

Theory of Gravity Dependent on Entropy

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Gravitational redshift decreases the resolution of measurements, causing entropy by increasing the possible microstates of position and momentum for objects within a gravitational field. This is the first paper with a quantitative equation for gravitational entropy, which extends the concept of black hole entropy to any object within a gravitational field. It also provides the first calculation of “inertial entropy,” the entropy of moving objects, by correlating changes in the scale of spacetime due to gravity with those of special relativity. The entropy scale factor (ESF) combines gravitational entropy and inertial entropy to propose that all changes in the scale of spacetime are due to entropy. In the ESF, gravity is due to entropy, not the mass of Newtonian gravity or the energy and momentum of general relativity. The key difference between Newtonian gravity and the ESF is that mass is a property of an object, while gravitational entropy is a property of a field. This means that in the ESF the entropy of objects can be increased by the gravitational fields of nearby objects. This increase in entropy results in the ESF predicting more gravitational force than Newtonian gravity does for complicated systems. This increase in force may be able to explain the phenomena attributed to dark matter, like galaxy rotation dynamics, without the need for dark matter. The same changes in scale predicted by the ESF may be able to explain the expansion of the universe, both in the inflationary epoch and the accelerating universe, without the need for an inflaton field or dark energy. The similarity of the ESF to Newtonian gravity for systems with one or two gravitational bodies explains why it has not been ruled out by prior tests of gravity.

Keywords: Gravity; entropy; general relativity; special relativity; dark matter.

1. Introduction

General relativity improved on Newtonian gravity in countless ways, including refining orbital predictions for objects in high gravitational fields, predicting black holes and predicting gravitational time dilation. Many of these improvements have been experimentally verified, but it is possible that Newtonian gravity was wrong in other ways as well. While both general relativity and Newtonian gravity have been

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well studied in systems with one or two dominant masses, they perform less well in complicated systems. For example, in galaxy rotation, galaxy cluster dynamics and the expansion of the universe, both general relativity and Newtonian gravity have performed poorly. To explain these discrepancies multiple theories have been proposed, including dark matter, inflation and dark energy. Could both Newtonian gravity and general relativity be wrong enough to explain the above phenomena, while still meeting the rigid requirements of tests of gravity?

There are two unexplored facets of gravity that deserve further scrutiny.

- (1) Gravity could be due to entropy, instead of the energy and momentum of general relativity. The entropy of an object is closely associated with its mass.¹ To the author's knowledge no test of general relativity has specifically evaluated if entropy or energy/momentum is more closely correlated with gravitational force. On the other hand, the entropy of the universe has been correlated with the expansion of space, both in the inflationary epoch (due to the formation of particles and radiation) and in the accelerating universe (due to the growth of black holes).^{2,3} This is not true of the universe's total energy, because energy is conserved. If the same change in scale that causes gravity also causes the expansion of the universe, then the correlation between expansion and entropy may indicate that spacetime changes scale due to entropy.
- (2) Gravitational fields may combine differently than predicted by Newtonian gravity or general relativity. To the author's knowledge all rigorous tests of gravity have been in systems with one or two dominant masses. Systems with three or more objects may behave differently than either Newtonian gravity or general relativity predicts.

The entropy scale factor (ESF) is a theory of gravity that proposes that gravity depends on entropy instead of mass, and because of this, gravitational fields combine differently than what is predicted by Newtonian gravity or general relativity.⁴⁻⁶ In this theory, all changes in the scale of spacetime depend on entropy, including those due to gravity, the Lorentz transformations of special relativity and the expansion of the universe. Previous papers have defined the ESF and gravitational entropy, and related both of these concepts to astrophysical phenomena. This paper provides a mathematical framework for the ESF, with quantitative predictions of entropy due to gravity and relative motion.

2. Gravity Dependent on Entropy

Newton's law of universal gravitation is given in Eq. (1), where F is the force, G is Newton's gravitational constant, m_1 and m_2 are the masses of two different objects, and d is the distance between the two objects

$$F = G \frac{m_1 m_2}{d^2}. \quad (1)$$

Black hole thermodynamics gives a precise way to convert mass to entropy. The entropy of a black hole is given by the Hawking formula, where A is the areas of the event horizon, k_B is Boltzmann's constant and l_P is the Planck length as⁷

$$S_{BH} = \frac{k_B A}{4l_P^2}. \quad (2)$$

The mass of a Schwarzschild black hole is directly proportional to its radius. Since entropy is proportional to area, mass is proportional to the square root of entropy. This relationship can be used to modify Newtonian gravity so that it depends on entropy, instead of mass, as in Eq. (3). In this equation, S_1 and S_2 represent the entropy of each of two objects, and w is a new gravitational constant dependent on entropy.

