Proton-Boron ($p - B^{11}$) colliding beam fusion reactor

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Abstract

In the proton-boron colliding beam fusion reactor, the power which must be supplied to maintain an optimal colliding beam configuration is estimated to be at least 5.1 times greater than the fusion power. This implies that effective power conversion efficiencies to electrical power in excess of 84% will be required. Furthermore, if the transverse collisional spread of the proton beam is to be limited by electron drag, the boron density is constrained to have magnitudes well below the optimal value at which the fusion power is maximised.

I. INTRODUCTION

Rostoker, Binderbauer, and Monkhorst [1] (RBM) have proposed a conceptual fusion reactor design which combines the environmentally attractive neutron-free property of the proton-boron (p-B) fusion reaction with the compact magnetic field geometry of the Field Reversed Configuration (FRC). The reaction products are energetic charged alpha particles which can be confined by the magnetic field. Since no neutrons are produced in the primary reaction, radiative activation of the containing walls by particle bombardment is not a major concern. This design concept offers the vision of a small fusion power source with the possibility for high-efficiency direct energy conversion into electricity and with minimal radiation shielding requirements.

In the FRC, beams of large orbit fuel ions propagate across the magnetic field, with enough electrons present for the system to be quasi-neutral. The energetic fuel ions follow confined orbits in an annular field-reversed layer with annular width small compared to the annular radius, and they carry a significant fraction of the current required for magnetic field reversal.

Two modes of operation are envisaged:

(1) Fuel ions move with essentially the same cross-field velocities and with effective perpendicular temperatures in "betatron" motion of $\sim 235\,\text{KeV}$ where the fusion cross-section for a thermal plasma is close to its maximum value. The beam energies of protons and borons are of the order of 300 KeV and 3.3 MeV respectively.

(2) Cold beams of fuel ions move at different cross-field velocities with relative energy of collision at 600 KeV. At this "resonant" energy, the fusion cross-section is a maximum which is approximately three times larger than the fusion cross-section for a thermal plasma of protons and borons. The perpendicular temperature of the beams must be substantially less than 140 KeV while the sum of the beam energies of the fuel ions need not be larger than 1 MeV.

The proposed fusion reactor concept presents formidable challenges when compared to more conventional design concepts such as the Tokamak. The interaction energies of the fuel ions are higher: $\sim 235\,\mathrm{KeV}$ in the case of mode 1, and $\sim 600\,\mathrm{KeV}$ in the case of mode 2. The energy gain per fusion reaction is smaller: the energy output per fusion reaction is $\sim 8.7\,\mathrm{MeV}$ ($H + B^{11} = 3\ He^4 + 8.68\,\mathrm{MeV}$), and thus in mode 1 the energy gain factor is ~ 2.4 if the energy invested in the beam energy of the fuel ions is $\sim 3.6\,\mathrm{MeV}$, while in mode 2 the energy gain factor can be no more than ~ 14.5 for a resonant collisional energy of interaction of $\sim 600\,\mathrm{KeV}$. Furthermore, in mode 2 operation, an optimal colliding beam configuration must be maintained in the face of ion collisional effects: collisional heating and expansion of the beam ions, and deviations from the resonant interaction energy due to collisional drag.

In our review and evaluation of the physics issues involved in (p-B) fusion reactors, we will focus on mode 2 operation since the reactor design concept based on colliding beams of protons and borons has been the subject of considerable controversy.

Lampe and Manheimer [2] (L and M) have emphasized that the collisional rate ν_{pB} for (p-B) momentum exchange scattering is 37 times faster than the fusion reaction rate ν_F, and that the collisional (p-B) drag rate although slower than ν_{pB} by a factor of twice the mass ratio m_p/m_B is still faster than ν_F by a factor 7. They discussed a variety of collisional processes which would destroy the colliding beam equilibrium on time scales much faster than the fusion reaction time. They considered an explicit equation for the time evolution of the transverse spread of the proton beam, based on the Fokker-Planck kinetic equation, and their calculations clearly showed the heating of the proton beam due to p-B collisions in the limit $\nu_{\mathrm{pB}} > \nu_{pe}$, where ν_{pe} is the collisional rate for proton-electron (p-e) momentum transfer. They concluded that the required colliding beam equilibrium "cannot be sustained for long enough to provide fusion gain."

RBM responded that the criticisms of L and M are relevant "if one considers only collisions and omits the rest of the physics of magnetic confinement which L and M proceed to do in the balance of their report." As stated by RBM, the problem with the calculation of L and M is that: "there is no current, no magnetic field, and no inductance, and they consider only an initial value problem"; "not a steady-state problem"; "there are no external sources for the protons, and the sink due to burning is also not considered." They suggested that it may be possible to maintain the fuel ion velocities by the injection of pulsed beams.

The comments of L and M underline the difficult problem posed by particle collisions. The fusion reactivity decreases if the temperatures of the fuel ion beams acquire a thermal spread and/or if the interaction energy deviates from the resonant energy. Efficient operation therefore requires that optimal conditions be maintained for times long enough for a significant number of fusion reactions to occur. The technological issues involved in forming and maintaining an appropriate colliding beam configuration are indeed very serious. Nevertheless, the "rest of the physics," alluded to by RBM, does in principle allow for the possibility of quasi-stationary colliding beam configurations with lifetimes longer than collisional times, and it may be somewhat premature for L and M to declare, without further investigations, that "the concept is fundamentally flawed."

The ion beam velocities can be maintained constant by counterbalancing the particle drag forces with the forces due to an inductive electric field and/or by transferring momentum to the ion beams by some appropriate mechanism (e.g. injecting additional beams). The control of the transverse velocity spread ("temperature") of the proton beam presents a greater challenge. However, the rate of increase of the proton "temperature" is proportional to the boron density, and at sufficiently low boron density, the proton "temperature" can be limited by electron drag when $\nu_{\mathrm{pB}} < \nu_{pe}$.

Given that quasi-stationary colliding beam configurations are in principle not precluded, the essential issue becomes that of determining whether or not there are beam configurations which could provide the basis for a viable fusion reactor.

RBM have analyzed a set of multifluid equations obtained by taking moments of the Fokker-Planck equation. The particle distribution functions were taken to be drifted Maxwell distributions where the temperature and mean velocity may be time dependent. These equations were used to obtain design parameters for fusion reactors based on mode 1 operation. These equations are however inadequate for the investigation of reactor designs based on mode 2 operation since Maxwellian distributions are not solutions of the lowest order Fokker-Planck kinetic equations for colliding ion beam configurations. The merits of fusion reactor designs based on mode 2 operation are still being evaluated by RBM.

