Space-time software: Computer graphics utilities in special relativity

Edwin F. Taylor
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 25 March 1988; accepted for publication 19 July 1988)

No one can experience directly the world of the very fast, described by special relativity. Interactive graphics displays have been developed for personal computers that help students visualize this world. They are in the form of interactive graphics utilities that students use to carry out homework exercises and take-home projects. Sequential versions of these programs have been used for 3 years in classes in various institutions. This article describes the programs and reports on the educational outcomes of these computer uses.

I. INTRODUCTION

In 1905, the fastest vehicle ridden by most people was the railroad train. Today, the commercial jet airliner moves ten times as fast, but still only at approximately one-millionth of the speed of light. Although the consequences of special relativity are everywhere around us, we cannot experience directly the fundamental phenomena that it describes.

This article describes programs for the personal computer that model the world of the very fast, programs that make up in some measure for the fact that we cannot experience this world directly. It is crucial for learning that students manipulate the models themselves. This is ensured by embodying the models in interactive graphics utilities with which students carry out assigned homework and take-home projects.

II. THE COMPUTER PROGRAMS[1]

Here are brief descriptions of the computer programs in their developed form and a summary of student reactions to them in the fall of 1987.

VISUAL APPEARANCE (Fig. 1) is a demonstration program showing a one-eyed view of a rudimentary landscape seen through the windshield of a rocket ship moving with any speed up to that of light. The landscape consists of four stick figures: a large cube, a small cube, a pyramid, and a skyscraper. At high relative speeds, these objects appear to distort and rotate. Doppler-shifted colors are shown on the IBM EGA display. No current version attempts to present predicted changes of intensity. Although the operator can accelerate, decelerate, change altitude, and turn right or left, this is clearly a demonstration program rather than a utility. MIT students felt the "classroom explanation of what it did was good." Otherwise, they found it "interesting, but not particularly useful."

SPACETIME combines four diplays, all of which share the same data. First is the "position versus velocity" view of a multilane highway (Fig. 2) on which move clocks, rods, light flashes, and a shuttle that can change lanes. A second display is the "position versus time" view of a conventional space-time diagram (Fig. 3—horizontal axis space, vertical axis time), showing events and the world lines of objects that move along the highway. The operator places objects and events on the highway and places events, light cones, and invariant hyperbolas on the space-time diagram, steps time forward and backward, rides on any object on the highway (except a light flash), and transforms the space-time diagram from one reference frame to another. A third display (Fig. 4) splits the screen to show both the highway and the space-time diagram as time is changed. The final display is a table of numerical data on events and objects (not shown). The entries in this table can be annotated by the operator.

Students were very enthusiastic about the SPACETIME program. They reported using it a lot and felt strongly that it helped them understand relativity. ("Ingenious. Something every physics student shouldn't be without. Extremely useful.") They singled out for praise the ability to jump quickly from one object to another (in HIGHWAY), from one display to another, and from one reference frame to another (in SPACETIME DIAGRAM). Readers wishing to see

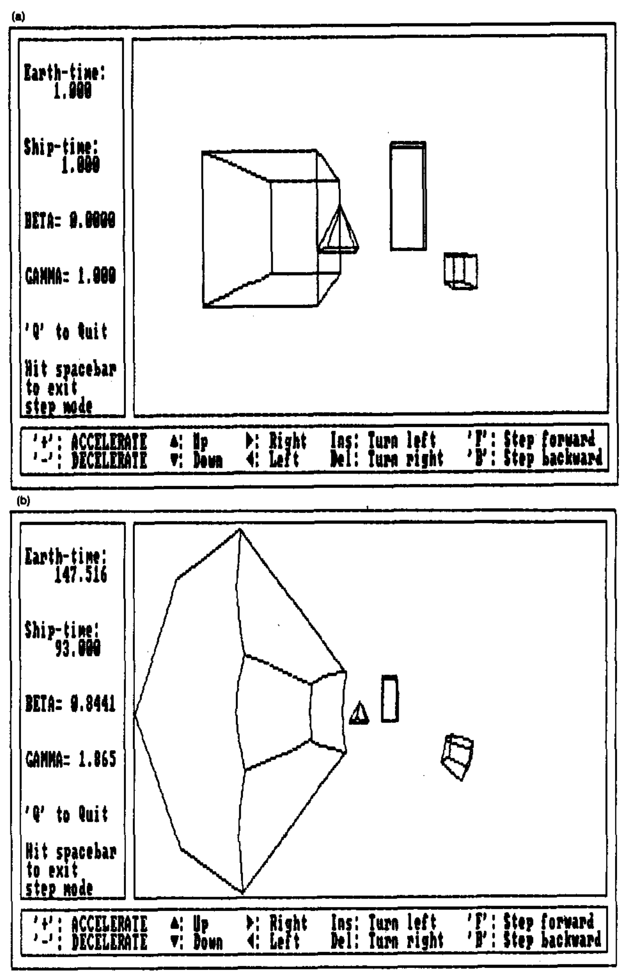

Fig. 1. VISUAL APPEARANCE program, showing point of view of (a) an observer at rest with respect to a simple landscape and (b) point of view of observer moving at 0.85 c with respect to the same landscape.

