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^{a)} Permanent address.

The amazing many colored relativity engine

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

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A simple microcomputer program is described that makes beginning students think relativistically about elementary space and time measurements. The program allows such students to perform their own *gedanken* experiments to learn the consequences a clock synchronization convention has for measurements of the lengths or speeds of moving objects and the rates of moving clocks. To use the program requires no knowledge of physics, algebra, or geometry.

I. AN ENGINE FOR TEACHING RELATIVITY

I describe below a simple method for teaching the elementary (special) relativistic properties of space and time measurements. This approach was developed and refined during several teachings of a "general education" course on special relativity for nonscientists. Such students are not at home with elementary algebraic manipulations and a direct derivation of the Lorentz transformation along conventional lines is beyond their conceptual powers (nor would they be able to use the equations if they had them). Even a step by step derivation of length contraction, time dilation, and the relativity of simultaneity requires a sophistication with algebraic formalism beyond the attainments of most of these students, and they are entirely distracted from genuine physical and philosophical subtleties by their marginally successful efforts to cope with the mathematics.

One can remove much of this irrelevant but overpowering obscurity by replacing algebra with arithmetic. To make many of the conceptual points it is not necessary to derive everything for general values of v/c . Almost everything is illustrated just as well by special cases, for example $v/c = \frac{3}{4}$. Many students who strain mightily over $(1 - v/c)(1 + v/c) = 1 - v^2/c^2$ will swallow $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12}$ with relative ease.

But the arithmetical approach cannot avoid all algebra, and for some students even arithmetic distracts from the real issues. Stimulated by this chronic source of frustration, I was led to design what might be characterized as a hands-on *gedanken* experiment. In this *gedanken* laboratory length contraction, time dilation, relativity of simultaneity, velocity addition, and the like are extracted directly from measurements made with one simulated set of instruments to determine the properties of another such set. No calculations are required beyond the arithmetical manipulation of data. The mutual consistency of all measurements is self-evident. And all the major relativistic effects are directly revealed.

My original idea, in the late 1960s, was to build this pe-

dagogical device into something like a relativistic slide rule—an analog computer, with various sets of gear-linked counter-rotating concentric cylinders representing moving meter sticks and the readings on attached clocks. I imagined the whole thing in brass on a mahogany base, driven by a large ebony-handled crank: a Relativity Engine.

They laughed when I brought them drawings. Nobody would make it for me. So I had to make do with sheets of paper schematically depicting various slices of this gorgeous space-time Relativity Engine.

I got by with pieces of paper throughout the next decade and a half until I noticed that slide rules had vanished, and the campus was suddenly teeming with little computers. The time had come to simulate my Engine on the display screen; students would be able to turn the crank at any of hundreds of conveniently located keyboards. Mahogany and brass were out, but color was in.

This article describes my digital Relativity Engine.

Section II gives a general view of the Engine. It is written with teachers of physics in mind, but it should be intelligible to students with some prior acquaintance with special relativity who might also enjoy playing with the Engine.

Section III is an introduction to the Engine for students who have no acquaintance whatever with special relativity. It is in effect a user's manual. It will also provide teachers with a more detailed picture of how the Engine operates and the pedagogical possibilities it gives rise to. Nowhere in Sec. III do I attempt to put the Engine into the context of an introductory course in special relativity: There are too many different ways one might try, and it seemed to me this was really a matter for the judgment of the individual teacher.

Some suggestions along these lines are given in Sec. IV A, which is again addressed to teachers. Section IV B gives a brief formal analysis of the Engine, which might help those wishing to design Engines of their own with parameters different from mine. Section IV C discusses a few technical features of my computer program and tells interested readers how to get a copy for themselves.

II. INTRODUCTION FOR SOPHISTICATES

Place yourself on a space platform somewhere out in the void and imagine two long straight trains (one red, one blue) of identical rockets moving past you and past one another in opposite directions at the same speed. Occupants of the red train are going to measure various properties of the blue one, and vice versa.

The first thing to notice about the Relativity Engine is that it presents both trains from the point of view of a *third* observer (on the space platform), symmetrically situated between the two.¹ Since, except for their directions of motion, the blue and red rocket trains appear entirely equivalent from the third (platform) point of view, it is evident and unavoidable that whatever conclusions the blue observers reach about the red rockets, the red observers will conclude the same about the blue. Reciprocity is built in.

Each rocket in each train has one porthole window halfway along its length. Alongside the window is the number of the rocket. (The rocket at the front of each train is number 1, the second rocket, number 2, etc.)

Each rocket carries a clock, which can be read by somebody inside the rocket, or just as easily, through the window, by somebody outside. All the clocks on either set of rockets are identical and, in particular, they all run at the same rate. Indeed, as observed from the space platform, there is only one thing at all peculiar about either the rocket trains or the clocks: On neither train are the clocks synchronized. The clock in any rocket (of either train) is ahead of the clock in the rocket just in front of it (and behind the clock in the rocket just behind it). The amount of this disparity is the same from one rocket to the next, and is the same for both rocket trains.

This asynchronization can be viewed in two ways. One can regard it as the ordinary relativistic asynchronization (of order v/c) to be expected in the platform frame, if the clocks are properly synchronized in the frames of the trains on which they reside. Alternatively, however, one can regard the speed of the trains as being so small compared with the speed of light that all relativistic effects are utterly negligible. One can nevertheless imagine the clocks to have been deliberately asynchronized by somebody, either a practical joker, or a professor of physics, out to make a pedagogical point.

The *second* point of view is greatly to be preferred, at least until the students have acquired good *gedanken* experimental technique. One can, at this stage, guiltlessly retain one's Newtonian sense of absolute time, taking the position that the clocks on each train really are asynchronized. *But*—and here is where the fun begins—the people in the rockets do not know that their clocks disagree. On the contrary, they have been lied to, and told that the clocks in each rocket are in perfect agreement with each other. People in different rockets of the same train have no way of communicating with each other to check this—the rockets have no radios, or provisions for space walking from one to the next. In any event, the people on each train have no reason to be suspicious. Having been assured that their clocks are properly synchronized, they will act on this assumption in interpreting such information as they are able to gather about the other train.

Each train has just one way to gather information about the other. When any two rockets are directly opposite each other, then (and only then) people in the window of one

rocket can see the serial number and clock of the opposite rocket. Each rocket is equipped with a camera that can record the moment when its window faces the window of a rocket on the opposite train. The resulting photographs (which, of course, contain the same information regardless of which train they are taken from) record the numbers of the two adjacent rockets and the readings of the two clocks they are carrying.

As the two trains pass each other, people in each rocket are free to take photographs whenever the window of a rocket of the other train is outside their own window. When the two trains have completely passed each other the game ends. The rockets land at Edwards Air Force base, and everybody goes off with their photographs. The people from the blue rockets take their pictures to a conference center in Houston, to discuss what they can conclude about the red train. The people from the red rockets retire to a motel near Cape Canaveral to study the blue train from their own photographs.

Since whenever people on both of two adjacent rockets take photographs of each other's clocks and rockets, the two photographs agree, the data available to one group contains no photographs that contradict the other group's data. Indeed, had each group (somewhat wastefully) chosen to take photographs at all possible opportunities, both groups would have identical data to analyze. This is evident from the operation of the Engine, since the way in which students use it to take photographs is quite symmetric between the two trains.