In this report, we limit the scope of our discussion to two topics which have a direct bearing on the the design of colliding beam fusion reactors:

(1) We consider two quasi-stationary colliding beam scenarios subject to the constraint that the relative (p-B) energy $\mathcal{E}_{\rm pB}$ is equal to the resonant energy of 600 KeV, one involving an inductive electric field and the other non-inductive "external" forces acting on the fuel ions. From momentum balance we estimate the minimum power which must be supplied to sustain "steady state." We find that the power required is significantly smaller when non-inductive "external" forces are used rather than inductive electric fields. Nevertheless, even with non-inductive "external" forces, the required power is still quite large when compared to the fusion power; it is estimated that the required power must be at least > 5 times the fusion power. In addition, it is a major problem in itself to sustain highly efficient "external" forces acting on the ion beams, and we have not addressed this difficulty.

(2) We explore the conditions required for electron drag to be effective in limiting the transverse velocity spread of the proton beam. Electron drag is effective if $\nu_{\rm pB}/\nu_{pe} < 1$. At "steady state," the transverse "temperature" $T_\perp$ of the proton beam is approximately given by $T_\perp/\mathcal{E}_{\rm pB} \sim \nu_{\rm pB}/\nu_{pe}$. Since $T_\perp/\mathcal{E}_{\rm pB}$ is required to be small, we must have $\nu_{\rm pB}/\nu_{pe} < 1$, an inequality which can be satisfied if the electron temperature and/or the boron to electron density ratio n_B/n_e are sufficiently small. However, the fusion power is proportional to the product of the proton and boron densities $n_p n_B$, and with quasi neutrality $n_e = n_p + Z n_B$ (where $Z = 5$), the fusion power is a maximum when $n_B/n_e = 0.1$; hence it would be undesirable to have the density ratio n_B/n_e of borons to electrons much below 10%.

This report is organized into 3 sections. In Sec. II, we describe two quasi-stationary colliding beam configurations, and we estimate the minimum power required to sustain these quasi-stationary states. In Sec. III, we discuss the effect of electron drag in limiting the increase in the transverse velocity spread induced by proton-boron Coulomb collisions.

In Sec. IV, we state our conclusions.

II. POWER REQUIRED TO MAINTAIN QUASI-STATIONARY COLLIDING BEAM CONFIGURATION

A. Inductive Electric Field

An inductive electric field accelerating electrons and ions in opposite directions can be used to balance the collisional drag forces which act to decrease the relative velocity between the particle species.

Let us consider a simplified model of a colliding beam configuration in which beams of electrons, protons and borons are in motion relative to each other with velocities $\boldsymbol{V}_e = V_e\hat{\boldsymbol{z}}$, $\boldsymbol{V}_p = V_p\hat{\boldsymbol{z}}$ and $\boldsymbol{V}_B = V_B\hat{\boldsymbol{z}}$ respectively. The direction of beam propagation is taken to be parallel to the z-axis ($\hat{\boldsymbol{z}}$ is the unit vector in the z-direction).

The beam configuration is assumed to be in steady state on the collisional time scale. The components of the momentum equation in the z-direction with inclusion of an inductive electric field $\boldsymbol{E} = E\hat{\boldsymbol{z}}$ are

$$eE = m_p\nu_{pe}(V_p - V_e) + m_B\nu_{Bp}(V_p - V_B)\frac{n_B}{n_p} \tag{1}$$

$$ZeE = m_pZ^2\nu_{pe}(V_B - V_e) + m_B\nu_{Bp}(V_B - V_p) \tag{2}$$

$$-eE = m_p\nu_{pe}(V_e - V_p)\frac{n_p}{n_e} + m_pZ^2\nu_{pe}(V_e - V_B)\frac{n_B}{n_e} \tag{3}$$

where $Ze = 5e$ is the boron charge, m_a and n_a are the mass and density of species "a" (a=e,p,B), ν_{pe} and $\nu_{\rm pB}$ are the proton-electron (p-e) and proton-boron (p-B) Coulomb collision frequencies respectively, and $m_an_a\nu_{ab} = m_bn_b\nu_{ba}$.

Note that these equations are consistent with quasineutrality $n_e = n_p + Zn_B$. From Eq. (1), Eq. (3), and quasi-neutrality, we obtain the following equation relating V_e, V_p, V_B:

$$V_B - V_e = \left(\frac{1+S}{Z+S}\right)(V_p - V_e)$$

where the parameter S is defined by:

$$S = \frac{m_B}{m_p}\frac{n_e}{Zn_p}\frac{\nu_{Bp}}{\nu_{pe}} \tag{4}$$

and is proportional to the ratio of collision frequencies ν_{Bp}/ν_{pe}. Substituting in Eq. (4) the collision frequencies given by (see reference [2]):

$$\nu_{pe} = 2.6 \times 10^{-14} n_e \left(10 \text{ KeV}/T_e\right)^{3/2} \ \text{sec}^{-1}$$

$$\nu_{\text{pB}} = 1.09 \times 10^{-12} n_B \left(100 \text{ KeV}/\mathcal{E}_{\text{pB}}\right)^{3/2} \ \text{sec}^{-1}$$

$$\nu_{Bp} = 9.9 \times 10^{-14} n_p \left(100 \text{ KeV}/\mathcal{E}_{\text{pB}}\right)^{3/2} \ \text{sec}^{-1}$$

where T_e (in *kev*) is the electron temperature and $\mathcal{E}_{\text{pB}}$ is defined to be

$$\mathcal{E}_{\text{pB}} = \frac{1}{2} m_p \left(V_p - V_B\right)^2 = 600 \text{ KeV}$$

we obtain:

$$S = 120.0 \frac{m_B}{Z m_p} \left(\frac{T_e}{\mathcal{E}_{\text{pB}}}\right)^{3/2}.$$

Expressing the relative streaming velocities of ions to electrons $\{(V_p - V_e), (V_b - V_e)\}$ in terms of the relative streaming velocity of the ions $(V_p - V_B)$, we obtain:

$$V_p - V_e = \left(\frac{Z+S}{Z-1}\right)(V_p - V_B)$$

$$V_B - V_e = \left(\frac{1+S}{Z-1}\right)(V_p - V_B).$$

In order to sustain the steady state, the inductive electric field acting on ions and electrons must supply energy at a rate sufficient to replace the streaming energy transformed into thermal energy by particle collisions. This power constitutes circulating power in a reactor that is dissipated into thermal energy.

The magnitude of the plasma current J is

$$J = e\left(n_p V_p + Z n_B V_B - n_e V_e\right) = e n_e \left\{\left(\frac{Z+S}{Z-1}\right) - \frac{Z n_B}{n_e}\right\}(V_p - V_B)$$

and the power P_D dissipated by collisions is therefore given by:

$$\begin{aligned} P_D &= EJ \\ &= \frac{2 m_B n_B \nu_{Bp}}{m_p}\left[\frac{(Z+S)^2}{(Z-1)^2}\frac{n_e}{n_B Z S} + \frac{(1+S)^2}{(Z-1)^2}\frac{Z n_e}{S n_p} + 1\right]\mathcal{E}_{\text{pB}}. \end{aligned} \tag{5}$$

The first and second terms on the right hand side of Eq. (5) are due to electron-ion collisions, while the third term is due to proton-boron collisions.