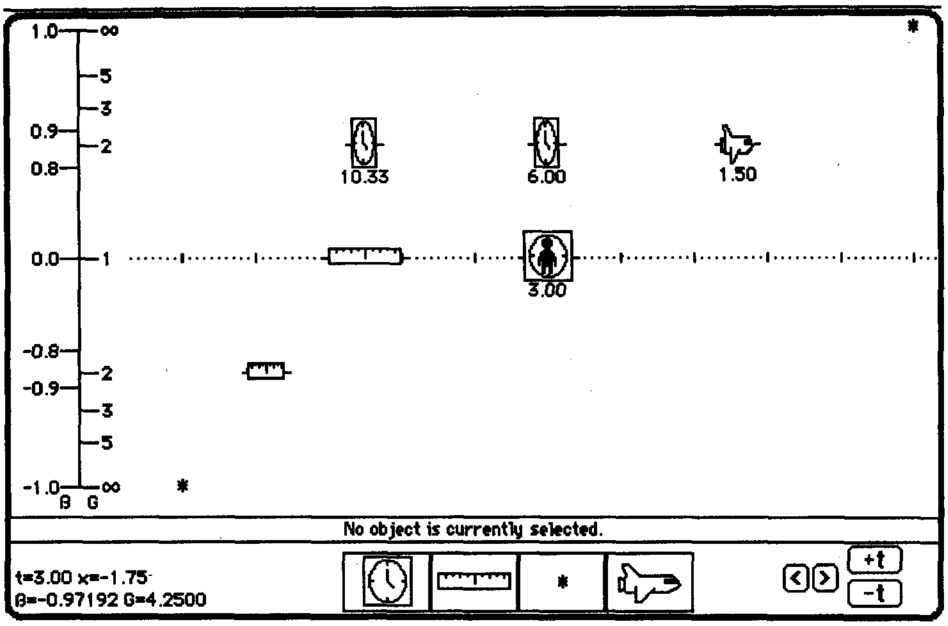

Fig. 2. The HIGHWAY display, showing clocks, rods, light flashes, and the shuttle, the only object that can change lanes. The two scales at the left show speed beta $= v/c$ and time stretch factor gamma for different lanes of the highway, as observed in the current frame.

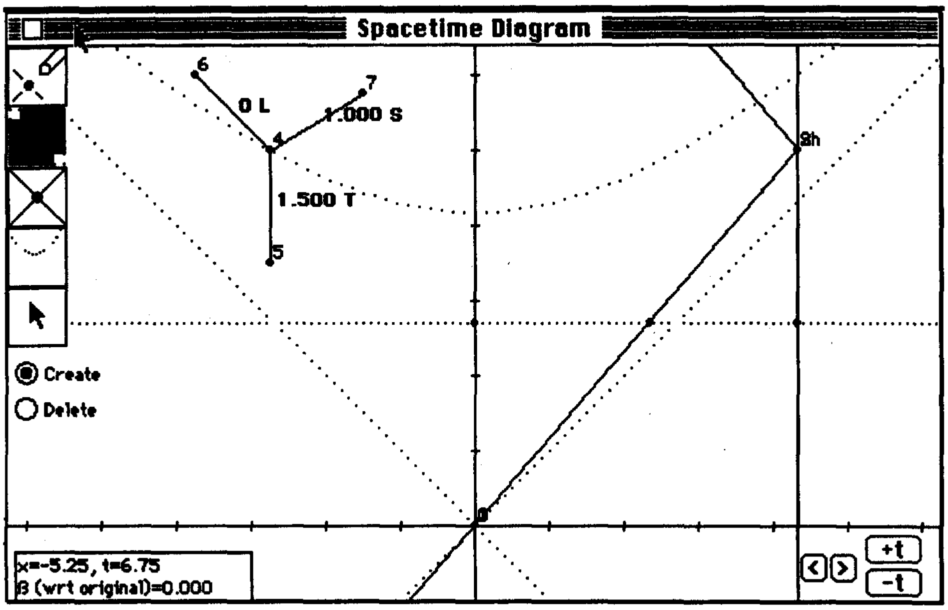

Fig. 3. The SPACETIME DIAGRAM display, showing the reversing world line of the shuttle, a light cone, an invariant hyperbola, and events with timelike, lightlike, and spacelike intervals with respect to one another.

