on the null cone by emitting photons from P in all the space-time directions permitted to photons. If one of these photons includes Q in its history, then PQ is null and we have $ds = 0$.
>
> Suppose now that neither material particle nor photon can include both P and Q in its history. Then the space-time displacement PQ is spacelike (24).

But doubts have been raised concerning Synge's chronometric approach in an illuminating paper on the foundations of the space-time metric in the GTR by Ehlers, Pirani, and Schild.[11] These three

[11] J. Ehlers, A. E. Pirani, and A. Schild, "The Geometry of Free Fall and Light Propagation," in L. O'Raifeartaigh, ed., *General Relativity* (New York: Oxford, 1972).

authors offer an important alternative to Synge's chronometric approach which will concern us below in the context of GMD. Hence it will be useful to quote *in extenso* from their paper. Speaking of Synge's method, they write:

> This procedure has two advantages. First, it uses as primitive a physical quantity that can, in fact, be measured locally and with extreme precision, and, secondly, it introduces as the primary geometric structure the metric, from which all the other structures can be obtained in a straightforward manner.
>
> If the aim is a *deduction* of the theory from a few axioms, the *chronometric* approach is indeed very economical. If, however, one wishes to give a *constructive* set of axioms for relativistic space-time geometries, which is to exhibit as clearly as possible the physical reasons for adopting a particular structure and which indicates alternatives, then the chronometric approach does not seem to be particularly suitable, for the following three reasons. It seems difficult to derive *from the behaviour of clocks alone,* without the use of light signals, the *Riemannian* form for the separation,
>
> $$ds = |g_{ij}dx^i dx^j|^{1/2} \qquad (1)$$
>
> rather than some other, first-degree homogeneous, functional form in the dx^i (as, for instance, the Newtonian form $ds = g_i dx^i$). Postulating this form axiomatically, one foregoes the possibility of understanding the reason for its validity. The second difficulty is that if the g_{ij} are defined by means of the chronometric hypothesis, it seems not at all compelling—if we disregard our knowledge of the full theory and try to construct it from scratch—that these chronometric coefficients should determine the [geodesic] behaviour of freely falling particles and light rays, too. Thus the geodesic hypotheses, which are introduced as additional axioms in the chronometric approach, are hardly intelligible; they fall from heaven like eqn (1). Finally, once the geodesic hypotheses have been accepted, it is possible, in the theories of both special and general relativity, to construct clocks by means of freely falling particles and light rays, as shown by Marzke [12] and, differently, by Kundt and Hoffman.[13] Thus, these hypotheses alone already imply a physical interpretation of the metric in terms of time. The chronometric axiom then appears either as redundant or, if the term "clock" is interpreted as "atomic clock", as a link between macroscopic gravitation theory and atomic physics: it claims the equality of gravitational and atomic time. It may be better to test this equality experimentally or to derive it eventually from a theory

[12] R. F. Marzke, "The Theory of Measurement in General Relativity," A.B. senior thesis, Princeton University, 1959.

[13] W. Kundt and B. Hoffman, "Determination of Gravitational Standard Time," in *Recent Developments in General Relativity* (New York: Pergamon Press, 1962).

> that embraces both gravitational and atomic phenomena, rather than to postulate it as an axiom.
>
> For these reasons, we reject clocks as basic tools for setting up the space-time geometry [14] and propose to use light rays and freely falling particles instead. We wish to show how the full space-time geometry can be synthesized from a few assumptions about light propagation and free fall (64/5).

Ehlers, Pirani, and Schild no more explain than Synge himself on what grounds it is assumed that the chronometrically ascertained *ds* values are those of a Finslerian metric. But they object that in Synge's account the *speciation* of a physically unproblematic Finslerian space-time metric into a uniquely Riemannian metric is gratuitous: Like manna, it "falls from heaven." I must refer the reader to chapter XXII of my *Philosophical Problems of Space and Time* [15] for a discussion of whether their own alternative ontology of the GTR space-time metric can avoid a counterpart to Synge's "manna" in its logical edifice. In any case, as I have already pointed out, by eschewing the solid rods of the more traditional rods-and-clocks method (i), Synge's chronometric method forfeits such justification as the presumably Pythagorean infinitesimal metric behavior of rigid rods can furnish for adopting the Riemannian species of space-time metric. But a corresponding justification is available to Synge, if it be granted that his clocks exhibit the special-relativistic clock *retardation* in local inertial frames.

(iii) *The Geodesic Method.* As I have explained elsewhere,[16] for surfaces in Euclidean 3-space, the prescription that a certain family of lines qualify as geodesics of the surface does *not* determine the metric of the surface up to a constant positive scale factor, or even the Gaussian curvature of the surface modulo such a factor. For example, I show there that, on an ordinary table top, the familiar straight lines qualify as geodesics with respect to various metrics which can differ *other than* by a scale factor and which generate not only *different* partitions into equivalence classes of congruent intervals, but also *different* partitions of the class of *angles* into metrical equivalence classes.[17] More generally, consider the case of

[14] See also Marzke and Wheeler, "Gravitation as Geometry-I: The Geometry of Space-Time and the Geometrodynamical Standard Meter," in H. Y. Chiu and W. F. Hoffman, eds., *Gravitation and Relativity* (New York: W. A. Benjamin, 1964).

[15] Second revised edition (Boston & Dordrecht: Reidel, 1973), hereafter "*PPST*, 2d ed.", p. 745.

[16] *Ibid.*, chapter III, Section B.

