CHAPTER 2 – MACH EFFECT THEORY

GRAVITATIONAL ABSORBER THEORY & THE MACH EFFECT

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The origins of mass can be described in terms of Mach's principle, which states that the mass of a body is determined by its interaction with the rest of the mass-energy in the universe. However, if a body undergoes a sudden acceleration, you may ask, "How can the universe respond immediately in a way to conserve momentum?" In order to explain this, we introduce the concept of advanced waves, which have been used successfully in both classical and quantum physics for the last 70+ years.

1. INTRODUCTION

The work of Hoyle and Narlikar (HN) is a "masterpiece" in general relativity (GR) theory because it is fully covariant and incorporates fully the idea of Mach's principle. It is what Einstein dearly wanted to do, but didn't think he quite managed with standard general relativity. But perhaps he did? This paper shows that the results of HN, or what I would prefer to call *gravitational absorber theory* (GAT) can be obtained from Einstein's GR with the addition of a mass fluctuation in time. In Section 2, I show that adding $m(t)$ is all that is needed. I have renamed the theory to emphasise that I am not interested in the static universe model; I do not include the HN mass creation (C)-field. I am only interested in the Machian development of the theory through the use of retarded and advanced waves.

The local field around a mass particle can still be thought of as the overlapping of the many retarded and advaced waves, which themselves carry energy and momentum. This field will have a potential anywhere in space-time and constitutes the background vacuum. The mass particle transfers energy and momentum with the "field" here and now, which is basically th vacuum. However, when the particle accelerates the universe as a whole reacts to the acceleration, causing changes in the local field, which can be transmitted to the particle conserving mometum on a universal scale.

Einstein began his work on general relativity by seeking a concordance with Mach's principle. That is, to explain inertia of a test mass in terms of other masses in the universe. Sciama, Nordtvedt, and others have shown that masses in motion exert non-radial gravitational forces on nearby masses (frame dragging). In particular, Sciama showed that just this frame-dragging effect from the rest of the universe can account for inertia. Woodward has exploited the result of Sciama to design a propellantless propulsion device that depends on such forces.

In spite of its prediction of frame-dragging, and apparent ability to account for inertia, some researchers feel that general ralativity does not provide a fully self-consistent Machian picture. While Sciama and Nordtvedt can calculate the inertial force on an accelerated object due to the rest of the mass in the universe, we feel that general relativity does not account for the effect of the accelerated mass back on the rest of the universe.

To properly account for the effect of the accelerated mass back on the rest of the universe, we employ the concept of advanced waves, made famous by Wheeler and Feynman. Hoyle and Narlikar developed a theory of general relativity that incorporated advanced waves. While Hoyle and Narlikar are well-known for their steady-state cosmology work, and they use HN theory in that work, we feel their theory stands as a fine extension to general relativity, ignoring the parts regarding mass creation.

The beauty of gravitational absorber theory (GAT) is that it allows one to think of a mass, here and now, being influenced by the rest of the matter in the universe via gravitational signals travelling at speed c and does not rely on some (old fashioned Newtonian) notion of "action-at-a-distance" or faster than light propagation of signals. Real gravitational signals travelling at speed c carry information from every part of the universe to a mass here and now. The only trick is, to have mass react instantaneously, you must invoke the advanced wave solution to the relativistic wave equation. This advanced wave travels backward in time from the distant reaches of the universe, to convey momentum to the here and now, allowing back reaction to appear instantaneous.

The reason that HN theory did not catch on in the 1960's is twofold.

1. Hoyle was looking for a static universe cosmology theory. He introduced the "C" field as a creation field to keep the mass density constant as the universe expanded. This C field can be removed without loss of the underlying theory.

2. Hawking raised an objection to the HN theory in 1965 which basically put the last nail on the coffin. He suggested that by integrating out into the distant future, the advanced wave integrals would diverge. That is correct. However, since the universe is not only expanding but accelerating in that expansion, there is a cosmic horizon beyond which you cannot integrate. That cutoff prevents the advanced wave integrals from diverging and therefore re-establishes the HN theory as a good working theory.

3. Now is the time to look at gravitational absorber theory in a new light. Forget the static cosmology and move forward.

Standard GR has the problem that masses are treated as static. That is in general not the case. The background gravitational potential can be nonzero even in a flat spacetime. The GAT allows for a dynamic communication of signals from every part of the universe to the here and now to conserve momentum. *Furthermore, and this is conjecture, the superposition of retarded and advanced waves throughout the universe could be a mechanism to understand dark energy and matter. For example, dark matter might just be the manifestation of the gravitational potential at a location in space where the retarded and advanced waves do not perfectly overlap. For example at the location of an accelerating mass. As an electromagnetic analogy, consider photons appearing near an accelerating mirror in the dynamic Casimir effect or equivalently Unruh radiation.*

There is sufficient reason to reconsider the gravitational absorber theory of Hoyle and Narlikar. In section 2, we allow for a mass fluctation in the Einstein equation of motion (geodesic) and obtain the HN equation of motion, which is a new result. In section 3, we give a very brief history of the HN paper sequence and rewrite their notation to assist the reader. In section 4, we compare the Einstein action and field equation with the HN field equation. In section 5, we show that the mass fluctuation frmula calculated by Woodward from the precepts of general relativity can also be obtained from HN theory. This is the main result of the paper.

Advanced waves were introduced by Dirac in 1938 to describe radiation reaction. His radiation reaction force equation is still in use today and can be found in most standard electrodynamics text books. The advanced wave concept was given a physical interpretation by Wheeler and Feynman in 1945 [1]. The idea has since been used successfully in quantum mechanics by John Cramer and later in the theory of gravitation by Hogarth 1962 [2] and Hoyle and Narlikar 1964 [3,4] whose work we will summarize for convenience below.

1.1 Electron Radiation Reaction in Electrodynamics

Dirac [5] first introduced the idea of advanced waves in electromagnetism in order to derive the radiation reaction of an accelerating electron.

The idea is as follows. Consider a single electron undergoing acceleration. The field surrounding the electron can be thought of in two parts, the outgoing and incoming. The actual field surrounding the electron is the usual retarded Lienard-Wiechert potentials and any incident field on the electron.

$$F^{\mu\nu}_{\text{act}} = F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{in}} \tag{1}$$

Furthermore, the Maxwell 4-potential wave equation allows for advanced solutions, which are the same form as retarded, only they go backward in time. The advanced solutions also satisfy the wave equation in Lorentz gauge (below, with $c = 1$):

$$\Box A_\mu = 4\pi j_\mu \tag{2}$$

$$\frac{\partial A_\mu}{\partial x_\mu} = 0 \tag{3}$$

We could equally well describe the actual field surrounding the electron by

$$F^{\mu\nu}_{\text{act}} = F^{\mu\nu}_{\text{adv}} + F^{\mu\nu}_{\text{out}} \tag{4}$$

where the $F^{\mu\nu}_{\text{out}}$ is the total field leaving the electron. The difference between the outgoing waves and the incoming waves is the radiation produced by the electron due to its acceleration.

$$F^{\mu\nu}_{\text{rad}} = F^{\mu\nu}_{\text{out}} - F^{\mu\nu}_{\text{in}} = F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}} \tag{5}$$

In the appendix of Dirac's paper, it is shown that this equation gives exactly the well known relativistic result for radiation reaction which can be found in standard text books on electromagnetism, for example Jackson [6].

1.2 Wheeler & Feynman: Absorber Theory

Wheeler and Feynman [1] accept Dirac's result but wish to give a physical explanation as to where the advanced electromagnetic field comes from. They resort to a suggestion made by Tetrode [7] and later by Lewis [8] which was to abandon the concept of electromagnetic radiation as a self interaction and instead interpret it as a consequence of an interaction between the source accelerating charge and a distant absorber. The absorber idea has the four following basic assumptions, which we quote directly from Wheeler-Feynman [1],

(1) An accelerated point charge in otherwise charge-free space does not radiate electromagnetic energy.
(2) The fields which act on a given particle arise only from other particles.
(3) These fields are represented by 1/2 the retarded plus 1/2 the advanced Lienard-Wiechert solutions of Maxwell's equations. This force is symmetric with respect to past and future.
(4) Sufficiently many particles are present to absorb completely the radiation given off by the source.

Now, Wheeler-Feynman considered an accelerated charge located within the absorbing medium. A *disturbance* travels outward from the source. The absorber particles react to this disturbance and themselves generate a field half advanced and half retarded waves. The sum of the advanced and retarded effects of all the charged particles of the absorber, evaluated near the source charge, give an electromagnetic field with the following properties [1]:

(1) It is independent of the properties of the absorbing medium.
(2) It is completely determined by the motion of the source.
(3) It exerts on the source a force which is finite, is simultaneous with the moment of acceleration, and is just sufficient in magnitude and direction to take away from the source the energy which later shows up in the surrounding particles.
(4) It is equal in magnitude to 1/2 the retarded field minus 1/2 the advanced field generated by the accelerated charge. In other words, the absorber is the physical origin of Dirac's radiation field
(5) This field combines with the 1/2 retarded, 1/2 advanced field of the source to give for the total disturbance the full retarded field which accords with experience.

The Wheeler-Feynman paper presents four derivations of the relativistic radiation reaction of an accelerated charge, each successive derivation increasing in generality. The first three derivations proceed by adding up all the electromagnetic fields due to the absorber particles. The fourth is the most general derivation, which only assumes that the medium is a complete absorber and so outside the medium the sum of all the retarded and advanced waves is zero. Each yields the well-known relativistic radiation reaction as given in text books [6].