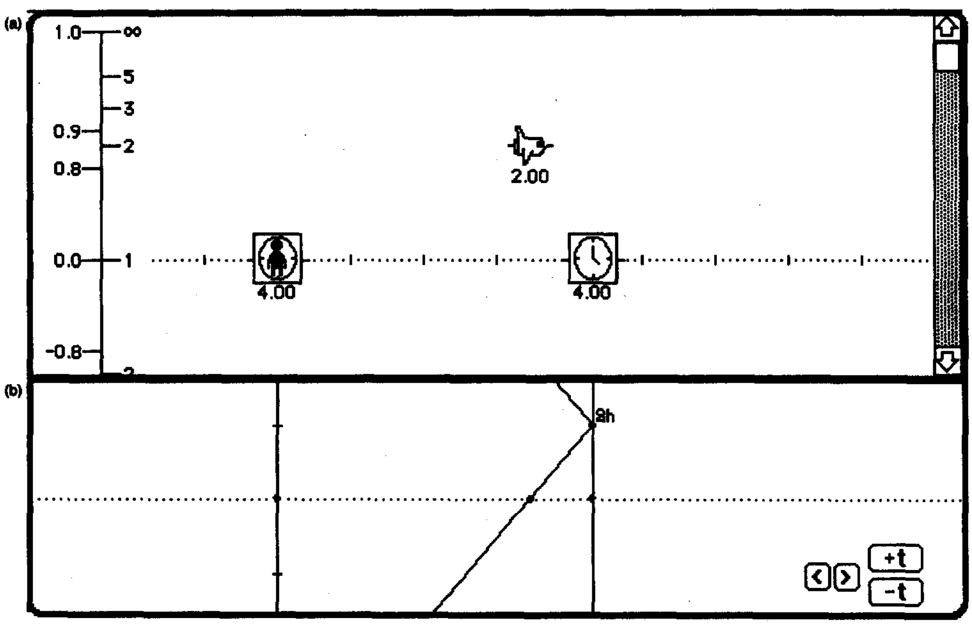

Fig. 4. The split screen, showing (a) HIGHWAY and (b) SPACETIME DIAGRAM, with two clocks and a reversing shuttle. As time is stepped forward or backward, objects move along the highway and the space-time diagram scrolls vertically past the horizontal dotted line of simultaneity, which stays vertically centered in the lower display.

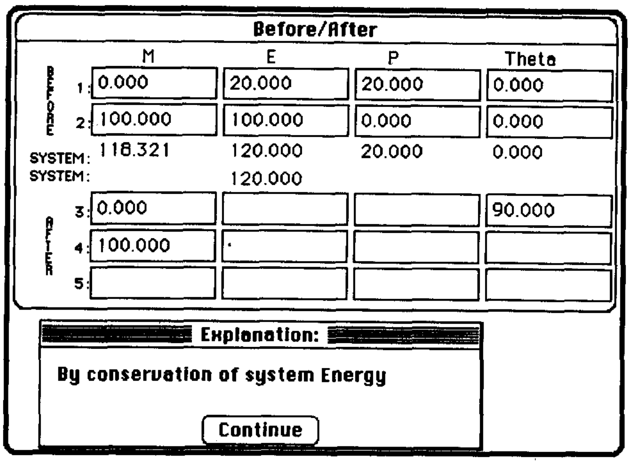

Fig. 5. The Table display in COLLISION, showing one step in the table completion routine analyzing the Compton scattering event.

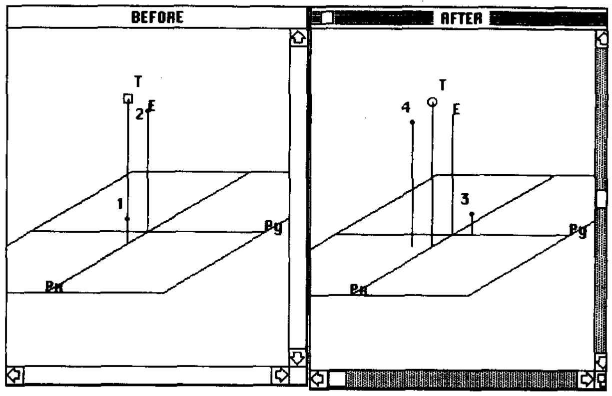

Fig. 7. The *before* and *after* energy-momentum plots for the Compton scattering event. The total energy and momentum of the system, labeled T, is the same before and after the collision, as required by the laws of conservation of momentum and energy.

a typical student exercise that uses the SPACETIME display may refer to the Appendix.