[17] For further details, see T. J. Willmore, *Differential Geometry* (New York: Oxford, 1959), pp. 87/8.

the *positive definite* Riemann metrics, familiarly encountered in the geometries of three-dimensional physical space. The mere specification of those curves in a given space which are to count as geodesics of *such* a Riemannian metric does not determine a partition of the *angles* into metrical equivalence classes *and* does not suffice to single out a metric up to a constant positive scale factor k. The reason is that a mere *geodesic mapping* of the space onto itself does *not* determine the metric tensor g_{ik} even up to a conformal transformation, let alone up to a similarity transformation or modulo k. On the other hand, if we prescribe both the partition of angles into metrical equivalence classes—via a suitable conformal mapping of the surface onto itself—*and* what paths are to be geodesics, then the metric (tensor) is prescribed to within k. In short, the combination of the conformal and geodesic structures does circumscribe the metric (tensor) modulo k. The geodesic structure is often called "the *projective* structure," because the term 'projective differential geometry' denotes the study of those properties of the geodesic paths themselves which are independent of any and all arc lengths defined on them.

The metrics of the space-times of the GTR are *indefinite* Riemannian metrics of Minkowskian signature. As we shall see shortly, in the context of the GTR's indefinite Riemannian space-time metric, Hermann Weyl [18] was able to impose two compatible requirements which generated conformal and geodesic ("projective") space-time structures, respectively. And in this way he effected a new theoretical circumscription of the GTR's space-time metric modulo k.

We saw in our quotation above from Ehlers, Pirani, and Schild that these authors are concerned to provide a rationale for the Minkowskian kind of indefinite space-time *Riemann* metric used in the GTR, by deriving it from assumptions and requirements which they regard to be more basic, as it were. By contrast, Weyl assumes at the outset, without any more rudimentary motivation, that in any GTR universe, the space-time metric (hereafter "ST-metric") has this particular character.[19] In essence, Weyl then goes on to obtain a *compatible* trio of assumptions by imposing the

[18] *Space-Time-Matter* (New York: Dutton, 1922; Dover, 1950), hereafter *"STM,"* pp. 179, 228/9, 313/4; *Raum-Zeit-Materie* (Berlin: Springer, 1923), 5th ed., hereafter *"RZM,"* pp. 228/9; *Mathematische Analyse des Raumproblems* (Berlin: Springer, 1923), hereafter *"MAR,"* pp. 18/9; *Philosophy of Mathematics and Natural Science* (Princeton: University Press, 1949), hereafter *"PMNS,"* p. 103.

[19] For a more detailed statement by Ehlers, Pirani, and Schild of the divergences of their construction from Weyl's, see *op. cit.*, p. 68 and fn.

following two further conditions as to how the ST-metric is to be chosen: First, at each world point, light rays and only light rays are to be metrically *null* space-time trajectories, and second, the physical space-time trajectories of any and all freely falling mass particles are to count geometrically as *geodesics* lying *inside* the light cone at each point. The *compatibility* of these two further conditions rests on the GTR assumption that the totality of all free-fall mass particle trajectories passing through an event determines the light cone as its space-time boundary, since a free particle of positive rest mass, though always slower than light, can pursue a light signal arbitrarily closely. Let us see what contribution is made by each of these two compatible requirements to singling out, for any GTR world of given mass-energy distribution, a ST-metric that is unique modulo k and whose metric tensor has a *non*-zero determinant.

1. The Requirement of the Metrical Nullity of Light Trajectories in Space-Time ("ST"). Let us formulate more precisely Weyl's GTR requirement (*STM* 228, *RZM* 228) that the ST trajectories of light pulses be coextensive with metrically null ST lines. At any given world-point P, consider the class D of ST directions of any and all infinitesimally neighboring distinct events Q, whose respective relative coordinates are given by a set dx^i in each case. Then Weyl first requires the metric tensor g_{ik} to be so chosen that within the class D of directions dx^i at any given world-point, all and only those ST directions which are physically permitted to light signals (either "incoming" or "outgoing") satisfy an equation of the form

$$ds^2 = g_{ik}\, dx^i\, dx^k = 0.$$

Letting the term 'photon' function interchangeably with the pre-quantum-theoretic term 'light ray' in this context, let us designate this metrical nullity requirement for photons by the abbreviation 'photon-$ds = 0$'.

Thus, Weyl first requires g_{ik} to be so chosen that the particular directions singled out by infinitesimal light propagation at any given point P all be metrically null vectors. Since the infinitesimal light rays at P combine into a "double" conical hypersurface in space-time, the latter's two lobes are thus *null* hypercones. By coupling the stated nullity stipulation as to the permissible kind of metric tensor with the GTR assumption concerning the behavior of photons at any one world-point, Weyl assures that infinitesimal light propagation determines an infinitesimal "double" *null* cone at each point. Since photons have a vanishing rest mass, we shall also speak of mass*less* (test) particles as generators of null cones.

