

Domain wall universe in the Einstein-Born-Infeld theory

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In this Letter, we discuss the dynamics of a domain wall universe embedded into the charged black hole spacetime of the Einstein-Born-Infeld (EBI) theory. There are four kinds of possible spacetime structures, i.e., those with no horizon, the extremal one, those with two horizons (as the Reissner-Nordström black hole), and those with a single horizon (as the Schwarzschild black hole). We derive the effective cosmological equations on the wall. In contrast to the previous works, we take the contribution of the electrostatic energy on the wall into account. By examining the properties of the effective potential, we find that a bounce can always happen outside the (outer) horizon. For larger masses of the black hole, the height of the barrier between the horizon and bouncing point in the effective potential becomes smaller, leading to longer time scales of bouncing process. These results are compared with those in the previous works.

PACS numbers: 04.50.+h, 98.80.Cq

Keywords: Einstein-Born-Infeld theory, Cosmology

Introduction: The idea that our Universe corresponds to a topological defect existing in a higher-dimensional spacetime has rather long history. The pioneering phenomenological models have been proposed in Refs. [1, 2]. Recently, motivated by the recent developments in string theory, especially by the discovery of objects (branes), where the ordinary particles and interactions can be trapped, these models have been extensively studied to explain various issues in theoretical physics. Phenomenologically interesting models were especially those discussed in [4, 5] (see e.g., [3] for the review). In the context of *brane world*, restrictions on the size of extra dimensions become much weaker. This idea has brought our interests to the detection of the presence of the extra dimensions, through collider experiments or table-top tests on gravitational law.

The (2nd) model of Randall and Sundrum (RS) [6], where a domain wall (=brane) universe is placed at the orbifold fixed point of the anti-de Sitter (AdS) space, has interesting properties for cosmology. After the proposal of RS, its application to cosmology has been studied by many authors. In the thin domain wall approximation, the dynamics of the domain wall can be traced by the junction conditions. The resulting cosmology strongly depends on the contents of the matter on the wall and the external geometry. In the simplest AdS black hole spacetime, with the Z_2 symmetry across the wall, the effective Friedmann equations are the same as the conventional ones, except for two important modifications [7, 8]. The first one is the term proportional to the square of the energy density on the wall and the other is a radiation-type

contribution caused by a geometrical effect. These two effects could induce modifications of cosmology in high energy regimes and have been constrained from observations. It can be naturally expected that in the more general spacetime, there would be new geometrical effects, which may explain the origins of dark matter and dark energy. The solutions of inflationary domain walls in the setup of the RS model were obtained in Ref. [9, 10, 11].

In this Letter, we will reconsider the problem about the dynamics of a domain wall universe in the charged black hole spacetime. This topics has been discussed not only in the context of Einstein-Maxwell (EM) theory in Ref. [12, 13, 14], but also in that of the Einstein-Born-Infeld (EBI) theory in [15, 16], where electromagnetic fields with the BI Lagrangian are coupled to gravity. The BI theory is a kind of generalizations of the Maxwell theory [17] and contains infinite number of higher derivative terms of gauge potential, which can be written in terms of the square root form. The special property of the BI theory is that the electrostatic energy of a point charge becomes finite. Thus, it would be a good candidate for the UV complete theory of the gauge field (see e.g., [18] for reviews). The black hole solutions in the EBI theory have been studied in [19, 20, 21]. Thermodynamical properties of EBI black holes solutions have been studied in Refs. [22, 23, 24, 25]. The vortices and monopoles were studied, in Ref. [26] and Ref. [27], respectively. In string theory, the BI term describes the electromagnetic fields living on the worldvolumes of D -branes. Applications of AdS-BI black hole solutions to the gauge-gravity duality have been discussed in [28].

In the previous works, it has been pointed out that a domain wall universe would experience a regular bounce in both of the EM [12, 13, 14] and the EBI models [15, 16, 20]. However, in these works, the electrostatic energy stored on the wall charge has not been taken into account.

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In the charged black hole spacetime, the domain wall should be charged to generate the jump of the electric field across the wall. The electrostatic energy of the wall can contribute to its dynamics. $U(1)$ -junction condition, which can be derived via the variation of total action with respect to the bulk gauge field, determines the amount of the charge stored on the wall.