COLLISION is a program that helps students analyze relativistic collisions, creations, transformations, decays, and annhilations of particles that move in one or two spatial dimensions. The program shows three interrelated displays. The first is a table (Fig. 5) on which the operator enters what is known of the mass, energy, and momentum of each incoming and outgoing particle (all three quantities measured in the same units). The operator can enter numbers or simple algebraic relations between entries (e.g., $3x$ for the energy of one particle and x for the energy of another particle). On command, the program attempts to complete the table, giving messages about what law or equation is being used at each step. The completed interaction can then be played as a movie (Fig. 6), either run continuously or stepped forward or backward frame by frame. Perspective three-dimensional plots (Fig. 7) show energy versus x and y momentum of each particle and their total before and after the interaction. The operator can rotate these plots to make the display clearer. All three displays can be transformed to the rest frame of any particle (except a light flash), to the zero-total-momentum frame, or to a frame moving with an arbitrary less-than-light velocity in the two-dimensional spatial plane of the collision.

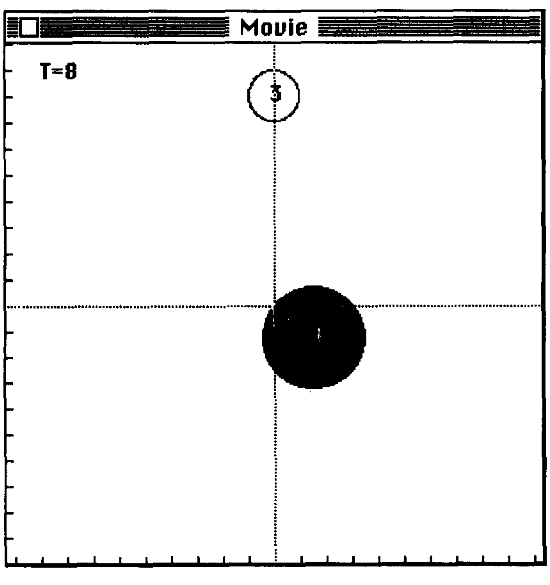

Fig. 6. A frame of the Movie showing photon (open circle) and electron (filled circle) leaving the Compton scattering event.

Students appreciated COLLISION ("Great program..." "I didn't need to do thousands of pages of algebra"), especially particular features ("The movie is great. Transformations are helpful"). COLLISION is conceptually more advanced and "technical" than SPACETIME, and this was reflected in student comments ("It's a momentum-energy calculator, nothing more"). They reported less use and on average found it less helpful in understanding relativity than SPACETIME ("Didn't make me *understand* any more").

III. THE CLASSES[2]

We report on three semesters of use of sequential versions of these graphics utilities programs in a relativity class at MIT, two semesters at Harvard University Extension School, and one semester at Franklin and Marshall College. Harvard Extension is a degree-granting continuing education school whose students have a wide variety of backgrounds and preparations. Most Harvard Extension students are employed full time, some of them in technical fields.

There were between 26 and 51 students in each of the five classes at MIT and Harvard. The largest fraction of MIT students were physics majors, forming 27% to 67% of the total, even though this is not a required course for physics majors. There were no prerequisites for the class at MIT, and students came from all four undergraduate years, with an occasional graduate student. Harvard Extension students were told they would need algebra and a little trigonometry, but no screening of potential students took place.

The class at Franklin and Marshall College, called "Relativity, Particles, and Waves," was taught by Professor Gregory Adkins. The 14 students in the class had already taken a year of introductory physics and a year of calculus. Relativity was taught in a 5-week segment at the beginning of the term.

The textbook used at MIT and Harvard, an unpublished revised draft of *Spacetime Physics* by Taylor and Wheeler, shares with the first edition[3] a tendency to use less algebra and other mathematical formalisms than most other relativity textbooks. The textbook used at Franklin and Marshall was *Concepts of Modern Physics* by Beiser.[4]

IV. INCORPORATING COMPUTER USE INTO THE CLASS

Each student in the Harvard and MIT classes received a disk with the programs on it and was given access to institutional computer facilities. The computers provided for student use at MIT were IBM AT's with graphics displays and 80287 math coprocessor chips. At Harvard Extension, the computer facility provided Macintosh 512E machines. In both cases, the facilities were for use by students in several classes and were available essentially around the clock.