To what extent does Weyl's first requirement photon-$ds = 0$ *restrict* the allowable metric tensor of the ST-metric? Let us elaborate on Weyl's own quite cryptic statement of the reasoning that issues in his well-known answer to this question. At any given ST point P, the equation $g_{ik}\, dx^i\, dx^k = 0$ generally contains ten independent values g_{ik}; i.e., there are at most ten such independent values. And at any point P, the nine sets of relative coordinates dx^i for nine nearby events, respectively, located on as many different photon trajectories through P furnish nine independent equations

$$g_{ik}\, dx^i\, dx^k = 0$$

which together suffice to determine the nine independent *ratios* of the ten independent values g_{ik}. It will be noted that the nine *null* directions of as many photons determine only the *ratios* of the g_{ik} values at P, because $ds = 0$ for each of these directions. Furthermore, the ST directions dx^i allowed to photons at P are restricted to a double conical hypersurface. Hence it turns out that taking additional *null* directions would yield equations which are merely redundant with those corresponding to the initial nine null directions and would *not* restrict the g_{ik} values any further than their nine independent ratios do. Thus Weyl's first requirement photon-$ds = 0$ determines only the nine independent *ratios* of the g_{ik} at any given ST point P. In particular, the fixation of these ratios does *not* suffice to determine the ten independent *values* g_{ik} at P up to a positive factor k of proportionality *which is the same constant from point to point*. Contrast this with the feasibility of the determination of the ten individual *values* g_{ik} at P, at least up to a positive scale factor k which is the *same* constant from point to point, in Synge's aforementioned case of ten *positive* values ds for ten known *time-like* directions dx^i.

Since Weyl's first requirement photon-$ds = 0$ determines *only* the *ratios* of the g_{ik} at any given point, this requirement *allows*, in any *one* coordinate system, alternative functions g_{ik} as follows: These differ *other than* by a positive constant scale factor k that is the same from point to point. Hence, instead of determining the metric tensor or the metric ds up to a positive *constant* scale factor k, Weyl's first requirement determines the metric ds only up to *conformal* transformations of the metric tensor g_{ik}. The invariance of $ds = 0$ for photon trajectories under conformal transformations of the metric tensor and only under such transformations means the following: If g_{ab} is a metric tensor that assures the metrical nullity $ds = 0$ of photon ST trajectories, then this same nullity will be assured by just those non-zero metric tensors g_{ab}^* which are related to g_{ab} by a positive multiplying function $f(x^i)$ of the coordinates. And clearly the case in which the conformal factor $f(x^i)$ is a positive *constant* k is only a very special case. Thus, Weyl's requirement photon-$ds = 0$ defines a conformal structure for space-time and thereby allows an infinite class of *non*trivially different metric tensors. Therefore, if, in this context, the allowed metric tensors are to be further restricted up to a positive constant scale factor k, one must demand more than the coextensiveness of the ST trajectories of light pulses with metrically null world-lines. And, of course, the further restriction to be imposed must be compatible with Weyl's first requirement. In the statement of this restriction, let it be understood that a ST-trajectory lying inside the null cone at each of its points will be called "timelike."

2. The Requirement that the Physical ST Trajectories of Freely Falling Mass Particles Qualify Geometrically as Timelike Geodesics of the Indefinite Riemannian ST Metric. Let it be granted for now that all freely falling (test) particles of *positive* rest mass, i.e., "massive" (test) particles, have determinate ST trajectories, and that there are at least two such world-lines through every event.[20] Then Weyl compatibly supplements the conformal structure with a so-called *"projective"* structure on space-time by imposing the following further requirement (*MAR* 18/9; *PMNS* 103): The metric tensor g_{ik} of the indefinite Riemannian ST metric ds of Minkowskian signature is to be *so chosen* that all ST trajectories of freely falling massive (test) particles are turned into timelike GEODESICS. In other words, Weyl also stipulates that the metric ds be so chosen as to enable the ST trajectories of massive particles to qualify or count as timelike geodesics via the equation $\delta \int ds = 0$.

Having also imposed this important *geodesicity* requirement, Weyl is able to show that the class of conformally related metric tensors singled out by the prior requirement photon-$ds = 0$ is restricted further to a particular proper subclass whose members are pairwise related by any constant positive factor k. The factor k is constant in the sense of being the same from point to point rather than varying with the coordinates. One often speaks of any particular choice of this scale factor k as a trivial matter, on the

[20] Weyl, "Zur Infinitesimalgeometrie: Einordnung der projektiven und konformen Auffassung," *Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen* (1921): 99–112; reprinted in K. Chandrasekharan, ed., *Gesammelte Abhandlungen,* II (New York: Springer, 1968), p. 196 (page reference is to the reprint). See also *Space-Time-Matter,* Appendix I, p. 314.

presumed ground that the laws of nature are invariant under all choices of a metrical unit of "length." But Weyl (*PMNS* 83) remarks that in a quantum world the laws of atomism restrict the allowable values of k to some extent. Regardless of whether k is thus restricted, however, Weyl's construction—hereafter "Weyl's geodesic method"—circumscribes the metric (tensor) modulo k. And since all the other geometric structures can be obtained from the metric (tensor) in a straightforward manner, Weyl's geodesic method generates a unique metric geometry for the space-time of any GTR world of given mass-energy distribution.

As Weyl has been concerned to stress, if his geodesic method is not circular logically or epistemically—about which more below—then it determines the ST-metric modulo k both logically and epistemically, at least in principle, "without reliance on clocks and rigid rods" (*MAR* 19). By thus dispensing with the atomic clocks of Synge's method and with the rigid rods of Einstein's, Weyl has offered an important alternative foundation for the ST-metric of the GTR.

As we have seen, the construction of the GTR ST-metric by Ehlers, Pirani, and Schild—hereafter "E&P&S"—differs from Weyl's at least as follows: The former three authors set themselves the task of deriving or providing a rationale for the Minkowskian kind of indefinite Riemann ST-metric, rather than merely circumscribing it to within k after it first "fell from heaven," as they put it. But in carrying out their task, E&P&S define compatible conformal and projective structures on space-time by appealing, just as Weyl does, to the behavior of both massless and massive test particles, as postulated by the GTR.[21] And after having defined the latter compatible structures, E&P&S specify additional necessary and sufficient conditions which then yield a Riemannian metric that is unique modulo k. In this way, they aim to improve on Weyl's version of the geodesic method by showing that the existence of a unique Riemannian metric structure need not be postulated beforehand in order to generate it by means of the joint behavior of massless and massive test particles.