As it turns out, appropriately chosen *pairs* of photographs provide all of the interesting information. The relativity Engine permits students to take and study such pairs of photographs.

The trains appear on the upper half of the display screen. Each rocket has its serial number stamped on it. The reading of its clock is displayed next to the rocket. Touching the up-arrow key lets time advance by a small step: Each train moves forward a bit, and the reading of each clock advances. One can undo the passage of time by touching the down-arrow key, which restores the previous configuration. Thus by holding down first the up- and then the down-arrow key one can search forward and backward in time for suitably informative moments in the history of the two rocket trains.

When one finds a moment of particular interest one can make a photograph. This can only be done when the rockets are directly opposite one another (since the information can only be collected when the windows face each other). By touching either the right- or left-arrow key one calls up a pointer between the two trains. Further pressing of these keys moves the pointer to the right or left along the line of rockets until it reaches the pair of rockets whose photograph one wishes to record. Pressing a designated shutter key then records the photograph taken in (either of) those rockets at the lower left on the screen (safely out of the way of the two trains). One can then search further through time (with the up- and down-arrows) and space (moving the pointer with the right- and left-arrows) until one finds another suitable pair of juxtaposed rockets. Pressing the shutter results in a second photograph appearing at the lower right.

The student is invited to use the Engine to take suitable pairs of photographs from which, for example, the people on the red train, assuming their clocks are correctly syn-

chronized, can conclude (a) how fast the blue train is going, (b) the extent of the asynchronization of the blue clocks, (c) how fast the blue clocks are running compared with their own red clocks, or (d) how long the rockets of the blue train are compared with their own red rockets. It is evident from the symmetry of the situation (but students should be invited to verify it directly anyway) that people from the blue train, assuming that *theirs* are the correctly synchronized clocks, will reach the same conclusions about the red train.

One can also call up a meteor that moves parallel to the two trains in the space between them at a third speed, specified in the symmetric platform frame by the user. Should two rockets be positioned for a photograph while the meteor happens to be directly outside of their windows, then the meteor appears in the photograph. One can use pairs of photographs in both of which the meteor appears, to measure its speed from either train. By introducing meteors of appropriate speeds one discovers some interesting things. There is, for example, a special speed with the peculiar property that if a meteor is given that particular speed, then the speeds observers on either train find for the meteor are the same (even though the trains are moving in opposite directions). If one puts down a meteor moving sufficiently faster than the special speed one finds that observers on the two trains can disagree on the order in which two photographs of such a meteor were taken, with interesting consequences if the meteor is behaving in an irreversible manner (for example, burning up). By putting down meteors with a range of other speeds one can try to discover (or verify) the relativistic velocity addition law.

In this way, after several hours of playing with the Engine, students will learn for themselves that under the assumption that their own clocks are synchronized, observers on either train will conclude that clocks on the other run slower than theirs and that rockets on the other are shorter than theirs, with specific numerical measures of the extent of the slowing down and shrinkage. They will produce numerical measures of the speed of one train, accord-

ing to the other, and measurements of the extent to which the clocks of one are out of synchronization according to the other. They will discover an invariant velocity, and learn that objects moving faster than the invariant velocity that behave in a visibly irreversible manner can provide irrefutable photographic evidence that the clocks on one of the trains cannot be correctly synchronized.

The reason one and the same set of data can lead the passengers on either of the two trains to such apparently contradictory conclusions is entirely evident: The conclusions one chooses to draw from the data depend critically on which set of clocks (blue or red) one assumes to be correctly synchronized. Failure fully to grasp this point is the single most stubborn obstacle to a clear understanding of special relativity. Using the relativity Engine, the point is impossible to miss, since it is evident to the student (who sees things from the vantage point of the space platform) that *neither* set of clocks is correctly synchronized,² and the red and blue conclusions are crucially dependent on assuming that either the red or the blue clocks are the ones that are synchronized (in spite of the evidence that neither set is synchronized, which is, however, available only to those who can see the entire screen as a whole, and not to the people in Houston or Cape Canaveral, who only have isolated photographs of small parts).

III. INTRODUCTION FOR INNOCENTS: A USER'S MANUAL FOR THE RELATIVITY ENGINE

A. Rockets and clocks at various times

Pictured on the screen are two trains of identical rockets. [See Fig. 1(a).] We will call the upper train (that points to the left) the red train and the lower train (that points to the right) the blue train. (If you are using the color version of the Engine, then the red train *is* red and the blue train, blue. If you are using the version that runs on a monochromatic display think politically: The red train goes to the left.)

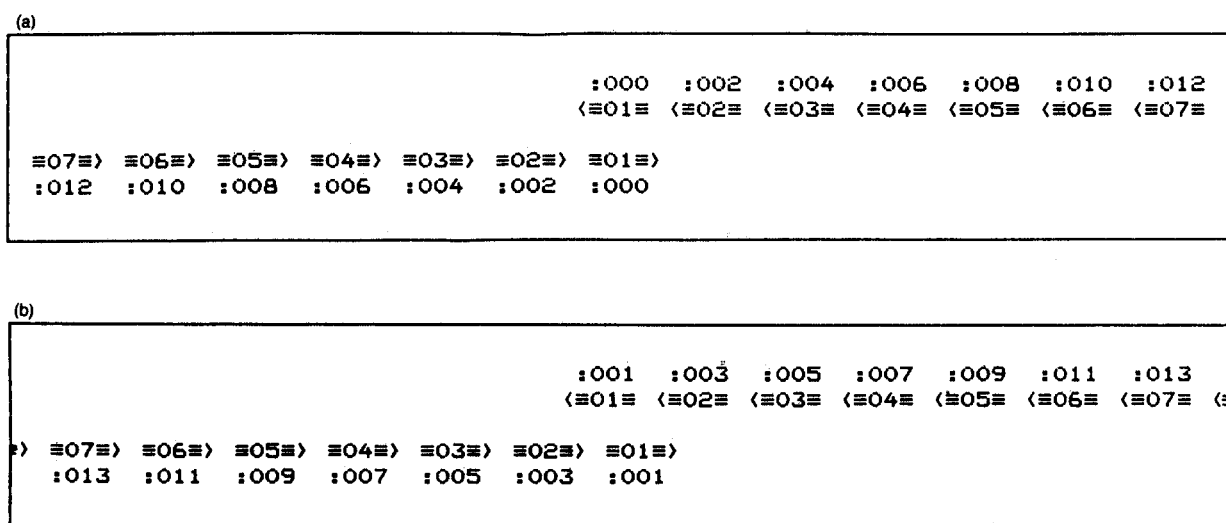


Fig. 1. (a) The Relativity Engine in its initial configuration. (On the screen the rockets of each train are solid rocket-shaped blocks, with no internal features other than their serial numbers, and each rocket has a flashing exhaust plume behind it.) (b) The Relativity Engine one time step after the configuration shown in (a). Note that the upper rocket has moved a bit to the left, the lower, to the right, and that each clock on each rocket has advanced by one tick. To get from the picture in (a) to the picture in (b) one touches the up-arrow key; to return from (b) to (a) one touches the down-arrow key.