Between five and eight students in each class used their own or a friend's computer. Slightly more students did this at Harvard Extension than at MIT.

At MIT and Harvard Extension, about one-third of the homework exercises required the use of the computer. In each case, computer displays were printed out and handed in with the homework. In addition, two rather lengthy take-home projects were carried out: one on space-time, the other on momentum energy. These projects made extensive use of the computer programs.

At Franklin and Marshall College, the class as a whole spent weekly 2-h sessions together in the computing center, where the program disks were kept. Students could also use the programs at the center on their own time.

At both Harvard and MIT, the computer programs were demonstrated in class with a portable computer and screen projector. The same equipment was used throughout the term to demonstrate relativistic effects under discussion and the solutions of exercises. The Harvard classes received a full user's manual for the programs. MIT students received little or no written documentation.

In the fall of 1987, students in all three classes—at MIT, Harvard, and Franklin and Marshall College—reported that they spent an average of about 1 h per week using the computers.

V. EVALUATION

Results of the first year of trials at MIT have been reported elsewhere.[5] That report incorporated some conclusions of Dr. Karen Cohen, an educational evaluator employed by MIT Project Athena, who observed the class, tabulated student computer logs, interviewed groups of students, helped to design a student questionnaire and study its results, and compared outcomes in this class with those of other classes using computers at MIT.

In addition to the regular student course evaluations, in all 3 years at MIT and for both years at Harvard Extension the students were asked to fill out in class a 10-page questionnaire designed by the instructor and containing questions about all aspects of the course, including general questions about use of computers and specific questions about individual computer programs. More than half of the students in the class returned completed questionnaires. A similar but shorter questionnaire was filled out by every student in the class at Franklin and Marshall. Student responses on these questionnaires concerning the separate computer programs were summarized above under the descriptions of these programs.

Table I shows responses to some general questions concerning computer use in the class for MIT and Harvard Extension students in the fall of 1986 and 1987 and Franklin and Marshall students in the fall of 1987. (These questions were not asked in this form in the first year at MIT, fall of 1985, but the responses for that year are consistent with those shown in the table.) The table shows that, on average, students felt the programs helped in their understanding of the subject, found the software friendly, and felt the amount of computer use was about right.

When asked to state the *best* feature of computer use, about half of the 1987 MIT students, about one-third of the 1987 Harvard Extension students, and two of the Franklin and Marshall students spontaneously used the words *see, view, visual,* or *visualize* to speak about the help the computer gave them in understanding relativity ("made it easier to visualize"). An additional one-quarter of the MIT students, more than half of the Harvard Extension students, and one Franklin and Marshall student mentioned reduction of the time spent on computation or algebra. Most of the remainder of the students in all three classes had a variety of positive comments.

When asked to state the *worst* feature of the computer use, one-third of the 1987 MIT students and 2 (out of 14) Franklin and Marshall students mentioned difficulty managing the programs. An additional one-third of MIT students worried that the programs "allowed for plugging in numbers without understanding the concepts." The final third of MIT students complained about the printers, the location of the computer facility, or the lack of a user's manual (not then written for the IBM version). Fewer of the Harvard students than the MIT students responded to the question asking about the worst feature; two of the Franklin and Marshall students said there was no worst feature and six more left the answer to this question blank.

The results shown in the table and other student responses seem quite similar over the 3 years of the trial, in spite of the fact that the computer programs became much more powerful and "finished" as time went on. This uniformity may be due to the fact that students in each class had no alternative program with which to compare the ones they were currently using. Evidently the programs were all sufficiently easy to use and trouble-free that their operation caused no major difficulties for most students.

My motivation for teaching the Harvard Extension classes was to try out the programs with students who had a wider variety of experience, backgrounds, mathematics skills, and interests than do students at MIT. As shown in the table, on average the Harvard students found the programs more helpful in understanding the subject than did MIT or Franklin and Marshall students. On average, more Franklin and Marshall students recommended increased use of the programs than did those in the other two groups. No other results significantly distinguish the three groups from one another.

Professor Gregory Adkins, who tried the programs at Franklin and Marshall College, concludes his summary report with this comment:

> I think the programs are very useful to the students in the study of relativity. They help visualization, allow relativistic manipulations...and are fun to use. I will use them the next time I teach this course, and I will recommend them to my colleagues.

Table I. Student responses to some questions about computer use.