Despite the stated divergences of the E&P&S method from Weyl's method, the E&P&S method is a *geodesic* method in essentially the same sense as Weyl's. Furthermore, Weyl, like Ehlers, Pirani, and Schild, denies rigid rods any role in the physical foun-

[21] The otherwise admirable presentation by E&P&S expositorily glosses over points that make it less perspicuous than it should be if its similarities and differences from Weyl's version of the geodesic method are to be readily discernible. For details, see the discussion on pp. 774/5 of *PPST,* 2d ed.

dations of the GTR ST-metric. Hence they all forfeit, no less than Synge does, such justification as the presumably Pythagorean infinitesimal metric behavior of rigid rods can furnish for adopting the Riemannian species of ST-metric.

We must now devote some attention to the logical and epistemic status of the concept of a *free* massive particle in the geodesic method as such.

In his 1921 paper, Weyl appeals to Einstein's principle of equivalence of inertial and gravitational mass as the basis for the physical assumption of his geodesic method that a freely falling massive particle has a determinate inertial motion in the sense of a determinate time-like ST-trajectory, regardless of its other characteristics such as mass and (chemical) composition. He writes: "In the theory of relativity, the projective and conformal structure have an immediately palpable significance. The former, the inertial tendency of the ST-direction of a moving material particle which imparts to it a determinate 'natural' motion upon its release in a given ST-direction, is that unity of inertia and gravitation by which Einstein replaced both, but which has so far lacked a suggestive name" (1968 reprint, 196; translation is mine). Since Weyl first introduced his geodesic method, it has become known that upon its release in a given ST-direction, the ST-trajectory of a freely falling gravitationally *multi*pole particle will not be the same as that of a gravitationally *mono*pole particle which is falling freely under otherwise relevantly identical conditions. And if the geodesic method is to yield the results demanded by the full-blown GTR, the time-like ST-trajectories of free massive particles having a gravitational *multi*pole structure *cannot* count as time-like *geodesics,* whereas the world-lines of the *mono*pole particles should so count.

But John Stachel has pointed out that this complication threatens the construction of the ST-metric by the geodesic method with logical circularity, and derivatively with epistemological circularity, as we shall now see. Let us assume, for argument's sake, that one has succeeded in assuring that the massive particles are free to the extent that they are effectively insulated from any and all KNOWN influences which, according to the GTR, would alter their ST-trajectory and render the geodesic method inoperative. This is explicitly assumed to be feasible by Graves (*op. cit.,* p. 171). And it is recognized as a practical problem by Kundt and Hoffman, who speak of the emission of "geodesic test particles" by an observer and remark: "This will pose practical problems since the effects of spin motion and of the electromagnetic field must be made negligible" (*op. cit.*, p. 305). Furthermore, let us assume that even if there are as yet unknown perturbing influences in our GTR universe, our massive particles are effectively free of them as well by some good fortune. Even then the multipolarity of spinning particles poses more than just practical problems.

Specifically, the geodesic method is faced by the following problem: It must be able to avail itself, in a logically *non*circular way, of the distinction between gravitationally monopole and multipole free massive test particles. For it must *exclude* the ST-trajectories of the latter when stipulating that only the world-lines of the former are to be coextensive with the time-like geodesics of the desired Riemannian metric. And, of course, the latter metric is first going to be made available logically in the geodesic method by combining this geodesicity stipulation with the conformal structure of light rays. Hence no part of the GTR that is predicated on the resulting metric may be presupposed, on pain of vicious logical circularity, in order to distinguish at the outset between the two species of free test particles when imposing the geodesicity requirement on only one of them. Yet, as Stachel has explained, it would appear that precisely such a presupposition is needed, much as for method (i) above.

This logical circularity seems to beset the E&P&S version of the geodesic method no less than Weyl's. Consequent upon this logical circularity, there is also the epistemological one of knowing how to identify without (tacit) appeal to the resulting metrical theory, the gravitationally *right kind* of free particle, when first trying to ascertain the metric by the geodesic method. The geodesic method would become vitiated by patent epistemic circularity, if it were to seek to identify gravitationally monopole free particles by first attempting to *ascertain* whether their ST-trajectories are, in fact, geodesics! That Weyl was quite generally sensitive to the risk of possible epistemic circularity is attested by the fact that he wrote as follows: "Thus, if in the real world it is possible for us to discern the propagation of effects, and of light propagation in particular, and if moreover we are able to recognize as such the motion of free mass points which obey the metrical field, and to observe that motion, then we are able to read off the metric field from this alone, without reliance on clocks and rigid rods" (*MAR* 19, translation is mine).