So far, we have shown that the advanced wave idea has been used successfully in classical physics. Now we proceed to show that it can also be advantageously used within quantum mechanics. The transactional interpretation of quantum mechanics was written by John Cramer [9,10] in the 1980's. It is a way to view quantum mechanics which is very intuitive and easily accounts for all the well known paradoxes, EPR, which-way detection and quantum eraser experiments. We refer the reader to his paper, which is a very interesting read. All the usual quantum results hold, and it is simply an alternative point of view from the Copenhagen interpretation and collapsing-wave-function way of thinking.

Hoyle-Narlikar (HN) theory is a kind of absorber theory (with advanced waves) for gravitation rather than electrodynamics. HN theory agrees with Einstein's theory of gravitation in the limit of a smooth fluid mass density distribution in the rest frame of the fluid. All of the tests of Einstein's gravitation still apply to HN theory.

2. DERIVATION OF THE EINSTEIN GEODESIC EQUATION

One method used to derive the geodesic equation, also known as the equation of motion of a particle, is extremizing (minimizing) the line element. This will give us the shortest distance between two points. Taking the general line-element,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{6}$$

varying both sides (a similar derivation can be found in Dirac [11] p16), we get

$$\begin{aligned} 2ds\delta(ds) &= dx^\mu dx^\nu \delta(g_{\mu\nu}) + g_{\mu\nu} dx^\mu \delta(dx^\nu) + g_{\mu\nu} dx^\nu \delta(dx^\mu) \\ &= dx^\mu dx^\nu g_{\mu\nu,\lambda}(\delta x^\lambda) + 2 g_{\mu\lambda} dx^\mu \delta(dx^\lambda) \\ \delta(dx^\lambda) &= d(\delta x^\lambda) \\ dx^\mu &= \left(\frac{dx^\mu}{ds}\right) ds = v^\mu ds \end{aligned} \tag{7}$$

In order to extremize the action $\int \delta(mds)$, treat mass as a constant. Then we consider the following,

$$\begin{aligned} \int \delta(ds) &= \int \left[\frac{1}{2}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} g_{\mu\nu,\lambda}\delta x^\lambda + g_{\mu\lambda}\frac{dx^\mu}{ds}\frac{d}{ds}(\delta x^\lambda)\right] ds \\ &= \int \left[\frac{1}{2} g_{\mu\nu,\lambda} v^\mu v^\nu (\delta x^\lambda) + g_{\mu\nu} v^\mu \frac{d}{ds}(\delta x^\lambda)\right] ds \end{aligned} \tag{8}$$

Integrating the second term by parts we find

$$\int \delta(ds) = \int \left[\frac{1}{2} g_{\mu\nu,\lambda} v^\mu v^\nu - \frac{d}{ds}(g_{\mu\lambda} v^\mu)\right] (\delta x^\lambda) ds = 0 \tag{9}$$

For this to be true for any variation δx^λ we find that the term inside the square bracket must be zero hence,

$$\frac{d}{ds}(g_{\mu\lambda} v^\mu) - \frac{1}{2} g_{\mu\nu,\lambda} v^\mu v^\nu = 0 \tag{10}$$

Furthermore

$$\begin{aligned} \frac{d}{ds}(g_{\mu\lambda} v^\mu) &= g_{\mu\lambda}\frac{dv^\mu}{ds} + g_{\mu\lambda,\nu} v^\mu v^\nu \\ &= g_{\mu\lambda}\frac{dv^\mu}{ds} + \frac{1}{2}(g_{\mu\lambda,\nu} + g_{\lambda\nu,\mu}) v^\mu v^\nu \end{aligned} \tag{11}$$

By substitution of Eq. (11) into Eq. (10) we find

$$\begin{aligned} g_{\mu\lambda}\frac{dv^\mu}{ds} + \frac{1}{2}(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}) v^\mu v^\nu &= 0 \\ \frac{1}{2}(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}) &= \Gamma_{\lambda\mu,\nu} \\ \frac{dv^\sigma}{ds} + \Gamma^\sigma_{\mu\nu} v^\mu v^\nu &= 0 \end{aligned} \tag{12}$$

where the last equation, which is the usual geodesic equation, follows when you multiply throughout by $g^{\lambda\sigma}$. We did not start with a true particle Lagrangian, only a line element. The Lagrangian includes a mass of the particle. Note that we have left the mass entirely out of the variation since at present it is treated as a constant.

Let us compare this with what happens when you vary the rest mass of the particle.

2.1 Allow the Mass to change in Equation of Motion derivation

To see how similar the HN equation of motion is to the Einstein Geodesic, simply repeat the above calculation but allow the mass to change. The result is the Equation of Motion for the HN-theory. It is a little unclear as to why the usual Einstein geodesic does not contain the same mass variation. Is it because the mass is held constant in the Einstein case? If so **why** is the mass held constant?

Starting from the line element,

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ 2ds\delta(ds) &= \delta g_{\mu\nu} dx^\mu dx^\nu + 2g_{\mu\nu} dx^\mu \delta(dx^\nu) \\ \delta(ds) &= \left[\frac{1}{2}\delta g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + g_{\mu\nu}\dot{x}^\mu\frac{d}{ds}(\delta x^\nu)\right] ds \end{aligned} \tag{13}$$

Now, the action for mass m at position x can be simply written as,

$$\begin{aligned} I &= -\int m ds \\ \delta I &= -\int [\delta(m)ds + m\delta(ds)] \\ &= -\int \left[\frac{\partial m}{\partial x^\lambda}\delta x^\lambda + \frac{m}{2}\delta g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + m g_{\mu\nu}\dot{x}^\mu\frac{d}{ds}(\delta x^\nu)\right] ds \end{aligned} \tag{14}$$

Integrate the last term by parts and switch dummy variable $\nu \to \lambda$ we get,

$$\begin{aligned} \delta I &= -\int \left[\frac{\partial m}{\partial x^\lambda}\delta x^\lambda + \frac{m}{2}\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\dot{x}^\mu\dot{x}^\nu\delta x^\lambda - \frac{d}{ds}(m g_{\mu\lambda}\dot{x}^\mu)\delta x^\lambda\right] ds \\ &= -\int \left[\frac{\partial m}{\partial x^\lambda} + \frac{m}{2}\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\dot{x}^\mu\dot{x}^\nu - \frac{d}{ds}(m g_{\mu\lambda}\dot{x}^\mu)\right]\delta x^\lambda ds = 0 \end{aligned} \tag{15}$$

For this integral to be zero for any arbitrary δx^λ then the term in the square brackets must be zero, hence

$$\begin{aligned} \frac{d}{ds}(m g_{\mu\lambda}\dot{x}^\mu) &= \frac{m}{2}\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\dot{x}^\mu\dot{x}^\nu + \frac{\partial m}{\partial x^\lambda} \\ \frac{dm}{ds}g_{\mu\lambda}\dot{x}^\mu + m\left(g_{\mu\lambda}\frac{d\dot{x}^\mu}{ds} + g_{\mu\lambda,\nu}\dot{x}^\mu\dot{x}^\nu\right) &= \frac{m}{2}g_{\mu\nu,\lambda}\dot{x}^\mu\dot{x}^\nu + \frac{\partial m}{\partial x^\lambda} \end{aligned} \tag{16}$$

where we may make the $g_{\mu\lambda,\nu}$ term symmetric in μ, ν as follows,

$$g_{\mu\lambda}\frac{d}{ds}(m\dot{x}^\mu) = \frac{m}{2}(g_{\mu\nu,\lambda} - g_{\mu\lambda,\nu} - g_{\nu\lambda,\mu})\dot{x}^\mu\dot{x}^\nu + \frac{\partial m}{\partial x^\lambda} \tag{17}$$

Then using the definition for the Christoffel symbol $\Gamma_{\lambda\mu,\nu}$ and multiplying throughout by $g^{\sigma\lambda}$ we get,

$$\begin{aligned} \frac{d}{ds}(m\dot{x}^\sigma) + m(g^{\sigma\lambda}\Gamma_{\lambda\mu,\nu})\dot{x}^\mu\dot{x}^\nu - g^{\sigma\lambda}\frac{\partial m}{\partial x^\lambda} &= 0 \\ \frac{d}{ds}(m\dot{x}^\sigma) + m\Gamma^\sigma_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - g^{\sigma\lambda}\frac{\partial m}{\partial x^\lambda} &= 0 \end{aligned} \tag{18}$$

Written for mass m_a at position x_a the equation of motion becomes,

$$\frac{d}{d\tau}\left(m_a\frac{dx_a^\mu}{d\tau}\right) + m_a\Gamma^\mu_{\nu\lambda}\frac{dx_a^\nu}{d\tau}\frac{dx_a^\lambda}{d\tau} - g^{\mu\nu}\frac{\partial m_a}{\partial x_a^\nu} = e_a\sum_{b\neq a}F_\nu^{(\mathrm{b})\mu}\frac{dx_a^\nu}{d\tau} \tag{19}$$

where the Lorentz force has been included on the right for completeness. The world-lines of particles are not in general geodesics in the new theory. This equation agrees with the HN result in their book [12] p125 Eq.(138). In the HN book this equation of motion was derived directly from the gravitational field equation.