1. In assisting my understanding of the subject, computer programs were:						
not helpful	1	2	3	4	5	very helpful
MIT 86→	1	6	6	8	7	Average: 3.5
Harvard 86→	0	2	3	2	7	Average: 4.0
MIT 87→	1	4	3	9	7	Average: 3.7
Harvard 87→	0	1	0	5	5	Average: 4.3
F & M 87→	1	2	5	5	1	Average: 3.2
2. Learning how to use the computer display programs:						
took a long time	1	2	3	4	5	was very quick
MIT 86→	1	6	4	10	7	Average: 3.6
Harvard 86→	1	1	3	4	5	Average: 3.8
MIT 87→	2	3	5	5	8	Average: 3.6
Harvard 87→	0	2	1	2	6	Average: 4.1
F & M 87→	0	2	2	8	2	Average: 3.7
3. By and large, the computer programs were:						
hostile	1	2	3	4	5	user-friendly
MIT 86→	0	1	3	14	10	Average: 4.2
Harvard 86→	0	1	3	4	6	Average: 4.1
MIT 87→	0	2	2	11	8	Average: 4.1
Harvard 87→	0	1	4	4	2	Average: 3.6
F & M 87→	0	0	5	5	4	Average: 3.9
4. On balance, I would like to see the use of computers in this class[a]						
eliminated	1	2	3	4	5	increased substantially
MIT 86→	1	5	10	11	1	Average: 3.2
Harvard 86→	0	1	9	3	1	Average: 3.3
MIT 87→	1	8	14	1	0	Average: 2.6
Harvard 87→	0	0	9	2	0	Average: 3.2
F & M 87→	1	1	4	5	3	Average: 3.6

[a] We tabulate results of No. 4 using the 1986 version of the question. In 1987 it was slightly different, asking whether the amount of computer use was "much too great" or "not nearly enough."

VI. FURTHER CONCLUSIONS

Here are some less formal conclusions that I have drawn from this experience as developer and instructor–user of these programs:

(1) Initial use of the programs at the beginning of the course brings students into immediate contact with the "basic phenomena" (e.g., time stretching, Lorentz contraction, relative synchronization of clocks), while the logical and experimental bases for these phenomena are being more slowly developed in class and readings. The explanations, when they arrive, apply to phenomena that are by then "old friends," and, therefore, are much more convincing.

(2) Few students will use the programs unless the exercises that require them are not only assigned but also collected and graded. Printouts of computer displays are an important part of this accountability. In the second year of the 3-year trial, homework was assigned but not collected or graded.[2] Few students did the homework—a sad conclusion, but repeatedly verified, both here and in other settings.

(3) Computers should be used as part of a "balanced diet" of teaching methods.[6] The one-third of the MIT students who worried that overuse of computers can encourage button pushing instead of understanding are right to worry. That is why no more than one-third of the exercises and none of the examinations are done on the computer. Computers are used to reduce the usual pathological dependency on algebraic models of the subject, not to encourage dependency on computer models of the subject!

(4) Graphics and algebra can work together. We have found it to be very important that the computer provide not only graphic displays but also numerical output. One of the most productive uses of the programs is to parallel computer solutions of specific cases with algebraic solutions of general cases. Then the solution of an exercise becomes a three-stage process: (a) visualizing the phenomenon by setting up a particular case on the computer, viewing alternative graphic representations of the solution, and watching animations of the physical process; (b) carrying out an algebraic solution of the general case; (c) checking the algebraic solution by substituting the numbers used in the specific computer case and verifying equality of results. In this way, the rigor and generality of an algebraic solution is combined with the concreteness of a visualized outcome. Preliminary trials of similar programs in quantum physics confirm the importance of numerical output from the graphics utility programs.

(5) In our work with graphics utilities, we have found an important parameter to be what Schwartz calls "the size of the sandbox"—how large a region of the subject-matter field can be explored with a single display or set of interrelated displays. If this region is too small, exploration is

limited; the student using the program continually runs into the sides of the sandbox. In contrast, one can get lost in a sandbox as big as a desert; powerful progams with many features are difficult to learn and confusing to operate. Aside from the demonstration program, VISUAL APPEARANCE, we have just two graphics utility programs that cover the entire course: one for space-time and one for momentum energy.

(6) Substantial resources of time, attention, and money are required to develop computer utilities and associated users' guides, teachers' guides, and sample problems. The relativity progams described here (along with others that proved to be blind alleys) have taken 3 years to develop. This plus a parallel effort in quantum physics has so far required a total of approximately one-half time for the author during the 3 years. Wages for student programmers for this and the related quantum effort, provided by grants from MIT Project Athena and Apple Computer, approximate $70 000, with an additional $7 000 for materials and services. These financial figures do not include benefits, overhead, costs of administration, partial salary and benefits for the author, cost of the personal computers on which the development took place or of the computer facilities used by students in classes. They also do not include large amounts of programming time provided by unpaid physics senior thesis students.

