arXiv:0912.0520v1 [hep-ph] 2 Dec 2009

Electroweak stars: how nature may capitalize on the standard model's ultimate fuel

De-Chang Dai[1], Arthur Lue, Glenn Starkman[2] and Dejan Stojkovic[1]
[1]*Department of Physics, SUNY at Buffalo, Buffalo, NY 14260-1500 and*
[2]*CERCA, Department of Physics, Case Western Reserve University, Cleveland, OH 44106-7079*

We study the possible existence of an *electroweak star* – a compact stellar-mass object whose central core temperature is higher than the electroweak symmetry restoration temperature. The source of energy of the electroweak star is standard-model non-perturbative baryon number (B) and lepton number (L) violating processes that allow the chemical potential of $B+L$ to relax to zero. The energy released at the core is enormous, but gravitational redshift and the enhanced neutrino interaction cross section at these energies make the energy release rate moderate at the surface of the star. The lifetime of this new quasi-equilibrium can be more than ten million years. This is long enough to represent a new stage in the evolution of a star **if stellar evolution can take it there**.

PACS numbers: 04.50.Kd

Introduction. The last stage of the evolution of a star whose original mass was below the Chandrasekhar limit is believed to be a neutron star. A neutron star has no active energy source which balances the force of gravity. Instead, it is supported by degeneracy pressure, which arises due to the Pauli exclusion principle. However, this pressure can support only neutron stars lighter than about $2.1M_{\odot}$, which is known as the Tolman-Oppenheimer-Volkoff limit. Neutron stars heavier than this limit eventually become black holes.

Recent studies point out the existence of a state between the neutron star and black hole. It is called a quark star. This state owes its existence to the QCD phase transition in which the original nuclear matter becomes quark matter. This process can release 10^{53} erg in about 10^{-3} to 10^{-2}s [1, 2] in the form of $\nu\bar{\nu}$ bursts. While this a huge amount of energy, it is released in too short a time to provide the pressure to prevent gravitational collapse. After this energy release, a star containing effectively only three quark flavors (u,d,s) can exist in a stable equilibrium where the pressure is provided by the Pauli exclusion principle. However, at higher densities where four or more quark flavors are present, quark matter cannot avoid gravitational collapse.

Since in the gravitational collapse matter gets compressed to ever increasing densities/temperatures, it is natural to explore what could happen at the electroweak phase transition, the next (and last) within the standard model. In the standard model, both baryon and lepton number $U(1)$ global symmetries are accidental, conserved perturbatively, but violated by non-perturbative processes such as those mediated by instantons. At temperatures well below the electroweak symmetry-breaking scale, $T \lesssim 100$GeV, these non-perturbative processes are suppressed by the extremely small factor $e^{-8\pi/\alpha}$ (giving, for example, a proton life-time of 10^{141}yrs [3]). (Note, unless otherwise indicated we shall employ natural units where $\hbar = c = k_B \equiv 1$.) However, above this scale, baryon and lepton number violating processes are expected to be essentially unsuppressed (although the combination $B-L$ remains conserved). Quarks can then be effectively converted into leptons. In this process, which we call *electroweak burning*, huge amounts of energy can be released.

Let us assume that in the core the temperature is above, or at least very near, the weak scale, and the matter density is also comparable $\rho \gtrsim (100\mathrm{Ge}V)^4$. At these densities, the matter is opaque even to neutrinos and the energy released at the center cannot stream freely out of the star. Nevertheless, the energy can be carried out, mostly by neutrinos and anti-neutrinos. These can also carry out any excess anti-lepton number generated in the electroweak burning, and thus prevent the lepton number chemical potential from halting the consumption of the baryon number. This mechanism can likely provide a stable energy source which can counteract gravity for a while. In this paper, we study the inner structure of such a star and calculate its lifetime. We find that this new phase can last up to 10^{15}s, which is long enough to give a new name to this type of stars - electroweak stars. Future work will consider questions of stability, of evolution – if and when the electroweak star arises in the late-phases of evolution of ordinary stars – of the structure of the outer cooler layers, and of observational signatures of these objects.

