

Mr. Tompkins in Java: Interactive Lorentz transformations

Prasenjit Saha^{a)}

Astronomy Unit, Queen Mary and Westfield College, University of London, London E1 4NS,
United Kingdom

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This paper describes a simple applet for illustrating Lorentz transformations. The user specifies stationary and moving objects and light pulses, and sees animations of two reference frames. Even with minimal graphics (the objects are colored dots and the light flashes are expanding circles), such animations can make the concepts of redshift, length contraction, time dilation, and non-simultaneity more intuitive than traditional spacetime diagrams. © 2003 American Association of Physics Teachers.

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I. INTRODUCTION

Lorentz transformations are one of the enduring fascinations of doing physics. The heart of our fascination is the transformation of time. We can cope with length contraction and velocity-dependent inertia, but our ordinary intuition balks at the relativity of “meanwhile.”

Acquiring a good understanding of relativity requires developing new intuition for time transformations. For this, spacetime diagrams are helpful. But viewing time as another spatial dimension diminishes the full impact of time transformation. It is much more striking if we can perceive the time coordinate with the aid of an animated visualization. In this paper I will argue that a very simple but interactive visualization can make nonsimultaneity and other Lorentzian effects easy to appreciate and make the resolution of some well-known paradoxes more obvious.

Visualizations of the Lorentz transformations have a long history. In Gamow’s *Mr. Tompkins* stories¹ from the 1940s, the hero dreams of a city where light travels at about 10 mph. Textbooks often illustrate thought experiments: Taylor and Wheeler² is a fine example. Mermin used computer graphics to illustrate relativity;³ an older ink-and-paper incarnation of it appears in his book on special relativity.⁴ More software for teaching relativity may be found in the relativity series of *Physics Academic Software*.⁵ The internet has several relativistic “flight simulators,” of which *Seeing Relativity*⁶ is noteworthy for its superb graphics. Hamilton has reviewed several flight simulators in addition to providing animations and detailed explanations.⁷ And yet computer graphics aren’t everything: *Relativity Visualized* by Epstein⁸ shows just how much can be accomplished with simple ink drawings. This little-known book overflows with insights and original ways of explaining both special and general relativity, such as a type of spacetime diagram with proper-time as an axis and coordinate-time as arc length; it has a half-genius-half-buffoon quality that surpasses even the Feynman lectures.

This paper describes a simple computer program that shows $\{t, x, y\}$ in two inertial frames related by a boost, with t represented by an animation rather than a spatial axis as in a spacetime diagram. It does not represent what we might see from a relativistic ship (and in particular does not attempt perspective, Terrell rotation, or color shifts). Despite this modest scope, however, it has two advantages that help give insight: (i) it is highly interactive, and users can pose problems that the author might not have thought of, and (ii) it

runs in a Web browser. The Java applet and source code are available at the author’s web site⁹ and EPAPS.¹⁰

In the following, I explain the ideas implemented, and show some examples illustrating redshift, time dilation, length contraction, and nonsimultaneity in a simple way. Viewing the animations over the Web is recommended, but not essential.

II. THE PROGRAM

Figure 1 shows how the program appears in a browser. There are two graphical panels, representing two inertial frames at a specified relative velocity (which is $0.8c$ for all the examples in this paper). On the screen, the red dots are fixed in the left frame and move to the rightward in the right frame; for blue dots the converse is true. But this paper uses a different scheme (because animations have been replaced by still frames): fixed dots in whatever frame are dark gray, moving dots are light gray. Circles represent wavefronts from light flashes.

A light flash is a convenient way of marking a spacetime event. The user specifies a light flash by clicking with the mouse while the animation is running, initiating an expanding circle. Let us represent an event by $\{t_l, x_l, y_l\}$ in the left frame and $\{t_r, x_r, y_r\}$ in the right frame. The coordinates are related by (using $c = 1$)

$$\{t_l, x_l, y_l\} = \{\gamma(t_r + vx_r), \gamma(x_r + vt_r), y_r\} \quad (1)$$

and its inverse

$$\{t_r, x_r, y_r\} = \{\gamma(t_l - vx_l), \gamma(x_l - vt_l), y_l\}. \quad (2)$$

Animations show the frames while keeping $t_l = t_r$.

