Feasibility of a Colliding Beam Fusion Reactor

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N. Rostoker, M. W. Binderbauer, and H. J. Monkhorst (*1*) revisit several ideas—the use of the reactions of protons with boron-11 (p-^{11}B) to produce fusion energy, (*2*) the use of the field-reversed configuration (FRC) for magnetic confinement of particles and energy (*E*) (*3*), and the use of nonthermal ion distributions to enhance the fusion reactivity (*4*)—in their proposal for a colliding beam fusion reactor (CBFR). While there are unresolved issues with each of these choices, I focus here on nonthermal ion distributions.

The fundamental difficulty with nonthermal ions is apparent when one compares the fusion cross section ($\sigma_{fusion} \approx 1$ barn for p-^{11}B at $E_{cm} = 580$ keV, where E_{cm} is the energy in the center-of-mass frame) to the effective cross section for many small-angle Coulomb scattering events that combine to produce a scattering angle of 90° rms in an incident beam ($\sigma_{eff} \approx 60$ barns for protons scattering on ^{11}B at $E_{cm} = 580$ keV). Highly nonthermal systems, like the colliding beam reactor proposed by Rostoker *et al.* would relax to local thermal equilibrium before a significant amount of fusion power could be produced. Alternatively, the nonthermal ion distribution could be maintained by cycling sufficient power through the system.

With this possibility in mind, it is instructive to examine the operating point presented by Rostoker *et al.* ($n_p = 4 \times 10^{15}$/cm^3, $n_B = 1 \times 10^{13}$/cm^3, $T_p = 25$ keV, and $T_e = 20$ keV), where n_p is the proton density, n_B is the boron density, T_p is the temperature of the protons, and T_e is the temperature of the electrons. We consider the problem in the frame-of-reference of the protons. Maintaining $E_{cm} = 580$ keV would require that the boron beam velocity be $u_B \approx 1.1 \times 10^9$ cm/s. Coulomb collisions with the boron beam would heat the proton distribution at a rate (*5*) of $1/2 m_B u_B^2 n_B \upsilon_\varepsilon^{B/p} \approx 500$ W/cm^3, where $m_B = 11$ AMU is the boron mass and $\upsilon_\varepsilon^{i/j}$ is the energy transfer rate from collisions between particles of species "i" and "j." As Rostoker *et al.* suggest, heat would be removed from the protons by collisions with the (colder) electrons at a rate of $3/2 n_p(T_p - T_e)\upsilon_\varepsilon^{p/e} \approx 350$ W/cm^3. The relatively small net heating of the proton distribution could be eliminated if T_e were reduced to 18.6 keV.

The largest term in the electron power balance is direct heating by the boron beam, which would proceed at the rate of $1/2 m_B u_B^2 n_B \upsilon_\varepsilon^{B/e} \approx 2.0$ kW/cm^3. Maintaining the electrons at $T_e \approx 18.6$ keV would require that $P_{electron} \approx 2.5$ kW/cm^3 be extracted from the electron distribution. While bremsstrahlung could provide only 44 W/cm^3 of this electron cooling, experience suggests that it would not be difficult to find other channels for electron heat loss. The 2.5 kW/cm^3 extracted from the electrons must be balanced by ion heating.

The fusion power (P_{fusion}) calculated by Rostoker *et al.* at the operating point is $P_{fusion} = n_p n_B Y_{p\text{-}^{11}B}\langle\sigma_{p\text{-}^{11}B} v\rangle \approx 45$ W/cm^3, where $Y_{p\text{-}^{11}B} = 8.7$ MeV is the energy yield per fusion event (and v is the velocity of the proton and boron nuclei). The difference between $P_{electron}$ and P_{fusion} sets a lower limit on the external ion heating power. Thus, the fusion gain (which is normally defined as the ratio of the fusion power to the external heating power), $Q \equiv P_{fusion}/P_{heat} \leq P_{fusion}/(P_{electron} - P_{fusion}) \approx 0.02$ would be much lower than the value of 2.7 stated by Rostoker *et al.* (*1*).