Density, pressure, and particle number density inside the electroweak star. We separate the core of the electroweak star into three regions, as shown in Fig. 1. The central region is hotter and denser than the electroweak symmetry restoration temperature ($T \gtrsim 100$GeV). This region is very dense ($\rho > 10^8\mathrm{GeV}^4$), but small (several cm). We find that the total mass inside this region is $\sim 5\times 10^{-6}M_{\odot}$. Since the lepton and baryon number are not conserved in this region, but $B-L$ is, baryons are freely transformed to anti-leptons so long as it is thermodynamically favorable to do so. A $SU(2)$-preserving instanton interaction can convert 9 quarks into 3 anti-leptons [4], for example:

$$udd \;\; css \;\; tbb \; \rightarrow \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau \tag{1}$$

and all $B-L$ and electromagnetic charge conserving variations involving quarks, anti-quarks, neutrinos, antineutrinos and charged leptons.

At these temperatures each particle carries about 100GeV of energy, so this process can release about 300GeV per neutrino. This energy flows out of the central core, and eventually out of the star. However, in the central region, the mean free paths of all particles are short compared to the size of the star. The energy must therefore diffuse out of the central core. Similarly, the non-perturbative B-violating processes, while reducing B, reduce L. Initially $B - L > 0$ in stars, because neutrinos escape, and thus there are more neutrons plus protons than there are electrons. Thus, the B-violating processes would cause an anti-lepton number to build up. This must either stop the burning, or be carried out of the core by a flux of anti-leptons. Outside the central core, baryon and lepton number are conserved. The density falls with increasing radius, and particle mean free paths increase, however, they are still too short for particles to stream freely out of the star. Eventually, however we reach the star's neutrino-sphere, where the neutrino (and anti-neutrino) mean free path is long enough for these particles to flow freely. (This is well within the radius at which the same is true of photons.) The neutrinos and anti-neutrinos carry off not just most of the energy produced in the core burning region (the rest is carried off by photons), but also the lepton number, thus permitting the burning to continue. Since the fuel is predominantly baryons not anti-baryons, resulting in the production of an excess of anti-leptons over leptons, there will be a greater flux of anti-neutrinos than neutrinos from the star. But the excess is very small. This may be a signature of this phase – albeit a rather difficult one to detect. An eventual discovery of anti-neutrino flow from a compact star can be a hint for the electroweak star.

The size of a quark star is about 10km, while its mass is about $1M_\odot$. The electroweak core cannot significantly change the region outside the core. We use the so-called "bag model" [5] as an approximate solution of the region outside the core. The pressure and energy density are [6]

$$\begin{aligned} P &= \sum_i p_i - B \qquad (2)\\ \epsilon &= \sum_i \epsilon_i + B. \qquad (3)\end{aligned}$$

Here, P is the total pressure, p_i is the partial pressure of particle i, B is the bag energy density, ϵ is the total energy density, and ϵ_i is the energy density of particle

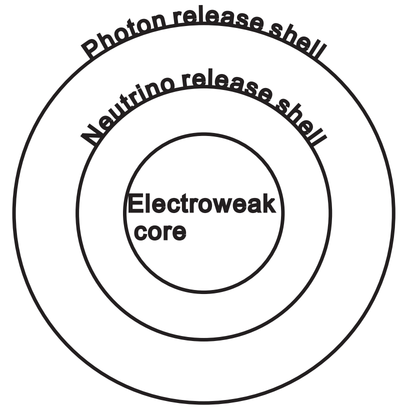

FIG. 1: The structure of an electroweak star. We separate the core of the star into three regions: I. central core where the electroweak symmetry is restored, II. region where the high energy neutrinos are trapped, III. region from which high energy neutrinos can escape. Neutrinos will be emitted between the regions II. and III. Photons are emitted at the surface of the star. We find the size of the core to be at most several cm.