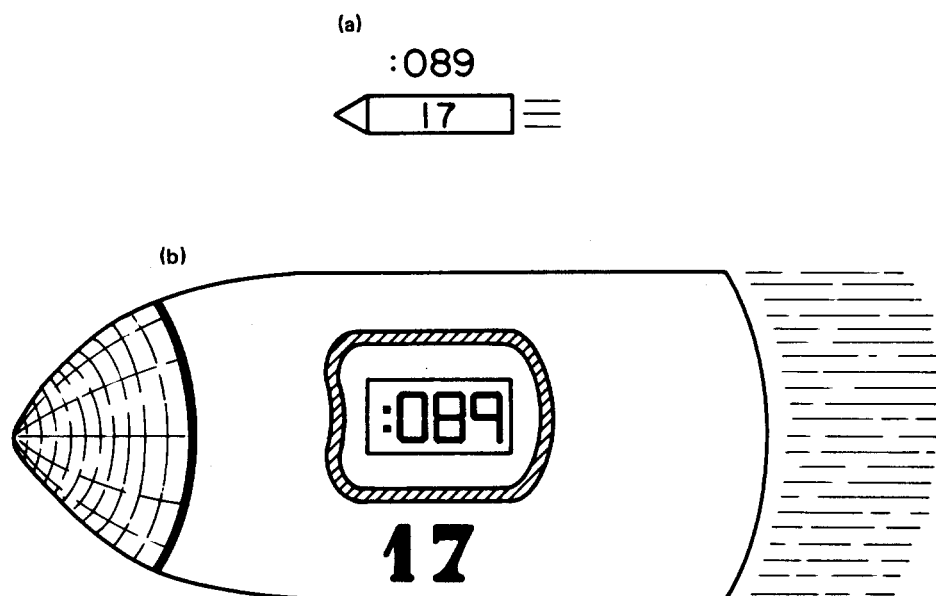


Fig. 2. A somewhat less schematic picture of what each rocket in a train represents: (a) shows rocket #17 as it appears on the screen, when the clock it carries reads :089 ticks; (b) gives a somewhat more realistic drawing, to emphasize that the clock itself is inside the rocket, but visible through the window just above the serial number.

There are two numbers associated with each rocket of each train:

(1) At the center of every rocket is its identification number. The rocket at the front of each train is numbered 1, and subsequent rockets in each train are numbered 2, 3, 4,..., depending on how far down the train they are.

(2) Alongside each rocket (a little above the red rockets and a little below the blue ones) is a three digit number preceded by a colon. The front rockets both show :000, the second rockets in each train show :002, the third, :004, etc. The number attached to a rocket is the reading (in a unit of time we will call a "tick") of a timer (or a stopwatch—we will call it simply a clock) that is inside the rocket. If the screen were big enough to show more detailed pictures of each rocket, then this could have been made more vividly evident by showing a window in the middle of each rocket, under which was the rocket's identification number and behind which (i.e., inside the rocket) was a clock showing the number with the colon. Figure 2 shows the big picture that a little picture represents.

Notice that none of the clocks on the red train (we will call them the "red clocks") agree with each other: They all have different readings. These different readings are, however, simply related: As you move toward the rear of the train, the clocks get ahead by *two ticks per rocket*. Thus the clock in rocket 1 reads :000; the clock in rocket 2 reads :002; the clock in rocket 3 reads :004; etc.

The clocks on the blue train (the "blue clocks") are set in exactly the same way: The clock in rocket 1 reads :000, and the readings of clocks in subsequent cars increase by two ticks from each rocket to the one behind it.

Although they are not synchronized with one another, all the clocks on both trains are good clocks of identical construction. In particular, they all run at the same rate. To check this, push the up-arrow key, which produces a new picture, showing what things look like a little later. Notice that two things have changed as a result of pushing the up-arrow [Fig. 1(b)]: (a) each train has taken a small step forward (one-sixth of a rocket length, to be precise—a distance we shall in fact define to be one "step") and (b) the

reading on every clock has advanced by exactly one tick.

By repeatedly pressing the up-arrow key (or just holding it down if you are in a hurry), you can survey the trains at various times; with each press of the key every clock advances by one tick, and both trains move forward another step. To recapture what things looked like at an earlier time, press the down-arrow, which results in each train moving backward a step and each clock losing a tick.

Thus the up-arrow takes things forward in time, and the down-arrow, backward. It is clear from the sequence of views you get by pressing the arrows that all the clocks on both trains are indeed running at the same rate, and that the trains are simply moving steadily along at the same speed in opposite directions. There is nothing at all peculiar about any of this except for the fact that the clocks are not synchronized.

B. The clock conspiracy

A group of scientists (not on either train) are interested in investigating the kinds of conclusions reached by people using such sets of unsynchronized clocks *if those people are under the false impression that their various clocks actually do agree with each other*. It is these scientists who, unknown to the occupants of either train, have carefully arranged for the clocks in the different rockets of each train to be out of agreement with each other.

The scientists have lied to the occupants of each train about their clocks, telling them that the clocks in the different rockets of their train are in perfect agreement with each other. They have also arranged things to ensure that the occupants of each train cannot learn that they have been lied to. First of all, nobody can move down the train from one rocket to the next, making a direct comparison of the clocks. They are all locked into their own rockets. Furthermore, no mechanism is provided for communicating between different rockets in the train. No radios, no telephones, no leaning out the window and taking a look. The occupants of each rocket are completely isolated from the people in any of the other rockets of the train. The flaming

	:015	:017	:019	:021	:023	:025	:027	:029	:031	:0
	<01=	<02=	<03=	<04=	<05=	<06=	<07=	<08=	<09=	<1
0=)	=09=)	=08=)	=07=)	=06=)	=05=)	=04=)	=03=)	=02=)	=01=)	
33	:031	:029	:027	:025	:023	:021	:019	:017	:015	

Fig. 3. The two trains, shown at a moment when the rockets are lined up directly opposite each other, so that the occupants of any rocket can look through their window and see the serial number and clock reading of the rocket opposite them. Only at such moments of perfect alignment can useful information be collected or can photographs be taken.

exhaust³ prevents them from crawling into the nose cone of their own rocket and tapping out a message on the rear of the one in front of them, or vice versa. Any other means of communicating between rockets you might try to invent has been anticipated and made impossible by these clever, determined, and unscrupulous scientists.

The occupants of each train, having no reason to expect deception, therefore believe that the clocks in the various rockets making up their own train are indeed synchronized.

C. Watching the other train

It is a boring journey. There is nothing to do but look out the window, and there is very little to see. With one exception (noted below in Sec. III F), the only time people in either train have anything at all to look at, is when the other train passes by. Even this is not very exciting for the occupants of any one rocket, except when a rocket on the other train is directly opposite so that the windows of the two rockets are perfectly lined up. Then, and only then, it is possible for the occupants of each rocket to see the identification number of the other rocket, and note, through the window of the other rocket, what its clock is reading.

At the moment depicted in Fig. 3, for example, the occupants of rocket #5 on the red train are able to look at the clock in rocket #2 of the blue train and verify that at time :023 (according to their own clock) the blue clock in rocket #2 reads :017. In exactly the same way, of course, the occupants of the blue rocket #6 will be noting that at time :025 (according to their own clock) the clock in red rocket #1 reads :015. Shifting our attention up and down the trains, we see that at this same moment other people in other rockets will be reaching other conclusions of the same general character. Note that the occupants of red

rocket #8 have nothing to report at this moment, since no blue rocket is opposite them.