Set-up: We consider the dynamics of an n -dimensional domain wall Σ embedded into the $(n+1)$ -dimensional spacetime M_{\pm} , where the indices (\pm) denote the left and right sides with respect to the wall. The bulk action contains the matter as well as the gravitation with a negative cosmological constant while the wall action can contain the arbitrary kind of matter as well as the tension. Therefore, the total action is given by

$$S = \sum_{I=\pm} \int_{M_I} d^{n+1}X \sqrt{-G_I} \left[\frac{1}{2\kappa_{n+1}^2} \left({}^{(n+1)}R_I - 2\Lambda_I \right) + \mathcal{L}_I \right] \\ + \int_{\Sigma} d^n x \sqrt{-g} \left(-\sigma + \mathcal{L}_m \right) \\ + \frac{1}{\kappa_{n+1}^2} \int_{\Sigma} d^n x \sqrt{-g} \left(K_+ + K_- \right), \quad (1)$$

where \mathcal{L}_I and \mathcal{L}_m are matter and fields living in each bulk region and on the wall, respectively. The final term represents the Gibbons-Hawking term [30].

We assume \mathcal{L}_I as the electromagnetic field with the BI Lagrangian. Note that only the electric components of the gauge field will appear in our problem. Thus, the bulk matter Lagrangian is chosen to be

$$\mathcal{L}_I = 4\alpha^2 \left(1 - \sqrt{1 + \frac{F_{MN}^{(I)} F_{(I)}^{MN}}{8\alpha^2}} \right), \quad (2)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the $U(1)$ field strength and A_M is the corresponding vector potential. The BI theory is a kind of generalization of Maxwell theory and the parameter α controls the degree of deviation from the Maxwell theory. It is known that in the BI theory the electrostatic energy of a point particle is finite and is a good for successful UV completion to the Maxwell theory. In $\alpha \rightarrow \infty$ limit, the Maxwell theory is recovered $\mathcal{L}_{\pm} \approx -F_{MN}^{(\pm)} F_{(\pm)}^{MN}/4$. The electric flux density is given by the nonvanishing components of the tensor defined by

$$E_{AB} = \frac{F_{AB}}{\sqrt{1 + \frac{F_{MN} F_{MN}}{8\alpha^2}}}. \quad (3)$$

By defining this, one can generalize Gauss's law in the Maxwell theory, to the case of the BI theory. For our purpose, it is rather convenient to introduce the rescaled parameter and gauge field by $\beta := \sqrt{2}\kappa_{n+1}\alpha$ and $\bar{A}_M = \sqrt{2}\kappa_{n+1}A_M$, respectively.

Domain wall universe: Then, we discuss the dynamics of a domain wall universe. It is assumed that the wall is infinitely thin and the Israel junction conditions can be applied. The bulk metric has a maximally symmetric, external $(n-1)$ -dimensional space and one static extra dimension:

$$ds_{\pm}^2 = -f_{\pm}(R)dT_{\pm}^2 + R^2\gamma_{ij}dx^i dx^j + \frac{dR^2}{f_{\pm}(R)}, \quad (4)$$

where γ_{ij} is the metric of a maximally symmetric $(n-1)$ -dimensional space and the subscripts (\pm) specify the bulk region.

One may choose the common spatial coordinates $R = R_+ = R_-$ and $x_+^i = x_-^i = x^i$. The domain wall is located at $(R = a(\tau), T_{\pm} = T_{\pm}(\tau))$, where τ represents the proper time on the wall, defined by $-f_{\pm}(a)\dot{T}_{\pm}^2 + \dot{a}^2/f_{\pm}(a) = -1$. $\epsilon_{\pm} = +1(-1)$ represents the outward (inward) direction, i.e., that of increasing (decreasing) a with respect to the wall. The induced metric on the wall becomes the Friedmann-Robertson-Walker form of curvature K with the scale factor a .

