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PARTICLE AND ENERGY TRANSPORT IN A FIELD REVERSED CONFIGURATION

By

ARTAN QERUSHI

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2000

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To

my family

and

my close friends

who always believed in me,

and supported me morally

all along the way.

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ACKNOWLEDGMENTS

First of all, I would like to express my gratitude to Norman Rostoker, who led me through all the work toward my degree. I have appreciated every second of his role as a scientific advisor and as a friend. His scientific genius and his wisdom have been a continuous guidance and inspiration for me since I joined the CBFR project; his wit and sense of humor have given me many hours of intellectual joy and heartfelt laughter. Many thanks also to Hendrik J. Monkhorst, who introduced me to the CBFR project and offered me the great opportunity to spend most of my time learning about plasma physics at UC Irvine. I have appreciated his scientific advice all these years; his continuous friendship helped me to get through many a difficult moment.

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Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

PARTICLE AND ENERGY TRANSPORT IN A FIELD REVERSED CONFIGURATION

By

Artan Qerushi

May 2000

Chairman: H. J. Monkhorst
Major Department: Physics

This work has been part of a collaboration between the University of Florida and the University of California, Irvine, aimed at building a fusion reactor which is compact, environmentally friendly, and easy to maintain. The specific work of this dissertation concerns theoretical issues about equilibrium and particle transport in such a reactor, which is based on magnetic configurations known as Field Reversed Configurations (FRC).

The equilibrium study shows how to obtain solutions for various physical quantities of interest in the case of fusion reactions with one or many types of ions. The main attribute that comes out of this study is the existence of mixed confinement states in a Colliding Beam Fusion Reactor (CBFR). In these mixed confinement states ions are confined magnetically while the electrons are confined electrostatically. A whole range of electric fields of different strengths can be

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accessed by tuning the externally applied magnetic field. A strong electric field that is confining for electrons can avoid their anomalous transport.

The next part of this work concerns the effect of collisions on particle orbits in a CBFR. Simulations of particle orbits show that there are two main types of orbits: (1) betatron orbits, which can be thought of as sine waves propagating along a circle, and (2) drift orbits, which can be thought of as small circles rolling over larger circles. It is shown that large-angle collisions between ions can change a betatron orbit to a drift orbit. The direction of rotation of the drift orbit is in the diamagnetic direction in all cases where the electric field is confining for electrons and the $\vec{E} \times \vec{B}$ drift dominates over the gradient drift $\vec{B} \times \nabla B$. This is an important finding for the ion transport in a CBFR. Simulations show also that small angle collisions between electrons and ions do not change the topology of betatron orbits, but only increase the amplitude of their radial oscillations with time.

The last part of this work is related to the development of a formal diffusion theory that enables the calculation of the diffusion rates of betatron orbits and drift orbits due to small angle collisions. This diffusion theory is based on test particle methods used in kinetic theory of plasmas. It has the merit of being applicable to particle orbits of any size and rapidly varying magnetic fields that pass through zero.

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CHAPTER 1
INTRODUCTION

We start the chapter by describing the main problems of present-day Tokamaks. Then we introduce the concept of a Colliding Beam Reactor (CBR), which in principle solves all these problems. Of crucial importance in the theoretical developments related to the CBR is the establishment of a transport theory that describes how particles transport energy across the magnetic field. The existing transport theories are inadequate for describing particle transport in a CBR. The reason is that they apply to adiabatic plasmas, i.e., to plasmas where particles have small gyroradii compared to typical lengths in the system, which is not the case for a CBR, where the majority of ions have large gyroradii. The chapter ends with a statement of the research problem of this dissertation and a brief outline of the chapters that follow.

1.1 Tokamaks

The present-day fusion effort is concentrated on Tokamaks [1]. Experiments with these devices have been going on for over 30 years in most industrialized countries of the world culminating with the International Thermonuclear Experimental Reactor [2] (ITER). From them has come the main body of plasma physics that we know today. However, current Tokamak research faces several problems [3, 4] related to

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- size,
- radiation damage, and
- maintenance.