3. HOYLE-NARLIKAR THEORY DEVELOPMENT

There is some motivation for looking into the HN-theory in detail. We begin from the first of the Hoyle-Narlikar papers, through to the writing of their book. The notation they use is very unfortunate and difficult to read. There are too many similar letters being used for different parameters. Here we attempt to rewrite the theory in a more familiar notation, using Greek letters for 0,1,2,3 and Roman letters only to distinguish particle “a” from particle “b”. We do not use their European style of 4-vector 1,2,3,4. Rather we use the 0,1,2,3 numbering which has become fairly standard throughout the world. The flat metric will be taken as $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. Where ever possible we leave c not equal to unity which helps with dimensional analysis. We tackle the papers in order starting with the first published.

Paper 1: The first paper in the sequence, in 1962, [13] was entitled “Mach’s Principle and the creation of matter”. The main point of the paper was to argue that although Einstein was very much influenced by Mach’s ideas, he did not quite manage to get the full spirit of Mach’s main idea embedded into the field equations... *mass depends on interaction with the rest of the mass-energy in the universe.*

We would argue that Mach’s principle has several definitions and several of those are in fact already contained in Einstein’s general relativity theory.

According to HN, they take the Einstein field equations, written as,

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \lambda g^{\mu\nu} = -\kappa T^{\mu\nu} \tag{20}$$

and plug in the well known Robertson Walker line element,

$$ds^2 = c^2dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{21}$$

where $k = 0, \pm 1$ and where r can be chosen for an observer attached to any particular particle. For the stress-energy tensor, they assume

$$T^{\mu\nu} = \left(\rho + P + \frac{4}{3}u\right)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} - \left(P + \frac{u}{3}\right)g^{\mu\nu} \tag{22}$$

where ρ is the matter density, P is the gas pressure and u is the radiation density. Hoyle and Narlikar set out, in a series of papers, to formulate a gravitational theory (which encompasses Einstein’s equations) which included Mach’s principle from the start. This theory would have both retarded and advanced waves. Essentially this would be the gravitational equivalent of Wheeler-Feynman absorber theory for electrodynamics.

Paper 2: In 1963 [14], we see HN play around with the Einstein action and add in their C-field, to add matter to the universe in an attempt to preserve the density as the universe expands. This is not really of interest for our work. Sciama [15] publishes work in the same journal on Wheeler-Feynman absorber theory and mentions Hogarth’s work [2].

Paper 3: In January 1964 we see the first attempt at something new. The paper is entitled, “Time symmetric electrodynamics and the arrow of time in cosmology”, [16]. Here we see the first introduction of the Fokker-Schwarzschild-Tetrode action (FST action) [17,18,7], a discussion of the Wheeler–Feynman absorber theory [1], and reworking time-symmetric electrodynamics in a flat and Riemannian space-time. Here we rewrite these familiar equations for convenience since it will set up the new notation for their later work.

We start with a summary of the first few HN equations which we then “translate” into better notation below. We quote directly from the paper [16]:

.. we consider space-time to be given by the co-ordinates x^i and by the line-element

$$ds^2 = \eta_{ik}dx^i dx^k \tag{23}$$

where $\eta_{ik} = \mathrm{diag}(-1, -1, -1, +1)$. *The charges are labelled by letters $a, b, c....$ The a^{th} particle has co-ordinates a^i, mass m_a, charge e_a and proper time a given by*

$$da^2 = \eta_{ik}da^i da^k \tag{24}$$

We have chosen the velocity of light to be unity so that the time units are the same as the space units. The Schwarzschild-Tetrode-Fokker action function is then defined by

$$J = -\sum_a m_a \int da - \sum_a \sum_{b\neq a} \frac{1}{2}e_a e_b \int\int \delta(ab_i ab^i)\eta_{lm} da^l da^m \tag{25}$$

where

$$\delta(ab_i ab^i) = \eta_{ik}(a^i - b^i)(a^k - b^k) \tag{26}$$

The equations of motion can be obtained from (25) by requiring that J be stationary with respect to variations of the world lines of particles. If we define the 4-potential of b at a point x by the function

$$A_m^{(b)}(x) = \int e_b \delta(xb_i xb^i)\eta_{mk} db^k \tag{27}$$

the equations of motion take the form

$$m_a \frac{d^2 a^k}{da^2} = e_a \sum_{b \neq a} F_l^{k(b)} \frac{da^l}{da} \tag{28}$$

where

$$F_{kl}^{(b)} = \frac{\partial A_l^{(b)}(x)}{\partial x^k} - \frac{\partial A_k^{(b)}(x)}{\partial x^l} \tag{29}$$

represents the 'field' of charge b at point x.

Note that a better notation of the FST action and derivations for the potential and Maxwell's equations, can be found at the very end of the book by Barut on electrodynamics [19]. Our notation is similar to Barut's only we use x instead of z. Also we use a and b to distinguish particles rather than α and β since these could easily be mistaken for summation variables. We start by rewriting the above notation as follows:
For flat space-time.

Define the metric as $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. The charges are labelled by a, b, c as before. The a^{th} particle has coordinates x_a^μ , mass m_a, charge e_a, and proper time τ given by the line element as

$$ds^2 = c^2 d\tau^2 = dx_a^\mu dx_{a\mu} \ , \tag{30}$$

with $c = 1$ we get,

$$\begin{aligned} d\tau^2 &= \eta_{\mu\nu} dx_a^\mu dx_a^\nu \\ d\tau &\to \eta_{\mu\nu} \dot{x}_a^\mu \dot{x}_a^\nu d\tau \ . \end{aligned} \tag{31}$$

where differentiation w.r.t τ is represented by the dot above the symbol. The action can be written as,

$$I = -\sum_a \int \frac{1}{2} m_a (\dot{x}_a^\nu)^2 d\tau - \sum_a \sum_{b \neq a} e_a \int A_\mu^{(b)}(x_a^\nu) \dot{x}_a^\mu d\tau \tag{32}$$

$$A_\mu^{(b)}(x_a^\nu) = \int e_b D(x_a - x_b) \eta_{\mu\nu} dx_b^\nu \equiv \int e_b D(x_a - x_b) \eta_{\mu\nu} \dot{x}_b^\nu d\tau' \tag{33}$$

$$D(x_a - x_b) = \left[\eta_{\alpha\beta}(x_a^\alpha - x_b^\alpha)(x_a^\beta - x_b^\beta)\right]$$

Note that the 4-potential of particle b ($A_\mu^{(b)}$) is evaluated at the location of particle a. The proper time for particle b is given by τ' and $\dot{x}_b = dx_b/d\tau'$. The Lagrangian can be written as

$$I = \sum_a \int L(x_a^\nu, \dot{x}_a^\nu) d\tau \tag{34}$$

$$L(x_a^\nu, \dot{x}_a^\nu) = -\frac{1}{2} m_a (\dot{x}_a^\nu)^2 - e_a \sum_{b \neq a} A_\mu^{(b)}(x_a^\nu) \dot{x}_a^\mu \tag{35}$$

with equation of motion given by

$$\frac{\partial L}{\partial x_a^\nu} - \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}_a^\nu}\right) = 0 \ . \tag{36}$$

Differentiating the Lagrangian we get

$$\begin{aligned}
\frac{\partial L}{\partial x_a^\nu} &= -\sum_{b\neq a} e_a \frac{\partial A_\mu^{(b)}}{\partial x_a^\nu}\dot{x}_a^\mu \\
\frac{\partial L}{\partial \dot{x}_a^\nu} &= -m_a\dot{x}_a^\nu - \sum_{b\neq a} e_a A_\nu^{(b)} \\
\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}_a^\nu}\right) &= -\frac{d}{d\tau}(m_a\dot{x}_a^\nu) - e_a\frac{\partial A_\nu^{(b)}}{\partial x_a^\mu}\frac{dx_a^\mu}{d\tau} \\
m_a\ddot{x}_a^\nu &= e_a\dot{x}_a^\mu \sum_{b\neq a} F_{\nu\mu}^{(\mathrm{b})}(x_a^\nu) \qquad (37)
\end{aligned}$$

where the last equation is the equation of motion of particle a. The mass m_a is taken to be constant. Finally we define the electromagnetic field tensor as

$$\begin{aligned}
F_{\nu\mu}^{(\mathrm{b})}(x_a^\nu) &= \left(\frac{\partial A_\mu^{(b)}}{\partial x_a^\nu} - \frac{\partial A_\nu^{(b)}}{\partial x_a^\mu}\right) \\
F_{\nu\mu}^{(\mathrm{b})} &= \frac{1}{2}\left(F_{\nu\mu}^{\ (\mathrm{b})\mathrm{ret}} + F_{\nu\mu}^{(\mathrm{b})\mathrm{adv}}\right) \quad . \qquad (38)
\end{aligned}$$

Now we follow the paper but write only in the new notation. Using Dirac's identity,

$$\begin{aligned}
\eta^{\mu\nu}\frac{\partial^2}{\partial x^\mu \partial x^\nu}D(x-x_b) &= -4\pi\delta(x^0-x_b^0)(x^1-x_b^1)(x^2-x_b^2)(x^3-x_b^3) \\
&= -4\pi\delta^4(x-x_b) \quad . \qquad (39)
\end{aligned}$$

The 4-potential satisfies [19],

$$\begin{aligned}
\Box^2 A^{(\mathrm{b})\mu}(x) &= \sum_{b\neq a} e_b \int \dot{x}_b^\mu(\tau')\Box^2 D(x-x_b)d\tau' \\
&= -4\pi\sum_{b\neq a} e_b \int \dot{x}_b^\mu \delta^4(x-x_b)d\tau' \qquad (40)
\end{aligned}$$

which is the same as writing,

$$\Box^2 A^{(\mathrm{b})\mu}(x) = \eta^{\mu\nu}\frac{\partial^2}{\partial x^\mu \partial x^\nu}A_\sigma^{(b)}(x) = -4\pi j_\sigma^{(b)}(x) \qquad (41)$$

where the current density $j_\sigma^{(b)}(x)$ is given by

$$j_\sigma^{(b)}(x) = e_b\int_{-\infty}^{\infty}\eta_{\sigma\lambda}\dot{x}_b^\lambda\delta^4(x-x_b)d\tau' \qquad (42)$$

It can be shown that [19], the 4-potential satisfies the gauge condition

$$\begin{aligned}
\frac{\partial A^{(b)\mu}}{\partial x^\mu} &= \sum_{b\neq a} e_b \int \dot{x}_b^\mu \frac{\partial}{\partial x^\mu}D(x-x_b)d\tau' \\
&= -D(x-x_b)\mid_{\tau'=-\infty}^{+\infty} = 0 \quad . \qquad (43)
\end{aligned}$$

We may derive the following:

$$\frac{\partial F_\nu^{(b)\mu}}{\partial x^\mu} = -4\pi j_\nu^{(b)}(x) \quad , \qquad (44)$$

which are Maxwell's inhomogeneous equations.