Marking objects is a little more complicated, because it involves specifying a world line rather than just an event. The user can “set the scene” with objects at $t_l = t_r = 0$ and then start the animation. (As mentioned above, light flashes can be marked while an animation is already running.) Suppose the user wants to mark an object stationary in the right frame. To do so, she/he would select the blue button and then click in either frame, whereupon a blue dot appears in both frames. At $t_l = t_r = 0$, we have from Eq. (2)

$$\{x_r, y_r\} = \{\gamma x_l, y_l\}, \quad (3)$$

or equivalently

$$\{x_l, y_l\} = \{\gamma^{-1} x_r, y_r\}. \quad (4)$$

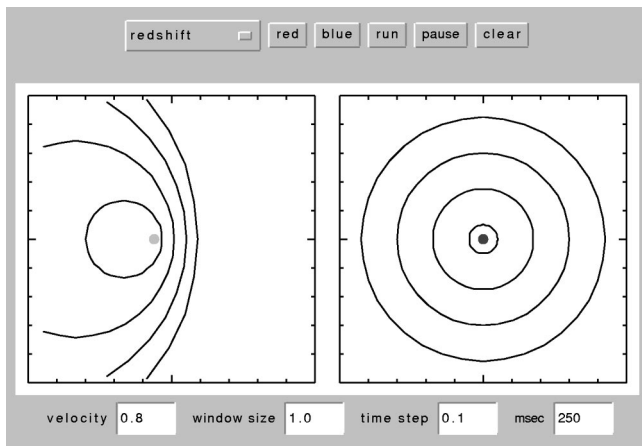


Fig. 1. A screen shot of the applet. Common browsers will run the program if Java is enabled, but the exact appearance will vary. Subsequent figures will show only the two main graphical panels. In all the figures, dark gray dots are stationary, light gray dots are moving, and circles represent expanding wavefronts.

For objects stationary in the left frame, replace blue by red and swap the subscripts l and r . Note that clicking in either frame specifies coordinates in both frames.

The asymmetry between Eqs. (3) and (4) might seem puzzling. However, note that these equations represent not just boosts, but boosts followed by a mapping to zero time coordinate. Because $\{x_r, y_r\}$ is stationary, mapping to $t_r=0$ in Eq. (3) makes no difference; whereas because $\{x_l, y_l\}$ is moving, the mapping to $t_l=0$ changes the appearance of Eq. (4). From the user's point of view, Eqs. (3) and (4) simply illustrate length contraction.

III. EXAMPLES

Users can set up their own scenarios, but four examples are included. Unfortunately, the program cannot illustrate velocity addition or the twin paradox because those problems require at least two Lorentz transformations, whereas the program is restricted to one.

A. Redshift

Figure 2 shows wavefronts of light flashes for a stationary and moving source. The moving source produces nonconcentric circles, illustrating how light spreads around the event of emission, not around the worldline of the source. It becomes obvious that observers behind will see a redshift and observers ahead will see a blueshift. Moreover, it is clear that observers perpendicular to the source motion will also see a redshift. (Incidentally, sound spreads with respect to the worldline of the medium. So a wind can produce a Doppler shift with neither source nor listener moving.)

B. Time dilation

Figure 3 represents light moving forward and back along a moving rod. The rod is stationary in the right frame and has length ℓ ; in the left frame it is moving and has length ℓ/γ . In the right frame, light takes time ℓ on each of the forward and return trips (note that we are using $c=1$). In the left frame, light takes a time $\ell/[\gamma(1-v)]$ going forward (much longer