The size of a fusion reactor is determined by the requirement that the plasma confinement time be greater than or equal to the fusion burn time. For a Tokamak plasma a rough estimate of the confinement time is [5]

$$\tau = \frac{R^2}{2D}, \tag{1.1}$$

where R is the minor radius of a Tokamak plasma and D is the diffusion coefficient.

For *classical diffusion*

$$D_c = a_i^2 \nu_{ie}, \tag{1.2}$$

where a_i is the ion gyroradius and ν_{ie} is the ion-electron scattering frequency. For *anomalous diffusion* the diffusion coefficient is given by the Bohm formula

$$D_B = \frac{1}{16} a_i^2 \Omega_i = \frac{1}{16} \frac{cT_e}{eB}, \tag{1.3}$$

where $\Omega_i = eB/m_i c$. Roughly $D_B/D_{cl} = \frac{1}{16}\frac{\omega_i}{\nu_{ie}}$. For fusion conditions $D_B/D_{cl} \approx 10^8$.

The diffusion in Tokamaks is anomalous. Experiments with Tokamaks suggest that the confinement time can be as big as $1000R^2/2D_B$; this value is currently assumed in the design of ITER where plasma has a minimum radius of 2.8 m (see Figure 1.1). ITER has an enormous size. The solution to the size problem is

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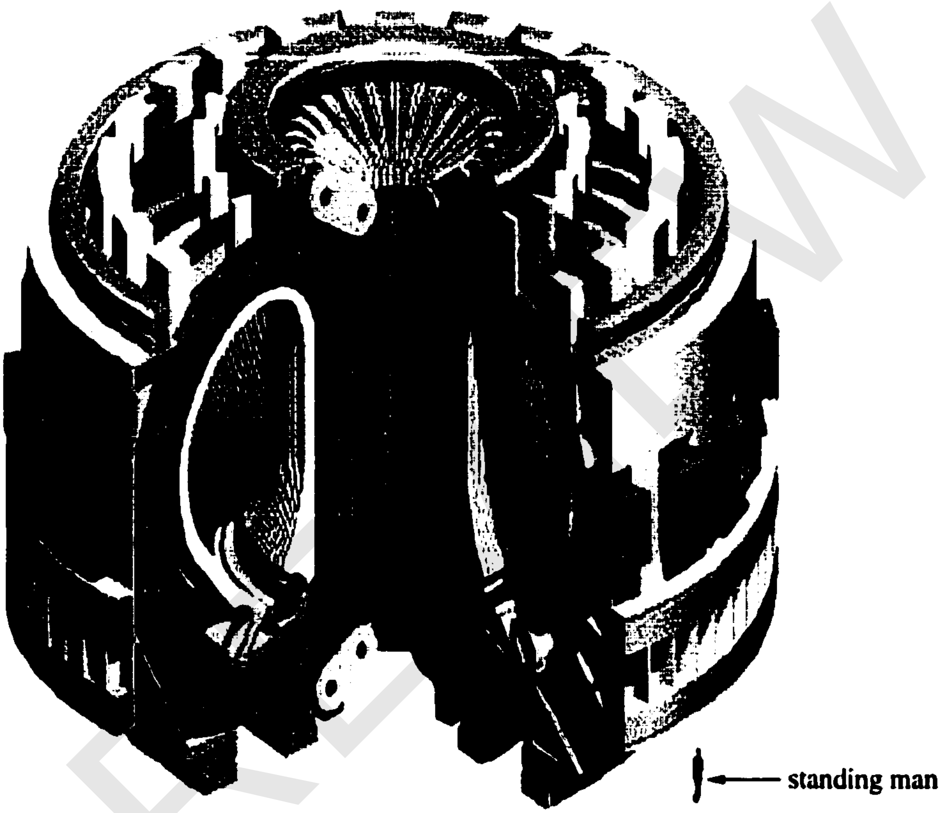
a reactor where the diffusion is classical; in such a reactor the confinement time would be $\tau \approx 10^8 R^2/2D_B$ and the size of the plasma would be reduced by several orders of magnitude.