As we noted, Graves (170) was likewise aware of the risk of epistemological circularity in the geodesic method. But he overlooked the problem of logical *and* epistemic circularity posed by gravitational multipolarity of free particles. And he took no cognizance of the role of the conformal structure in the geodesic method, but assumed without argument that this method can achieve its objective by means of the projective structure alone, writing: "The choice of geometry, and the geodesic hypothesis, is derived from the central empirical conclusion from the equivalence principle: a class of space-time paths is observationally singled out as the trajectories of *all* freely falling bodies. The geometry is selected so that these are its geodesics, not because of some procedure with special rods and clocks." [22] Having thus greatly oversimplified the ontological and epistemic merits of the geodesic method vis-à-vis both the method of rods-and-clocks and Synge's chronometric method, Graves felt entitled to write:

> . . . Reichenbach, Grünbaum, and their disciples are correct in arguing against the conventionalists that once a standard has been chosen, the geometry is determined. . . . However, they completely miss the point by speaking as if the standard were some *particular* kind of rods and clocks. The crucial fact is rather that *any* body will do. Furthermore, it is not any assumed metrical properties of the standard (such as invariance under transport) that are relevant, but simply its path in free fall. Once we can trace all these paths we have all the information we need to determine the geometry. We have not yet *measured* the intervals and curvatures, to be sure; but measurement is a separate operation. In measurement we *discover* the metrical features of the already determined geometry; whereas on Reichenbach's theory, we *invent* a geometry as an attempt to make the results of our various measurements consistent (*Conceptual Foundations,* p. 172).

But, in view of multipolarity, it is *not* the case, as Graves would have it, that "*any* [free] body will do." And as against Graves's oversimplified verdict as to the relative merits of the geodesic method (152–164, 170–172), it seems that our analysis sustains a different conclusion: Each of the three methods (i), (ii), and (iii) above has its own logical and epistemological "dirty linen," as it were. And this conclusion points up anew the likely moral that I stated by means of a quotation from Hilary Putnam àpropos the difficulties besetting method (i).

In this latter vein, John Stachel has commented on the E&P&S version of the geodesic method, writing:

[22] "Reply to Stein and Earman," this JOURNAL, LXIX, 19 (Oct. 26, 1972): 647–649, p. 648. This published paper is the truncated form of a longer unpublished reply to Stein and Earman, which Graves presented orally at the December 27, 1972, Boston meeting of the Eastern Division, APA.

> Since this method of test particles has recently come into favor, it is very satisfying to see it fully developed. Of course, each of the three methods currently proposed for explicating the significance of the metric structure of space-time (rods and clocks, paths of massive particles and clocks, and paths of massless and massive particles) has advantages and drawbacks. Thus, it is perhaps better to regard them as alternative ways of looking at the implications of a metrical structure for space-time than to claim absolute superiority for one.[23]

Coupled with Graves's neglect of the geodesic method's specified "dirty philosophical linen" is his blithe assumption that ontologically *this* method must be preeminently foundational not only for standard 1915 GTR, but especially for the Riemannian metric of Wheeler's *empty* curved space-time. To assess this assumption, I shall now disregard the aforementioned "dirty linen" when I ask: In an empty GMD universe, what is the ontic status of the massive free test particles of the geodesic method? Clearly, in the GMD ontology, these geodesic material particles are *not* primitive entities, any more than are the much maligned rods-and-clocks of method (i) or Weyl's photons or Synge's atomic clocks. Instead, in the empty world of GMD, the geodesic material particles are themselves held to be literally constituted out of empty, curved, metric space-time to begin with, no less than any and all other traditionally used physical metric devices! It is a consequence of GMD's all-out *geometrical* reductionism and absolutism that the metric of empty space-time can no more *first* be *induced* in the ST-manifold (in part or whole) by geodesic particles than by rigid rods. In other words, the GMD ST-metric cannot consistently first be grounded ontologically even partly on the inertial behavior of geodesic particles, although that behavior *can* physically *realize* or single out ST-trajectories which qualify as (time-like) geodesics with respect to the GMD ST-metric. For in GMD no entities other than the structure of empty space-time itself are *ontologically* necessary for endowing space-time with metric ratios or with such curvature properties as are determined by these ratios. Hence, according to GMD, any and all particles or radiation that serve to specify the Riemannian metric of the empty GMD space-time can do so at best *only epistemically* as a means of our discovering that metric.

Surely it cannot legitimately be assumed tacitly without any further ado that in the context of GMD the highly derived or non-

[23] "Space-Time Problems," a review of the Synge *Festschrift* entitled *General Relativity* in which the Ehlers, Pirani, and Schild paper cited above appears, *Science*, CLXXX (Apr. 20, 1973): 292–293, p. 292.

primitive ontological status of the geodesic particles can be rendered innocuous or mitigated in this context by simply appealing to their smallness qua so-called "test" particles. Why then should we accept Graves's undaunted declaration that vis-à-vis both the rods-and-clocks of method (i) and the atomic clocks of method (ii), the freely falling material geodesic particles of Weyl's method (iii) play a preeminently foundational role *ontologically,* not only in standard 1915 GTR but in GMD??

So far, we have only partially articulated the ontic status of the metric in GMD as distinct from the ontologically more non-committal standard 1915 GTR. We must now turn to the ramifications of that articulation.

II. THE ONTOLOGY OF EMPTY CURVED METRIC SPACE IN THE GMD OF CLIFFORD AND WHEELER

The empty space(-time) of the Clifford-Wheeler GMD is both metric and curved. Here we shall deal only with the first of these two fundamental entities. I have explained why I cannot share Graves's somewhat dogmatic belief that the geodesic method enjoys foundational ontic preeminence both in standard 1915 GTR and in GMD. I shall nonetheless find it very useful to employ Weyl's geodesic method as a framework for treating the issues that will now concern us. Wheeler's GMD assumes a Riemannian kind of ST-metric at the outset no less than Weyl's version of the geodesic method does. Hence the latter is fully as germane to our ontological analysis of GMD as the E&P&S version of that method. Furthermore, I believe that, *mutatis mutandis,* the conclusions we shall reach by reference to Weyl's geodesic method are obtained not only via the E&P&S version of the geodesic method but also via either the rods-and-clocks method or Synge's chronometric method. These forthcoming ontological conclusions fully allow but do not require that there is *concordance* of the metric results furnished by the three major metrical methods (i), (ii), and (iii). As Kundt and Hoffman have noted, it is, of course, a matter of experimental fact whether there is such concordance, i.e., whether these three methods are actually alternative specifications or physical realizations of one and the same ST-metric modulo k.