By varying the action (1) with respect to the metric degrees of freedom, the dynamics of a wall is determined by the Israel junction condition [29]: $[K_{\mu\nu} - g_{\mu\nu}K] = -\kappa_{n+1}^2 S_{\mu\nu}$, where the stress-energy tensor on the wall is defined by

$$\sqrt{-g}S^{\mu\nu} = 2\frac{\delta}{\delta g_{\mu\nu}} \int d^n x \sqrt{-g} \left(-\sigma + \mathcal{L}_m \right), \quad (5)$$

and $[A] := A_+ - A_-$ represents the jump of a bulk quantity A across the wall. In general, the wall induced stress-energy tensor in general has the form $S^{\mu}_{\nu} = \text{diag}(-\rho - \sigma, p - \sigma, \dots, p - \sigma)$. σ , ρ and p are the wall tension, the energy density and pressure of the time-dependent matter, respectively. The nonvanishing components of Israel junction conditions are given by

$$-\frac{n-1}{a} \sum_I \epsilon_I \sqrt{f_I + \dot{a}^2} = \kappa_{n+1}^2 (\rho + \sigma), \\ \sum_I \epsilon_I \left(\frac{n-2}{a} \sqrt{f_I + \dot{a}^2} + \frac{f_{I,a}}{2} \frac{1}{\sqrt{f_I + \dot{a}^2}} \right) \\ + \frac{\ddot{a}}{\sqrt{f_I + \dot{a}^2}} = \kappa_{n+1}^2 (p - \sigma), \quad (6)$$

where the index I runs (\pm) and we defined $\bar{F}_{(I)}^2 := \bar{F}_{MN}^{(I)} \bar{F}_{(I)}^{MN}$.

Then, we derive the $U(1)$ junction condition across the wall. The variation of the total action (1) with respect

to A_M gives

$$\begin{aligned}
\delta_{\bar{A}} S &= \frac{1}{2\kappa_{n+1}^2} \sum_I \int_{M_I} d^{n+1} X \sqrt{-G_{(I)}} \\
&\times \nabla_C \left(\frac{G_{(I)}^{AB} G_{(I)}^{EC} \bar{F}_{AE}^{(I)}}{\sqrt{1 + \bar{F}_{(I)}^2 / (8\beta^2)}} \right) \delta \bar{A}_B^{(I)} \\
&- \frac{1}{2\kappa_{n+1}^2} \int_{\Sigma} d^n y \sqrt{-g} \sum_I \frac{n_C^{(I)} G_{(I)}^{AB} G_{(I)}^{EC} \bar{F}_{EA}^{(I)} \delta \bar{A}_B^{(I)}}{\sqrt{1 + \bar{F}_{(I)}^2 / (8\beta^2)}} \\
&+ \int_{\Sigma} d^n y \sqrt{-g} \frac{\delta \mathcal{L}_m}{\delta \bar{A}_\tau} \delta \bar{A}_\tau. \tag{7}
\end{aligned}$$

Here on the boundary, we must impose the continuity condition of the gauge potential: $\bar{A}_M^{(+)} u_{(+)}^M = \bar{A}_M^{(-)} u_{(-)}^M =: \bar{A}_\tau$ on the wall Σ . The bulk parts of the variation $\delta \bar{A}_{(\pm)}$ in $M_{(\pm)}$ give rise to the equation of motion

$$\nabla_C \left(\frac{\bar{F}^{(I)BC}}{\sqrt{1 + \bar{F}_{(I)}^2 / (8\beta^2)}} \right) = 0. \tag{8}$$

Then, we derive the condition that the bulk gauge field satisfy on the boundary. In our spacetime, on Σ , $n_R = \epsilon \dot{T}$, $u^T = \dot{T}$ and $n_E^{(\pm)} G_{(I)}^{AB} G_{(I)}^{CE} \bar{F}_{EA}^{(I)} \delta \bar{A}_B^{(I)} = \epsilon_I \bar{F}_{T_I R}^{(I)} \delta \bar{A}_\tau$. Thus $U(1)$ -junction condition on Σ is given by

$$\sum_I \frac{\epsilon_I \bar{F}_{T_I R}^{(I)}}{\sqrt{1 + \frac{\bar{F}_{(I)}^2}{8\beta^2}}} = 2\kappa_{n+1}^2 \frac{\delta \mathcal{L}_m}{\delta \bar{A}_\tau}. \tag{9}$$