ITER PARAMETERS

Fusion Power	1.5 GW	Plasma Current	21 MA
Neutron Loading	1 MW/m^2	Safety Factor, q	3
Inductive Burn Time	1000 sec	B_0 (at 8.1 m radius)	5.7 T
Plasma Major Radius*	8.1 m	B_{max} (at TF coil)	12.5 T
Plasma Minor Radius	2.8 m	Heating Power	100 MW

*radius measured from the center of the last closed flux surface

Figure 1.1: The International Thermonuclear Experimental Reactor. Note the enormous size of the reactor and its complicated design.

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The radiation damage problem comes from the fact that the fusion products in the reaction

$$\mathrm{D} + \mathrm{T} \rightarrow \alpha(3.6\ \mathrm{MeV}) + \mathrm{n}(14.1\ \mathrm{MeV}) \tag{1.4}$$

are neutrons which hit the reactor walls and cause radiation damage to their materials. In order to reduce the flux of particles causing radiation damage, the first wall of the plasma has to be placed to a certain distance from the plasma edge and this increases the size of the reactor. Also, the fusion energy in the D-T reaction is released in the form of fast neutrons, which means that thermal processes with low efficiency ($\sim$ 30 %) have to be used to collect this energy.

The D-T reaction is favored in Tokamaks because, of all the known fusion reactions, it has the largest reactivity, at relatively low ion temperature (around 10's of keV), and the smallest atomic number, Z [6]. This means that the amount of heating for a D-T reaction to take place is lower than for the other fusion reactions, and a Tokamak, to achieve a specific burnup fraction, will be much smaller for D-T than for other fuels. Furthermore, low Z means less energy lost to radiation such as synchrotron radiation and Bremsstrahlung. In fact, in the lowest β devices[1], such as most Tokamaks, synchrotron radiation losses are so significant that D-T is the only reaction where ignition[2] can take place [7].

The problems associated with radiation damage and activation of the material walls can be solved by using *advanced fuels* in which fusion products are not neutrons but charged particles. The advanced fuels are

---

[1] β is defined as the ratio of plasma pressure to magnetic field pressure, i.e., $\beta \equiv 8\pi nkT/B^2$.

[2] Ignition means the fusion power exceeds the Bremsstrahlung radiation power. It does not mean that no input power is then required to maintain the fusion burn conditions. In a Tokamak that has reached ignition it is still necessary to maintain the current which means there are I^2R dissipative losses. It can only be an energy amplifier.

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$$\mathrm{D} + \mathrm{He}^3 \rightarrow \alpha(3.7\ \mathrm{MeV}) + \mathrm{p}(14.7\ \mathrm{MeV}) \tag{1.5}$$

or

$$\mathrm{p} + \mathrm{B}^{11} \rightarrow 3\alpha(8.7\ \mathrm{MeV}). \tag{1.6}$$

These aneutronic reactions have higher Z, lower thermal reactivity and require higher temperatures.[3] Therefore, in order to burn advanced fuels a "high β" device and adequate heating is needed. On the other hand, when advanced fuels are used, the fusion energy is released in the form of charged particles whose energy can be collected by direct electromagnetic converters [8] with an efficiency as high as 90 % [9].

Tokamaks present many engineering problems of which the most serious is radiation damage from 14 MeV neutrons. In addition they would be very expensive to maintain because of radioactivity from activation and the toroidal design.

1.2 Non-Adiabatic Plasmas

Magnetic confinement systems in plasma physics mostly involve particles where the orbit radius and orbit period are small compared to the characteristic scales of length and time. Plasmas consisting of such particles are called *adiabatic plasmas.* In such plasmas the individual particles closely follow the magnetic field lines by tightly circling around them. The particle motion in adiabatic plasmas can be described by the drift formalism and gyration centers. A sketch of a typical adiabatic drift motion is shown in Figure 1.2. The magnetic moment, μ, of the orbiting particle is roughly conserved along the magnetic field line, i.e., μ is an

---

[3] For D-He^3, for example, the temperature has to be higher by a factor of about 10 compared to D-T.