(7) Developing the programs changes the developers more than using the programs changes the users. While working on these graphics utility programs during the past 3 years, I have come to think about relativity quite differently than I did during the preceding 20 years. My current view of the subject is much more visual, more fluid, more process oriented, covering a wider range of phenomena. I am continually surprised at what the pictures show me and at the major and minor misconceptions on my part that become graphically apparent as I manipulate these pictures. The way I make progress in learning and teaching the subject now is more interactive than it used to be, harnessing a wider variety of more powerful tools, involving trial and error to a greater extent than before. In short, both my professional life and my view of physics have been transformed. I see similar changes in the student programmers who develop the displays. Few students taking the course report a similar transformation, possibly because their view of the subject is influenced from the beginning by computer displays.

These computer graphics utilities have been tried out in courses organized in fairly conventional ways. One goal was to disturb existing organization as little as possible. My overall conclusion from trials under these circumstances is a modest one: that computer graphics utilities provide a useful additional tool that helps students engage and master the subject. The degree of influence is commensurate with the hour or so a week that students used the software. *Graphics* utilities are particularly valuable in visualizing relativity, where no one has direct experience of the primary phenomena under study. Graphics *utilities* are important because they require student manipulation of the models, an involvement necessary for learning.

ACKNOWLEDGMENTS

The author is grateful to MIT Project Athena for primary financial support of this project as well as for technical advice. The International Business Machines Corporation contributed most of the computers on which development and class trials took place at MIT. Apple Computer Company provided one Macintosh computer and some funds for converting programs to that machine. Student projects and pay were arranged through the MIT Undergraduate Research Opportunities Program, and Brenda Nace of the Department of Physics supervised expenditures. James Abbott and Eric Berman were superb teaching assistants in classes at MIT and Harvard Extension. Professor Gregory S. Adkins carried out the trial at Franklin and Marshall College and provided a helpful evaluation of the results. Student programmers are the backbone of the project: Glen Myers programmed the IBM version of SPACETIME. Chris Mayer programmed the most recent IBM version of COLLISION, following earlier versions by Shawn Gaither and Kenney Ng. Eric Berman programmed the Macintosh versions of SPACETIME and COLLISON. Edward Hontz, Jr., programmed the most recent version of VISUAL APPEARANCE, developed originally by Thomas LeCompte. True colleagues in a difficult task.

APPENDIX: SAMPLE EXERCISE—THE TSAO PARADOX

A letter sent to the Massachusetts Institute of Technology by Hsien-Yen Tsao of Los Angeles poses the following paradox, which he asserts disproves the theory of relativity.

The setting: A train travels at high speed. A runner on the train sprints toward the back of the train with the same speed (with respect to the train), as the train moves forward (with respect to the Earth).

The paradox: We know that the clocks on the train run slow compared to Earth clocks. We also know that the runner's clock runs slow compared to the train clocks. Therefore, the runner's clock should run "doubly slow" with respect to the Earth. *But the runner is not moving with respect to the Earth!* Therefore, the runner's clock must run at the same rate as the Earth clock. How can it possibly be that the runner's clock runs "*doubly slow*" with respect to the Earth clock and also runs *at the same rate* as the Earth clock?

Use the SPACETIME program to analyze the Tsao para-

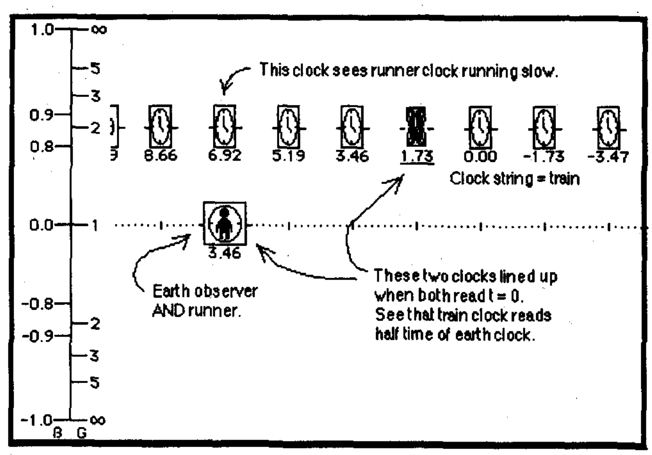

Fig. 8. Printout of HIGHWAY display showing the Tsao paradox, including possible student notation showing that, as usual, the paradox rests on the relativity of simultaneity.

dox. Let the train be moving with a gamma of 2. HINTS: Use a clock string for the train, select the central clock in the string by clicking on it so you can identify it visually, and change the time step to edge the train into the position you want. Notice that the runner, though "on the train," shows up in the HIGHWAY display standing on the zero-velocity center strip, since he is not moving with respect to the Earth. So let the reference clock represent *both* the runner *and* the Earth clock.