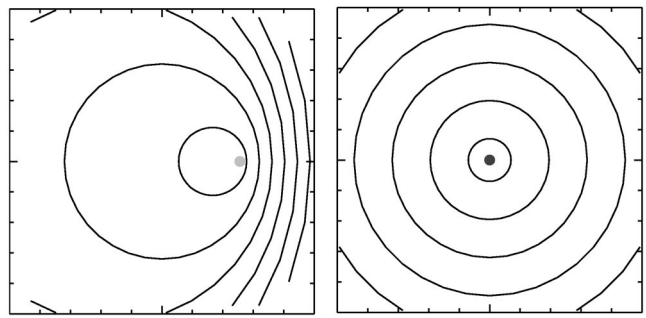


Fig. 2. Illustration of redshift. The circles represent light flashes from a source stationary in the right frame but moving to the right in the left frame. (Under Galilean transformation, the left frame would have concentric circles all moving with the source.)

than in the right frame) and $\ell/[\gamma(1+v)]$ going back (shorter than in the right frame), resulting in a total time of $2\ell/[\gamma(1-v^2)]$ or $2\gamma\ell$.

Playing with this animation invites the user to think about the following paradox. If the user sets off a light flash by

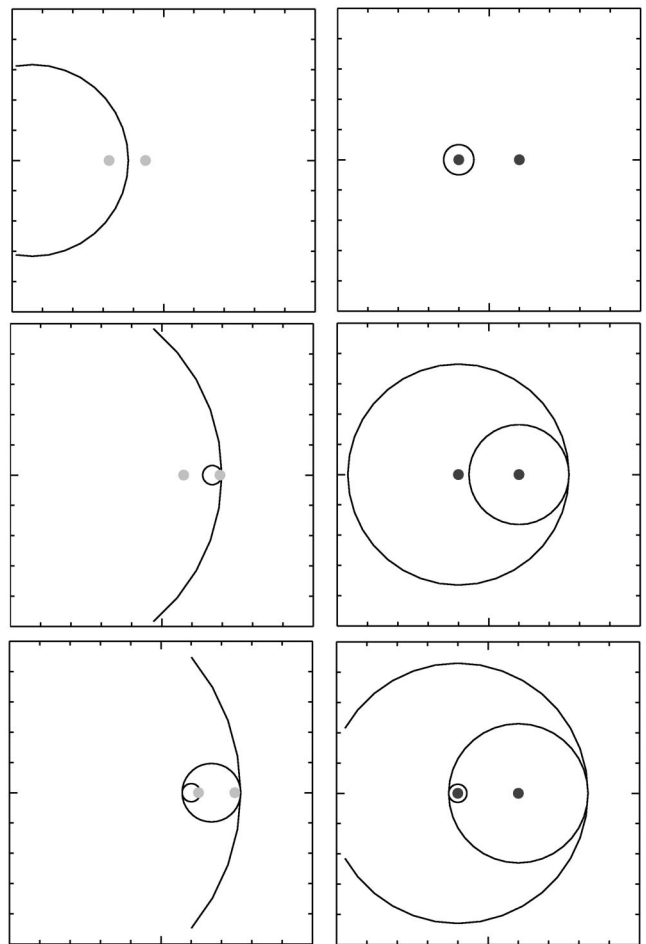


Fig. 3. Illustration of time dilation. The two dots (say A and B) are stationary in the right frame and moving in the left frame. Light travels from A to B and back again. In the upper panels, A has emitted a light flash for which $t_l < t_r$. In the middle panels the light has reached B and been reflected, at $t_l > t_r$. In the lower panels the light reached A and has been reflected again, and this event has $t_l = t_r$.