i. The subscript i runs over all types of particles. We choose the specific value $B^{1/4} = 145$MeV. The condition for electric charge neutrality is $\sum_i q_i n_i = 0$. Here, q_i is the charge of the particle i. The pressure, energy density, and number density of particles can be well approximated from an ideal gas distribution:

$$\begin{aligned} p_i &= \frac{g_i}{6\pi^2}\int_{m_i}^{\infty} dE(E^2-m_i^2)^{3/2} f_i(E) \qquad (4)\\ n_i &= \frac{g_i}{2\pi^2}\int_{m_i}^{\infty} dE(E^2-m_i^2)^{1/2} E f_i(E)\\ \epsilon_i &= \frac{g_i}{2\pi^2}\int_{m_i}^{\infty} dE(E^2-m_i^2)^{1/2} E^2 f_i(E).\end{aligned}$$

Here, g_i is the number of degrees of freedom of particle i (e.g. $g = 2$ for leptons and $g = 6$ for quarks), m_i is the mass of the particle, μ_i is the chemical potential of the particle and T is the temperature. The distribution function is

$$f_i(E) = \frac{1}{1 \pm e^{(E-\mu_i)/T}} \tag{5}$$

for bosons and fermions.

Outside the central region I, baryon and lepton numbers are conserved. We assume for simplicity that the chemical potentials of the left- and right-handed fermions are the same, and that all fermions of the same flavor have the same chemical potential. An antiparticle has the chemical potential of the opposite sign. The interaction $d \rightarrow u + e + \bar{\nu}_e$ indicates that $\mu_d = \mu_u + \mu_e - \mu_\nu$. We also assume $\mu_\nu = 0$ outside the electroweak core, so that neutrinos or anti-neutrinos can propagate away, and use 1 to add the condition, $\mu_u + 2\mu_d + \mu_\nu = 0$, inside the

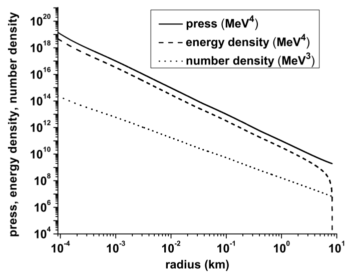

FIG. 2: The pressure, energy density and particle number density change with the radius of the star. The total radius of the star is about 8.2km, while the mass is about $1.3 M_\odot$.

electroweak core. We then calculate the structure of the star according to the Oppenheimer-Volkoff equation [7]

$$\frac{dP}{dr} = -\frac{(\epsilon+P)(M+4\pi P r^3)}{r^2(1-2M/r)}, \tag{6}$$

$$\frac{dM}{dr} = 4\pi\epsilon r^2. \tag{7}$$

$$g_{tt} = (1-\frac{2M_{\rm star}}{R_{\rm surface}})e^{-\int_0^{P(r)}\frac{2dP}{P+\epsilon}} \tag{8}$$

$$g_{rr} = \frac{1}{1-\frac{2M(r)}{r}} \tag{9}$$

$R_{\rm surface}$ is radius of the star. $M_{\rm star}$ is the mass of the star. Since we are dealing with very dense objects, general relativistic effects described by the metric components g_{tt} and g_{rr} will be important. We assume that the energy density is much higher than B^4. The equation of state does not change much and is approximately $\epsilon = 3P$.