There are also moments when nobody on either train has anything of interest to report. In Fig. 4, for example, the rockets are not lined up with each other, and the occupants of adjacent rockets cannot look through each other's windows and determine the reading of the clocks in the other rocket. All the information we shall collect and make use of comes from moments of perfect alignment, when each rocket can reveal the reading of its clock to the rocket on the other train that is directly outside its window.

D. Taking pictures of yourself and others

Occupants of the rockets can, whenever they wish, record the information they collect when the window of another rocket appears outside theirs. Thus (Fig. 5) the occupants of blue rocket #8 can note that at "blue time" :041 red rocket #3 is directly opposite and its clock reads :031. This information can be recorded in a sentence, as we have just done, or in some kind of table of data, or, most vividly, in a little picture of the juxtaposed rockets and clocks (Fig. 6) that shows the two rocket numbers and the two clock readings.

Note that such a picture can be used by occupants of either rocket. Thus occupants of the red rocket #3 would conclude from the data summarized in Fig. 6 that at "red time" :031 blue rocket #8 is directly opposite and its clock reads :041. This statement is based on exactly the same data as the statement in the preceding paragraph, but there is an important difference in how the data are interpreted. The people in the red rocket use their clock reading to establish what the time is (since they believe their clocks to tell correct time) and record the reading of the blue clock as mere-

	:020	:022	:024	:026	:028	:030	:032	:034	:036	:038	:
	<01=	<02=	<03=	<04=	<05=	<06=	<07=	<08=	<09=	<10=	<=
=)	=10=)	=09=)	=08=)	=07=)	=06=)	=05=)	=04=)	=03=)	=02=)	=01=)	
0	:038	:036	:034	:032	:030	:028	:026	:024	:022	:020	

Fig. 4. The two trains, shown at a moment when the rockets are not lined up directly opposite each other. At such moments the occupants of any rocket can see neither the serial number nor the clock reading of any other rocket, and photographs cannot be taken.

	:027	:029	:031	:033	:035	:037	:039	:041	:043	:045	:047	:049
	<=01=	<=02=	<=03=	<=04=	<=05=	<=06=	<=07=	<=08=	<=09=	<=10=	<=11=	<=12=
2=>	=11=>	=10=>	=09=>	=08=>	=07=>	=06=>	=05=>	=04=>	=03=>	=02=>	=01=>	
49	:047	:045	:043	:041	:039	:037	:035	:033	:031	:029	:027	

Fig. 5. The two trains, shown at another moment when the occupants of the rockets can learn something about the other train by looking out of their window.

ly a number displayed on a certain timepiece whose relevance to correct time remains to be established. For people on the red train red clocks tell time; blue clocks are objects to be investigated. For people on the blue train, it is the other way around.

Let us suppose, then, that the people in each rocket have cameras that can take such pictures (or sketch pads in which they draw them) as a convenient way of recording the information acquired when a rocket of the other train is directly opposite. *You too have been provided with such a camera, to aid in your investigation of the data collected by the train people.*

Press the left- or right-arrow key, and notice the vertical double arrow (↑) that appears between the two trains [Fig. 7(a)]. We shall refer to it as the “camera.” If you hold down the left- (or right-) arrow key the camera moves to the right or the left. Should it pass an interesting place release the key and it will stop there.

To take a picture press the F9 key (hereafter referred to as the “shutter”). Should the trains not be lined up, you will be reprimanded by a message at the bottom of the screen reminding you that rockets can only take pictures of each other when they are aligned. To make the camera work, let time run forward (or backward) (with the up- or down-arrow keys) until you find an interesting configuration in which the rockets are aligned. Then move the camera (with the right- and left-arrow keys) until it is anywhere between the two rockets whose numbers and clock readings you wish to record. Now, when you press the shutter, a picture representing the data collected by people in either of the rockets you have indicated appears at the lower left of the screen [Fig. 7(b)]. This picture contains precisely the same information as the pictures that people in either of the two rockets might have made.

As it happens one cannot learn much of interest from a single picture. But *pairs* of pictures, as we shall see, can be very informative indeed. So take another picture: Search

around in time (up- or down-arrows) until you find another (or the same) lined up configuration, move the camera right and left (right- or left-arrows) until you find another enticing pair of lined up rockets, press the shutter (F9), and notice that the new picture you have just taken appears at the lower right (Fig. 8).

You can continue taking further pictures in this way, but only two of them will be displayed on the screen at a time. If you take a third picture, the first two will be erased, and the third (and a subsequent fourth) will be displayed in the same parts of the screen where the first and second were shown. It is, therefore, a good idea to note the information contained in a pair of pictures, before going on to take another pair.

The reason only two pictures are shown at a time is partly to avoid cluttering up the screen but, more importantly, because *all* interesting information can be gathered by considering appropriate *pairs* of pictures. It is never necessary to have more than two in front of you at any one time.

E. Drawing conclusions from pictures

Imagine now that the trains are loaded with at least one passenger in each rocket, and that they pass by each other, as pictured on the display screen, with passengers in each rocket photographing and noting down what they see as their window passes directly opposite each rocket window of the other train. After the trains pass each other, they return to their base, and all the passengers disembark with their pictures (without being allowed to examine the clocks in any of the other rockets of either train). They are immediately rushed off to two debriefing centers—one for the red passengers and another for the blue, where they can pool all their photographs, and use them to answer a series of questions about the nature and behavior of the other train.

Both groups of passengers have the same collection of photographs at their disposal. But their answers to certain questions can differ, because the red passengers will extract information from the pictures under the assumption that the red clocks were synchronized, while the blue passengers will assume that the blue clocks agreed with each other. You can use the Relativity Engine to answer questions put to the passengers. For example:

Question: How fast do the red passengers say the blue train was moving?

Answer: To answer this you need to follow a particular piece of the blue train as it moves along. Any rocket will do—say rocket #7. Take any two pictures that show rock-

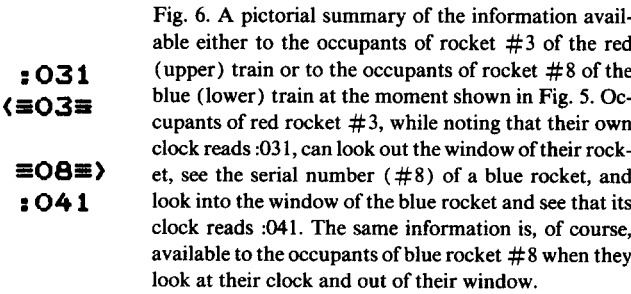
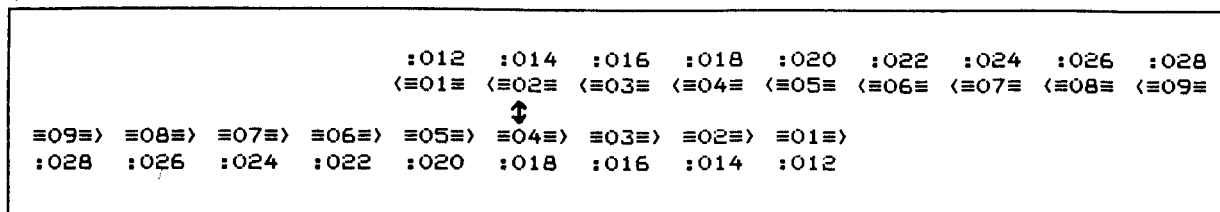


Fig. 6. A pictorial summary of the information available either to the occupants of rocket #3 of the red (upper) train or to the occupants of rocket #8 of the blue (lower) train at the moment shown in Fig. 5. Occupants of red rocket #3, while noting that their own clock reads :031, can look out the window of their rocket, see the serial number (#8) of a blue rocket, and look into the window of the blue rocket and see that its clock reads :041. The same information is, of course, available to the occupants of blue rocket #8 when they look at their clock and out of their window.