$$F = w \frac{\sqrt{S_1} \sqrt{S_2}}{d^2}. \quad (3)$$

The new gravitational constant, w , can be found by taking Newton's gravitational constant, multiplying by the mass of two black holes and dividing by the square root of each black hole's entropy. The value for w is given as

$$w \approx 1.8 \times 10^{-4} \text{ Nm}^2 \text{ KJ}^{-1} = 1.8 \times 10^{-4} \text{ mK}. \quad (4)$$

3. Gravitational Entropy

The physical meaning of black hole entropy is a subject of debate, but the ESF interprets black hole entropy as uncertainty about the position and momentum of a black hole, due to its gravitational field.^{6,8} The measure of an object's microstates of position and momentum due to a gravitational field can be termed as "gravitational entropy."

The key difference between Newtonian gravity and the ESF is that the mass is a property of objects, while gravity is a property of a field. The mass of a black hole is confined within its event horizon, but its gravitational field extends into space. This gravitational field causes gravitational redshift, which decreases the resolution of measurements of any object within the field, not just the black hole. The decrease in resolution increases the microstates of position and momentum for objects within the field, resulting in gravitational entropy.⁶

The entropy for an object outside the event horizon can be found by looking for a physical meaning to the change of scale. For a system of two black holes, if the product of the scale at each black hole was proportional to the gravitational force this would give a compelling meaning to the changes in scale. In one sense this change is more compelling than the product of the two black holes' masses, used in Newtonian gravity, which is only proportional to force when the square of distance is accounted for.

As shown in Eq. (5), if the change in the scale falls off linearly with distance, then the product of the changes in scale would be proportional to gravitational force, as

seen in Eq. (5). In this equation r_{S1} is the Schwarzschild radius of black hole 1, and r_{S2} is the Schwarzschild radius of black hole 2

$$F = w \frac{\sqrt{S_1} \sqrt{S_2}}{d^2} \propto \left(\frac{r_{S2}}{d}\right) \left(\frac{r_{S1}}{d}\right). \tag{5}$$

Inside the event horizon the entropy of a black hole is proportional to the Schwarzschild radius squared (Eq. (2)). Keeping this same relationship outside the event horizon means that gravitational entropy would depend on the square of the quantity Schwarzschild radius over distance, as in Eq. (6). In this equation, S_G is gravitational entropy and S is the Hawking entropy of a black hole.

$$S_G = S \left(\frac{r_S}{d}\right)^2. \tag{6}$$

In Eq. (6), when r_S equals d , S_G equals S , so the gravitational entropy of a black hole is the same as its Hawking entropy.

In the ESF, the entropy of a black hole causes a change in the scale of length in the radial direction, without changing the scale of the circumferential directions.⁴ By changing the space in this way, the volume of a black hole can change without changing its surface area. Maintaining the surface area of black holes is important because this value is essential to black hole thermodynamics. By changing the lengths in this way, the volume of a black hole decreases compared to that of a similar sphere in Euclidean geometry, but the volume of a black hole is undefined in general relativity.⁹

The ESF goes on to predict that the length dilation caused by the black hole's entropy is enough to expand the Planck length in empty space to the Schwarzschild radius within the black hole.⁴ This increase in a standard length can be termed as "length dilation." This degree of length dilation means that there is no way to make measurements within the black hole, a fact that is true in both general relativity and the ESF.

In the ESF, entropy also affects the scale of time.⁴ In this theory, the entropy of a black hole increases the Planck time to the time it would take light to cross the distance of the Schwarzschild radius in empty space.

For a spherically symmetric system, these changes are time dilation and length dilation in the direction of the radius, but not in the circumferential directions, just like within the event horizon. For non-spherically symmetric systems, the relative level of length dilation and time dilation depends on the arrangement of the gravitational objects. This is because length dilation is in the direction of changing entropy, but time dilation affects all spatial dimensions.

Figure 1 shows the measurement notation used in this paper. For an observer at object 1, the levels of time dilation and length dilation at location 3 depend on the entropy of object 2, θ and the distances between objects 1 and 2 (d_{1-2}) and between objects 2 and 3 (d_{2-3}).

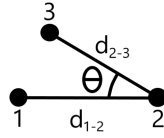


Fig. 1. Measurement Notation for a three-body system.