We now study a particular case with the total size of the star of about 8.2km and its mass $1.3M_\odot$. Because of the properties of fermi statistics, fermions can have large energies at $T = 0$. In that case the energy of neutrinos is not proportional to the temperature, instead it is proportional to the chemical potential μ_ν. At such a small radius, the solution for the profile function of the energy density is very close to Tolman's static solution of Einstein's equations for a sphere of fluid [8], i.e. $\epsilon \propto r^{-2}$. We can use this profile to estimate the size of the central core. From Fig. 2, we can see that the size of the central electroweak core can be at most several cm (this is the distance at which the density falls below 100GeV^4)). Such a dense star cannot be stable and would collapse in the absence of an internal energy source. [6]. The non-perturbative electroweak interactions provide that source of energy. Meanwhile as the baryons are burned, gravitational collapse brings more material to the central part of the core feeding the fire. We assume that eventually a quasi-equilibrium between the burning and the refueling is achieved. If so, the star enters a quasi-stable state that can last for a while. The question of stability will be addressed in future work.

Location of the neutrino-sphere Neutrinos near the central region have very high energies (but below the weak scale) and the local density is extremely high. Their mean free path in this region is thus very short, and they cannot stream freely from the star. However, as the local density falls with radius, and the neutrinos' average energy decreases, their mean free path increases. We define the neutrino escape radius (as a function of neutrino energy) as the distance from the center at which a neutrino with that energy has a mean free path greater than the thickness of the overlying matter and can therefore escape the star. The neutrino-sphere is the set of such spherical shells for neutrinos of energies characteristic of the local temperature.

Now we study the neutrino mean free path in dense media as presented in [9]. We assume that the cross section of neutrino scattering is [10]

$$\sigma_i \sim \frac{G_F^2 s}{\pi} \sim \frac{G_F^2 E_\nu E_i}{\pi}. \tag{10}$$

Here, $G_F \sim 1.166\times10^{-5}\mathrm{GeV}^{-2}$ is Fermi constant, s is the center of mass energy, E_i is the energy of the particle i, while E_ν is the neutrino energy. The mean free path is

$$\frac{1}{\lambda} = \sum_i \sigma_i n_i \tag{11}$$

Unlike ordinary stars (even neutron stars), particles propagating through the electroweak stars suffer large gravitational redshift. Thus E_ν is not a constant, but changes as

$$E_\nu(r) = \frac{\sqrt{g_{tt}(r_0)}}{\sqrt{g_{tt}(r)}}E_\nu(r_0). \tag{12}$$

The redshift ratio is shown in Fig 3. Obviously, the redshift effect near the center is much larger than at the surface. Since inside the star $\epsilon \sim 3P$, the redshift can be estimated from Eq. (8). It is

$$E_\nu \sim \left(\frac{P(r)}{P(r_0)}\right)^{1/4} E_\nu(r_0) \tag{13}$$

The pressure is about (100GeV)^4 near the center, and about (100MeV)^4 near the surface (before the bag energy changes the pressure a lot). Therefore, a particle retains only a fraction 10^{-3} of its original energy. Though instanton processes release huge amounts of energy at the center, the star emits much more moderate amounts. The result is shown in Fig. 4. A particle with the original energy of 100GeV near the center indeed carries only 165MeV as it leaves the surface.

We now use this energy corrected for the redshift to calculate the mean free path on the surface. From Eq. (10) and Eq. (11) the mean free path on the surface is

$$\frac{1}{\lambda} \sim \frac{G_F^2 \epsilon}{\pi} E_\nu(R_{\rm surface}) \tag{14}$$

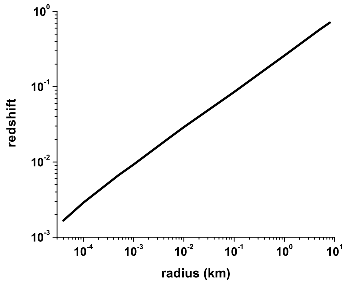

FIG. 3: The redshift factor $\sqrt{g_{tt}(r)}$ vs. radius of the star. The redshift near the center is much larger than at the surface. A particle with the original energy of 100GeV near the center carries away only 165MeV as it leaves the surface.