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adiabatic invariant of the motion, hence, the name "adiabatic" plasma (usually there are also other invariants). On average such a plasma can readily be described by some kind of fluid theory - in particular magnetohydrodynamics (MHD). This area of plasma physics is well developed, and almost all fusion concepts are based on adiabatic plasmas.

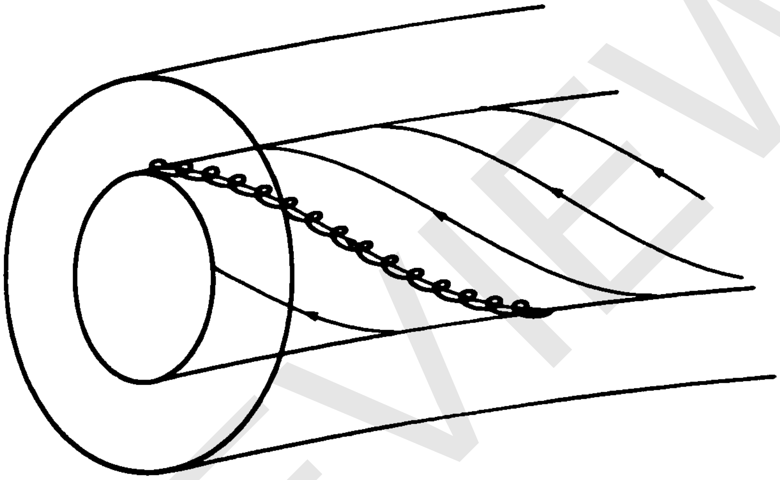

Figure 1.2: Sketch of an adiabatic particle orbit in a Tokamak. The particle tightly orbits the magnetic field lines.

In contrast, non-adiabatic particles have orbits on a scale comparable to the plasma radius and orbit periods on the same scale as the characteristic times involved in the system. Further, they usually have high kinetic energy and do not follow field lines - nor do they in general conserve their orbital magnetic moments. To describe the motion of such a particle one has to solve the particular set of individual particle orbit equations. A typical orbit of a non-adiabatic particle is depicted in Figure 1.3. To describe the macroscopic effects of such a plasma one, generally, has to resort to a treatment based on kinetic theory.

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High energy non-adiabatic particles have only been studied and employed in fusion plasmas as minority particles for heating, and lately because of the necessity to include reaction products (which are of high energy). Fusion plasmas where non-adiabatic ions are the majority particle, however, have previously been studied many years ago in the DCX program at Oak Ridge [10] and MIGMA [11]. The particle densities achieved were too small for fusion power applications by many orders of magnitude.