The fusion power P_f due to (p-B) collisions is :

$$P_f = n_p n_B \langle \sigma v \rangle \mathcal{E}_f \tag{6}$$

where $\langle \sigma v \rangle$ is the fusion reactivity and $\mathcal{E}_f = 8.68\,\mathrm{MeV}$ is the fusion energy released by a fusion reaction. For polarized fuel ions with energy of collision equal to the "resonant" energy $\mathcal{E}_{\mathrm{pB}} = 600\,\mathrm{KeV}$, the fusion reaction rate is (see reference [2]):

$$\langle \sigma v \rangle = 2.0 \times 10^{-15}\ \mathrm{cm}^3\,\mathrm{sec}^{-1}\,.$$

The ratio of the power dissipated by collisions to the fusion power is therefore

$$\frac{P_D}{P_f} = 5.1 \left\{ \frac{n_e}{n_B Z S} \frac{(Z+S)^2}{(Z-1)^2} + \frac{n_e Z}{n_p S} \frac{(1+S)^2}{(Z-1)^2} + 1 \right\}. \tag{7}$$

For a given value of n_B/n_e, the power ratio P_D/P_f has a minimum at $S = \frac{Z\{1-(Z-1)n_B/n_e\}^{1/2}}{\{1+Z(Z-1)n_B/n_e\}^{1/2}}$, while for a given value of S the minimum occurs at $n_B/n_e = \frac{Z+S}{Z(Z^{1/2}+S)(Z^{1/2}+1)}$.

The power ratio P_D/P_f has an absolute minimum at $n_B/n_e = 1/(2Z)$ and $S = Z^{1/2}$, and its minimum value $(P_D/P_f)_{\mathrm{min}}$ is :

$$\left(\frac{P_D}{P_f} \right)_{\mathrm{min}} = 5.1 \left\{ 1 + \frac{4Z^{1/2}}{(Z^{1/2}-1)^2} \right\}.$$

For $Z = 5$ (borons), this ratio is equal to ~ 35.

In Fig. 1 we plot P_D/P_f as a function of the electron temperature T_e for $n_B/n_e = 0.05, 0.1, 0.15$. It has an absolute minimum of 35 for $n_B/n_e = 0.1$ at $T_e \sim 25\,\mathrm{KeV}$. The required circulating power, much of which is dissipated into thermal energy, will therefore be a large multiple of the fusion power.

The power available for conversion into electrical power is $P_f + P_D$. If the effective conversion efficiency is η, net electrical power is available only if $\eta(P_f + P_D)$ is greater than the power P_D required to maintain steady state, $\eta(P_f + P_D) > P_D$. For $P_D/P_f = 35$, the conversion efficiency must satisfy the inequality $\eta > 35/36 = 0.97$. Thus, a steady state established by inductive electric fields is not attractive as the basis for a fusion reactor producing net electrical power.

B. Non-inductive External Forces

In an alternative "steady state," the ion streaming velocities can be held constant by "external" forces, while the electrons have an intermediate mean velocity determined by equality of the opposite proton and boron drag forces on the electrons. It is here assumed that there is some appropriate and efficient means of transferring momentum to the proton and boron beams to make up the losses due to collisional drag.

Let $\mathcal{F}_p$ and $\mathcal{F}_B$ represent the equivalent "external" forces which maintain the streaming velocities of the ions. The modified momentum equations are:

$$\mathcal{F}_p = m_p \nu_{pe}(V_p - V_e) + m_B \nu_{Bp}(V_p - V_B)\frac{n_B}{n_p} \tag{8}$$

$$\mathcal{F}_B = m_p Z^2 \nu_{pe}(V_B - V_e) + m_B \nu_{Bp}(V_B - V_p) \tag{9}$$

$$0 = m_p \nu_{pe}(V_e - V_p)\frac{n_p}{n_e} + m_p Z^2 \nu_{pe}(V_e - V_B)\frac{n_B}{n_e}. \tag{10}$$

The relative streaming velocity of the ions to electrons are now related to $V_p - V_B$ by:

$$V_p - V_e = \frac{Z^2 n_B}{n_p + Z^2 n_B}\left(V_p - V_B\right)$$
$$V_e - V_B = \frac{n_p}{n_p + Z^2 n_B}\left(V_p - V_B\right).$$

The magnitude of the plasma current J is

$$J = en_p\left(V_p - V_e\right) + Zen_B\left(V_B - V_e\right) = \frac{en_p n_B Z(Z-1)}{n_p + Z^2 n_B}\left(V_p - V_B\right).$$

The dissipated power, equal to the rate at which work is done by the forces $\mathcal{F}_p$ and $\mathcal{F}_B$, is therefore given by:

$$P_D = n_p V_p \mathcal{F}_p + n_B V_B \mathcal{F}_B \frac{2 m_B n_B \nu_{Bp}}{m_p}\left[1 + \frac{Z n_e}{S(n_p + Z^2 n_B)}\right]\mathcal{E}_r$$

and the ratio of dissipated power to fusion power is

$$\frac{P_D}{P_f} = 5.1\left[1 + \frac{Zn_e}{S(n_e + Z(Z-1)n_B)}\right]. \tag{11}$$

In Fig. 2 we plot P_D/P_f as a function of the electron temperature T_e for $n_B/n_e = 0.01, 0.1, 0.15$.

The dissipated power is lower than that for a steady state with inductive electric fields. For density ratio $n_B/n_e = 0.1$ and electron temperature $T_e = 25\,\text{KeV}$, the dissipated power is a factor of ~ 9 times the fusion power. The power ratio P_D/P_f decreases with increasing electron temperature but increases with decreasing density ratio n_B/n_e.

In the limit $S > \frac{Z}{1+Z(Z-1)n_B/n_e}$ (or $T_e > 42.6/(1 + 20n_B/n_e)^{2/3}\,\text{KeV}$), the dissipated power tends to its minimum value and it is then due essentially to proton-boron collisions. Nevertheless, it is at least ~ 5 times the fusion power (see also reference [2]), corresponding to a required effective efficiency of conversion to electrical power of $\eta = 5/6 = 0.83$.

In Appendix A we establish that the inclusion of an additional inductive electric field always results in an increase in the dissipated power. It is preferable therefore to maintain the ion velocities only by non-inductive "external" forces in order to reduce the dissipated power to the lowest level possible.