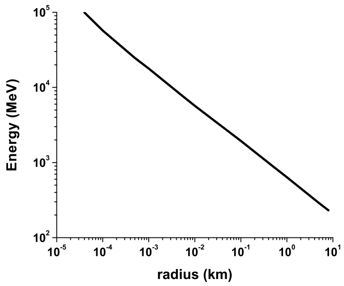

FIG. 4: Neutrino energy as a function of the distance from the center where only effects of gravitational redshift are taken into account (no energy loss due to interactions). The energy on the surface is about 1000 times smaller than the original energy at the center.

Eq. (14) yields $\lambda \sim 10^{12}\text{MeV}^{-1} \sim 10^{-4}\text{km}$. A neutrino with this energy cannot leave the star freely, and must lose its energy while propagating from the center to the surface. However, when we take the redshift into account, the neutrinos have roughly the same energy as the background they are propagating in. The consequence will be that the neutrinos do not lose much energy in the energy transport process. This is true in general for the other types of particles. If a particle has the energy density ϵ, its energy can be estimated as $\sim \epsilon^{1/4} \sim P^{1/4}$. Comparing this with Eq. (13), the energy of a neutrino decreases with the distance the same way as the background. Thus, the neutrinos coming from the center and arriving to a certain point inside the star have roughly the same energy as the neutrinos that were emitted by thermal processes at that point.

We would like now to estimate the (neutrino) luminosity of the electroweak star. The maximum luminosity can be obtained from the black-body radiation at the neutrino-sphere

$$\mathcal{L}_\nu \leq \mathcal{L}_{bb}\epsilon_\nu \pi R_{\rm nrs}^2 \tag{15}$$

This places an upper bound of about $3\times 10^{41}\text{MeV}^2 = 7\times 10^{56}erg/s$. At this rate it would take less than a second to release $1M_\odot$. Fortunately, 15 is a severe over-estimate of the luminosity. It does not take general relativity into account, nor does it allow for the fact that the luminosity depends not just on the temperature and density at the neutrino-sphere but also on their gradients, since those determine the *net* outward flux of energy.

First we will estimate the energy release rate when we include redshift effects in calculating the ratio dE/dt. An upper bound on the energy release rate can be obtained from the free fall time of the incoming quark shell into the electroweak-burning core. This determines the maximum rate at which the electroweak core burning can be fed. The free fall acceleration over a short distance given by the mean free path λ is roughly given by $a = M_{\rm ew}/r_{\rm ew}^2$, where $r_{\rm ew}$ refers to a radius one mean free path (λ) away from the electroweak core (relativistic corrections will not change the result significantly). Therefore, the energy release rate can be no more than

$$\left(\frac{dE}{dt}\right)_{\rm max} \sim 4\pi r_{\rm ew}^2 \sqrt{\frac{2\lambda M(r_{\rm ew})}{r_{\rm ew}^2}}\, \epsilon(r_{\rm ew})\, g_{tt}(r_{\rm ew}), \tag{16}$$

This is just 10^{27}MeV^2, which is already much lower than the earlier upper limits obtained from 15. We compare this rate with the electroweak baryogenesis interaction rate [11]

$$\frac{dE_w}{dt} \sim 4\pi r_{\rm ew}^2 \times 20\alpha_{\rm ew}^5 T^4 \tag{17}$$

This rate is about 10^{34}MeV^2, which is much larger than the rate in (16). Therefore we can reasonably assume that as quarks reach the electroweak core, they are converted into neutrinos instantaneously. Otherwise, the infalling matter would pile up and form a black hole. We note however, that a more detailed analysis must be performed to follow the time evolution of this system.

One intriguing possibility is that the authors of [14] and [15] are correct that gravitational collapse of a stellar mass does not result in a black hole but in an object of very high density and temperature toward its center. In this case, the ignition of core baryon-burning will indeed result in an electroweak star.