We shall see that two quite different verdicts will be reached as to the ontic status of the GMD ST-metric according as one rejects or accepts one of the cardinal tenets of Riemann's own conception of the foundations of the metric of continuous n-dimensional physical space. Indeed, the latter tenet will be seen to be incompatible with the ontological commitments of GMD. In view of the important ramifications of this incompatibility, it behooves us now to recall this explicit and fundamental idea of Riemann's, which pertains to the basis for the metric equality or "congruence" of intervals in a continuous n-dimensional spatial or temporal manifold. Weyl (*STM* 97/8) discussed this idea and then expressed it very concisely as follows: "according to Riemann, the conception 'congruence' leads to no metrical system at all, not even to the general metrical system of Riemann, which is governed by a quadratic differential form" (101).

Here Weyl is alluding to the fact that, in Riemann's 1854 inaugural dissertation, an important distinction is drawn between two kinds of metrics: a (nontrivial) metric which is "implicit" in, or intrinsic to, the space on which it is defined, on the one hand, and a metric which is correspondingly nonimplicit or extrinsic. This distinction of Riemann's emphatically must NOT be confused with the very different *Gaussian* contrast that is unfortunately also expressed by means of the terms 'intrinsic' and 'extrinsic'. Since Riemann set forth his distinction only illustratively and intuitively, I have attempted in an earlier article [24] to give a relatively much more precise explication of it.

On the basis of this explication, I gave a more precise formulation (547) of the following thesis of Riemann's, which I dubbed "Riemann's Metrical Hypothesis" (RMH) and whose attribution to Riemann I documented there in detail: In a continuous n-dimensional physical space, there is no kind of *intrinsic* basis at all for any nontrivial metric that would qualify as *implicit* in that space to within a constant positive scale factor k. Hereafter I shall refer to this latter hypothesis by the abbreviation 'RMH'.

Let us pause briefly to illustrate the intellectual ubiquity of the intuition underlying Riemann's claim that the presumedly differentiable manifold of physical space is devoid of any nontrivial "implicit" or intrinsic metric. Thus, the mathematician Morris Kline spoke of the metric as *imposed* on the continuous spatial manifold when he wrote: "Strictly speaking, Riemann's curvature, like Gauss's, is a property of the metric imposed on the manifold rather than of the manifold itself." [25] Note here the contrast between "a property . . . of the manifold itself," i.e., a property intrinsic to the manifold, on the one hand, and, on the other hand, "a property

[24] "Space, Time, and Falsifiability, Part I," *Philosophy of Science*, XXXVII (1970): 469–588, part A, § 2.

[25] *Mathematical Thought from Ancient to Modern Times* (New York: Oxford, 1972), p. 892.

of the metric imposed on the manifold," i.e., a property extrinsic to the manifold because it is first bestowed upon it by an imposed and hence extrinsic metric.

It is a corollary of RMH, as extended to continuous four-dimensional physical space-times, that entities *extrinsic* to these differentiable manifolds are needed ontologically to generate, in a GTR universe of given mass-energy distribution, the GTR's *particular* metric ratios of ST-intervals, ratios by which the metric geometry is determined to be what it is in the given case. By contrast, according to GMD, the space-time in which we live is both empty *and* metric (Riemannian), so that *no* entities other than four-dimensional empty space-time itself are *ontologically* necessary for endowing that physical manifold with a particular set of Riemannian metric ratios! Thus, in virtue of the emptiness of the ST-manifold, its Riemannian metric must be "implicit" in it or intrinsic to it modulo k in the sense of at least *not* being imposed on it, but of being grounded solely in the very structure of that four-dimensional physical manifold itself.

Furthermore, as we saw at the end of section I, according to GMD's all-out *geometrical* reductionism and absolutism, it cannot be held that the ST-metric is FIRST *induced* in its four-dimensional manifold by the behavior of such entities as (atomic) clocks, rods, light rays, geodesic particles, or the like. For though in GMD these agencies are claimed *not* to be "foreign entities immersed" in space-time—to use Wheeler's parlance—they *are* asserted to be *reducible* to the ontologically and logically *prior* metric geometry of space-time. Yet *Riemann* deemed the behavior of at least some devices of this kind to be *ontologically necessary* for first conferring a metric structure upon continuous physical space or time. Since, contrariwise, GMD holds all such devices to be themselves ontologically reducible to curved empty metric space-time, in that theory the ST-metric cannot be ontologically grounded on their behavior. Instead, in GMD such devices can at best *physically realize* the intrinsic metric equalities with which GMD claims empty space-time to be endowed. In this way, such devices can function *epistemologically* in measurement.

We see that if GMD's program of reducing all of physics to metric ST-geometry were to be successful empirically—a possibility which Wheeler himself has now discounted as unlikely—then this putative explanatory success could redound to the empirical *discredit* of RMH. Conversely, in the absence of such massive empirical success or pending such success, the assumption of RMH may be warranted and has the consequence of impugning the ontology of GMD. In my 1970 paper (521–524), I pointed out this logical consequence after having following Weyl (*STM* § 12) in calling attention to the prior scientific fruitfulness of RMH for standard 1915 GTR.