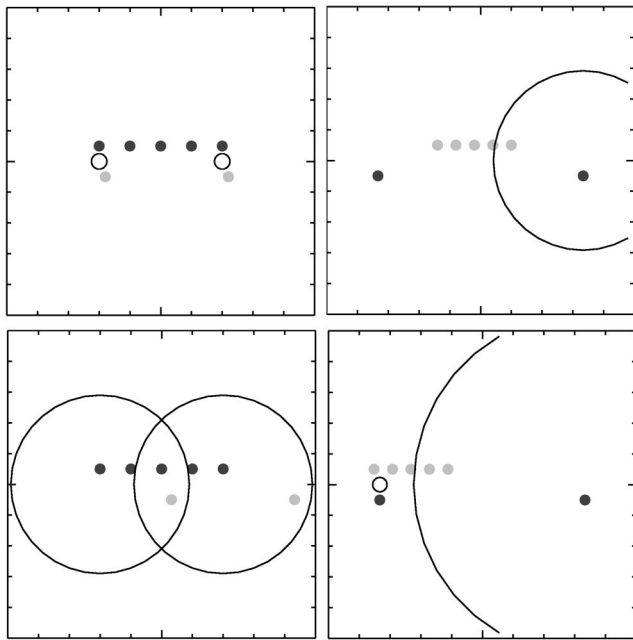


Fig. 4. Illustration of the javelin problem. The row of five dots represents the barn and is stationary in the left frame, while the other two dots represent the javelin and are stationary in the right frame. Light flashes go off when the head of the javelin reaches the right end of the barn and when the tail of the javelin reaches the left end of the barn. These light flashes are simultaneous in the left (barn) frame. But in the right (javelin) frame, the tail event occurs after the head event but *before* a light flash from the head event can reach the tail.

clicking at some t_r in the right frame, the left frame may have the same flash at $t_l < t_r$. But how does the left frame “know” at t_l what the user will do in the right frame at $t_r > t_l$?

C. Length contraction in one dimension: The javelin paradox

The third example is a well-known length-contraction paradox. One version of it involves a long javelin, a short barn, and a relativistic sprinter. The sprinter runs with the javelin into the barn fast enough to shorten the javelin to the barn’s length; as the rear of the javelin enters the barn, someone closes the barn door. Does the javelin really fit into the barn?

Figure 4 resolves the paradox. In the javelin frame, the barn door does close. But the tip of the javelin has reached the far wall of the barn before that. The tip must either cut through the wall or get stopped by the wall, but the news will not have gotten to the rear of the javelin before the barn door closes.

D. Length contraction in two dimensions: The frisbee paradox

You are in the back seat of a car, and having nothing better to do you open the sunroof and throw a frisbee up and forward through it. The frisbee is actually wider than the sunroof, but thanks to your relativistic frisbee-throwing skills, length contraction shortens the frisbee to fit. But in the frisbee frame it is the sunroof that has contracted, so how can the frisbee possibly fit?

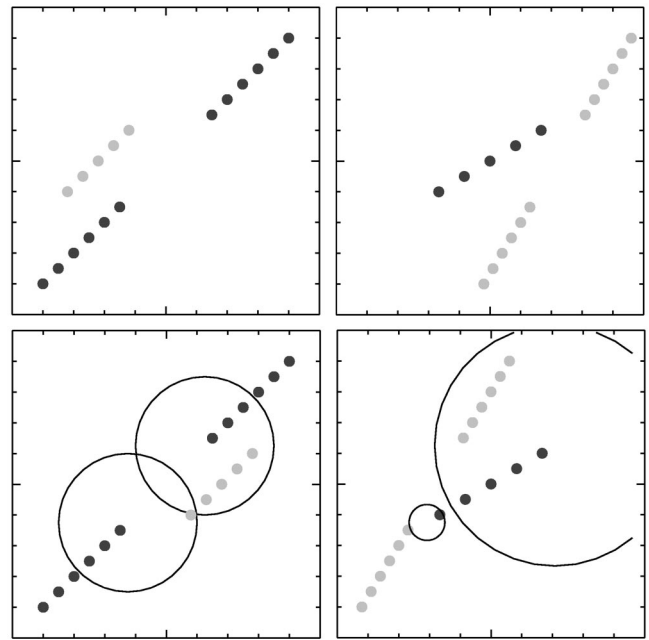


Fig. 5. Illustration of the frisbee problem. The frisbee is represented by five dots stationary in the right frame and the sunroof by ten dots stationary in the left frame. (The orientation is awkward in this figure; the “road” is to the upper left.) Light flashes go off as the edges of the frisbee cross the edges of the sunroof. Because length contraction affects only the direction of motion, the orientations of frisbee and sunroof depend on the frame. Consequently the edge-crossings are simultaneous in the left (sunroof) frame but not in the right (frisbee) frame.

This is one version of a less well-known length-contraction paradox. (See Ref. 8 for more versions.) Figure 5 illustrates it. (We neglect problems concerning the rotation of the frisbee here.) The point is that a moving object that is not aligned with the direction of motion will be tilted as well as contracted.