Formally, all Maxwell's equations and the Lorentz force equation are derivable from the action principle, except radiation reaction terms (self force terms). The radiation reaction becomes a force due to advanced waves coming from the absorbing universe mass-energy. The time symmetry is emphasized by rewriting the 4-potential as a sum of retarded and advanced parts.

$$
\begin{aligned}
A_\mu^{(b)} &= \frac{1}{2}\left(A_\mu^{(\mathrm{b})\mathrm{Ret}} + A_\mu^{(\mathrm{b})\mathrm{Adv}}\right) \\
A_\mu^{(\mathrm{b})\mathrm{Ret}} &= e_b \frac{\eta_{\mu\nu}\dot{x}_b^\nu}{\eta_{\alpha\beta}(x^\alpha - x_b^\alpha)\dot{x}^\beta}
\end{aligned}
\tag{45}
$$

An alternative approach to the first term in the Lagrangian is to is to vary it directly and derive the equations of motion from scratch rather than using the Euler-Lagrange formula. The notation has now been introduced so we will not continue with the Riemannian Space-time summary.

Paper 4 & 5: These two papers [20,21] are referring entirely to the C-field, which was an addition of matter in order that the mass-density of the universe ρ remain constant as the universe expands. We are not interested in the C-field, since we do not require a static universe, and wish to treat the universe as not only expanding but accelerating in that expansion. We skip these two papers.

Paper 6, 7, & 8: Now we jump ahead to the full HN-theory and the fully Machian action, or in their words, the *full action*. The first of these [222] is a short paper including the C-field. This is not of so much interest. The next two papers [3,4] are the main papers with the theory we wish to use. These two papers should be read together. The HN-theory is given in *A new theory of gravitation* 1964 [3] with extra details in the 1966 paper [4] entitled *A conformal theory of gravitation*.

A summary of the new theory [3] follows with reference also to the extra detail in [4]. Particle interactions are propagated along null geodesics (at no distance in a 4 dimensional or light-like sense). According to HN the action developed thus far is of the form

$$
I = \frac{1}{16\pi G}\int R\sqrt{(-g)}d^4x - \sum_a m_a \int d\tau - \sum_a \sum_{b\neq a} 4\pi e_a e_b \int\int G_{\alpha\beta} dx_a^\alpha dx_b^\beta \tag{46}
$$

the first two terms looking very different that the direct particle interaction representing the electromagnetic last term. The term in m_a is derived from Galileo's concept of inertia and has been present since before Newton. Einstein retained this traditional term. Neither of the first two terms is correct, the first being a field or energy density the second being attributed to matter only. Only terms of the form using a double integral should be present. The first two terms have been artificially separated by traditional thinking. In what follows we construct a purely gravitational theory with the first and second terms combined into a single mass-energy term. It may also be possible to combine the electromagnetic term into the same term but we leave that for a later discussion. In order to convert the line integral $\int m_a d\tau$ into a sum of double line integrals we make the following assumptions:

(1) The mass $m_a = m(x_a)$ (mass at position x_a) must become a direct particle field, it must arise from all the other mass in the universe.
(2) Since mass is scalar we expect it to arise through a scalar Greens function.
(3) The action must be symmetric between any pairs of particles, [3].

Let each particle b give rise to a mass-field (*spherical monopole type g-waves*). Denote this field at a general point x by $m^{(\mathrm{b})}(x)$. At any point x_a on the path of particle a, we have $m^{(\mathrm{b})}(x_a)$ as the contribution of particle b to the mass of particle a at the position x_a. Summing for all b particles ,

$$
m(x_a) = m_a = \sum_b m^{(\mathrm{b})}(x_a) \tag{47}
$$

this gives the mass at point x_a due to all particles including those at position x_a. For electromagnetism we avoided positions where $x_a = x_b$ but for gravity we need not do this, [12] p109 Eq(46). The non-electromagnetic part of the action I_{mat} for many particles a,b... can be written in the form,

$$
I_{\mathrm{mat}} = -\frac{1}{2}\sum_a \int m(x_a)d\tau = -\sum_a\sum_b \int m^{(\mathrm{b})}(x_a)d\tau \tag{48}
$$

In order that (48) be symmetric for any particle pair a,b we must have $m^{(\mathrm{b})}(x_a)$ in the form

$$
m^{(\mathrm{b})}(x_a) = \int G(x_a, x_b)d\tau' \tag{49}
$$

so that

$$\int m^{(\mathrm{b})}(x_a)d\tau = \int\int G(x_a, x_b)d\tau d\tau' \tag{50}$$

where $G(x_a, x_b) = G(x_b, x_a)$ is a scalar Greens function. The mass function at a point x due to the world-line of particle a, at position x_a, is defined by

$$m^{(\mathrm{a})}(x) = \int G(x, x_a)d\tau \quad . \tag{51}$$

The mass function varies from point to point. *Before we plunge into the depths of HN-theory, let us first have a brief aside on the development of the field equations for the Einstein action, which is considerably easier!*

4. COMPARISON OF THE ACTIONS AND FIELD EQUATIONS.

4.1 The Einstein Action

For comparison we write down the basic Einstein Action, without the electromagnetic field,

$$I_{\mathrm{Einstein}} = \frac{1}{16\pi G}\int R[-g]^{1/2}d^4x - \sum_a m_a \int d\tau \quad . \tag{52}$$

The field equations are derived by varying the action and setting the variation equal to zero, [12] p112. The metric tensor will be varied according to $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$ in a volume V with $\delta g_{\mu\nu} = 0$ at the boundaries. Varying the above action yields,

$$\begin{aligned}\delta I_{\mathrm{Einstein}} &= \frac{1}{16\pi G}\int \delta(R[-g]^{1/2})d^4x - \sum_a m_a \int \delta(d\tau) \\ &= \frac{1}{16\pi G}\int \delta(g^{\mu\nu}R_{\mu\nu}[-g]^{1/2})d^4x - \sum_a m_a \int \delta(d\tau) \quad .\end{aligned} \tag{53}$$

Using

$$\begin{aligned}-\sum_a m_a \int \delta(d\tau) &= \frac{1}{2}\int T_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\ \delta\left([-g]^{1/2}\right) &= -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}\end{aligned} \tag{54}$$

Next we expand out the first term with the Ricci tensor,

$$\begin{aligned}\delta I_{\mathrm{Einstein}} &= \frac{1}{16\pi G}\int \delta(g^{\mu\nu}R_{\mu\nu})[-g]^{1/2}d^4x - \frac{1}{16\pi G}\int \frac{1}{2}Rg_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\ &\quad + \frac{1}{2}\int T_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\ &= \frac{1}{16\pi G}\int \left[R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8\pi G T_{\mu\nu}\right]\delta g^{\mu\nu}[-g]^{1/2}d^4x \\ &\quad + \frac{1}{16\pi G}\int g^{\mu\nu}\delta R_{\mu\nu}[-g]^{1/2}d^4x\end{aligned} \tag{55}$$

the last term is zero since the variation vanishes on the boundary. Hence by setting $\delta I_{\mathrm{Einstein}} = 0$ for any arbitrary variation $\delta g_{\mu\nu}$ we obtain the Einstein's field equations,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{56}$$

The familiar energy momentum tensor is easily derived, see Hoyle and Narlikar's book [12] p112. This follows from using $d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$ and thus $\delta(d\tau) = \delta g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu d\tau$ which leads to

$$\begin{aligned}
-\sum_a \int m(x_a)\delta(d\tau) &= -\sum_a \int m(x)\delta g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\delta^4(x-x_a)d\tau \\
&= -\int_V T^{\mu\nu}\delta g_{\mu\nu}[-g]^{1/2}d^4x = +\int_V T_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\
\text{where } \ T^{\mu\nu} &= \sum_a \delta^4(x-x_a)[-g]^{-1/2}m(x)\dot{x}^\mu\dot{x}^\nu d\tau \ . \qquad (57)
\end{aligned}$$

The energy-stress tensor $T^{\mu\nu}$ is a sum over all the mass-energy in the universe, excluding the electromagnetic field which is treated separately. This is exactly the same calculation that will appear in the HN-theory later.