Born-Infeld black hole: We briefly review the static black hole solutions in the EBI theory. We will focus on the case of the asymptotically AdS bulk spacetime $\Lambda < 0$. For the moment, we omit the subscripts (\pm) . The metric is given by

$$ds^2 = -f(R) dT^2 + \frac{dR^2}{f(R)} + R^2 \gamma_{ij} dx^i dx^j, \tag{10}$$

where

$$\begin{aligned}
f(R) &:= K - \frac{m^2}{R^{n-2}} + \left(\frac{4\beta^2}{n(n-1)} + \frac{1}{\ell^2} \right) R^2 \\
&- \frac{2\sqrt{2}\beta}{n(n-1)R^{n-3}} \sqrt{2\beta^2 R^{2n-2} + (n-1)(n-2)q^2} \\
&+ \frac{2(n-1)q^2}{nR^{2n-4}} \\
&\times {}_2F_1 \left[\frac{n-2}{2n-2}, \frac{1}{2}, \frac{3n-4}{2n-2}; -\frac{(n-1)(n-2)q^2}{2\beta^2 R^{2n-2}} \right] \tag{11}
\end{aligned}$$

and ${}_2F_1[a, b, c; x]$ is Gauss's hypergeometric function. The AdS curvature length scale is related to the bulk cosmological constant through $\ell := \sqrt{-(n-1)(n-2)}/\Lambda$.

In the Maxwell limit $\beta \rightarrow \infty$ and/or for larger R , the solution reduces to the AdS Reissner-Nordström (AdS-RN) one:

$$f(R) = K + \frac{R^2}{\ell^2} + \frac{q^2}{R^{2n-4}} - \frac{m^2}{R^{n-2}} + O\left(\frac{1}{R^{4n-6}}\right). \tag{12}$$

In the regime $\beta = O(1)$, however, the behavior of the metric function can be modified because

$$\begin{aligned}
f(R) &= K - \frac{m^2 - A(n, \beta, q)}{R^{n-2}} \\
&- \left[\frac{2c(n)\beta}{n} - B(n, \beta, q)(2n-1)q \right] \frac{q}{R^{n-3}} \\
&+ \left[\frac{4\beta^2}{n(n-1)} + \frac{1}{\ell^2} \right] R^2 + O(R^{n+1}), \tag{13}
\end{aligned}$$

where

$$\begin{aligned}
A(n, \beta, q) &:= \frac{2(n-1)q^2}{n\sqrt{\pi}} \left(\frac{2\beta^2}{(n-1)(n-2)q^2} \right)^{(n-2)/(2n-2)} \\
&\times \Gamma\left(\frac{3n-4}{2n-2}\right) \Gamma\left(\frac{1}{2n-2}\right), \\
c(n) &:= \sqrt{\frac{2(n-2)}{n-1}} \\
B(n, \beta, q) &:= \frac{4\beta}{cn(2n-1)q} \frac{\Gamma\left(\frac{3n-4}{2n-2}\right) \Gamma\left(\frac{-1}{2n-2}\right)}{\Gamma\left(\frac{n-2}{2n-2}\right) \Gamma\left(\frac{2n-3}{2n-2}\right)}, \tag{14}
\end{aligned}$$

which are simply function of the charge q other than the parameters of the theory. The nonvanishing component of $U(1)$ field strength is given by

$$\bar{F}^{RT} = \frac{2\sqrt{(n-1)(n-2)}\beta q}{\sqrt{2\beta^2 R^{2n-2} + (n-1)(n-2)q^2}}. \tag{15}$$

The black hole is now positively charged. The corresponding gauge potential is given by

$$\begin{aligned}
\bar{A}_T &= \Phi + \sqrt{\frac{n-1}{2(n-2)}} \frac{q}{R^{n-2}} \\
&\times {}_2F_1 \left[\frac{2(n-2)}{(n-1)}, \frac{1}{2}, \frac{3n-4}{2(n-1)}, -\frac{(n-1)(n-2)q^2}{2\beta^2 R^{2n-2}} \right] \tag{16}
\end{aligned}$$

where Φ represents the possible constant shifts of the vector potential. In the vicinity of the black hole, \bar{A}_T is regular in contrast to the Maxwell theory, while in the far region, it falls off as $1/R^{n-2}$. The modified Gauss's law determines the black hole charge

$$Q = \int \left(4\mathcal{L}'(F^2) \right) \bar{F}^{RT} R^{n-1} d\Omega_{(n-1)}. \tag{17}$$

By performing the integration, the total charge is found to be $Q = \sqrt{2(n-1)(n-2)}q\Omega_{n-1}$ where Ω_{n-1} is the volume of $(n-1)$ -sphere. For the cases that $K = 0$ and $K = -1$ Gauss's law can be satisfied per unit volume.