(a)



(b)

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:014
<=02=

=04>
:018

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Fig. 7. A camera, represented by a double arrow (\updownarrow), appears on the screen whenever the right- or left-arrow key is touched, and can be moved right or left by pushing down the appropriate key. When the camera is between a pair of rockets *and the rockets are perfectly aligned* then a picture can be taken by pressing the shutter key (F9). The camera is shown at a configuration of such perfect alignment in (a), and the resulting picture is shown in (b).

et #7 at two different moments. Such a pair is shown in Fig. 9. From these two pictures, the red passengers would reason as follows:

Blue rocket #7 was opposite red rocket #2 at red time :023, and a little later, at red time :083 it was opposite red rocket #14. It therefore moved 12 rocket lengths in a time of 60 ticks, so its speed is 12/60 or 0.2 red rocket lengths per tick.⁴ (*Problem 1: Find other pictures of blue rocket #7 at other moments that confirm the conclusion that its speed is 0.2 red rocket lengths per tick. Find pictures of other blue rockets that demonstrate to the red passengers that those blue rockets are also moving at a speed of 0.2 rocket lengths per tick. Find still other pairs of pictures that demonstrate to the blue passengers that various red rockets are all moving past them at 0.2 blue rockets per tick.*)

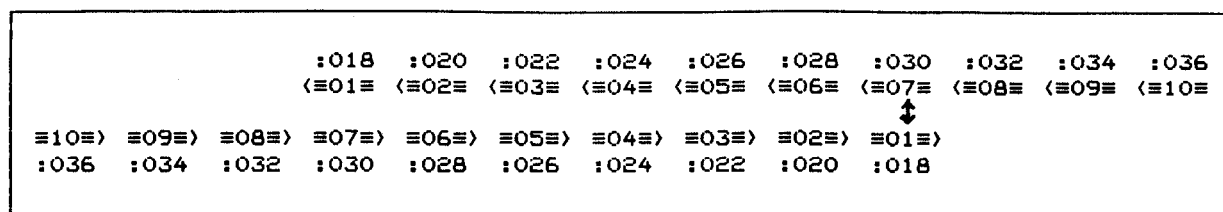
But this is not the only information about the blue train that the red passengers can extract from the two pictures of Fig. 9.

Question: How fast are the clocks on the blue rocket running? (We, of course, know that the clocks on both trains are running at the same rate. But because the clocks on neither train are correctly synchronized, it turns out

that occupants of each train reach an interesting conclusion about the rate at which the clocks on the other train are running.)

Answer: Occupants of the red train can study the rate at which the blue clock in blue rocket #7 is running by comparing it with their own clocks in the two pictures. Of course the two pictures show two *different* red clocks—one in rocket #2, the other in rocket #14. The red passengers, however, believe that their clocks are all properly synchronized, and therefore that one clock is as good as another for telling correct time. They, therefore, note from the red clocks that 60 ticks (:083 – :023) elapsed, during which the blue clock in blue rocket #7 only changed its reading by 36 ticks (:069 – :033). They conclude that the blue clock in blue rocket #7 is running at a rate that is only 36/60 or 0.6 of the rate of their own clocks. (*Problem 2: Find other pairs of pictures that permit you to reach the same conclusion about the clock in car #7 and various other clocks on the blue rocket. Find pairs of pictures that lead people on the blue train to conclude that clocks on the red train are running slowly.*)

A somewhat more subtle question is this:



```

:014          :030
<=02=        <=07=

=04>          =01>
:018          :018

```

Fig. 8. After the picture in Fig. 7(b) was taken, time was advanced (by holding down the up-arrow key), another moment of perfect alignment was found, the camera was moved (by pressing the right-arrow key) to another pair of rockets, and a second photograph was taken (by pressing F9). Both pictures are shown in the figure in the actual positions they occupy on the display screen.

:023
 <≡02≡

:083
 <≡14≡

:056
 ≡11≡

:056
 ≡05≡

≡07≡>
 :033

≡07≡>
 :069

≡03≡
 :040

≡13≡
 :072

Fig. 9. A pair of photographs, from which occupants of the red (upper) train will conclude that rocket #7 of the blue (lower) train is moving at a speed of 0.2 red rockets per tick, and that the clock on blue rocket #7 is running at only 60% the rate of the clocks on the red train.

Question: How long do the red passengers say the blue rockets are?

Answer: You can answer this from any pair of pictures in which the red clocks read the *same* time. Consider, for example, the pair of pictures shown in Fig. 10. These reveal (1) that at a red time of :056, the center of blue rocket #3 was directly opposite the center of red rocket #11 and (2) that at the *same* red time of :056 ticks, the center of blue rocket #13 was directly opposite the center of red rocket #5. Consequently, at red time :056 ten full blue rockets (half of #3, half of #13, and all of the nine rockets from #4 to #12) stretched across the same space as only six full red rockets (half of #11, half of #5, and all of the rockets from #10 to #6.) But if a line of ten blue rockets stretched across the same distance as a line of six red rockets, then the blue rockets must be only 6/10 or 0.6 the length of the red rockets.

Please note two points, the first profoundly important, and the second a technical detail about using and interpreting the Engine:

(1) The fact that both pictures were taken at the *same* time, according to the red clocks, is absolutely essential in permitting the occupants of the red train to conclude that the blue rockets are shorter than the red rockets. If they did not believe that the two pictures were taken at the same time, then they would realize that the blue rockets had moved between the two pictures, and they then could not draw conclusions about the length of the segment of blue train from the middle of rocket #3 to the middle of rocket #13 without correcting for this motion.

(2) What about the fact that Fig. 10 shows two rockets, one red and one blue, that are clearly the same length? That, dear reader, is an artifact of the way in which the

:056
 <≡11≡
 ≡03≡>
 :040

:056
 <≡05≡
 ≡13≡>
 :072

Fig. 10. A pair of photographs, from which occupants of the red (upper) train will conclude that the blue (lower) rockets are only 60% as long as the red rockets, and that the clocks on the blue train are out of synchronization by 3.2 ticks per rocket. Note that all the actual pictures show are the serial numbers of the rockets and the readings of their clocks; they do *not* show the ends of the rockets. [More accurate but less beautiful photographs ("unretouched") can be taken by using F5 rather than F9 as the shutter key.] The lengths of the blue rockets are *inferred* from noting that it takes 10 of them to fill up the space occupied by only 6 red rockets.