4.2 Quick note on scalar densities

Using J as the Jacobian (see Dirac's book on gravitation [11] p37),

$$\begin{aligned}
dx^{\mu'} &= dx^\mu J \quad \text{or} \quad d^4x' = Jd^4x \\
J &= \frac{\partial x^{\mu'}}{\partial x^\alpha} \\
g_{\alpha\beta} &= \frac{\partial x^{\mu'}}{\partial x^\alpha} g_{\mu'\nu'} \frac{\partial x^{\nu'}}{\partial x^\beta} \ . \qquad (58)
\end{aligned}$$

The determinants satisfy,

$$\begin{aligned}
g &= Jg'J \\
g &= J^2 g' \\
\Rightarrow \sqrt{-g} &= J\sqrt{-g'} \qquad (59)
\end{aligned}$$

since $g = \|g_{\alpha\beta}\|$ is negative. That makes $\sqrt{-g}$ a positive quantity. Hence we may define the following invariant quantity for any scalar density, for example $H \to T_{\mu\nu}\delta g^{\mu\nu}$,

$$\begin{aligned}
\int_V H\sqrt{-g}d^4x &= \int_V H\sqrt{-g'}Jd^4x = \int_V H'\sqrt{-g'}d^4x' \\
\text{hence } \int_V T_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}d^4x &= \quad \text{invariant} \ . \qquad (60)
\end{aligned}$$

4.3 The HN-Theory Action

Omitting the electromagnetic field for now, using the definitions (47) and (50), the action can be written, following Hoyle-Narlikar "A New Theory of Gravitation", [3], as:

$$I = -\sum_a \frac{1}{2}\int m(x_a)d\tau = -\sum_a\sum_b \int\int G(x_a,x_b)d\tau d\tau' \qquad (61)$$

There is just one term, a sum over all the masses in the universe. The energy is not separated out, because of mass-energy equivalence. This requires that a "universe" consist of at least two particles for them to interact and create a space-time between them. The factor 1/2 comes in because each $G(x_a, x_b)$ is shared by two particles a and b. This makes no difference to the equations of motion. The paper has no factor of 1/2 in front of the double sum, whereas the HN book does have the factor of 1/2. The most general wave equation is

$$g^{\mu\nu}G(x,x_a)_{;\mu\nu} + \mu RG(x,x_a) = -[-g]^{-1/2}\delta^4(x-x_a) \qquad (62)$$

in which R is the Ricci scalar and μ is a constant taken later to be 1/6 since the wave equation is then conformally invariant [23]. The next step is to vary the geometry in a finite volume V, $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$ with $\delta g_{\mu\nu} = 0$ on the boundary. It will be shown that

$$\delta I = \int P_{\mu\nu} \delta g^{\mu\nu} [-g]^{1/2} d^4x \tag{63}$$

where $P_{\mu\nu}$ is a symmetric tensor. The formalism becomes a theory when we assert that $\delta I = 0$ which requires

$$P_{\mu\nu} = 0 \tag{64}$$

which are the field equations of the new theory.

4.4 Field equation for HN-theory

Now for the field equations, [3]. Consider the change in $G(x_a, x_b)$ due to an infinitesimal change $\delta g_{\mu\nu}$ in $g_{\mu\nu}$ over a finite volume V, with $\delta g_{\mu\nu} = 0$ on the boundary of V. By dividing throughout by $[-g]^{-1/2}$, the equation for the Greens function $G(x, x_a)$ can be written as,

$$\frac{\partial}{\partial x^\mu}\left[[-g]^{1/2} g^{\mu\nu} \frac{\partial G(x, x_a)}{\partial x^\nu}\right] + \mu R [-g]^{1/2} G(x, x_a) = -\delta^4(x - x_a) \tag{65}$$

The variation can be made by setting $G \to G + \delta G$ and $g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$, and this becomes,

$$\frac{\partial}{\partial x^\mu}\left[[-g]^{1/2} g^{\mu\nu} \frac{\partial \delta G}{\partial x^\nu}\right] + \mu R [-g]^{1/2} \delta G = -\frac{\partial}{\partial x^\mu}\left[\delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G}{\partial x^\nu}\right] - \mu \delta(R[-g]^{1/2}) G \tag{66}$$

This agrees with Eq (71) in the HN book, [12] p113-114. It appears that δG satisfies the same differential operator as $G(x, x_a)$ itself, but with a distributed source term, not a δ-function at point x_a. The solution for δG can be written down as follows, (see [26] for first use of this solution on the scalar Greens function p186)

$$\begin{aligned}
\delta G(x_a, x_b) &= \int_V \frac{\partial}{\partial x^\mu}\left[\delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G(x_a, x)}{\partial x^\nu}\right] G(x_b, x) d^4x \\
&\quad + \mu \int_V \delta(R[-g]^{1/2}) G(x_a, x) G(x_b, x) d^4x \\
&= -\int_V \delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G(x_a, x)}{\partial x^\nu} \frac{\partial G(x_b, x)}{\partial x^\mu} d^4x \\
&\quad + \mu \int_V \delta(R[-g]^{1/2}) G(x_a, x) G(x_b, x) d^4x
\end{aligned} \tag{67}$$

where we have integrated the first term by parts and set $\delta g^{\mu\nu} = 0$ at the boundary of the volume. This agrees with Eq (12) in [3] (and Eq (72) in the HN book). The variation of the action then becomes

$$\begin{aligned}
\delta I &= -\frac{1}{2}\sum_a \int m(x_a) \delta(d\tau) - \frac{1}{2}\sum_a \int \delta m(x_a) d\tau \\
&= -\frac{1}{2}\sum_a \int m(x_a) \delta(d\tau) - \sum_a \sum_b \int\int \delta G(x_a, x_b) d\tau d\tau' \\
&= -\frac{1}{2}\sum_a \int m(x_a) \delta(d\tau) - \sum_a \sum_b \int_V \int\int \delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G(x_a, x)}{\partial x^\nu} \frac{\partial G(x_b, x)}{\partial x^\mu} d^4x d\tau d\tau' \\
&\quad + \mu \sum_a \sum_b \int_V \int\int \delta(R[-g]^{1/2}) G(x_a, x) G(x_b, x) d^4x d\tau d\tau'
\end{aligned} \tag{68}$$

Using the earlier definitions of mass at point x due to the world-lines of particle a and particle b, Eq.(236),

$$m^{(a)}(x) = \int G(x_a, x) d\tau$$
$$m^{(b)}(x) = \int G(x_b, x) d\tau'$$

we arrive at

$$\delta I = -\frac{1}{2}\sum_a \int m(x_a)\delta(d\tau) - \sum_a \sum_b \int_V \delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial m^{(a)}(x)}{\partial x^\nu} \frac{\partial m^{(b)}(x)}{\partial x^\mu} d^4x$$
$$+ \mu \sum_a \sum_b \int_V \delta(R[-g]^{1/2}) m^{(a)}(x) m^{(b)}(x) d^4x \qquad (69)$$

This agrees with Eq (13) in [3]. There are typos in the papers making these look like covariant derivatives when they are only partial derivatives.

The first term in the variation of the action, Eq. (69) is the familiar energy momentum tensor for mass-energy. This follows from using $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$:

$$-\sum_a \int m(x_a)\delta(d\tau) = -\sum_a \int m(x)\delta g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \delta^4(x - x_a) d\tau$$
$$= -\int_V T^{\mu\nu} \delta g_{\mu\nu} [-g]^{1/2} d^4x = \int_V T_{\mu\nu} \delta g^{\mu\nu} [-g]^{1/2} d^4x$$
$$\text{where } T^{\mu\nu} = \sum_a \delta^4(x - x_a)[-g]^{-1/2} m(x) \dot{x}^\mu \dot{x}^\nu d\tau \qquad (70)$$

This is exactly the same as for the Einstein action treated earlier. This does not include the electromagnetic fields which are treated separately.

At this point rather than follow the paper [3], it appeared quicker to follow the book [12]. We take up the derivation there. In order to compare the older paper [3] with the more recent text book [12] we return to the variation of the Greens function Eq (66). The book uses $\mu = 1/6$ and has a factor of $1/2$ in front of the double sum, so the following terms will have a multiplicative factor $1/2$ throughout. We may split the Green function into advanced and retarded parts, [12] p114 Eq (73),

$$G(x, x_b) = \frac{1}{2}\left[G^{\text{ret}}(x, x_b) + G^{\text{adv}}(x, x_b)\right] \ . \qquad (71)$$

The retarded part gives the following contribution to $\delta G(x_a, x_b)$, see earlier Eq (67),

$$\delta G^{\text{ret}}(x_a, x_b) = -\frac{1}{2}\int_V G^{\text{ret}}(x_a, x) \frac{\partial}{\partial x^\mu}\left[\delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G^{\text{ret}}(x, x_b)}{\partial x^\nu}\right] d^4x$$
$$-\frac{1}{12}\int_V \delta(R[-g]^{1/2}) G^{\text{ret}}(x_a, x) G^{\text{ret}}(x, x_b) d^4x \qquad (72)$$

where $G^{\text{ret}}(x_a, x) = G^{\text{adv}}(x, x_a)$.