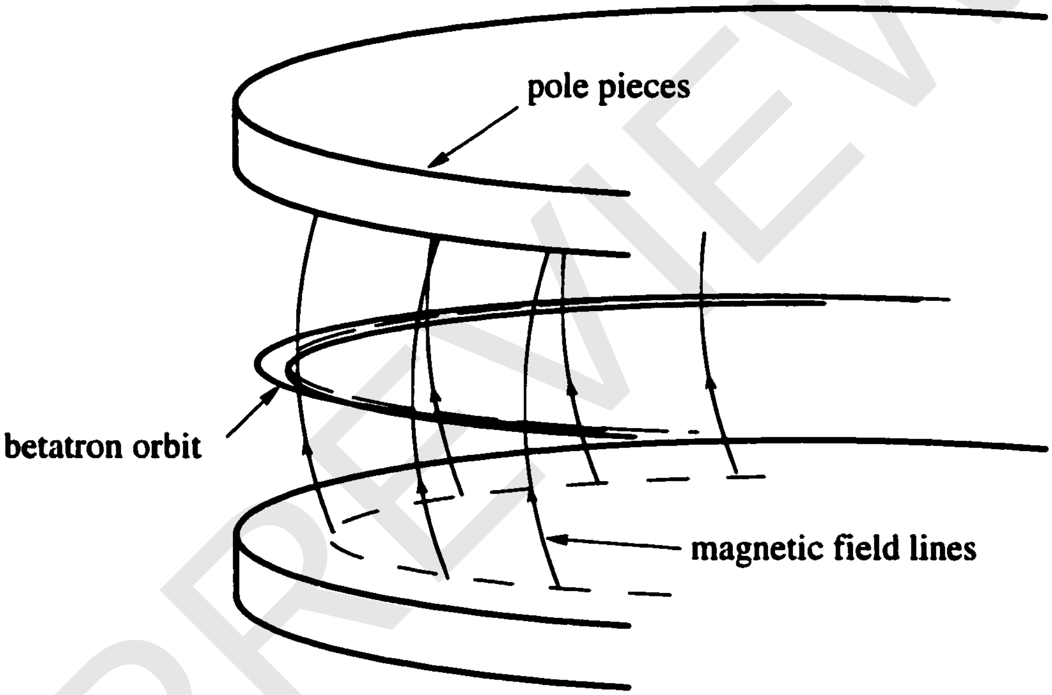

Figure 1.3: Sketch of a non-adiabatic orbit in a betatron. The scale of the particle orbit is comparable to the size of the system and does not follow magnetic flux surfaces.

Beyond the differences discussed with regards to the orbit dynamics, there are also profound differences between adiabatic and non-adiabatic plasmas with respect to stability behavior [12] and transport.

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1.3 Reversed Field Configurations

The magnetic field of a Field Reversed Configuration (FRC) is shown in Figure 1.4. There are two regions of field lines: open and closed. They are separated by a surface called the *separatrix*. In the region of closed field lines there is a cylindrical surface where the magnetic field vanishes; it is called the *null surface* (see Figure 1.5).

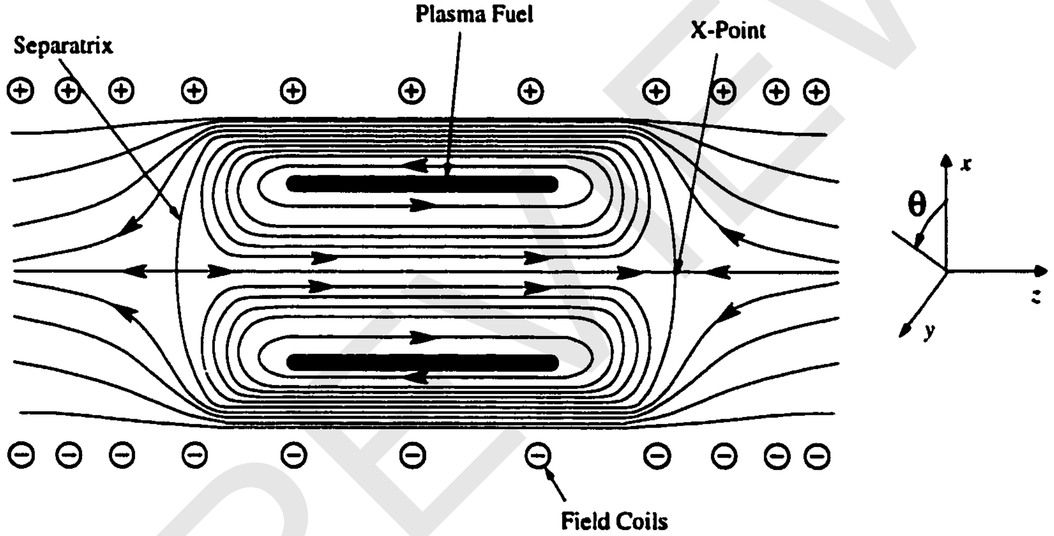

Figure 1.4: Magnetic field of FRC.

If energetic particles are injected at the null surface radius with velocities in the azimuthal direction $\hat{\theta}$, they will go around the axis oscillating radially about the null surface: their orbits, called ***betatron orbits***, are depicted in Figure 1.5. The cylindrical shell shown in Figure 1.5 represents an energetic ion beam that has been injected into an FRC at the radius of the null surface. Another class of possible orbits are the ***drift orbits*** which have a much smaller radius of gyration, but are also non-adiabatic.