Fig. 11. The "unretouched" version of the photographs shown in Fig. 10, to emphasize that a single photograph does not show enough of a rocket to permit a direct determination of its length. The picture on the left, for example, contains no information beyond the fact that at the instant the middle of red rocket #11 was directly opposite the middle of red rocket #3, the clock in the middle of red rocket #11 read :056 ticks and the clock in the middle of blue rocket #3 read :040 ticks. The pictures in Fig. 10 also contain no more information than this, but the full rockets have nevertheless been drawn, simply to make them prettier.

Engine presents the data for your subsequent use. All a camera actually photographs are the serial numbers and clocks attached to the middle of the two rockets. Both ends of the rockets are too far away to appear in the pictures. Because a list of four numbers is rather abstract, and to make it evident that those numbers are associated with rockets moving in definite directions, the Engine has simply added to each photograph in as nonprejudicial a way as possible (i.e., symmetrically) a schematic representation of the nose cones and exhaust fumes. If this offends or confuses you, you can take more literal pictures using F5 as the shutter key rather than F9. The two pictures of Fig. 10, taken in the unretouched form given by the F5 shutter key are shown in Fig. 11. Evidently they are less obviously associated with rockets, but they do make it quite clear that you cannot learn from a single photograph anything about the comparative lengths of the two rockets.

(Problem 3. Find other pairs of pictures showing moments of time that are simultaneous according to the red clocks, and confirm that they all lead to the conclusion that the blue rockets are 0.6 the length of the red ones. Find pairs of pictures showing moments simultaneous according to the blue clocks, and confirm that those pictures lead to the contrary conclusion that the red rockets are 0.6 the length of the blue ones.)

The two photographs in Fig. 10 also make it evident to occupants of the red train that the clocks on the blue train are not synchronized, for the pictures portray two blue clocks that read :040 and :072 ticks at a single moment of time (:056 ticks) according to the red clocks.

Question: We know that the clocks on the blue train are out of synchronization by two ticks per blue rocket. How do the occupants of the red train, thinking that their own red clocks are synchronized, characterize the lack of synchronization they observe on the blue train (in ticks per blue rocket)?

Answer: They note that the two blue clocks in Fig. 10 disagree by 32 ticks (:072 - :040) and are ten rockets apart (one is on rocket #13 and the other on rocket #3), so the asynchronization is $32/10 = 3.2$ ticks per blue rocket. (Problem 4. Find other pairs of pictures leading occupants of the red train to the same conclusion and still other pairs that lead occupants of the blue train to conclude that clocks on the red train are out of synchronization by 3.2 ticks per red rocket.)

F. A meteor passes by

One other thing occasionally relieves the boredom of the passengers. Every now and then a meteor moves along the lengths of both trains, and if a photograph is taken at one of those rare moments when it happens to be moving past a pair of windows just as the windows are opposite one another, then the meteor actually appears in the photograph. By comparing pairs of such photographs it is possible to determine the speed of one and the same meteor, according to the occupants of either (or both) of the two trains. You can produce such meteors for subsequent investigation as follows:

First move the rockets and the camera about until you have the camera (which is now functioning as a site for the creation of a meteor) at a part of the picture where you would like the meteor to appear. Then press F7. You will be asked to specify how many "spaces" the meteor (called an "object") moves in each "time step." (A space is just the amount of screen equal to $\frac{1}{6}$ of a rocket length; a time step is the interval between one picture and the one produced from it by a single touch of the up-arrow.) This has no obvious relation to the speeds people on either train will assign to the meteor, but it gives you a way to control how fast the meteor moves. If you want a fairly fast meteor, when asked how many spaces per time step type, for example, "5/1" and press the carriage return. The meteor will appear wherever you left the camera, and as time moves forward and backward it will zip back and forth between the trains at a brisk pace.

To get a slow meteor you can enter something like "1/3." Note that now as you advance time you only see the meteor in every third picture. This is a limitation of the display screen (more precisely, the monochrome display screen, but the program written for the graphics display does not take advantage of its more sophisticated capabilities), which cannot move the meteor by a fraction of a space. Consequently, if its speed is a third of a space per time step, all the screen can show is that every three time steps the meteor has moved over another space.

The game is now to try to put down meteors in suitable places and with suitable speeds so they can be caught in pairs of photographs. (Once you have such a pair, you can use them to determine the speed assigned to the meteor by occupants of either train.) Some experimentation is required to produce such a meteor. Sometimes you will find that your meteors never appear opposite rocket windows when the windows themselves are opposite each other, and

such meteors cannot be photographed at all. But by adjusting the position of the pointer camera at the moment you create the meteor, you can make meteors that do appear at windows when photography is possible. (To get rid of an unsatisfactory meteor press F8, or, if you have immediate plans for introducing another one, just press F7 to override the earlier specifications.) A pair of such photographs of a meteor moving at one space per three time steps ($\frac{1}{3}$ of a space per time step) is shown in Fig. 12.

There are many things you can investigate with suitably chosen meteors, of which only two are mentioned here:

(1) It is possible to specify a special meteor speed that results in occupants of *both* the blue and the red trains, using their own clocks, assigning *exactly the same speed* to such meteors. You should determine, by experimentation, what that speed is, in spaces per time step, and you should also note the speed, in rockets per tick, that occupants of either train assign to meteors moving with that special speed.

(2) Interesting things happen if you introduce meteors with speeds significantly greater than the special speed. You should try, for example, to produce two photographs of such a meteor in which the time order in which the photographs were taken depends on whether one believes the red or the blue clocks agree with each other. Evidently if the meteor were a real meteor (which was burning up) it would be evident from its appearance which picture was *really* taken first. Thus, if meteors of sufficiently high speed, which were changing in a manner that revealed the direction of time, could be photographed by both ships, this would provide enough information to convince the occupants of one of the trains that there was something wrong with their clocks. (Such meteors, traveling in the opposite direction, would provide similar information to the occupants of the other train.)

(3) Convince yourself, by trying out several cases, that the above problem never arises with meteors moving slower than the special speed.

(Problem: Find pairs of pictures illustrating these points. Can you discover (this will be hard, without advice from informed people) a general formula relating the two speeds occupants of the two trains assign to one and the same meteor, that is valid for all the meteors you can create?)

G. Summary of input to the engine

I list here all the ways in which you can communicate with the Engine:

Up-arrow: One step forward in time. (One "time step.")

:019	:045
<≡03≡	<≡07≡
■	■
≡04≡>	≡06≡>
:021	:043

**To produce an object that moves M spaces in N time steps
enter M/N where M and N are integers between -20 and 20: 1/3**

Fig. 12. A pair of pictures capturing two moments in the history of a meteor (the squarish blob) moving at one space every three time steps. Evidently people on the red (upper) train believe that the meteor moves four rockets in $45 - 19 = 26$ ticks, or at a speed of $2/13$ rockets per tick. People on the blue (lower) train say its speed is $1/11$ rockets per tick. [Note the message that appeared when the meteor making key (F7) was pressed, and the information that was entered to produce the particular meteor subsequently photographed.]

Down-arrow: One step backward in time.

Right-arrow: Call up the camera and move it one space to the right.

Left-arrow: Call up the camera and move it one space to the left.

F1: Exit from the program.

F5: Take an unretouched photograph (in which the front and rear of the rockets are not in the picture).

F9: Take a retouched photograph (in which the front and rear of the rockets have been added by an artist, to make the direction of motion of each rocket evident from the picture).

F6: Erase any photographs currently on the screen.