We now calculate the energy release rate at infinity (or intensity) more accurately taking into account the transport of energy through the star. Gravitational redshift and time delay along with the enhanced neutrino interaction cross section at these energies will affect the energy transport through the star and make the energy release rate moderate at the surface of the star. The relativistic transport of energy can be described by

$$4\pi r^2 \lambda \frac{d(S(r) g_{tt}(r))}{\sqrt{-g_{rr}(r)}dr} = -L. \tag{18}$$

Here $S(r) \equiv dE/(dtdA)$, where A is area, is the energy flux at the radius r, while $L \equiv dE/dt$ is the energy intensity at infinity (which is practically about the same order

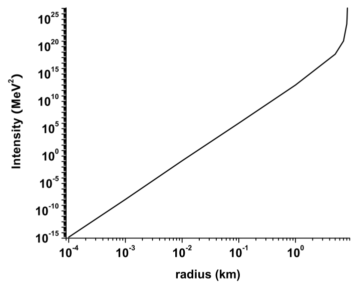

FIG. 5: The energy release rate vs. neutrino release radius: The energy release rate increases with the radius of the neutrino release shell. The maximum energy release rate is on the surface, but there it exceeds the limit from the quark shell free fall into the electroweak core (which minimizes the denominator in (15)). This implies that the neutrino release shell must be inside the star.

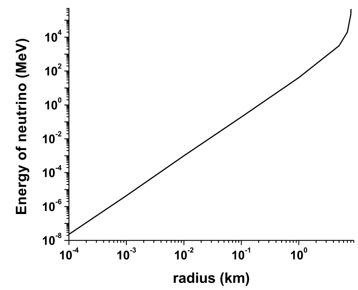

FIG. 6: The neutrino energy at the surface of the central electroweak core vs. the radius of the neutrino release shell. From Fig. 5 we that the energy release rate increases with the radius. Higher energy release rate needs the source of higher energy. The energy of neutrinos therefore increases with the radius of release shell. However, the energy is already larger than 500GeV at 8.2km. The star cannot support this amount of energy implying that its radius must be smaller than that.

as the intensity at the neutrino release shell). The factor $g_{tt}(r)$ describes both the redshift and time delay. The energy flux $S(r)$ can be found from the energy density ϵ_ν given by (4) knowing the energy (i.e. chemical potential) of neutrinos E_ν. The metric coefficients are given by (6). It is then straightforward to calculate L. Fig. 5 shows the intensity L as the function of the radius of the neutrino release shell. Larger radius implies lower energy density, which means that higher energy neutrinos can escape. The energy release rate therefore increases with the radius of the shell and it is maximal at the surface of the star. However, there it exceeds the limit from the free fall. This implies that the neutrino release shell must be inside the star. We now perform one more consistency check. At the neutrino release shell the following must be true, $4\pi r^2 S(r) g_{tt}(r) = L$. From there we can find the energy at the central electroweak core needed to support a certain intensity at the surface of the star. Fig. 6 shows the results. Higher energy release rate needs the source of higher energy. If the neutrino release radius is 8.1km, then the neutrino energy must be larger than 300GeV. We therefore conclude that the radius of the neutrino release shell must less than 8.1km. If the neutrino escape radius is about this size, the energy release rate is about 10^{24}MeV^2. Integrating this, we find that it takes about 10^{15}sec (10 million years) to release energy equivalent of $1M_\odot$. This is the minimal life-time of the electroweak star. This also implies that electroweak stars are not likely to be visible to us in neutrinos. At 1kpc a neutrino source of this luminosity emitting predominantly 100MeV neutrinos would produce a flux of just $10^3/m^2s$. This implies that we must reliably characterize the photon luminosity before we can determine how best to observe an electroweak star.