III. LIMITATION OF TRANSVERSE VELOCITY SPREAD OF PROTON BEAM BY ELECTRON DRAG

In addition to maintaining the energy of collision of the fuel ion beams close to the resonant energy, it is essential to control the increase in the transverse velocity spread of the ion beams. Otherwise, the annular width of the field-reversed layer will increase until the particle trajectories intersect the nearby walls and/or particles are lost axially through the x-points at the ends of the FRC.

In this section, we investigate the effect of electron drag on the transverse velocity spread of the proton beam induced by (p-B) Coulomb collisions. We first consider time dependent solutions of the Fokker-Planck equation (with and without an inductive electric field) for the distribution function of a proton beam which is injected into a plasma of boron ions and electrons. We establish that the velocity spread of the proton beam remains small if the (p-B) collision frequency is less than the (p-e) collision frequency, $\nu_{\rm pB}/\nu_{pe} < 1$. We then derive a beam envelope equation to describe the time dependence of the mean square width of the proton beam by taking appropriate moments of the Fokker-Planck equation. We confirm that essentially the same condition is required for small transverse velocity spread of the proton beam when a field reversed magnetic field and a transverse electrostatic field are included in the analysis.

We describe the time evolution of the proton beam distribution function F_p in the boron frame of reference where the boron ions are stationary. The boron ions have zero temperature while the electron distribution function is approximated by a drifting Maxwellian with temperature T_e and mean velocity $\boldsymbol{V}^e = V_e\hat{\boldsymbol{z}}$ which is taken to be parallel to the z-axis. The time evolution of $F_p(\boldsymbol{v}, t)$ is determined by the simplified Fokker-Planck equation (see Appendix B):

$$\frac{\partial F_p}{\partial t} + \frac{e}{m_p}\boldsymbol{E}\cdot\frac{\partial F_p}{\partial \boldsymbol{v}} = C_{pe}(F_p, F_e) + C_{\rm pB}(F_p, F_B) + \mathcal{S}(\boldsymbol{v}, t) \tag{12}$$

where the collision operators C_{pe} and $C_{\rm pB}$ are approximated by :

$$C_{pe} = \nu_{pe}\frac{\partial}{\partial v_\alpha}F_p(v_\alpha - V^e_\alpha))$$
$$C_{\rm pB} = \nu_{\rm pB}\, v_0^3\left\{\frac{m_p}{(m_p+m_B)}\frac{\partial}{\partial v_\alpha}F_p\frac{v_\alpha}{v^3} + \frac{m_B}{(m_p+m_B)}\frac{1}{2}\frac{\partial}{\partial v_\alpha}\frac{1}{v}(\delta_{\alpha\beta} - \frac{v_\alpha v_\beta}{v^2})\frac{\partial F_p}{\partial v_\beta}\right\}.$$

We include a source term $\mathcal{S}(\boldsymbol{v}, t)$ to model the injection of the proton beam in the z-direction and an inductive electric field $\boldsymbol{E} = E\hat{\boldsymbol{z}}$ to balance the collisional drag forces on the protons. We neglect proton-proton collisions. The subscripts α, β of v_α, v_β denote the cartesian components x, y, z of the velocity vector $\boldsymbol{v}$. A sum over repeated indices α, β is implied. v_0 is the characteristic speed of the proton beam.

We introduce spherical coordinates v, θ, ϕ where $v_x = v\sin\theta\cos\phi, v_y = v\sin\theta\sin\phi, v_z = v\cos\theta$, and we assume cylindrical symmetry about the z-axis. The

Fokker-Planck equation for F_p can then be expressed in the form :

$$\frac{\partial F_p}{\partial \tau} = \frac{1}{u^2}\frac{\partial}{\partial u}u^3 F_p - \left(\frac{eE}{m_p \nu_{pe} v_0} + \frac{V_e}{v_0}\right)\left\{\frac{1}{u^2}\frac{\partial}{\partial u}u^2\cos\theta F_p - \frac{1}{u\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta F_p\right\} \tag{13}$$

$$+\frac{\nu_{\mathrm{pB}}}{\nu pe}\frac{m_B}{(m_p+m_B)}\left\{\frac{m_p}{m_B}\frac{1}{u^2}\frac{\partial F_p}{\partial u} + \frac{1}{2u^3\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial F_p}{\partial\theta}\right\} + S(u,\theta,\tau) \tag{14}$$

where $\tau = \nu_{pe}t$, $u = v/v_0$.

We consider the injected proton beam to be monoenergetic and sharply peaked in the direction of propagation, and we model the source term S by :

$$S = S_0\Theta(\tau)\delta(u-u_i)2a_0\exp\left(-\frac{a_0\sin^2\theta}{2}\right)$$

where the injection speed is given by u_i, the parameter $a_0 \gtrsim 1$ determines the initial angular spread, and S is finite only for values of $\theta \lesssim 1$. $\Theta(\tau)$ is a step function of the time variable τ.

We are interested in solutions for which the proton distribution function remains sharply peaked in the direction of propagation, and we simplify the Fokker-Planck equation further by making the substitution $\sin\theta \to \theta$, $\cos\theta \to 1-\theta^2/2$.

A. Zero Inductive Electric Field $E = 0$ and $\boldsymbol{V}^e = 0$

In the case where there is no inductive electric field ($E = 0$) and the electron mean velocity $\boldsymbol{V}^e = 0$, we have:

$$\frac{\partial F_p}{\partial \tau} = \frac{1}{u^2}\frac{\partial}{\partial u}q(u)F_p + \frac{A}{2u^3\theta}\frac{\partial}{\partial\theta}\theta\frac{\partial F_p}{\partial\theta} + S_0\Theta(\tau)\delta(u-u_i)2a_0\exp(-a_0\theta^2/2) \tag{15}$$

$$q(u) = u^3 + \epsilon_m A$$
$$\epsilon_m = \frac{m_p}{m_B}$$
$$A = \frac{\nu_{\mathrm{pB}}}{\nu_{pe}}\frac{1}{(1+\epsilon_m)}.$$

The solution of this equation is [4] :

$$F_p(u,\tau) = \overline{F}_p(u,\tau)2a(u)\exp\{-a(u)\theta^2/2\} \tag{16}$$

where

$$\overline{F}_p(u,\tau) = \frac{S_0 u_i^2}{u^3+\epsilon_m A}\left\{\Theta(u-u(u_i,\tau)) - \Theta(u-u_i)\right\}$$
$$a(u) = \frac{a_0}{\left[1+\frac{a_0}{3\epsilon_m}\log\left\{\left(1+\epsilon_m A/u^3\right)/\left(1+\epsilon_m A/u_i^3\right)\right\}\right]}$$

and $u(u_i,\tau)$ is the solution of the characteristic equation $\frac{du}{d\tau} = -\frac{q(u)}{u^2}$ with initial conditions $u = u_i$ at $\tau = 0$:

$$u(u_i,\tau) = \left\{\left(u_i^3+\epsilon_m A\right)\exp(-3\tau) - \epsilon_m A\right\}^{1/3} > 0.$$

Note that the proton speed is reduced significantly below its initial value $u < u_i$ on a time scale of the order of the (p-e) collision time $\sim 1/\nu_{pe}$. The angular spread is determined by the magnitude of $a(u)$. In the limit $a_0 \to \infty$ and $u_i^3 > u^3 > \epsilon_m A$,

$$a(u) = \frac{3u^3}{A}\frac{1}{(1-u^3/u_i^3)}.$$

Hence, for $A < 1$ and $u \sim 1$, $a(u) > 1$ and the angular spread of the proton beam remains small.