IV. DISCUSSION

The initial motivation for writing two-frame animations of $\{t, x, y\}$ was to present standard examples in a simple yet appealing way. But after setting up and viewing a number of examples, such animations became quite interesting, independently of the particular examples.

Consider the following features of the animations:

- (1) A moving pattern of dots is length-contracted. The contraction factor noticeably increases with velocity, but the exact dependence is not obvious.
- (2) Light wavefronts are immune to length contraction. They remain circles in both frames.
- (3) An event can be marked in several ways: by a light flash, by a light wavefront crossing a dot, or by a moving dot crossing a stationary dot. If two event markers coincide in one frame (for example, a light flash goes off just as the tip of the javelin reaches the far end of the barn), they will coincide in both frames.

The assertions 1 and 3 (that is, length contraction and the event concept) imply time dilation. Gamow makes this point with beautiful simplicity. Mr. Tompkins in his dream city notices length contraction first: a passing cyclist appears unbelievably shortened, and when Mr. Tompkins gets on a bi-

cycle himself, the street appears shortened. Time dilation and the non-invariance of simultaneity follow as consequences of length contraction: because the street is shortened, Mr. Tompkins can cycle past five blocks in a few breaths.

We can translate Mr. Tompkins's experience into more formal language as follows. Suppose x_1 represents a dot stationary in the left frame and x_2 is stationary in the right frame (that is, x_{1l} constant, x_{2r} constant). If we write v for the velocity and γ for the length-contraction factor, assertion 1 implies that

$$x_{2l} = \gamma^{-1}x_{2r} + vt_l, \quad x_{1r} = \gamma^{-1}x_{1l} - vt_r \quad (5)$$

with the function $\gamma(v)$ as yet unspecified. Now let x_1 cross x_2 . By assertion 3, we may express this event as $x_{1l} = x_{2l} = x_l$ in the left frame and at $x_{1r} = x_{2r} = x_r$ in the right frame. If we substitute x_l and x_r in Eq. (5) and rearrange the terms, we obtain

$$\{t_r, x_r\} = \{\gamma(t_l - wx_l), \gamma(x_l - vt_l)\}, \quad w \equiv (1 - 1/\gamma^2)/v. \quad (6)$$

If we take $\gamma = 1/\sqrt{1-v^2}$, or equivalently $w = v$, then Eq. (6) is a boost. If we take a different $\gamma(v)$, then Eq. (6) still retains two of the familiar properties of a boost: (i) it can be inverted by replacing v by $-v$, as may be verified by direct substitution; (ii) it produces a time dilation of γ , that is, $(\partial t_r / \partial t_l)_{x_l} = \gamma$.

Assertions 2 and 3 (that is, the constancy of the speed of light) then imply that $\gamma(v)$ can only be $1/\sqrt{1-v^2}$. To see this, recall the time-dilation example (Fig. 3 and the accompanying discussion). In that example we used kinematics and

the constant speed of light to calculate the back-and-forth light travel time in the left frame as $2l/[\gamma(1-v^2)]$, and then used $\gamma = 1/\sqrt{1-v^2}$ to derive the dilated time $2\gamma l$. A different $\gamma(v)$ would give a time-dilation factor $\neq \gamma$, thereby contradicting the previous paragraph.

We have now expressed Lorentz transformations in "animation-oriented" language. Statements 1–3 carry all the quantitative information about a boost, but without equations and without mentioning time-transformation explicitly at all.

^{a)}Electronic mail: p.saha@qmul.ac.uk; <http://ankh-morpork.maths.qmul.ac.uk/~saha>

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⁹The author's Web site is (<http://ankh-morpork.maths.qmul.ac.uk/~saha/teach/lorentz/>).

¹⁰See EPAPS Document No. E-AJPIAS-71-013311 for the software, including examples, and related documentation. A direct link to this document may be found in the online article's HTML reference section. The document may also be reached via the EPAPS homepage (<http://www.aip.org/pubservs/epaps.html>) or from <ftp.aip.org> in the directory /epaps. See the EPAPS homepage for more information.