The equation for δG^{ret} above, can be written more symmetrically by integrating the first term by parts,

$$\delta G^{\text{ret}} = \frac{1}{2}\int_V \delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G^{\text{adv}}(x, x_a)}{\partial x^\mu} \frac{\partial G^{\text{ret}}(x, x_b)}{\partial x^\nu} d^4x$$
$$-\frac{1}{12}\int_V \delta(R[-g]^{1/2}) G^{\text{adv}}(x, x_a) G^{\text{ret}}(x, x_b) d^4x$$
$$(73)$$

This agrees with the book [12] p115, Eq. (77). The advanced part of δG is similar with the advanced and retarded G's switched

$$\delta G^{\text{adv}} = \frac{1}{2}\int_V \delta([-g]^{1/2} g^{\mu\nu}) \frac{\partial G^{\text{ret}}(x, x_a)}{\partial x^\mu} \frac{\partial G^{\text{adv}}(x, x_b)}{\partial x^\nu} d^4x$$
$$-\frac{1}{12}\int_V \delta(R[-g]^{1/2}) G^{\text{ret}}(x, x_a) G^{\text{adv}}(x, x_b) d^4x$$
$$(74)$$

The full expression for $\delta G(x_a, x_b)$ is the sum of the advanced and retarded parts. The next step is to find the variation of the action,

$$\sum_a \sum_b \int \int \delta G(x_a, x_b) d\tau d\tau' \ . \tag{75}$$

Here we introduce the mass field from p115 [112],

$$\begin{aligned} m(x) &= \frac{1}{2}\left[m^{(\mathrm{ret})}(x) + m^{(\mathrm{adv})}(x)\right] \\ m^{(\mathrm{ret})}(x) &= \sum_a \int G^{(\mathrm{ret})}(x, x_a) d\tau \\ m^{(\mathrm{adv})}(x) &= \sum_a \int G^{(\mathrm{adv})}(x, x_a) d\tau \end{aligned} \tag{76}$$

The variation of G becomes,

$$\begin{aligned} \sum_a \sum_b \int \int \delta G(x_a, x_b) d\tau d\tau' = &\frac{1}{2} \sum_a \sum_b \int_V \int \int \delta([-g]^{1/2} g^{\mu\nu}) \left[\frac{\partial G^{\mathrm{adv}}(x, x_a)}{\partial x^\mu} \frac{\partial G^{\mathrm{ret}}(x, x_b)}{\partial x^\nu} \right. \\ &+ \left. \frac{\partial G^{\mathrm{ret}}(x, x_a)}{\partial x^\mu} \frac{\partial G^{\mathrm{adv}}(x, x_b)}{\partial x^\nu} \right] d^4x d\tau d\tau' \\ &- \frac{1}{12} \sum_a \sum_b \int_V \int \int \delta(R[-g]^{1/2}) \left[G^{\mathrm{adv}}(x, x_a) G^{\mathrm{ret}}(x, x_b) \right. \\ &+ \left. G^{\mathrm{ret}}(x, x_a) G^{\mathrm{adv}}(x, x_b) \right] d^4x d\tau d\tau' \end{aligned} \tag{77}$$

Note that each term has one sum over a and one over b, so when we substitute in the mass fields all the summations are used.

$$\begin{aligned} \sum_a \sum_b \int \int \delta G(x_a, x_b) d\tau d\tau' = &\frac{1}{2} \int_V \delta([-g]^{1/2} g^{\mu\nu}) \left[\frac{\partial m^{\mathrm{adv}}}{\partial x^\mu} \frac{\partial m^{\mathrm{ret}}}{\partial x^\nu} + \frac{\partial m^{\mathrm{ret}}}{\partial x^\mu} \frac{\partial m^{\mathrm{adv}}}{\partial x^\nu} \right] d^4x \\ &- \frac{1}{12} \int_V \delta(R[-g]^{1/2}) \left[m^{\mathrm{adv}} m^{\mathrm{ret}} + m^{\mathrm{ret}} m^{\mathrm{adv}} \right] d^4x \end{aligned} \tag{78}$$

We may therefore simplify the δI to remove the summations. Here is a full summary so far:

$$\begin{aligned} \delta I &= -\frac{1}{2} \delta \left[\sum_a \int m(x_a) d\tau \right] \\ &= -\frac{1}{2} \sum_a m(x_a) \delta(d\tau) - \frac{1}{2} \sum_a \int \delta m(x_a) d\tau \\ &= -\frac{1}{2} \int_V T^{\mu\nu} \delta g_{\mu\nu} [-g]^{1/2} d^4x - \frac{1}{2} \sum_a \sum_b \int \int \delta G(x_a, x_b) d\tau d\tau' \\ &= +\frac{1}{2} \int_V T_{\mu\nu} \delta g^{\mu\nu} [-g]^{1/2} d^4x - \frac{1}{2} \int_V \delta([-g]^{1/2} g^{\mu\nu}) \left[\frac{\partial m^{\mathrm{adv}}}{\partial x^\mu} \frac{\partial m^{\mathrm{ret}}}{\partial x^\nu} \right] d^4x \\ &\quad + \frac{1}{12} \int_V \delta(R[-g]^{1/2}) \left[m^{\mathrm{adv}} m^{\mathrm{ret}} \right] d^4x \end{aligned} \tag{79}$$

where we have replaced the first term with the familiar energy-stress tensor expression and flipped from contravariant to covariant notation with a minus sign change. The second term in the above equation can be expanded to give;

$$
\begin{aligned}
\delta(\tfrac{1}{2}[-g]^{1/2}g^{\mu\nu})\frac{\partial m^{\mathrm{adv}}}{\partial x^{\mu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\nu}} =& \\
-\frac{1}{2}&\left[\delta[-g]^{1/2}g^{\alpha\beta}\frac{\partial m^{\mathrm{adv}}}{\partial x^{\alpha}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\beta}} + [-g]^{1/2}\delta g^{\mu\nu}\frac{\partial m^{\mathrm{adv}}}{\partial x^{\mu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\nu}}\right] \\
=\frac{1}{2}&\left[\left(\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}\right)g^{\alpha\beta}\frac{\partial m^{\mathrm{adv}}}{\partial x^{\alpha}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\beta}}\right] \\
&\quad -[-g]^{1/2}\delta g^{\mu\nu}\frac{1}{2}\left[\frac{\partial m^{\mathrm{adv}}}{\partial x^{\mu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\nu}} + \frac{\partial m^{\mathrm{adv}}}{\partial x^{\nu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\mu}}\right]
\end{aligned}
\tag{80}
$$

where we have used the following useful identity [12] p 113, in the last step

$$
\delta[-g]^{1/2} = -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2} \tag{81}
$$

The $\delta(R[-g]^{1/2})$ term in δI can be expanded also as follows;

$$
\begin{aligned}
\frac{1}{6}\delta(R[-g]^{1/2}) &= \frac{1}{6}\delta(R_{\mu\nu}g^{\mu\nu}[-g]^{1/2}) \\
&= \frac{1}{6}\left[\delta(g^{\mu\nu}R_{\mu\nu})[-g]^{1/2} + R\delta([-g]^{1/2})\right]
\end{aligned}
\tag{82}
$$

using the Eq (81) again for the last term we find,

$$
\begin{aligned}
\frac{1}{6}\delta(R[-g]^{1/2}) &= \frac{1}{6}\left[R_{\mu\nu}[-g]^{1/2}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}[-g]^{1/2} - \frac{1}{2}Rg_{\mu\nu}[-g]^{1/2}\delta g^{\mu\nu}\right] \\
&= \frac{1}{6}\left[\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\delta g^{\mu\nu}[-g]^{1/2} + g^{\mu\nu}\delta R_{\mu\nu}[-g]^{1/2}\right]
\end{aligned}
\tag{83}
$$

hence the contribution to δI becomes

$$
\begin{aligned}
\frac{1}{6}\int_V \delta(R[-g]^{1/2})\left[m^{\mathrm{adv}}m^{\mathrm{ret}}\right]d^4x &= \frac{1}{6}\int_V \left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\delta g^{\mu\nu}[-g]^{1/2}\left[m^{\mathrm{adv}}m^{\mathrm{ret}}\right] \\
&\quad + \frac{1}{6}\int_V g^{\mu\nu}\delta R_{\mu\nu}[-g]^{1/2}\left[m^{\mathrm{adv}}m^{\mathrm{ret}}\right]d^4x
\end{aligned}
\tag{84}
$$

We will treat the $\delta R_{\mu\nu}$ term separately, it does not go to zero as in the Einstein case *unfortunately!*.

$$
\begin{aligned}
\frac{1}{12}\int_V g^{\mu\nu}\delta R_{\mu\nu}[-g]^{1/2}\left[m^{\mathrm{adv}}m^{\mathrm{ret}}\right]d^4x &= \frac{1}{2}\int \theta_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\
\theta_{\mu\nu} &= \frac{1}{6}\frac{g^{\mu\nu}}{\delta g^{\mu\nu}}\delta R_{\mu\nu}m^{\mathrm{adv}}m^{\mathrm{ret}}
\end{aligned}
\tag{85}
$$

The shorthand for δI then becomes,

$$
\begin{aligned}
\delta I = &+\frac{1}{2}\int_V T_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x \\
&+\frac{1}{2}\left[\left(\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}\right)g^{\alpha\beta}\frac{\partial m^{\mathrm{adv}}}{\partial x^{\alpha}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\beta}}\right] \\
&\qquad -[-g]^{1/2}\delta g^{\mu\nu}\frac{1}{2}\left[\frac{\partial m^{\mathrm{adv}}}{\partial x^{\mu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\nu}} + \frac{\partial m^{\mathrm{adv}}}{\partial x^{\nu}}\frac{\partial m^{\mathrm{ret}}}{\partial x^{\mu}}\right] \\
&+\frac{1}{6}\int_V \left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\delta g^{\mu\nu}[-g]^{1/2}\left[m^{\mathrm{adv}}m^{\mathrm{ret}}\right] \\
&+\frac{1}{2}\int \theta_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x = 0\,.
\end{aligned}
\tag{86}
$$