Highly nonthermal fusion schemes generally suffer from an unattractive power balance (*4*), and so magnetic fusion research has focused on systems that are in local thermal equilibrium. In such systems, high fusion gain ($Q \gg 1$) remains a distinct possibility.

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References and Notes

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Recognizing the unfavorable power balance of a thermal proton-boron (p-^{11}B) plasma, N. Rostoker *et al.* (*1*) propose restricting the proton energy relative to the boron to be near the resonance in the fusion cross section at $E_0 \pm \Delta E = (580 \pm 140)$ keV. Although this beam-beam configuration would avoid the large power required (*2*) in the migma (*3*) to maintain the proton distribution against self-collisions, a large power input would nevertheless be required to replace the directed energy lost to frictional heating of the proton and boron beams. The classical formula (*4*) for this power density is

$$P_{fric} = (m_p^{-1} + m_B^{-1}) \times (Z_B^2 e^4 \ln\Lambda / 4\pi\varepsilon_0^2) n_p n_B / \upsilon_0$$

where $\ln \Lambda \approx 15$ (or larger) is the Coulomb logarithm and Z_B is the atomic number of boron, 5. Because the fusion power density is $P_{fusion} = (8.7 \text{ MeV}) \langle\sigma\upsilon\rangle n_p n_B$, if we set the reactivity equal to its value at the resonance, enhanced by a factor of 1.6 for spin polarization, $\langle\sigma\upsilon\rangle = 1.3 \times 10^{-21}$ m^3 s^{-1}, then we find the ratio to be $P_{fusion}/P_{fric} \approx 0.12$, which shows that the proposal for a reactor with net electrical power output is unrealistic.

The power balance would be at least another factor of three less favorable than this estimate because the coupling of the ions through the electrons would be stronger than the direct coupling if $T_e < E_0/15 = 40$ keV. If the electron coupling were decreased by raising T_e, T_p must also rise. Rostoker *et al.* suggest that the T_p has only to be less than 140 keV, but this is not sufficiently cool. In order for protons with velocity $\sqrt{2E_0/m_p} \pm \sqrt{k_B T_p/m_p}$ to lie within the resonance, T must be $\kappa_B T_p \lesssim (\Delta E)^2/2E_0 = 17$ keV. Rostoker *et al.* published a similar unworkable reactor design earlier (*5*) with $\kappa_B T_p = 200$ keV.

Next to power balance, a serious problem of the CBFR is equilibrium. The plasma volume envisaged would be a long, thin cylindrical shell with thickness/radius $\Delta r/r \approx 0.08$. Such a configuration would not be in axial equilibrium because the tension of the field lines curving around the shell at the ends would provide a powerful compressive force. For 2-dimensional equilibria, $\Delta r/r \approx 0.5$ (*6*), so the highly localized profiles required to prevent radial particle and energy losses would not be maintained.

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Response: The main point of the comment by Nevins is that direct heating of electrons from Coulomb scattering by the boron beam (2.0 kW/cm^3) is the largest term in the electron power balance, and that we neglected it in our article (*1*). The electrons would have to be maintained at a low *T* in order to cool the proton beam. The problem is with the calculation of the electron T, which is not detailed in our article or in the comment by W. M. Nevins. The calculation of heating of electrons by boron scattering does give a large result. However, electrons lose energy to the boron ions because their velocity would be higher and there would be a collisional drag effect. This energy loss would cancel the energy gain. The electron temperature T_e is defined by

$$\frac{3}{2} N_e T_e = \int_0^{r_B} 2\pi r\, dr\, n_e \int d\vec{\upsilon}\, \frac{1}{2} m(\vec{\upsilon} - \vec{V}_e)^2 \times f_e(\vec{r},\vec{\upsilon}) = N_e \frac{m}{2} [\langle \vec{\upsilon}^2 \rangle - \vec{V}_e^{\,2}]$$

where n_e is the electron density, $\vec{V}_e$ is the average electron velocity, $f_e(\vec{r}, \vec{\upsilon})$ is the electron distribution function, and N_e is the line density of electrons, which is constant. The calculation of dT_e/dt involves the Fokker-Planck equation and is complicated. If it is done in a straightforward way with the above definition, the result will be as Nevins describes. However, if we note that in a steady state $dV_e/dt = 0$ and calculate only $d\langle \upsilon^2 \rangle/dt$, the cancellation takes place, and electrons would only be heated by the protons. Similar considerations show that protons are cooled by electrons and boron.