In our calculations we neglected the fact that some fraction of energy is carried away by photons, since they have shorter mean free path. Further, though the net lepton number is conserved in this region, the number of neutrinos and antineutrinos are not conserved. We also ignored the effects of energy transport due to convection. Finally, neutrino energy was calculated from the chemical potential change, instead of the temperature change. However, we assume that these effects will not change the order of magnitude estimate.

Conclusions. We studied the possibility of the existence of a new phase in the stelar evolution. After all of the thermonuclear fuel is spent, and possibly after the supernova explosion, but before the remaining mass crosses its own Schwarzschild radius, the temperature of the central core may surpass the electroweak symmetry restoration temperature. At this point, non-perturbative baryon-number violating interactions in the ordinary $SU(2)xU(1)$ electro-weak Standard Model become unsuppressed. The consequent relaxation of the stellar baryon number chemical potential, i.e. the conversion of baryons to anti-leptons, may provide a source of pressure which can balance gravity. We constructed a solution to TOV equation whose central pressure is non-singular (and thus not a black hole). We showed that the in-falling matter gets converted into neutrinos at the rate much faster than the free-fall rate, which indicates that the mater burns before crossing its own Schwarzschild radius (assuming that the core is not within the Schwarzschild radius itself). Our analysis shows that lifetime of this new phase is at least 10 million years which we propose to call an electroweak star. We emphasize that the electroweak star (unlike the recently proposed dark star which would be supported by dark matter annihilation [13]) relies only on Standard Model physics for its existence.

Electroweak stars would be an exciting addition to the diverse menagerie of astrophysical bodies that the universe provides. Nevertheless, considerable work remains to be done before we can claim with confidence that such objects will form in the natural process of stellar evolution, or that they will indeed burn steadily for an extended period. Similarly, since it seems unlikely that the flux of neutrinos would be detectable, assessing the visibility of these fascinating new objects requires a careful modeling of their outer structure to determine the photon luminosity and spectrum.

DS acknowledges the financial support from NSF, grant number PHY-0914893. GDS and AL thanks CERN for its hospitality. GDS is supported by a grant from the US DOE to the particle astrophysics theory group at CWRU.

---

[1] A. Drago, A. Lavagno and I. Parenti, Astrophys. J. **659**, 1519 (2007) [arXiv:astro-ph/0512652].
[2] A. Drago, G. Pagliara, G. Pagliaroli, F. L. Villante and F. Vissani, arXiv:0809.0518 [astro-ph].
[3] F. C. Adams and G. Laughlin, Rev. Mod. Phys. **69**, 337 (1997) [arXiv:astro-ph/9701131].
[4] N. S. Manton, Phys. Rev. D **28**, 2019 (1983).
[5] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
[6] C. Kettner, F. Weber, M. K. Weigel and N. K. Glendenning, Phys. Rev. D **51**, 1440 (1995).
[7] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).
[8] R. C. Tolman, Phys. Rev. **55** (1939) 364.
[9] C. Shen, U. Lombardo, N. Van Giai and W. Zuo, Phys. Rev. C **68**, 055802 (2003) [arXiv:nucl-th/0307101].
[10] W. J. Marciano and Z. Parsa, J. Phys. G **29**, 2629 (2003) [arXiv:hep-ph/0403168].
[11] G. D. Moore, Nucl. Phys. B **568**, 367 (2000) [arXiv:hep-ph/9810313].
[12] D. Stojkovic, F. C. Adams and G. D. Starkman, Int. J. Mod. Phys. D **14**, 2293 (2005) [arXiv:gr-qc/0604072].
[13] D. Spolyar, K. Freese and P. Gondolo, Phys. Rev. Lett. **100**, 051101 (2008) [arXiv:0705.0521 [astro-ph]].
[14] C. Barcelo, S. Liberati, S. Sonego and M. Visser, Phys. Rev. D **77**, 044032 (2008) [arXiv:0712.1130 [gr-qc]].
[15] M. Visser, C. Barcelo, S. Liberati and S. Sonego, arXiv:0902.0346 [gr-qc].