On the other hand, for $A > 1$ and $u \sim 1$, $a(u) < 1$ and the above solution is no longer valid. L and M investigated this limit and observed growth in the transverse velocity spread of the proton beam [1].

B. Electron Drag Balanced by Inductive Electric Field $E \neq 0$ and $\boldsymbol{V}^e \neq 0$

In the case where an inductive field $E \neq 0$ (or an equivalent "external" force) is present to balance the drag forces on the protons, and $\boldsymbol{V}^e \neq 0$, we have

$$\frac{\partial F_p}{\partial \tau} = \frac{1}{u^2}\frac{\partial}{\partial u}\left(q(u)+\frac{u^2\theta^2}{2}\right)F_p + \frac{1}{u\theta}\frac{\partial}{\partial\theta}\theta^2 F_p + \frac{A}{2u^3\theta}\frac{\partial}{\partial\theta}\theta\frac{\partial F_p}{\partial\theta} \tag{17}$$
$$+S_0\Theta(\tau)\delta(u-u_i)2a_0\exp(-a_0\theta^2/2) \tag{18}$$

$$q(u) = u^3 - u^2$$

where $\tau = \nu_{pe}t$, $u = v/v_0$, and v_0 is now defined to be the proton beam speed at which the electron drag is balanced by the inductive electric field $eE = m_p\nu_{pe}(v_0 - V^e)$. We assume $A < 1$ and we ignore the boron drag $\epsilon_m A \lesssim 1$.

For values of u such that $q(u) > (u\theta)^2/2$, the solution for $F_p(u,\tau)$ is:

$$F_p(u,\tau) = \overline{F}_p(u,\tau) 2a(u) \exp\{-a(u)\theta^2/2\} \tag{19}$$

where

$$\begin{aligned}
\overline{F}_p(u,\tau) &= \frac{S_0 u_i^2}{q(u)} \{\Theta(u - u(u_i,\tau)) - \Theta(u - u_i)\} \\
a(u) &= \frac{a_0 h_2(u)}{(1 + a_0 A h_1(u))} \\
h_1(u) &= \int_u^{u_i} du \frac{h_2(u)}{u q(u)} \\
h_2(u) &= \exp\left(2 \int_u^{u_i} du \frac{u}{q(u)}\right)
\end{aligned}$$

and $u(u_i,\tau)$ is the solution of the characteristic equation $\frac{du}{d\tau} = -\frac{q(u)}{u^2}$ with initial conditions $u = u_i$ at $\tau = 0$:

$$u(u_i,\tau) = 1 + (u_i - 1)\exp(-\tau).$$

The proton velocity u decreases from u_i and approaches $u \to 1$.

In the limit $a_0 \to \infty$ and $1 > (u-1) > \theta^2/2$, we have $a(u) \sim 2/A > 1$, and the angular spread remains small.

For $(u-1) \sim \theta^2/2$, the above solution is no longer valid. The Fokker-Planck equation for a proton distribution sharply peaked about $u = 1$ can then be approximated by

$$\frac{\partial F_p}{\partial \tau} = \frac{\partial}{\partial \xi}\left(\xi + \frac{\overline{\theta}^2}{2}\right) F_p + \frac{1}{\overline{\theta}} \frac{\partial}{\partial \overline{\theta}} \overline{\theta}^2 F_p + \frac{1}{\overline{\theta}} \frac{\partial}{\partial \overline{\theta}} \overline{\theta} \frac{\partial F_p}{\partial \overline{\theta}} \} \tag{20}$$

where we have introduced new dimensionless variables, $\overline{\theta} = (2/A)^{1/2}\theta$ and ξ defined by $u - 1 = (A/2)\xi$.

We do not have an analytic solution of this equation. However, the averaged distribution function $\widehat{F}_p = \int d\xi F_p$, is determined by:

$$\frac{\partial \widehat{F}_p}{\partial \tau} = \frac{1}{\overline{\theta}} \frac{\partial}{\partial \overline{\theta}} \overline{\theta}^2 \widehat{F}_p + \frac{1}{\overline{\theta}} \frac{\partial}{\partial \overline{\theta}} \overline{\theta} \frac{\partial \widehat{F}_p}{\partial \overline{\theta}} \Big\}$$

and the time independent solution of $\widehat{F}_p$ is:

$$\widehat{F}_p \to \exp(-\overline{\theta}^2/2) = \exp(-\theta^2/A).$$

The transverse velocity spread $\overline{v}_{\perp}^2$ of the proton beam is therefore:

$$\overline{v}_{\perp}^2/v_0^2 \sim \frac{1}{A} = \frac{\nu_{\rm pB}}{\nu_{pe}}.$$

A similar result is also obtained from a consideration of the beam-envelope equation derived in the next section with the inclusion of a field-reversed magnetic field and a transverse electrostatic field.

C. Beam-envelope Equation

By taking appropriate moments of the Fokker-Planck equation, we derive a set of coupled equations to describe the mean square width of the proton beam [5].

Let the mean square width of the proton beam X^2 be

$$X^2 = \left\langle x^2 \right\rangle = \frac{\int d^3r \int d^3v F x^2}{\int d^3r \int d^3v F}. \tag{21}$$

Then, from the Fokker-Planck equation (see Appendix B), we have:

$$\frac{d}{dt}\left\langle \frac{x^2}{2} \right\rangle - \langle x v_x \rangle = 0 \tag{22}$$

$$\frac{d}{dt}\langle x v_x \rangle - \left\langle v_x^2 \right\rangle - \left\langle \frac{e}{m}\left(x E_x - \frac{x v_z B}{c} \right) \right\rangle = \langle x v_x C_{pe} \rangle + \langle x v_x C_{\rm pB} \rangle \tag{23}$$

$$\frac{d}{dt}\left\langle v_x^2 \right\rangle - \left\langle \frac{2e}{m}\left(v_x E_x - \frac{v_x v_z B}{c} \right) \right\rangle = \left\langle v_x^2 C_{pe} \right\rangle + \left\langle v_x^2 C_{\rm pB} \right\rangle \tag{24}$$

$$\frac{d}{dt}\left\langle v_y^2 \right\rangle = \left\langle v_y^2 C_{pe} \right\rangle + \left\langle v_y^2 C_{\rm pB}.+ \right\rangle \tag{25}$$