The field equations are then seen to be,

$$T_{\mu\nu} + \theta_{\mu\nu} + \frac{1}{6}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)m^{\text{adv}}m^{\text{ret}}$$
$$- \frac{1}{2}\left[\frac{\partial m^{\text{adv}}}{\partial x^{\mu}}\frac{\partial m^{\text{ret}}}{\partial x^{\nu}} + \frac{\partial m^{\text{adv}}}{\partial x^{\nu}}\frac{\partial m^{\text{ret}}}{\partial x^{\mu}} - g_{\mu\nu}g^{\alpha\beta}\frac{\partial m^{\text{adv}}}{\partial x^{\alpha}}\frac{\partial m^{\text{ret}}}{\partial x^{\beta}}\right] = 0 \tag{87}$$

What remains is to expand $\theta_{\mu\nu}$ in its full glory. See the Addendum for details.
After some trivial algebra, which is obvious to the most casual observer, and only takes a couple of pages of calculation we get...

$$\theta_{\mu\nu} = -\frac{1}{6}\left[g_{\mu\nu}\Box^2(m^{\text{adv}}m^{\text{ret}}) - (m^{\text{adv}}m^{\text{ret}})_{,\mu\nu}\right] \tag{88}$$

where $\Box^2$ is the wave equation $\partial_{\mu}\partial^{\mu}$.

5. DERIVATION OF WOODWARD'S MASS CHANGE FORMULA

5.1 Woodward's Power Equation → mass change formula

From Woodward's book [24], page 73 Eq(3.5), we find;

$$\delta m = \frac{1}{4\pi G}\left[\frac{1}{\rho_0 c^2}\frac{\partial P}{\partial t} - \left(\frac{1}{\rho_0 c^2}\right)^2\frac{P^2}{V}\right]$$
$$\delta m = \frac{1}{4\pi G}\left[\frac{V}{m_0 c^2}\frac{\partial^2 \varepsilon}{\partial t^2} - \left(\frac{V}{m_0 c^2}\right)^2\frac{1}{V}\left(\frac{\partial \varepsilon}{\partial t}\right)^2\right]$$
$$= \frac{1}{4\pi G}\left[\frac{V}{m_0}\frac{\partial^2 m}{\partial t^2} - V\left(\frac{1}{m_0}\right)^2\left(\frac{\partial m}{\partial t}\right)^2\right]$$
$$\frac{\delta m}{V} = \frac{1}{4\pi G}\left[\frac{1}{m_0}\frac{\partial^2 m}{\partial t^2} - \left(\frac{1}{m_0}\right)^2\left(\frac{\partial m}{\partial t}\right)^2\right] \tag{89}$$

where V is volume over the device, P is power to the device, and $P = d\varepsilon/dt$. Energy is $\varepsilon = mc^2$ and mass density $\rho_0 = m_0/V$. This agrees with the dimensions of $[G] = [FL^2/M^2]$.

5.2 HN-theory field equation → mass change formula

Let's define the HN-field equation (in a smooth fluid) as follows (which agrees with Eq.(16) in reference [4]) by grouping terms together;

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -8\pi G(T_{\alpha\beta} + \delta T_{\alpha\beta}) \quad \text{where}$$
$$-(8\pi G)\delta T_{\alpha\beta} = \frac{2}{m}(g_{\alpha\beta}g^{\mu\nu}m_{;\mu\nu} - m_{;\alpha\beta}) + \frac{4}{m^2}(m_{;\alpha}m_{;\beta} - \frac{1}{4}m_{;\gamma}m^{;\gamma}g_{\alpha\beta}) \tag{90}$$

Now we expand the terms out. Let us put back in c and not set it equal to one, which can be confusing. The terms in μ, ν mix the time and spatial derivatives in an unexpected way.

Consider first the T_{00} and T_{jj} terms separately, using flat metric (+1,-1,-1,-1),

$$
\begin{aligned}
-\frac{8\pi G}{c^4}\delta T_{00} &= \frac{2}{m}\left[g_{00}\left(\frac{g^{00}}{c^2}\frac{\partial^2 m}{\partial t^2}+g^{jj}\frac{\partial^2 m}{\partial x_j^2}\right)-\frac{1}{c^2}\frac{\partial^2 m}{\partial t^2}\right] \\
&\quad+\frac{4}{m^2}\left[\frac{1}{c^2}\left(\frac{\partial m}{\partial t}\right)^2-\frac{g_{00}}{4}\left(\frac{1}{c^2}\left(\frac{\partial m}{\partial t}\right)^2-\left(\frac{\partial m}{\partial x_j}\right)^2\right)\right] \\
&= \frac{2}{m}\left[\cancel{\frac{1}{c^2}\frac{\partial^2 m}{\partial t^2}}-\frac{\partial^2 m}{\partial x_j^2}-\cancel{\frac{1}{c^2}\frac{\partial^2 m}{\partial t^2}}\right] \\
&\quad+\frac{1}{m^2}\left[\frac{4}{c^2}\left(\frac{\partial m}{\partial t}\right)^2-\frac{1}{c^2}\left(\frac{\partial m}{\partial t}\right)^2+\left(\frac{\partial m}{\partial x_j}\right)^2\right] \\
&= -\frac{2}{m}\frac{\partial^2 m}{\partial x_j^2}+\frac{1}{m^2}\left(\frac{\partial m}{\partial x_j}\right)^2+\frac{3}{m^2c^2}\left(\frac{\partial m}{\partial t}\right)^2
\end{aligned}
\tag{91}
$$

where we treat the derivatives with respect to $\partial/\partial x_j$ as a 3 component gradient-like term.

$$
\begin{aligned}
-\frac{8\pi G}{c^4}\delta T_{jj} &= \frac{2}{m}\left[g_{jj}\left(\frac{g^{00}}{c^2}\frac{\partial^2 m}{\partial t^2}+g^{jj}\frac{\partial^2 m}{\partial x_j^2}\right)-\frac{\partial^2 m}{\partial x_j^2}\right] \\
&\quad+\frac{4}{m^2}\left[\left(\frac{\partial m}{\partial x_j}\right)^2-\frac{g_{jj}}{4}\left(\frac{1}{c^2}\left(\frac{\partial m}{\partial t}\right)^2-\left(\frac{\partial m}{\partial x_j}\right)^2\right)\right] \\
&= \frac{2}{m}\left[-\frac{1}{c^2}\frac{\partial^2 m}{\partial t^2}+\cancel{\frac{\partial^2 m}{\partial x_j^2}}-\cancel{\frac{\partial^2 m}{\partial x_j^2}}\right] \\
&\quad+\frac{1}{m^2}\left[4\left(\frac{\partial m}{\partial x_j}\right)^2+\frac{1}{c^2}\left(\frac{\partial m}{\partial t}\right)^2-\left(\frac{\partial m}{\partial x_j}\right)^2\right] \\
&= -\frac{2}{mc^2}\frac{\partial^2 m}{\partial t^2}+\frac{3}{m^2}\left(\frac{\partial m}{\partial x_j}\right)^2+\frac{1}{m^2c^2}\left(\frac{\partial m}{\partial t}\right)^2
\end{aligned}
\tag{92}
$$

where $j = 1,2$ or 3. Now take the trace of $T_{\alpha\alpha}$ where $\alpha = 0,1,2,3$ by adding the last two equations.

$$
\begin{aligned}
-\frac{8\pi G}{c^4}\,\mathrm{Tr}(\delta T_{\alpha\alpha}) &= -\frac{2}{m}\frac{\partial^2 m}{\partial x_j^2}+\frac{4}{m^2}\left(\frac{\partial m}{\partial x_j}\right)^2+\frac{4}{m^2c^2}\left(\frac{\partial m}{\partial t}\right)^2-\frac{2}{mc}\frac{\partial^2 m}{\partial t^2} \\
\frac{1}{c^2}\,\mathrm{Tr}(\delta T_{\alpha\alpha}) &= \frac{1}{4\pi G}\left[\left\{\frac{1}{m}\frac{\partial^2 m}{\partial t^2}-\frac{2}{m^2}\left(\frac{\partial m}{\partial t}\right)^2\right\}\right. \\
&\quad\left.+\left\{\frac{c^2}{m}\frac{\partial^2 m}{\partial x_j^2}-\frac{2c^2}{m^2}\left(\frac{\partial m}{\partial x_j}\right)^2\right\}\right]
\end{aligned}
\tag{93}
$$

where we assume we are summing over α and j. This last expression should be compared with the previous result Eq. (89) above. Note that there are also spatial terms here, which in previous papers I incorporated into the time derivatives [28]. I now think that was a mistake and have written them out explicitly here. This is the main result of the paper. Quoting from a paper [29] by R. Medina:

> "Unlike the inertia of energy, which is well known, many physicists are not aware of the inertia of pressure (stress). In many cases such an effect is negligible, but for the case of the stress produced by electrostatic interactions, it is comparable to the inertial effects of the electromagnetic fields. If the inertia of stress is neglected the calculations are inconsistent."

The spatial and temporal terms may be related, in the sense that in The Mach effect drive (or MEGA drive), PZT (lead zirconate titanate) expands and contracts. In a different device, that may not be the case.