Submit a single printout of the HIGHWAY diplay with clear notes on it that make the resolution of the Tsao paradox as obvious to the reader as it is to you.

Solution: As true of most pardoxes, the solution to the Tsao paradox hinges on the relativity of simultaneity. The train is represented by a string of clocks synchronized in its rest frame. The clocks in this string are not synchronized in the Earth frame, which is shown in the HIGHWAY display of Fig. 8. This lack of synchronization allows the central train clock to run slow as observed in the Earth frame, while the runner's clock appears to run slow according to the train clock currently passing it.

[1]The IBM versions of the programs described in this article are distributed by Physics Academic Software, American Institute of Physics, 335 East 45th Street, New York, NY 10017; phone: 800-247-7497. The Apple Macintosh version is distributed by Kinko's ACE, P.O. Box 8000, Ventura, CA 93002; phone: 800-235-6919.

[2]The class organization at MIT and Harvard Extension generally followed that described in James B. Gerhart, "Handling Numbers," Am. J. Phys. **54**, 493 (1986). We collected and marked homework, whereas he did not.

[3]Edwin F. Taylor and John Archibald Wheeler, *Spacetime Physics* (Freeman, San Francisco, 1966).

[4]Arthur Beiser, *Concepts of Modern Physics* (McGraw-Hill, New York, 1981), 3rd ed.

[5]Edwin F. Taylor, "Computer Graphics Utilities in Special Relativity," Proceedings of the 1986 IBM Advanced Education Projects Conference, San Diego, CA, 5–8 April 1986, p. III-41.

[6]Edwin F. Taylor, "Learning from Computers about Physics Teaching," Am. J. Phys. **56**, 975–980 (1988).

Gauge kinematics of deformable bodies

Alfred Shapere[a] and Frank Wilczek[a]
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138 and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 6 June 1988; accepted for publication 12 July 1988)

The treatment of the motion of deformable bodies requires a specification of axes for each shape. A natural kinematic formulation of this problem is presented in terms of a gauge structure over the space of shapes that the body may assume. Within this framework, one may compute the net change in orientation of a body with angular momentum zero, resulting from a given sequence of deformations.

I. INTRODUCTION

Gauge potentials figure prominently in the formulation of fundamental physical laws. The abstractness of these laws, however, does not easily lend itself to an intuitive understanding of the concepts involved. Here we argue that gauge potentials arise naturally in a much more mundane, but in return more readily visualized, context—the description of the motion of deformable bodies. We hope that our exposition will provide both an introduction to some of the basic concepts of gauge theories and a useful framework for discussing the kinematics of deformable bodies.

A cat, held upside down by its feet and released at rest from a suitable height, will almost always manage to land on its feet.[1,2] A diver leaving the board with no angular momentum may perform several twists and somersaults before hitting the water.[3,4] In both cases, by executing a sequence of deformations beginning and ending at the same shape, a deformable body with nothing to push against and no angular momentum has undergone a net rotation.

In this article, we will present a convenient and natural context for computing the net rotation of a body, in the absence of external forces and torques, due to a given sequence of deformations. Our starting point is the observation that such rotations have no dependence on the rate at which the deformations are made—the equations governing the motion are invariant under time reparametrizations. Only the geometry of the sequence of deformations matters. We shall show that the rotation of a self-deforming body may be naturally expressed in a purely geometric form, in terms of a gauge potential over configuration space.

A similar kinematic framework was devised recently for the description of another problem involving deformable bodies: swimming at low Reynolds number.[5] In that case, calculation of the gauge potential required the solution of a highly nontrivial hydrodynamic problem, while here we shall be able to write the complete solution in a simple, closed form.

The configuration space of a deformable body is the space of all possible shapes.[5] We should at the outset distinguish between the space of shapes located somewhere in space and the more abstract space of *unlocated* shapes. The latter space may be obtained from the space of shapes *cum* locations by declaring two shapes that differ only by a rigid rotation to be equivalent. When no external forces act upon a deformable body, then we may always work in its center-