The position of the horizon can be written as a function of other parameters $R_H = G(\ell, q, m^2)$, which can be inversely solved as $m^2 = F(\ell, q, R_H)$, where

$$\begin{aligned} F(\ell, q, R_H) &= KR_H^{n-2} + \left(\frac{4\beta^2}{n(n-1)} + \frac{1}{\ell^2} \right) R_H^n \\ &\quad - \frac{2\sqrt{2}\beta R_H}{n(n-1)} \sqrt{2\beta^2 R_H^{2n-2} + (n-1)(n-2)q^2} \\ &\quad + \frac{2(n-1)q^2}{nR_H^{n-2}} \\ &\quad \times {}_2F_1 \left[\frac{n-2}{2n-2}, \frac{1}{2}, \frac{3n-4}{2n-2}, -\frac{(n-1)(n-2)q^2}{2\beta^2 R_H^{2n-2}} \right]. \end{aligned} \quad (18)$$

If the equation $m^2 = F(\ell, q, R_H)$ has two, one and no roots of R_H for a given mass, then the black hole has two, one and no horizons, respectively. In the extremal case, two horizons are degenerating at R_H^{ext} , where $\partial F/\partial R_H = 0$ is satisfied. In the case that $K = 0$, the solution for the extremal condition is given by

$$R_H^{\text{ext}} = \left(\frac{\beta \ell^2 q \sqrt{8(n-2)}}{\sqrt{n} \sqrt{8\beta^2 \ell^2 + n(n-1)}} \right)^{1/(n-1)}, \quad (19)$$

whose corresponding mass is given by

$$\begin{aligned} m_{\text{ext}}^2 &= \frac{2(n-1)q^2}{n} \left(\frac{\sqrt{n(8\beta^2 \ell^2 + n^2 - n)}}{\beta \ell^2 \sqrt{8(n-2)q}} \right)^{\frac{n-2}{n-1}} \\ &\quad \times {}_2F_1 \left[\frac{n-2}{2n-2}, \frac{1}{2}, \frac{3n-4}{2n-2}, -\frac{n(n-1)(8\beta^2 \ell^2 + n(n-1))}{16\beta^4 \ell^4} \right]. \end{aligned} \quad (20)$$

In the limit $\beta \rightarrow \infty$,

$$m_{\text{ext}}^2 \rightarrow \frac{2(n-1)q^{n/(n-1)}}{n\ell^{(n-2)/(n-1)}} \left(\frac{n}{n-2} \right)^{\frac{n-2}{2(n-1)}}, \quad (21)$$

which is the result in the case of the RN solution. In the cases of curvature $K = \pm 1$, there is no analytic way to express m_{ext}^2 .

There would be at most two event horizons. Spacetime has two event horizons if its mass is in the range given by $m_{\text{ext}}^2 < m^2 < A(q)$, where A is defined in Eq. (14). It is straightforward to check that the ratio m_{ext}^2/A is always less than unity. Similarly, the spacetime has a single horizon for the larger masses $m^2 > A(q)$ and no horizon is formed for the smaller (even negative) masses $m^2 < m_{\text{ext}}^2$, i.e., a naked singularity at the center appears. In the case that $K < 0$, for some sets of parameters the mass of the black hole in the extremal case can be negative $m_{\text{ext}}^2 < 0$. In such a case, two horizons appear even for negative mass of black holes. In the limit that $\beta \rightarrow \infty$ we obtain $A \rightarrow \infty$ and there are always two horizons. If the extremal condition (19) is not satisfied at any mass of the black hole, only a single horizon appears. In the case that $m^2 < A$, no horizon is formed and a naked singularity appears. In the case that $m^2 > A$, a single horizon is formed.