A non-relativistic coordinate framework can be created by assuming that the changes in scale predicted by the ESF fall off linearly with distance, as shown in Eq. (5). Within this framework, spherically symmetric systems, like a single black hole, have equivalent increases for Planck time and Planck length in the radial direction at any distance from the black hole. The equivalent changes in scale outside the black hole mirror the changes in scale seen within the event horizon. Gravitational entropy is defined by these balanced changes of scale, which occur in spherically symmetric systems.

Interactions among three or more objects that are not in a line have an additional form of entropy, “constellation entropy,” because time dilation can exceed length dilation for objects in these systems. Constellation entropy is not present in low-acceleration systems with only two objects, because each object only interacts with the spherically symmetric field of the other object.

4. Constellation Entropy

The difference between the ESF and Newtonian gravity becomes apparent in systems with three or more gravitationally interacting objects. In Newtonian gravity, the force on one object from two gravitational objects is the sum of the two objects’ individual forces on the object, as in Eq. (7). In this equation, F_3 is the force on object 3, d_{1-3} is the distance between objects 1 and 3, and d_{2-3} is the distance between objects 2 and 3.

$$F_3 = G \frac{m_1 m_3}{d_{1-3}^2} + G \frac{m_2 m_3}{d_{2-3}^2}. \quad (7)$$

The ESF is different because each object’s total entropy is affected by the gravitational fields of the other objects.^{5,6} Again, this effect can be seen by examining Schwarzschild black holes. The Schwarzschild radius is given by Eq. (8), where M is the mass of the black hole and c is the speed of light

$$r_S = \frac{2GM}{c^2}. \quad (8)$$

Since the Schwarzschild radius depends on the speed of light, a decrease in the apparent speed of light from gravitational time dilation will increase the apparent radius of the black hole, and consequently its apparent entropy. For example, if the gravitational field from another object causes light to appear to move half as fast near a black hole, an observer will measure the black hole to have four times the

radius as it would have had in empty space. Since the black hole's entropy is proportional to the square of its radius, this means that the black hole will appear to have 16 times the entropy as it would have had without the additional time dilation from another object.

Using the non-relativistic coordinate framework described in the previous section, the apparent change in the speed of light due to the ESF is given by Eq. (9), where c_W is the speed of light at an object and θ is the angle between the observer, the black hole, and the object

$$c_W = c \left(1 - \frac{r_S |\sin \theta|}{d} \right). \tag{9}$$

The apparent speed of light does not change for two-object interactions, like those in the previous section, because $\sin 0$ is 0. This is to say that photons traveling in this direction appear to travel more slowly, due to time dilation, but they also appear to have less distance to travel, because standard lengths appear to be longer. For three-body systems that are not in a line, however, light will appear to move more slowly at each object, because of time dilation that has not been cancelled out by length dilation.

The Schwarzschild radius (Eq. (8)) can be combined with Eq. (9) to give the apparent radius of black hole 3 (r_{W3}), from the perspective of object 1, due to the gravitational field of black hole 2, as in Eq. (10). In this equation, r_{S3} is the Schwarzschild radius of black hole 3, r_{S2} is the Schwarzschild radius of black hole 2, and d_{2-3} is the distance between black hole 2 and 3. The apparent speed of light at black hole 3, due to the gravitational field of black hole 2, is (c_{w3-2}) .

$$r_{W3} = \frac{2GM}{\left(c \left(1 - \frac{r_{S2} |\sin \theta|}{d_{2-3}} \right) \right)^2} = \frac{r_{S3}}{(c_{w3-2})^2}. \tag{10}$$

This radius can be combined with the Hawking equation (Eq. (2)) to give the apparent entropy of black hole 3 (S_{W3-2}), from the perspective of object 1, as modified by the gravitational field of black hole 2, as

$$S_{W3-2} = \frac{k_B 4\pi \left(\frac{r_{S3}}{(c_{w3-2})^2} \right)^2}{4l_P^2} = \frac{S_3}{(c_{w3-2})^4}. \tag{11}$$

In the ESF, the force on black hole 3 would be given by the following equation, Eq. (12), in which S_{W1-2} is the entropy of object 1, modified the gravitational field of object 2, and S_{W3-2} is the entropy of object 3, modified by the gravitational field of object 2, and so on.

$$F_3 = w \frac{\sqrt{S_{W1-2}} \sqrt{S_{W3-2}}}{(d_{1-3})^2} + w \frac{\sqrt{S_{W2-1}} \sqrt{S_{W3-1}}}{(d_{2-3})^2}. \tag{12}$$

Systems with more than three bodies would require multiple interactions to calculate apparent entropy. Equation (13) gives the gravitational entropy of object 4, from the perspective of object 1, due to the fields of objects 2 and 3.

$$S_{W_{4-2,3}} = \frac{S_4}{(c_{w_{3-2}})^4 (c_{w_{4-3}})^4}. \quad (13)$$

5. Inertial Entropy

In the ESF all changes in the scale of spacetime are due to entropy, including changes from inertial reference frame.⁴ For this to be true, an increase in relative velocity must correlate with an increase in microstates of position and momentum. The number of microstates of position and momentum due to inertial reference frame is termed as “inertial entropy.”