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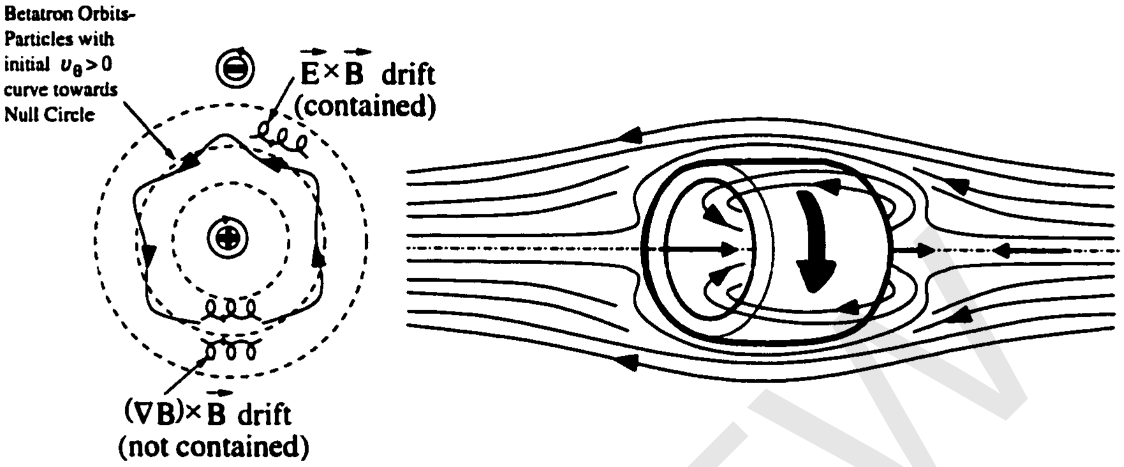

Figure 1.5: FRC with typical particle orbits. The cylindrical shell represents a beam of energetic ions injected at the null surface radius and rotating in its diamagnetic direction (increasing θ).

The structure of the magnetic about the null surface implies that axis-encircling particles with betatron orbits must encircle the axis in one direction only which is called the *diamagnetic* direction. Particles rotating in their diamagnetic direction are confined magnetically, i.e., they are pulled toward the null surface by the Lorentz force $e_j \vec{v} \times \vec{B}$. For ions the diamagnetic direction is that of increasing θ. The direction of rotation that is the opposite of the diamagnetic direction is called *counterdiamagnetic*. Particles rotating in their counterdiamagnetic direction are pushed away from the null surface by the Lorentz force which is in this case deconfining. The diamagnetic direction for electrons will be that of decreasing θ, due to their negative charge; their counterdiamagnetic direction will be that of increasing θ.

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1.4 Concept of Colliding Beam Reactor

The ideal solution to the size problem of Tokamaks would be a reactor in which the transport of particles is classical. Experiments with Tokamaks related to the diffusion of fusion products and plasma heating by means of energetic neutral beams [13] indicate that high energy ions diffuse and slow down classically while the background thermal plasma is subject to anomalous transport. The physical reason of the classical behaviour of energetic ions is that they have large gyroradius a_i and average the short-wavelength fluctuations so that only wavelengths larger than a_i cause anomalous transport. The large orbit ions are insensitive to short-wavelength fluctuations that cause anomalous transport. This interpretation is supported by computer simulations [14, 15]. These results lead to the conjecture that *if most of the ions in a plasma were energetic, classical transport would prevail if long wavelength modes were stable* [16].

In the Tokamak experiments [13] the ion beam density was initially about 1% of the thermal plasma density. When the ion density was increased it was found that there is a threshold beam density above which the beam drives Alfvèn modes [17] in the background plasma which produce anomalous losses of the beam. However, experiments with energetic beams in Field Reversed Configurations were surprisingly stable [18]. In fact, ideal MHD stability theory predicted that the tilt mode would destroy FRCs in a few microseconds but that was not observed [19, 20]. Typical FRC experiments lasted at least a few hundred microseconds, a clear indication that MHD is not applicable for an FRC where the plasma ions are non-adiabatic.