We now make the ansatz that the x-component of the particle velocity v_x be expressed in the form

$$v_x = \frac{x}{X}\frac{dX}{dt} + \delta v_x \tag{26}$$

and we assume that E_x and B_y vary linearly with x:

$$E_x = E_x' x \qquad\qquad B_y = B' x$$

Then from Eq (22),

$$\langle x \delta v_x \rangle = 0$$

and we have

$$\langle x v_x \rangle = X \frac{dX}{dt}$$

$$\left\langle v_x^2 \right\rangle = \left(\frac{dX}{dt}\right)^2 + \left\langle (\delta v_x)^2 \right\rangle$$

$$\left\langle \frac{e}{m}\left(x E_x - \frac{x v_z B_y}{c}\right)\right\rangle = -\Omega_\beta^2 X^2$$

$$\left\langle \frac{e}{m}\left(v_x E_x - \frac{v_x v_z B_y}{c}\right)\right\rangle = -\Omega_\beta^2 X \frac{dX}{dt}$$

where the betatron frequency Ω_β is

$$\Omega_\beta^2 = \frac{eB'}{m_p c} V_p - \frac{e}{m_p} E_x' \tag{27}$$

and the mean velocity of the proton beam is $\boldsymbol{V}^p = V_p \hat{\boldsymbol{z}}$.

From Eq (23) and Eq (24), we have

$$\begin{aligned} X\frac{d^2X}{dt^2} + \Omega_\beta^2 X^2 &= \left\langle (\delta v_x)^2 \right\rangle + \langle x v_x C_{pe} \rangle + \langle x v_x C_{\mathrm{pB}} \rangle \\ &= \left\langle (\delta v_x)^2 \right\rangle - (\nu_{pe} + \nu_{\mathrm{pB}}) X \frac{dX}{dt} + \cdots \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{d}{dt} X^2 \left\langle (\delta v_x)^2 \right\rangle &= X^2 \left\langle v_x^2 C_{pe} \right\rangle + X^2 \left\langle v_x^2 C_{\mathrm{pB}} \right\rangle - 2\langle x v_x C_{pe} \rangle X \frac{dX}{dt} - 2 \langle x v_x C_{\mathrm{pB}} \rangle X \frac{dX}{dt} \\ &= -2\nu_{pe} X^2 \left\langle (\delta v_x)^2 \right\rangle \quad + \nu_{\mathrm{pB}} X^2 \frac{m_B}{m_p + m_B} V_p^2 + \cdots \end{aligned} \tag{29}$$

$$\frac{d}{dt}\left\langle (v_y)^2 \right\rangle = -2\nu_{pe} \left\langle (v_y)^2 \right\rangle \quad + \nu_{\mathrm{pB}} X^2 \frac{m_B}{m_p + m_B} V_p^2 + \cdots \tag{30}$$

where we have assumed that $V_p^2 > \{\langle (\delta v_x)^2 \rangle, \langle (v_y)^2 \rangle\} > T_e/m_p$.

The square root of the mean square equilibrium width X, neglecting collisions, is:

$$X = \frac{\langle (\delta v_x)^2 \rangle^{1/2}}{\Omega_\beta}.$$

In the presence of collisions, the proton beam expands due to p-B collisions. However, a stationary state can be achieved if the heating of the proton beam due to p-B collisions is balanced by cooling due to electron drag. At steady state, we have:

$$\frac{\langle (\delta v_x)^2 \rangle + \langle (v_y)^2 \rangle}{V_p^2} = \frac{\overline{v}_\perp^2}{V_p^2} \sim \frac{\nu_{\mathrm{pB}}}{\nu_{pe}} = 1.3(10)^3 \left(\frac{T_e}{\mathcal{E}_{\mathrm{pB}}}\right)^{3/2} \frac{n_B}{n_e}.$$

In summary, the transverse velocity spread $\frac{\overline{v}_\perp^2}{V_p^2}$ of the proton beam remains small if $\nu_{\mathrm{pB}}/\nu_{pe} < 1$. For $\frac{\overline{v}_\perp^2}{V_p^2} \sim \nu_{\mathrm{pB}}/\nu_{pe} \leq 0.25$, the electron temperature and the density ratio of borons to electrons are bounded by the inequality

$$\left(\frac{T_e}{\mathcal{E}_{\mathrm{pB}}}\right)^{3/2} \frac{n_B}{n_e} \leq 1.9(10)^{-4}.$$

IV. SUMMARY AND CONCLUSIONS

In the proton-boron colliding beam fusion reactor design concept of Rostoker, Binderbauer, and Monkhorst, proton and boron beams collide with a relative resonant energy $\mathcal{E}_{\mathrm{pB}}$ of collision equal to $\mathcal{E}_{\mathrm{pB}} \sim 600\,\mathrm{KeV}$. The fusion reactivity decreases if the temperatures of the fuel ion beams acquire a thermal spread or if the interaction energy deviates from the resonant energy. Efficient operation therefore requires that optimal conditions be maintained for times long enough for a significant number of fusion reactions to occur.

Quasi-stationary colliding beam configurations can in principle be sustained for times longer than Coulomb collisional times. Collisional drag forces reduce the relative velocity between particle species. However, if the drag forces are balanced with inductive electric fields and/or "external" forces acting on the proton and boron beams, the energy of collision of the fuel ion beams can be maintained close to the resonant energy. It is also essential to control the transverse velocity spread of the proton beam induced by proton-boron collisions. This can be achieved by means of electron drag if the boron density is sufficiently low.

However, in order to sustain steady state, energy must be supplied at a rate sufficient to replace the streaming energy transformed into thermal energy by particle collisions. This energy supply constitutes circulating power in a reactor.

From the momentum balance equations, the magnitude of the inductive electric fields or the "external" forces necessary for steady state can be calculated. The power which must be supplied can then be readily estimated.

We find that in the case of inductive electric fields, the ratio of the circulating power

$P_c = P_D$ (required to maintain the quasi-stationary state) to the fusion power P_f is

$$\frac{P_c}{P_f} = 5.1\left\{\frac{n_e}{n_B Z S}\frac{(Z+S)^2}{(Z-1)^2} + \frac{n_e Z}{n_p S}\frac{(1+S)^2}{(Z-1)^2} + 1\right\}$$

where the parameter S is defined to be $S \equiv \frac{n_e}{Zn_B}\frac{\nu_{\rm pB}}{\nu_{pe}} \sim 264\left(\frac{T_e}{\mathcal{E}_{\rm pB}}\right)^{3/2}$, T_e is the electron temperature, Z the boron atomic number, $n_i\,(i = e, p, B)$ the species density, $m_i\,(i = p, B)$ the species mass, $\nu_{\rm pB}$ the proton-boron collision frequency, and ν_{pe} the proton-electron collision frequency.