The rationale for inertial entropy can be seen by considering Compton scattering. In this process, an electron starts in a rest frame and is accelerated by a photon. As the energy of the incident photon increases there is a slow decrease in the wavelength of the recoil photon, corresponding to a small decrease the number of possible positions for the electron, due to improved resolution. There is a faster increase in the momentum of the recoil electron, corresponding to a large increase in the number of possible momenta for the electron.⁴ This mismatch in rates results in an increase in the total number of possible microstates of position and momentum for the electron as its velocity increases. The increase in the total number of possible microstates of position and momentum can be interpreted as an increase in entropy.

Inertial reference frames can be described by Lorentz transformations, which are linear coordinate transformations depending on relative velocity. The Lorentz transformation for distance results in increased lengths in the moving frame, which could be termed as “length dilation.” Special relativity adjusts this value to account for time dilation in the moving frame, resulting in the more familiar length contraction. The underlying length dilation of Lorentz transformations is still useful, however, because they result in a coordinate framework similar to what the ESF proposes for gravity, in that changes in length are independent of time dilation.

The time dilation and length dilation of Lorentz transformations mean that light moves at c regardless of reference frame, which forms the basis of special relativity. With the speed of light independent of velocity toward or away from an observer, the apparent entropy of a black hole will not change depending on motion in this direction. This is analogous to how a two-body gravitational system in the ESF does not change the apparent entropy of the two bodies. Therefore, the total entropy of an object includes its apparent entropy, plus the inertial entropy in the direction toward or away from an observer. This inertial entropy does not change the entropy of the object, it is instead responsible for the Lorentz transformations of special relativity.

Unlike gravity, relative velocity is a property of objects, not of fields. Since the relative velocity of an object does not increase the microstates of position and

momentum for nearby objects, the entropy of these nearby objects does not change. Consequently, the change in scale due to inertial entropy is limited to the objects themselves and will not result in a force between objects.

The entropy of a black hole in the rest frame (S) is given by the Hawking equation (Eq. (2)). If, like a spherically symmetric gravitational system, the increase in entropy of relative velocity is proportional to the square of the change in scale, then the inertial entropy (S_I) can be calculated as

$$S_I = \frac{S}{1 - \frac{v^2}{c^2}}. \quad (14)$$

6. Discussion and Conclusions

This paper is the first to provide equations for calculating entropy due to gravity and due to relative velocity. It also modifies Newton's law of universal gravitation to depend on entropy, instead of mass. In doing so, it relies on the close correlation between mass and entropy for many forms of matter.¹

Previous papers on the ESF have lumped gravitational and constellation entropy together.^{5,6} While both are related to gravity, they were broken apart in this paper to emphasize the differences between two-body systems and those with three or more gravitational bodies.

The examples in this paper use black holes because their entropy is defined unambiguously. Defining the entropy of stars is more complicated because a star's total entropy involves a combination of the thermodynamic, gravitational and constellation entropy for the star's many components. Further work defining the total entropy of stars will be needed in order to refine the ESF's predictions. This work may also refine the value of the gravitational constant, w .

Tests of gravity, including those for general relativity, are typically done in simple systems with one or two gravitating bodies. These tests may not be sensitive ways to evaluate the predictions of the ESF, which (in low acceleration situations) varies most from the above theories for systems with three or more interacting bodies.

Constellation entropy can only add to an object's total entropy. This means that if the ESF makes similar predictions to Newtonian gravity in simple systems, in complicated systems the ESF will predict a stronger force of gravity than does Newtonian theory. This increase in gravity may be able to explain galaxy rotation dynamics, galaxy cluster dynamics, and other phenomena that have been attributed to dark matter, without the need for dark matter.

One reason the ESF deserves further study is that it does not just apply to the dark matter phenomena. The changes in scale predicted by the ESF may also be able to explain the expansion of the universe, both in the inflationary epoch and in the accelerating universe.²⁻⁴ The ESF may also explain other mysteries involving the space around entropic objects, like the solar corona heating problem and the unexplained power of relativistic jets.^{10,11}

A fully relativistic formulation of the ESF will be needed to explain the many tests of general relativity. Many additional studies are needed to tell if the ESF is a good theory of nature. This paper provides a framework for designing such studies.

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