These experiments with Tokamaks and FRCs suggest that *it is possible to build a reactor where the anomalous transport of both ions and electrons is avoided*

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by having large orbit ions and electrostatically confined electrons. This is the concept of a Colliding Beam Reactor [21, 22, 23] (CBR). The plasma in such a reactor consists of energetic ion beams (see Figure 1.5) injected and trapped in an FRC near the null surface. These energetic ions have large (betatron) orbits and are insensitive to the short wavelength fluctuations that cause anomalous transport. Hence, if long wavelengths are stable the ions should have classical transport. As for the electrons, their anomalous transport can be avoided by having a strong electric field[4] confining them. The layout of a CBR is shown in Figure 1.6.

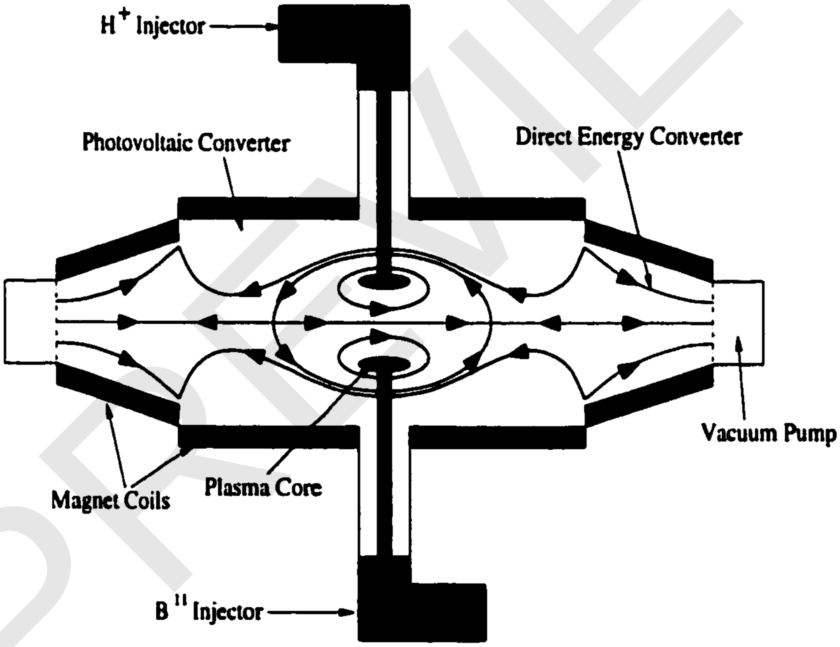

Figure 1.6: Layout of the Colliding Beam Reactor. Beams of energetic protons and B^{11} are injected at the null surface radius creating the plasma core, which has the shape of a cylindrical ring, and reversing the magnetic field created by the external coils. The fusion products are expelled at the ends of the FRC where they are collected by the Direct Energy Converters; here they are slowed down and their kinetic energy is converted into electricity.

---

[4] We will show how to set up such an electric field in the next chapter when we discuss a one-dimensional equilibrium model for the FRC.

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The CBR has the potential to solve all three problems associated with Tokamaks, i.e., size, radiation damage, and maintenance. The reactor is very compact (see Figure 1.7) due to the fact that the transport of particles is classical and hence the plasma minimum size is smaller by several orders of magnitude than a Tokamak. Also FRCs are "high β" devices and advanced fuels, such as $D - He^3$ or $p - B^{11}$, can be used with little or no energy in high energy neutrons. FRCs are also easy to maintain due to their simple linear geometry.