The absolute minimum value of P_c/P_f is 35, which corresponds to a density ratio of $n_B/n_e = 0.1$ and $S = Z^{1/2}$ (or equivalently $T_e = 25\,\text{KeV}$). The large magnitude of P_c/P_f is undesirable since it implies the need for very efficient means of converting the available power into electrical power. The power available for conversion into electrical power is $P_f + P_c$. Then, introducing an effective conversion efficiency parameter denoted by η, we find that net electrical power is available only if $\eta(P_f + P_c)$ is greater than the power P_c required to maintain steady state. Thus the effective conversion efficiency must be greater than 97%, that is, $\eta > 35/36 \sim 0.97$.

In the case of "external" forces, we find that the power required to maintain the quasi-stationary state is

$$\frac{P_c}{P_f} = 5.1\left[1 + \frac{Zn_e}{S(n_e + Z(Z-1)n_B)}\right].$$

This power is lower than that required for a quasi-stationary state with inductive electric fields. At an electron temperature of $T_e = 25\,\text{KeV}$ and for the optimal density ratio of $n_B/n_e = 0.1$ (at which value the fusion power is maximized), the circulating power is a factor of ~ 9 times the fusion power ($P_c/P_f \sim 9$). At higher electron temperatures, the magnitude of P_c/P_f is reduced and tends to a minimum of $P_c/P_f \rightarrow 5.1$ in the limit $T_e > 42.6/(1 + 20n_B/n_e)^{2/3}\,\text{KeV}$ (or $> \frac{Z}{1+Z(Z-1)n_B/n_e}$). The required effective efficiency of conversion to electrical power must therefore be greater than 84%, that is, $\eta > 5.1/6.1 \sim 0.84$.

Unfortunately, at the high electron temperatures necessary to keep the power ratio P_c/P_f close to its minimum value, electron drag will not be strong enough to cool the proton beam (i.e., suppress the transverse velocity spread) unless the boron density is considerably below its optimal value of $n_B/n_e = 0.1$. From steady-state solutions of the Fokker-Planck equation for the proton distribution function, we estimate that the transverse proton beam temperature $T_\perp$ is given by

$$\frac{T_\perp}{\mathcal{E}_{\mathrm{pB}}} \sim \frac{\nu_{\mathrm{pB}}}{\nu_{pe}} = \frac{Z n_B}{n_e} S = 1.32(10)^3 \frac{n_B}{n_e} \left(\frac{T_e}{\mathcal{E}_{\mathrm{pB}}} \right)^{3/2} .$$

At $T_e \sim 68\,\mathrm{KeV}$, electron drag will be effective in limiting the transverse proton beam temperature ($T_\perp/\mathcal{E}_{\mathrm{pB}} < 0.25$) only if the density ratio is $n_B/n_e < 0.005$. At this low boron density of $n_B/n_e = 0.005$, the fusion power is reduced to 10% of the maximum possible. The power ratio is $P_c/P_f \sim 7.4$ at $T_e = 68\,\mathrm{KeV}$ and $n_B/n_e = 0.005$, and the required effective conversion efficiency would then have to be greater than 88%, that is, $\eta > 7.4/8.4 = 0.88$.

The preferred scenario is the one involving "external" forces. It will be essential to have a highly efficient means of generating the "external" forces necessary to transfer momentum to the ion beams. It is as yet unclear how this can possibly be accomplished. But even if the means for efficiently generating the "external" forces were available, the ratio of the circulating power to the fusion power is still very high, $P_c/P_f > 5.1$, which implies that effective conversion efficiencies greater than 84% are required.

It should be noted that even this high value of $\eta \sim 0.84$ still underestimates the required effective conversion efficiency for the following reasons:

(1) In our simplified calculation of η, we assumed that the mechanism for transferring momentum to the proton and boron beams to make up the losses due to collisional drag is 100% efficient. It is unlikely that this level of efficiency can ever be achieved.

(2) We also assumed that the conversion efficiency is the same uniformly high value for all the various forms of the energy available for conversion into electricity (e.g. thermal energy, directed energy, bremsstrahlung). If the transverse spread of the proton beam is limited by electron cooling, the dissipated energy is eventually transferred to electron energy and must be removed through the electron loss channels. At the above nominal electron temperatures and boron densities, the bremsstrahlung loss rate is much smaller than the

dissipated power, and a large fraction of the electron energy will have to be removed as electron thermal energy. The conversion efficiency for thermal energy is ~ 0.4, well below the required effective conversion efficiency of $\eta > 0.84$. At the present time, direct energy converters (which convert energetic charged particle into electricity) have been designed with conversion efficiencies no higher than $\sim 75\%$ [6].

In addition, $P_c/P_f \sim 5.1$ is possible only at high electron temperatures. However, at such temperatures, the boron density will have to be well below its optimal value ($n_B/n_e < 0.1$) if electron drag is to be effective in limiting the transverse proton beam temperature. The fusion power P_f is then only a fraction of the maximum possible.

In summary, we conclude that the proton-boron colliding beam fusion reactor is not a viable concept unless technology capable of very high energy conversion efficiencies (no less than 84%) can be developed.

Appendix A: Mixture of Inductive Electric Field and Non-inductive "External" Forces

We now consider a "steady state" maintained by the presence of an inductive electric field E of arbitrary magnitude and by additional "external" forces forces $\mathcal{F}_p$ and $\mathcal{F}_B$ which act to maintain the streaming velocities of the ions at fixed prescribed values. We take E to be a parameter and we establish that the dissipated power is minimized when $E = 0$.