Thus I stated (470) that I question the Clifford-Wheeler statements of GMD in regard to "the compatibility of the theory [GMD] with the Riemannian metrical philosophy apparently espoused by its proponents." [26] But I trust that the logical incompatibility between the GMD ontology of the metric and RMH is now sufficiently clear. Hence I do not see that my earlier claim of such incompatibility deserves the following assessment made by John Earman: "Some philosophers are not content with mere empirical objections and can be satisfied only by a-prioristic refutations. In this vein, Adolf Grünbaum argued that GMD is incoherent." [27] According to Earman, my argument for incompatibility is "a prime example" (647, fn 13) of what he had described as "the tendency (all too prevalent in current philosophy) to view relativity theory as only a source for grist for some philosophical mill" (634). And Earman deplored the latter pernicious tendency at the end of the very same sentence which he began by extolling Graves's book in the following way: "I will not dwell here on . . . how loudly I applaud his [Graves's] attempt to describe relativity theory from the point of view of scientific realism" (634). Presumably, on Earman's view the espousal of scientific realism is free from the fetters of philosophical apriorism because a scientific realist interpretation of the GTR is vouchsafed in a straightforward way, much as one can read off the names from a telephone directory. I shall not enter here into objections to such uncritical ontological *literalism* for physical theory. Suffice it to invite the reader to note how much analysis it took in chapter XIX of *PPST*, second edition, to try to show that the coarse-grained entropy of classical statistical mechanics may be construed in scientific realist fashion instead of being a mere anthropomorphism.

Just as the presumed truth of RMH can serve to impugn the GMD ontology of the metric, so also it can provide a basis for dealing with the comment made by Clark Glymour [28] on my question "what serves to individuate the metrically homogeneous punctal event elements of the space-time manifold?" in GMD. If the

[26] The exegetical aspects of this expression of incompatibility are discussed in ch. XXII, § 3(b), pp. 783–788, *PPST*, 2d ed.

[27] "Some Aspects of General Relativity and Geometrodynamics," this JOURNAL, LXIX, 19 (Oct. 26, 1972): 634–647, p. 646.

[28] "Physics by Convention," *Philosophy of Science*, XXXIX (1972): 322–340.

aforementioned fruitfulness of RMH for classical 1915 GTR is taken to warrant the adoption of RMH as a working assumption, then it provides precisely the grounds for which Glymour asked when he said: "If metric relations are specifically excluded from the class of individuating relations then . . . we must ask for the grounds of this exclusion" (338). For, according to RMH, no intrinsic nontrivial metric relations are available, and surely the individuality of the fundamental world-point entities of the manifold cannot be made to rest on *extrinsic* metrical relations.

Glymour asked for the grounds for excluding metric relations from the class of individuating relations after having made the following statement by reference to a certain version of Leibniz's principle of the identity of indiscernibles: "we may then regard the entities of the manifold as individuated by their metric relatons" (*ibid.*). To this I say the following. If the empirical success of GMD's program of reduction were sufficient to sustain its dictum *"Physics is geometry,"* then RMH could no longer serve as a viable basis for impugning the GMD ontology of the metric. And *if,* moreover, the intrinsic metric relations postulated by GMD could be demonstrated to individuate the world-points of its ST-manifold, then I would agree with Glymour that the doubts I raised about GMD in regard to individuation are unwarranted. Indeed, Glymour overlooked that I had said as much in my 1970 paper (524) à propos the so-called "intrinsic coordinate systems" which obtain under conditions sufficiently *heterogeneous* to yield four *scalar* fields that can individuate *each* world point.

In any case, if such philosophical appraisal of GMD as inquiring into the adequacy of its principle of individuation is to be condemned with Earman as either invidiously aprioristic or "frivolous" (647), then one wonders what role, if any, Earman envisions for philosophical analysis and criticism of a scientific theory, as distinct from purely empirico-mathematical or technical appraisal of it. In the same paper, Earman (637) seeks to impugn a statistico-thermodynamic account of the "arrow of time" on the basis of the time orientation of an assumedly time-orientable relativistic space-time. But I have argued that the incompatibility alleged by Earman does not obtain, and that the time-orientation of space-time no more illuminates the past-future attributes than does the oppositeness of two senses on the time-axis of Newton's particle mechanics (*PPST,* 2d ed., 788–800).

Returning to the upshot of our discussion of the GMD ontology of the metric so far, we can say the following:

(a) If we accept the GMD claim that modulo k the four-dimensional ST-manifold is *intrinsically* and Riemannianly metric, then we must reject Riemann's own RMH.

(b) GMD postulates that there is an intrinsic basis for the metrical nullity of photon lines in space-time *and* for a particular set of metrical ratios among non-null ST-intervals. GMD then asserts that free massive particle trajectories qualify—via $\delta \int ds = 0$—as time-like geodesics with respect to any metric ds that generates both these intrinsic metric ratios and the stated metrical nullity for photons. Thus, in GMD the metrical time-like geodesicity of massive particle world-lines and the metrical nullity of light rays obtain as a matter of intrinsic fact. Having this presumed intrinsic foundation, the *geometrical* status of these two sets of world lines in GMD is *not* a matter of convention, stipulation, or human decree.

(c) Since GMD asserts massive free particles and photons to be *geometrically reducible* entities, the geodesic method of Weyl and of E&P&S does not provide an ontological foundation of the GMD ST-metric but only a specification or circumscription of that metric modulo k.