We have revisited ideas such as "the use of p-^{11}B as a fuel." However, we have developed new and systematic calculations and the conclusions are not the same as they were with the use of that proposed fuel.

Carlson employs a classical generic formula for the power density required to overcome the friction between proton and boron beams. This formula is inadequate for the Colliding Beam Fusion Reactor. The magnetic field is important, and it is distinguished by its absence in this formula. The complete formula can be derived by taking the appropriate moment of the Vlasov/Fokker-Planck equation. First, the equilibrium conservation of momentum equation is

$$n_i e_i E_r - T_i \frac{dn_i}{dr} = \frac{n_i e_i}{c} V_i |B_z| - \frac{n_i m_i V_i^2}{r}$$

with i = (1, 2) for protons and boron. Here n_i is the particle density; e_i is the charge; m_i is the mass; T_i is the temperature; E_r and B_z are the electric and magnetic field, respectively; and V_i is the velocity in the azimuthal direction. The kinetic equation obtained from the same moment, but including the collision operator, is

$$n_1 m_1 \frac{d\vec{V}_1}{dt} = n_1 e_1 \left[\vec{E} + \frac{1}{c} \vec{V}_1 \times \vec{B} \right] - T_1 \nabla n_1 + n_1 m_1 \frac{(\vec{V}_2 - \vec{V}_1)}{t_{12}}$$

with a similar equation for $\vec{V}_2$. The dot product of $\vec{V}_1$ with the above equations yields an energy (power) equation and the magnetic field seems to vanish. The power is

$$P_{fric} = \frac{d}{dt} \sum_{i=1,2} n_i m_i (V_i^2/2) = -\frac{n_1 m_1}{t_{12}} (\vec{V}_1 - \vec{V}_2)^2 + \sum_{i=1,2} n_i V_{ir} \left[e_i E_r - \frac{T_i dn_i}{n_i dr} \right]$$

The first term is precisely the expression for power density employed by Carlson. The second term involves the radial velocity V_{ir} and is positive definite. It can be estimated with some approximations; $V_{ir} \ll V_{i\theta}$ and $n_i V_{ir} = -D_i(\partial n_i/\partial_r)$, where D_i is the diffusion coefficient; $D_1 \simeq a_1^2/t_{12}$, where $a_1 = V_1/\Omega_1$ is the gyro-radius and $\Omega_1 = e_1|B_z|/m_1 c$. Similar expressions obtain for boron. Although V_{ir} may be neglected as compared with $V_{i\theta}$, it may not be neglected in the second term. This term can be estimated from the equilibrium equation. One can see that the magnetic field that previously seemed to vanish has returned. The second term is positive definite and the magnitude $n_1 m_1 (a_1/L_1)(V_1^2/t_{12})$ is similar to that of the first term. L_1 is the scale length of the equilibrium, that is $(1/L_1) = (1/n_1)|dn_1/dr|$. To determine this power quantitatively requires a considerable amount of work (*2*). It requires a new development in classical transport theory because earlier studies assume $a_i \ll L_1$, which would not be the case in the CBFR. This calculation should also include electrons and the fusion products. The result is that P_{fric} would be tolerable.

Concerning the resonance, we have made detailed calculations. If the beam temperatures are less than one-half of the half width of the resonance, the reactivity should be greater than one-half of the maximum reactivity for zero temperature beams. The result stated by Carlson that the beam temperature must be less than 17 keV seems to contradict this. However, no results for $\langle \sigma \upsilon \rangle$ are given for comparison.

The equilibrium calculations to which Carlson refers are not appropriate for the CBFR. Axial equilibrium requires an axial T, or the FRC will contract in the axial direction. It has been observed experimentally that FRCs have long axial equilibria. We have previously considered long, thin cylindrical shell models because they simplify many calculations.

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