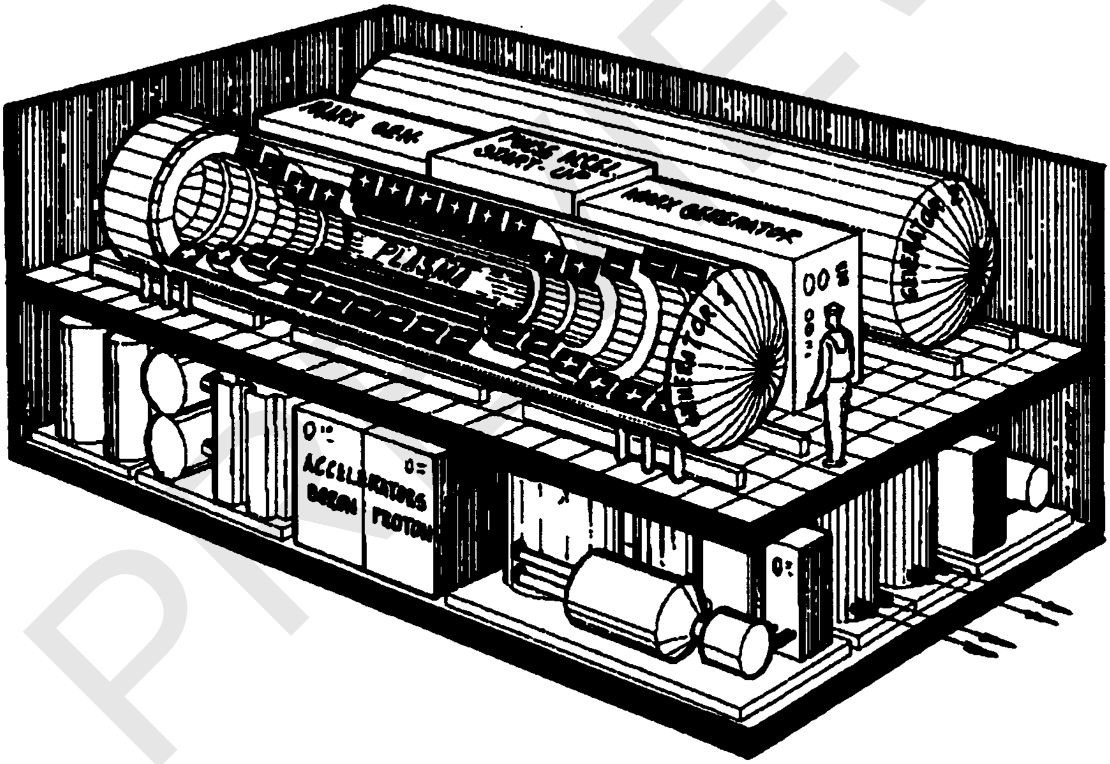

Figure 1.7: Colliding Beam Reactor. Note the compact size of the reactor and its simple linear geometry.

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1.5 Classical Diffusion

Using MHD equations it is possible to show[5] that the particle flow velocity across the magnetic field in a fully ionized plasma is

$$n\vec{V}_\perp = \left(\left(n\vec{E} - \frac{T_i}{e}\nabla n\right) \times \frac{\vec{B}}{B}\right) - (a_e)^2 \nu_{ei} \left(1 + \frac{T_i}{T_e}\right) \nabla n, \tag{1.7}$$

where $a_e = v_{Te}/\Omega_e$ is the electron gyroradius and ν_{ei} is the electron-ion collision frequency. Here $v_{Te} = (T_e/m_e)^{1/2}$ is the thermal velocity of electrons and $\Omega_e = eB/m_e$ is their cyclotron frequency.

If the first term in Eq. (1.7) can be neglected then the diffusion equation is

$$\Gamma = n\vec{V}_\perp = -D\nabla n, \tag{1.8}$$

where

$$D = a_e^2 \nu_{ei} \left(1 + \frac{T_i}{T_e}\right). \tag{1.9}$$

The *classical diffusion coefficient* is defined as

$$D_c \equiv a_e^2 \nu_{ei}. \tag{1.10}$$

The electron-ion collision frequency ν_{ei} is given by

$$\nu_{ei} = 6.3 \times 10^9 Z T_e^{-\frac{3}{2}} \left(\frac{n_e}{10^{20}}\right)^{-1}, \tag{1.11}$$

where T_e is in eV and n_e is in m^{-3}.

---

[5] See section 7.3a of [24].

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