The modified momentum equations are:

$$\mathcal{F}_p + eE = m_p\nu_{pe}(V_p - V_e) + m_B\nu_{Bp}(V_p - V_B)\frac{n_B}{n_p} \tag{A1}$$

$$\mathcal{F}_B + ZeE = m_p Z^2\nu_{pe}(V_B - V_e) + m_B\nu_{Bp}(V_B - V_p) \tag{A2}$$

$$-eE = m_p\nu_{pe}(V_e - V_p)\frac{n_p}{n_e} + m_p Z^2\nu_{pe}(V_e - V_B)\frac{n_B}{n_e} \tag{A3}$$

Solving for $(V_p - V_e), (V_B - V_e), \mathcal{F}_p, \mathcal{F}_B$ in terms of $V_p - V_B$ and E, we obtain

$$V_p - V_e = \frac{1}{(n_p + Z^2 n_B)}\left(Z^2 n_B(V_p - V_B) + \frac{eEn_e}{m_p\nu_{pe}}\right) \tag{A4}$$

$$V_B - V_e = -\frac{1}{(n_p + Z^2 n_B)}\left(n_p(V_p - V_B) - \frac{eEn_e}{m_p\nu_{pe}}\right) \tag{A5}$$

$$\begin{aligned}\mathcal{F}_p &= m_p\nu_{pe}\left\{\frac{Z^2 n_B}{n_p + Z^2 n_B} + \frac{SZn_B}{n_e}\right\}(V_p - V_B) - \frac{eEn_B Z(Z-1)}{n_p + Z^2 n_B} \\ &= -\frac{n_B}{n_p}\mathcal{F}_B.\end{aligned} \tag{A6}$$

The dissipated power is

$$\begin{aligned}P_D &= n_p F_p V_p + n_B F_B V_B + eE(n_p V_p + Zn_B V_B - n_e V_e) \\ &= m_B n_B \nu_{Bp}(V_p - V_B)^2\left[1 + \frac{Zn_e}{S(n_p + Z^2 n_B)}\right] + \frac{(eEn_e)^2}{m_p\nu_{pe}(n_p + Z^2 n_B)}.\end{aligned} \tag{A7}$$

Note that finite values of E increases the magnitude of P_D.

Appendix B: Fokker-Planck Equation

The Fokker-Planck which determines the time evolution of the distribution function $F_a(\boldsymbol{r}, \boldsymbol{v}, t)$ of the particle species in a collisional plasma is [3]:

$$\frac{\partial F_a}{\partial t} + \boldsymbol{v} \cdot \frac{\partial F_a}{\partial \boldsymbol{r}} + \frac{e_a}{m_a}\left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right) \cdot \frac{\partial F_a}{\partial \boldsymbol{v}} = \sum_b C_{ab}(F_a, F_b) \tag{B1}$$

where the collision operator C_{ab} for multiple small angle Coulomb scattering between species "a" and species "b" is given by:

$$C_{ab} = \Gamma_{ab}\left[-\left(1 + \frac{m_a}{m_b}\right)\frac{\partial}{\partial v_\alpha}\left(F_a \frac{\partial h_b}{\partial v_\alpha}\right) + \frac{1}{2}\frac{\partial^2}{\partial v_\alpha \partial v_\beta}\left(F_a \frac{\partial^2 g_b}{\partial v_\alpha \partial v_\beta}\right)\right]$$

and

$$\Gamma_{ab} = \frac{4\pi e_a^2 e_b^2 \lambda_{ab}}{m_a^2}$$
$$h_b = \int d^3v' \, \frac{F_b(\boldsymbol{v}')}{|\boldsymbol{v} - \boldsymbol{v}'|}$$
$$g_b = \int d^3v' \, F_b(\boldsymbol{v}')|\boldsymbol{v} - \boldsymbol{v}'|.$$

The subscripts α, β of v_α, v_β denote the cartesian components x, y, z of the velocity vector $\boldsymbol{v}$. A sum over repeated indices α, β is implied. e_a and m_a are the charge and mass of species "a" and λ_{ab} the Coulomb logarithm.

Particle number, total momenta, and total energy are conserved during collisions, $\int d^3v C_{ab} = 0$, $\int d^3v \, m_a \boldsymbol{v} C_{ab} + \int d^3v \, m_b \boldsymbol{v} C_{ba} = 0$, $\frac{1}{2}\int d^3v \, m_a v^2 C_{ab} + \frac{1}{2}\int d^3v \, m_b v^2 C_{ba} = 0$.

For proton-electron collisions, where F_e is taken to be a drifting Maxwellian $F_e(\boldsymbol{v}) = n_e\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\exp(-m_e(\boldsymbol{v} - \boldsymbol{V}^e)^2/2T_e)$ and the electron thermal velocity is much larger than typical proton velocities $v(m_e/2T_e)^{1/2} \lesssim 1$, we can approximate the collision operator C_{pe} by:

$$\begin{aligned} C_{pe} &= \Gamma_{pe}\frac{N_e}{3}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m_e}{T_e}\right)^{3/2}\left\{(1 + \frac{m_p}{m_e})\frac{\partial}{\partial v_\alpha}F_p u_\alpha + \frac{T_e}{m_e}\frac{\partial^2}{\partial v_\alpha \partial v_\alpha}F_p\right\} + \cdots \\ &= \nu_{pe}\left\{\frac{\partial}{\partial v_\alpha}F_p u_\alpha + \frac{T_e}{m_p}\frac{\partial^2}{\partial v_\alpha \partial v_\alpha}F_p\right\} \end{aligned}$$

where $u_\alpha = v_\alpha - V^e_\alpha$ and $\nu_{pe} = \Gamma_{pe}\frac{N_e}{3}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m_e}{T_e}\right)^{3/2}\frac{m_p}{m_e}$ is the proton-electron frequency.

For proton-boron collisions, where F_b is a drifting Maxwellian and the boron thermal velocity is much less than typical proton velocities $v(m_B/2T_B)^{1/2} \gtrsim 1$, we have:

$$C_{\rm pB} = \Gamma_{\rm pB} N_B \left\{ (1 + \frac{m_p}{m_B}) \frac{\partial}{\partial v_\alpha} F_p \frac{u_\alpha}{u^3} + \frac{1}{2} \frac{\partial^2}{\partial v_\alpha \partial v_\beta} \frac{F_p}{u} \left(\delta_{\alpha\beta} - \frac{u_\alpha u_\beta}{u^2} \right) \right\} + \cdots$$
$$= \nu_{\rm pB}\, v_0^3 \frac{m_B}{m_B + m_p} \left\{ \frac{m_p}{m_B} \frac{\partial}{\partial v_\alpha} F_p \frac{u_\alpha}{u^3} + \frac{1}{2} \frac{\partial}{\partial v_\alpha} \frac{1}{u} \left(\delta_{\alpha\beta} - \frac{u_\alpha u_\beta}{u^2} \right) \frac{\partial F_p}{\partial v_\beta} \right\} + \cdots$$

where $u_\alpha = v_\alpha - V^B_\alpha$, $\nu_{\rm pB} = \Gamma_{\rm pB} N_B (1 + m_p/m_B)/v_0^3$ is the proton-boron collision frequency, and v_0 is the characteristic speed $|\boldsymbol{V}^p - \boldsymbol{V}^B|$ of the protons relative to the borons.

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FIGURE CAPTIONS

FIG. 1. Quasi-stationary state maintained by inductive electric field; P_D/P_f vs T_e(KeV) for $n_b/n_e = 0.05, 0.1, 0.15$.

FIG. 2. Quasi-stationary state maintained by non-inductive "external" forces; P_D/P_f vs. T_e(KeV) for $n_b/n_e = 0.01, 0.1, 0.15$.