6. CONCLUSIONS

The main result we wish to emphasize is the mass fluctuation, Eq. (93). Compare this with Woodward's result Eq. (89) from his book [24] p73, Eq. (3.5). The consequences of this mass fluctuation are *astounding* as related to the Woodward effect and propellant-less propulsion methods. A following paper in this chapter, by Rodal, will describe how to calculate a force using the mass fluctuation calculated here. The calculated force and resonant frequency predictions will be compared to experimental data.

Hoyle-Narlikar gravitation , or gravitational absorber theory (GAT), is a valid theory that is fully consistent with Einstein's GR. It is a fully Machian theory of gravitation, which means that the mass of a body depends on its gravitational interaction with all the other masses in the universe. Text books on Einstein's GR rarely if ever mention advanced waves, yet the are necessary if interactions with distant matter are to be thought of as instantaneous.
Around 1965 Hawking voiced an objection to HN-theory [27], but that objection is no longer valid due to the accelerating expansion of the universe [28]. Hoyle-Narlikar theory is to gravitation what Wheeler-Feynman absorber theory is to electromagnetism. Einstein's General Relativity (GR) remains valid and all the tests of Einstein's GR also remain valid and carry over to the HN theory presented here. The real difference is in the highly symmetric and simplified Lagrangian, which treats a mass as being influenced by all the other masses in the universe, and that is all. A real universe must therefore be made up of at least two masses.

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Addendum

For those of you who just can't get enough algebra, here is the rest of the glorious details for the derivation of $\theta_{\mu\nu}$.

We need to expand out Eq. (85) and find the equation for $\theta_{\mu\nu}$;

$$\frac{1}{2}\int_V \theta_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x = \frac{1}{12}\int_V (\delta R_{\mu\nu}g^{\mu\nu})m^{\text{adv}}m^{\text{ret}}[-g]^{1/2}d^4x$$

The term in the round bracket on the RHS of the equation can be written as, [12], p118 Eqs (98,99).

$$\begin{aligned}(g^{\mu\nu}\delta R_{\mu\nu}) &= \frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\lambda}\left[[-g]^{1/2}w^\lambda\right]\\ w^\lambda &= (g^{\mu\nu}\delta\Gamma^\lambda_{\mu\nu} - g^{\mu\nu}\delta\Gamma^\nu_{\mu\nu})\end{aligned} \tag{94}$$

Hence we may write,

$$\begin{aligned}\frac{1}{12}\int_V (\delta R_{\mu\nu}g^{\mu\nu})m^{\text{adv}}m^{\text{ret}}[-g]^{1/2}d^4x &= \frac{1}{12}\int_V \frac{\partial}{\partial x^\lambda}\left[[-g]^{1/2}w^\lambda\right](m^{\text{adv}}m^{\text{ret}})d^4x\\ &= \frac{1}{12}\int_V w^\lambda\frac{\partial}{\partial x^\lambda}(m^{\text{adv}}m^{\text{ret}})[-g]^{1/2}d^4x\end{aligned} \tag{95}$$

where we have integrated by parts. This agrees with Eq (100) in Hoyle and Narlikar's book [12]. Using the following identities, from their book p118, w^λ can be expanded.

$$\begin{aligned}
\Gamma^{\nu}_{\mu\nu} &= \frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\mu}\left([-g]^{1/2}\right)\\
g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} &= -\frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\nu}\left([-g]^{1/2}g^{\lambda\nu}\right)\\
\delta([-g]^{-1/2}) &= +\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{-1/2}\\
\frac{\partial}{\partial x^\alpha}[-g]^{1/2} &= [-g]^{1/2}\Gamma^{\beta}_{\alpha\beta}
\end{aligned} \tag{96}$$

we will also be reusing the identity in Eq (81), which is also in reference [11] p50, Eq. (26.10).

Consider the following,

$$\begin{aligned}
\delta(g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}) &= \delta g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} + g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu}\\
\Rightarrow w^\lambda &= (g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu} - g^{\mu\nu}\delta\Gamma^{\nu}_{\mu\nu})\\
&= \delta(g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}) - \delta g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} - g^{\mu\nu}\delta\Gamma^{\nu}_{\mu\nu}
\end{aligned} \tag{97}$$

Now we need to consider the separate parts of the equation for w^λ and rewrite it;

$$\begin{aligned}
\delta\Gamma^{\nu}_{\mu\nu} &= \delta([-g]^{-1/2})\frac{\partial}{\partial x^\mu}([-g]^{1/2}) + ([-g]^{-1/2})\frac{\partial}{\partial x^\mu}(\delta[-g]^{1/2})\\
\delta(g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}) &= -\delta([-g]^{-1/2})\frac{\partial}{\partial x^\nu}([-g]^{1/2}g^{\lambda\nu}) - [-g]^{-1/2}\frac{\partial}{\partial x^\nu}[\delta([-g]^{1/2}g^{\lambda\nu})]
\end{aligned} \tag{98}$$

where we have differentiated the identities (96) above. Now we substitute these expression into the equation for w^λ to obtain,

$$\begin{aligned}
w^\lambda = &-\delta g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} - \frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\nu}[\delta([-g]^{1/2}g^{\lambda\nu})] - \frac{1}{[-g]^{1/2}}g^{\mu\lambda}\frac{\partial}{\partial x^\mu}[\delta[-g]^{1/2}]\\
&-g^{\alpha\lambda}\delta([-g]^{-1/2})\left[\frac{\partial}{\partial x^\alpha}[-g]^{1/2}\right] + \delta([-g]^{-1/2})[-g]^{1/2}g^{\alpha\beta}\Gamma^{\lambda}_{\alpha\beta}
\end{aligned} \tag{99}$$

Now we only need to substitute the following identities,

$$\begin{aligned}
\delta([-g]^{1/2}) &= -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}\\
\delta([-g]^{-1/2}) &= +\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{-1/2}\\
\frac{\partial}{\partial x^\alpha}[-g]^{1/2} &= [-g]^{1/2}\Gamma^{\beta}_{\alpha\beta}
\end{aligned} \tag{100}$$

to obtain the needed result for w^λ,

$$\begin{aligned}
w^\lambda = &-\delta g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} - \frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\nu}\left[\delta([-g]^{1/2}g^{\lambda\nu})\right] - \frac{1}{[-g]^{1/2}}g^{\mu\lambda}\frac{\partial}{\partial x^\mu}\left[\delta[-g]^{1/2}\right]\\
&-\frac{1}{2}g^{\alpha\lambda}\Gamma^{\beta}_{\alpha\beta}g_{\mu\nu}\delta g^{\mu\nu} + \frac{1}{2}\delta g^{\mu\nu}g_{\mu\nu}g^{\alpha\beta}\Gamma^{\lambda}_{\alpha\beta}
\end{aligned} \tag{101}$$

which agrees with Eq (102) p118 [12]. Now at this point, this wonderful expression for w^λ must be placed back inside the integral (276), because we need to find the result for $\theta_{\mu\nu}$. The three terms involving Christoffel symbols cancel out. You can integrate by parts and use the divergence theorem. The only remaining terms involve differentiations on the mass functions only.

$$\frac{1}{2}\int \theta_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}d^4x = -\frac{1}{6}\int\left(-\frac{1}{[-g]^{1/2}}\frac{\partial}{\partial x^\alpha}\left[\delta([-g]^{1/2}g^{\lambda\alpha})\right] - \frac{1}{[-g]^{1/2}}g^{\alpha\lambda}\frac{\partial}{\partial x^\alpha}\left[\delta[-g]^{1/2}\right]\right) \times \frac{\partial}{\partial x^\lambda}(m^{\mathrm{adv}}m^{\mathrm{ret}})[-g]^{1/2}d^4x \tag{102}$$

Expand out the first term and substitute for the $\delta[-g]^{1/2} = -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}[-g]^{1/2}$, then the $[-g]^{1/2}$ terms cancel out.

$$\Rightarrow \theta_{\mu\nu}\delta g^{\mu\nu} = \left(-\frac{1}{12}g^{\lambda\alpha}g_{\mu\nu}\delta g^{\mu\nu}\frac{\partial}{\partial x^\alpha} - \frac{1}{12}g^{\alpha\lambda}g_{\mu\nu}\delta g^{\mu\nu}\frac{\partial}{\partial x^\alpha}\right)\frac{\partial}{\partial x^\lambda}(m^{\mathrm{adv}}m^{\mathrm{ret}}) + \frac{1}{6}\delta g^{\lambda\alpha}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial x^\lambda}(m^{\mathrm{adv}}m^{\mathrm{ret}}) \tag{103}$$

Now we must substitute $\delta g^{\lambda\alpha} \to \delta g^{\mu\nu}$ to match the LHS of the equation in the last term. Performing contractions over α on the first two terms, leads to,

$$\theta_{\mu\nu} = -\frac{1}{6}\left(g_{\mu\nu}\frac{\partial^2}{\partial x^\lambda \partial x_\lambda}(m^{\mathrm{adv}}m^{\mathrm{ret}}) - \frac{\partial^2}{\partial x^\mu \partial x^\nu}(m^{\mathrm{adv}}m^{\mathrm{ret}})\right)$$
$$\text{thus } \theta_{\mu\nu} = -\frac{1}{6}\left(g_{\mu\nu}\Box^2(m^{\mathrm{adv}}m^{\mathrm{ret}}) - (m^{\mathrm{adv}}m^{\mathrm{ret}})_{,\mu\nu}\right) . \tag{104}$$

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