F7: Create a meteor at the position of the camera (\uparrow). The camera should be put in the desired position using the right- and left-arrow keys, before F7 is pressed. To get a meteor moving a certain number of spaces in some other number of time steps you must type the number of spaces (preceded by a minus sign if you want a meteor going to the left), then a slash (/), then the number of time steps, and then a carriage return.

F8: Remove the meteor.

IV. FURTHER REMARKS FOR SOPHISTICATES

A. Pedagogical matters

The above User's Manual (Sec. III) is not intended to be a complete list of all the measurements and discoveries students can make with the Engine, but it should give an idea of the kinds of questions they can be led to explore with its aid. There are two distinct pedagogical strategies for employing the Engine:

(1) Use it as a prologue to a discussion of relativity. Introduce the Engine from an entirely Newtonian point of view as a way of exploring the kinds of false conclusions people can be led to if they fail to realize that their clocks are not synchronized. Extract length contraction, time dilation, and the invariant velocity. Extract, if you want, the relativistic addition law. Get your students used to using the Engine to measure the lengths of moving objects with unsynchronized clocks. Then (and only then) introduce the real world through the constancy of the velocity of light. Point out the lesson of the Engine: that this would not cause problems if different observers used differently synchronized clocks. Prior experience with the Engine will go a long way toward alleviating that feeling of being led into a maze of contradictory assertions that can so easily paralyze even the brightest students on a first exposure to the subject.

(2) Use the Engine as an antidote to such paralysis after the subject has been developed from a conventional point of view. This has the disadvantage of failing to take full advantage of the Engine's therapeutic powers, but the advantage of enabling more capable students to approach the Engine with the relativistic formulas for length contraction, time dilation, relativity of simultaneity, and velocity addition, already at hand, thereby permitting them to undertake their *gedanken* experimental studies with the Engine with the aim of confirming specific quantitative expectations.

It would be futile for me to recommend either course of pedagogical action. The point of this essay is simply to call attention to the Engine as a novel and (in my experience) useful educational device for teaching special relativity.

People using it to teach a class will want to play with it themselves, develop their own sets of questions to be explored, and fit it into their syllabus as best meets their own aims.

B. Alternative engines

The particular version of the engine described here (and embodied in the program I have written) has the trains moving with respect to each other at the single speed of $\frac{1}{2}c$. One can design more sophisticated engines with adjustable relative speeds, for screens with greater resolution (and processors with greater speeds), although not many speeds are conveniently adapted to some of the constraints imposed by the discreteness of any visual display.

I am not persuaded that designing an Engine with variable speed is worth the effort. The most important pedagogical purpose served by the engine is forcibly bringing home the consequences of making measurements with improperly synchronized clocks. Any single speed can serve this purpose. The other advantage of the present engine is its widespread accessibility. Any student who uses a PC compatible machine to write essays and term papers can use that same machine to run the Engine. Should the age dawn (as no doubt it will) when students routinely prepare term papers on their own micro-Vaxes, then the time will perhaps have arrived to write a more flexible, powerful, and certainly more colorful Relativity Engine.

Nevertheless, should anybody wish to design a Relativity Engine with parameters different from mine, the following considerations might be of some help:

Regard a computer screen as composed of horizontal rows of discrete cells. The standard IBM monochromatic display, for which my version of the Engine was composed, contains 80 cells per row. For maximum verisimilitude, any version of the engine should have each train shift by just one cell in each time step. There is also no reason not to choose the time scale so that each clock advances by one unit (a "tick") in each time step. The free parameters in any realization of the Engine are therefore only two:

(1) The number of cells N making up a rocket. If every pair of red and blue rockets is to have a moment of perfect alignment for picture taking, it is necessary that N be even.

(2) The number of ticks Δ by which clocks on adjacent rockets disagree. If any clock is to be capable of showing the same readings as any other clock, then Δ must be an integer. If one is willing to use an Engine with an unpleasant artificial asymmetry in what different clocks can show, then Δ can be merely rational, but it had better have a fairly small denominator, if pairs of pictures taken at the same train time are to be readily available.

The two numbers N and Δ completely determine all the other parameters that characterize the engine (most importantly, the value of the invariant velocity c and the value of the speed v of one train with respect to the other). If one imposes the additional condition that an object moving at the invariant velocity should be displayed on the screen at every time step—i.e., that such an object should move a whole number of cells in a single time step—then the freedom in choosing the parameters is further reduced.

To work this out, consider the progress of the two pairs of rockets depicted in Fig. 13. Between Figs. 13(a) and 13(b), $\frac{1}{2}N$ time steps have elapsed (as indicated by the clock readings adjoining the rockets). Inspecting these two

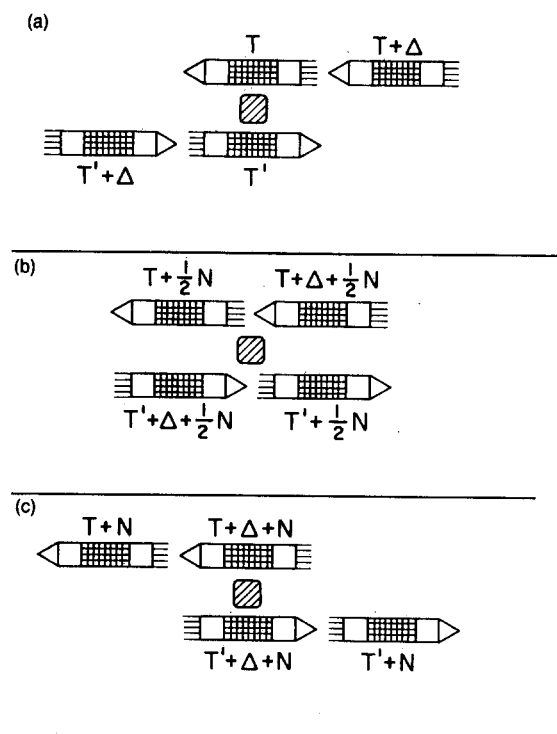


Fig. 13. A more abstract representation of three moments in the history of a pair of rockets from each train. Each rocket moves one cell per time step and stretches over N cells. Consequently $\frac{1}{2}N$ time steps elapse between parts (a) and (b) and between parts (b) and (c). This is confirmed by the fact that each clock advances by $\frac{1}{2}N$ ticks between parts (a) and (b) and between parts (b) and (c). Note also that the clocks on each train are out of synchronization by Δ ticks per rocket.

parts of the figure reveals that the speed of either train with respect to the other is

$$v = 1/(\Delta + \frac{1}{2}N) \quad (1)$$

rockets per tick. Further inspection reveals that in a time $\Delta + \frac{1}{2}N$ according to one train, a given clock on the other has only advanced by $\frac{1}{2}N$ ticks and, therefore,

$$\sqrt{1 - v^2/c^2} = \frac{1}{2}N / (\frac{1}{2}N + \Delta). \quad (2)$$

Equations (1) and (2) together give

$$c^2 = 1/[\Delta(N + \Delta)] \quad (3)$$

rockets per tick. An easy way to extract the condition that an object moving at speed c should cover a whole number of cells per time step is to note that the speed u of either train in the *screen* ("platform") frame is just one cell per time step so we require c/u in cells per time step to be an integer n . Now Eq. (3) gives us c in rockets per tick. We can get u in the same units, by noting that u is also just the speed of an object fixed on the screen in either *train* frame. It follows from Figs. 11(a) and 11(c) that in rockets per tick,

$$u = 1/(N + \Delta). \quad (4)$$

Combining this with Eq. (3) we have

$$n^2 = c^2/u^2 = 1 + N/\Delta. \quad (5)$$

The general Engine can therefore be characterized as follows.