Being mindful of this item (c), let us recall from our discussion of the rods-and-clocks method under (i) in section I above, Einstein's reservations about the foundational invocation of rigid rods as a (partial) physical basis for the GTR ST-metric. Then it becomes clear from item (c) that *qua ontological foundation* of the ST-metric, the geodesic method should probably be deemed even less satisfactory *for GMD* than rigid rods (and clocks) are for classical 1915 GTR.

I have stressed the logical incompatibility of the GMD ontology of the ST-metric with RMH. But I have yet to articulate the ramifications for the appraisal of that GMD ontology, if one assumes RMH and founds the ST-metric of classical (standard) 1915 GTR on the geodesic method.

Assuming RMH and the philosophical import of RMH as I have developed it, our earlier scientific statement of Weyl's geodesic method in section I (iii) of the present paper now requires philosophical supplementation in the form of a series of statements as follows:

1. Given the assumption of RMH, there is *no* intrinsic basis and hence no *initial* GEOMETRICAL *warrant* for singling out any particular classes of ST-trajectories as being, respectively, distinguished in regard to time-like *metrical geodesicity* and *metrical nullity*. Nor

is such an initial geometrical warrant created by the mere PRE-geometrical fact that the physical behavior of photons and of relativistically free massive particles—*pace* the gravitationally multipolar ones—does, respectively, single out two classes U_p and U_m of ST-trajectories and thereby determines the membership of their union U. For, given RMH, the objective physical determinateness of the membership of U does not detract one iota from the fact that humans selected U without intrinsic geometrical warrant, as against other classes of ST-trajectories *on the foundation of which a differently curved or even flat ST-structure would arise.*

2. The physically determinate membership of U assures the compatibility of the following stipulations laid down by the geodesic method: An indefinite Riemann metric (tensor) *is to be* SO CHOSEN that, with respect to *one and the same such metric ds,*

(a) The photon trajectories at any given world point are to BECOME *metrically null.*
(b) The ST-trajectories belonging to the proper subclass U_m of U are to be TURNED INTO time-like GEODESICS via the intratheoretic defining equation $\delta\int ds = 0$.

Graves, who confines his attention to the subclass U_m in this context, distinguishes with commendable clarity between the physical determinateness of the membership of U_m, on the one hand, and the geometrical status of that membership as metrical geodesics on the other, when he says: "These paths are real and observable, quite independent of any geometry in which they may be represented" (*Conceptual Foundations,* 171). Indeed, the crucial logical transition from the mere physical ST-trajectorihood of the members of U_m, which is PRE-geometric, to their geometrical status becomes evident when Graves says of the free massive particle trajectories: "The geometry [i.e., the ST-metric modulo k] is selected so that these are its geodesics" ("Reply," 648).

Given the assumption of RMH, the members of U_m are a kind of metrical geodesics by human convention: the metrical ratios of non-null intervals with respect to which the members of U_m so qualify are devoid—in Riemann's sense of RMH—of any "implicit" or intrinsic foundation. Similarly for the metrical nullity of the photon trajectories that constitute the subclass U_p of U. Thus, if RMH is true, all the physical trajectories belonging to U acquire their specific *geometrical* status in Weyl's method extrinsically by human convention.

3. It follows that, if RMH is true, the metrical structure of space-time—and thereby the very constitution of the only autonomous substance in the GMD monistic ontology—depends crucially for being what it is not only on the physically determinate membership of *U*, but also on the *intrinsically* UNFOUNDED, humanly stipulated ascriptions of metrical *geodesicity* and *nullity* to the appropriate members of *U*! As I have argued in great detail in my *Geometry and Chronometry*, and as Gerald Massey [29] also argues, the inevitable metrical conventionality consequent upon RMH is not at all of the merely trivial semantical kind which holds alike for any and all as yet semantically uncommitted vocabulary, including the as yet unpreempted WORDS or *noises* 'geodesic', 'metric *ds*', etc. For the latter trivial semantical kind of conventionality obtains regardless of whether the ascriptions of metrical geodesicity and nullity made by Weyl's geodesic method do have an intrinsic foundation, as GMD maintains, or fail to have such a foundation, as RMH claims! By contrast, if the GTR ST-metric does have the intrinsic kind of foundation claimed by the GMD ontology, then the stated nontrivial metrical conventionality does *not* inevitably obtain.

GMD asserts that all of physics, even if not psychology, is reducible to the metric geometry of space-time. I submit that, whatever one's views on the mind-body problem, attributes depending fundamentally, even if only partly, on particular human stipulations are not at all the kind of item that can reasonably be taken to be essential ontological elements in the very constitution of the only autonomous substance in the physical world. Yet, as we have seen, if RMH is true, precisely this is the case in the monistic geometrical GMD ontology: granted RMH, human stipulations enter ontologically—not just verbally!—into making the metric geometry of space-time be what it is, and thereby these stipulations paradoxically generate the character of the only autonomous "physical" substance recognized by GMD. For, on the assumption of RMH, human conventions are indispensable to the GMD metric geometry in the sense of entering essentially into the generation of those metrical properties of ST-intervals by which the GMD geometry is made to be uniquely what it is modulo *k* in our actual world.

ADOLF GRÜNBAUM

University of Pittsburgh

[29] "Is 'Congruence' a Peculiar Predicate?" in R. S. Cohen and R. C. Buck, eds., *Boston Studies in the Philosophy of Science*, vol. VIII (Dordrecht: Reidel, 1971), pp. 606–615.