Given the time shift Δ between adjacent rockets, the length of each rocket should be taken to be

$$N = \Delta(n^2 - 1), \quad (6)$$

where n is any integer greater than 1, and Δ (which is integral in the most attractive Engines) must be such as to make N an even integer. The invariant velocity is then

$$c = 1/n\Delta \quad (7)$$

rockets per tick, and the relative speed v of the two trains is given by

$$v/c = 2n/(n^2 + 1), \quad (8)$$

resulting in

$$\sqrt{1 - v^2/c^2} = (n^2 - 1)/(n^2 + 1). \quad (9)$$

My version of the Engine is one of the simplest possible, with $n = \Delta = 2$.

C. Computational details

The particular realization of the Engine I constructed runs on the IBM PC. There are two versions, designed for either the ordinary monochrome display or the color graphics display. Color helps to distinguish the different frames of reference and is much prettier to watch,⁵ but monochrome is more commonly available and quite manageable, thanks to the IBM extended character set and the graphical possibilities of inverse video.

The program is written in ordinary PC BASIC, which has to be compiled if it is to run at anything like a reasonable speed. I would be happy to send anybody a ready-to-run copy, upon receiving a formatted 5 $\frac{1}{4}$ -in. floppy disk and a suitable container, stamped and addressed, for returning it. In return for providing you with your own Engine, I would appreciate receiving any suggestions you have for its improvement, after using it with students.⁶

People interested in writing their own Engines for the PC might note that to achieve a decent speed I also found it necessary to execute the graphics by poking the appropriate data directly into the memory locations where the display screen is mapped. As an intelligible source of the arcane information necessary to achieve this feat, I found Peter Norton's *Inside the IBM PC* quite satisfactory.

ACKNOWLEDGMENTS

I received the computational wherewithal to develop my Relativity Engine through the IBM Project Ezra Grant to Cornell University. I am particularly indebted to Geoffrey Chester for encouraging me to pursue this realization of the Engine, and for providing me with the peace and quiet necessary for success.

¹In this respect, the Engine resembles the Brehme diagrams. It operates, however, at a very much more intuitive level, by keeping time rather than representing it as a second spatial direction.

²A very early asymmetric attempt at a Relativity Engine is described in N. D. Mermin, *Space and Time in Special Relativity* (McGraw-Hill, New York, 1968), Chap. 12, now lamentably about to drop out of print.

³A word to sophisticates: These rockets are moving with uniform velocity, so why are their engines on? Primarily because it makes the pictures prettier and more vivid. If this bothers you, then please regard the rockets

as moving not through the "void," but through a resistive medium. The reason they are not accelerating in spite of the blast is that the thrust of the rockets just balances the frictional force.

⁴The passengers on the red train do not, of course, use the term "red time," but simply say "time," since for them time is what they read from their clocks, which they believe to be synchronized. We, being better informed, may find it useful to use more accurate terms like "red time" or "blue time" to remind us that such statements actually refer only to the read-

ings of a particular set of clocks.

⁵Something with the resolution of the IBM Enhanced Color Display is required to make the Engine easily legible; the more primitive IBM color display also produces some unattractive noise between time steps.

⁶Unfortunately by the time I finished writing the program I had become trapped in an administrative job that prevents me from trying the Engine in the classroom until late 1991. My own experience comes entirely from the pieces of paper described in Sec. I.

The image of the physicist in modern drama

W. Brouwer

Department of Secondary Education,^{a)} University of Alberta, Edmonton, Alberta T6G 2J1, Canada

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While the role of the scientist in society has been a subject for playwrights from the time of the Renaissance onwards, quite a number of post-World War II plays have focused on the role physics and physicists play in modern society. In this article, the plays of Bertold Brecht, Friedrich Dürrenmatt, William Golding, and Heinar Kipphardt are reviewed, which focus especially on the difficult choices faced by physicists in the modern world in which the consequences of physics-based technology are immense, if not overwhelming. These plays also provide an attractive context for discussion on science and social responsibility.

I. INTRODUCTION

In 1882, the Norwegian playwright Henrik Ibsen wrote *An Enemy of the People*.¹ A day after completing the play, Ibsen was still not sure whether the play should be considered straight drama or a comedy. In the play, Dr. Thomas Stockman, a staff physician in a small town in Norway, discovers that the municipal baths, the town's major tourist attraction, are polluted by chemicals from the local pulp mill. Dr. Stockman is concerned that the chemical pollution could have serious consequences for the health of the bathers, and requests that the baths be closed until filters are installed to secure a source of clean water. After much discussion, the town council decides that financing for the purification of the water is not available for a few years and suppresses the report. At the risk of his municipal position, Dr. Stockman releases the report to the press. However, the newspaper's major advertisers, whose profits depend on the tourist trade, pressure the editor of the paper not to publish the report. Even the local labor leaders, whose workers' jobs would be affected by the report, are unenthusiastic about releasing the report. Finally, Dr. Stockman rents a hall and holds a public meeting to warn the townspeople. To his surprise, his report is not well received and the story ends with Dr. Stockman standing alone as an "enemy of the people."

The comic aspects of the play are primarily a result of Thomas Stockman's bounded optimism and his belief that his fellow citizens will naturally choose the best long-term solution to the problem. Again and again, Dr. Stockman presents his findings and proposed solution to his fellow citizens and, in each case, his expectations are dashed to the ground. The play shows both the moral dilemma faced by

scientists who find themselves in situations in which their allegiance to their employer conflicts with their preceived allegiance to the general public, or to a set of higher moral principles, and the political naivete scientists sometimes exhibit when pursuing their public goals.

The image of the scientist in modern society has also been treated in many of Shaw's plays, such as *Major Barbara*² and *The Doctor's Dilemma*,³ in Lawrence and Lee's *The Night Thoreau Spent in Jail*⁴ and *Inherit the Wind*,⁵ and in Sinclair's *The Enemy Had It, Too*.⁶ However, especially since 1945, physicists have been featured as central characters in a number of plays. In Brecht's *The Life of Galileo*,⁷ the conflict of Galileo with the Catholic Church has been treated from the perspective of the nuclear age and, in Dürrenmatt's *The Physicists*,⁸ the moral responsibility of physicists for the consequences of their research is discussed in a "comic" setting. In *The Brass Butterfly*,⁹ Golding contrasts the "rationality" of science with the basic irrationality of human nature. In the play, *In The Matter of J. Robert Oppenheimer*,¹⁰ Kipphardt dramatizes the "proceedings instituted against J. Robert Oppenheimer."

However, even plays in which actual historical characters appear should not be expected to be historically accurate. These plays are not intended to be historical documentaries but "criticisms of life or of society."¹¹ In each case, the author structures the play around a specific theme with the result that the situation portrayed is usually an oversimplification of the more complex real historical situation. For example, the Galileo story has been used in the past to illustrate such diverse themes as the subjugation of science to external authority, such as religion, the establishment of the *experimental* method in science, the intro-