Wall charge: The metric function f_{\pm} in Eq. (4) is given by Eq. (11), by replacing ℓ, q and m with ℓ_{\pm}, q_{\pm} and m_{\pm} , respectively. Now each bulk region is bounded by $0 < R < a(\tau)$ for $\epsilon = -1$ and $R > a(\tau)$ for $\epsilon = +1$. In the case $\epsilon = +1$, the graviton would not be localized on the wall, because the volume of the bulk is infinite and graviton zero mode is not nonavailable. The nonvanishing component of $U(1)$ -gauge field in each bulk side is obtained by replacing ℓ, q and m with ℓ_{\pm}, q_{\pm} and m_{\pm} in Eq. (15). The corresponding components of gauge potential is obtained from Eq. (16) by the same replacements with $\Phi \rightarrow \Phi_{\pm}$. Φ_{\pm} should be chosen to satisfy the continuity of the wall component of the gauge potential: $\bar{A}_{\tau} = \bar{A}_{T_+}^{(+)} \dot{T}_+ = \bar{A}_{T_-}^{(-)} \dot{T}_-$. The brane matter action is composed of the part of the ordinal matter, which is not coupled to the bulk electric field, and the one coupled to it:

$$\mathcal{L}_m = \mathcal{L}_0 + \frac{C}{a^{n-1}} \bar{A}_{\tau}, \quad (22)$$

where \mathcal{L}_0 represents the ordinary matter. Then, from the $U(1)$ -junction condition across the wall (9), we find

$$(2\kappa_{n+1}^2)C = \sqrt{2(n-1)(n-2)} (\epsilon_+ q_+ + \epsilon_- q_-). \quad (23)$$

In the Z_2 symmetric case $q_+ = q_- = q$ and $\epsilon_+ = \epsilon_- = \epsilon$,

$$(2\kappa_{n+1}^2)C = 2\sqrt{2(n-1)(n-2)}\epsilon q. \quad (24)$$

When $\epsilon_{\pm} = -1$, the wall is negatively charged to neutralize the positive charge of the black hole. In the case $\epsilon_{\pm} = +1$, both sides of the bulk do not contain the black hole horizons and electric field lines are extended from the wall toward the infinity. In the case $\epsilon_+ = -\epsilon_- = +1$, the spacetime is infinitely extended only into the (+)-direction. Then, the black hole charge is neutralized by that of the wall on the (-) side, and the charge on the (+)-side generates the electric field in the (+)-bulk. The opposite things happen in the case that $\epsilon_+ = -\epsilon_- = -1$.

Cosmology: We assume that the bulk spacetime is Z_2 -symmetric with respect to the wall: $\ell := \ell_+ = \ell_-$, $q := q_+ = q_-$ and $m := m_+ = m_-$. The energy density of the wall matter is given by

$$\rho = \frac{2C\bar{A}_T}{a^{n-1}f} (f + \dot{a}^2)^{1/2} + \rho_0 \quad (25)$$

where the first term represents the electrostatic energy induced by the wall charge and ρ_0 is that of the ordinary matter. Note that the continuity condition $\bar{A}_{\tau} = \bar{A}_T \dot{T} = \bar{A}_T (f + \dot{a}^2)^{1/2} / f$ and we may choose $\Phi = 0$. By squaring the first equation in Eq. (6), the effective cosmological equation can be derived in the form in analogy with the classical mechanics $\dot{a}^2 + V(a) = 0$, where we define the effective potential

$$V(a) := f(a) - \frac{\kappa_{n+1}^4 a^2 (\rho_0 + \sigma)^2}{4(n-1)^2 (1+G(a))^2}. \quad (26)$$

The function $G(a)$ is given by

$$G(a) = \sqrt{\frac{2(n-2)}{n-1} \frac{q\bar{A}_T}{a^{n-2}f}}, \quad (27)$$

which arises from the electrostatic energy. For $n = 4$, $G(a) \propto 1/a^7$ and at the later time the domain wall cosmology approaches the one in the neutral black hole background, including the RS cosmology.

For the numerical visualization of the potential, it is useful to introduce the dimensionless quantities $q = \hat{q}\ell^{n-2}$, $m = \hat{m}\ell^{(n-1)/2}$, $\beta = \hat{\beta}\ell^{-1}$, $t = \hat{t}\ell$ and $a = \hat{a}\ell$. The cosmological equations is reduced to $(\hat{a}, \hat{t})^2 + \hat{V}(\hat{a}) = 0$, where the potential is defined by

$$\hat{V}(\hat{a}) = \hat{f}(\hat{a}) - (x\hat{a})^2 \frac{(1 + \frac{\rho_0}{\sigma})^2}{(1 + \hat{G}(\hat{a}))^2}. \quad (28)$$

The dimensionless constant x is introduced by $x^2 := \kappa_{n+1}^2(\sigma\ell)^2/4/(n-1)^2$. The case of the RS tuning is that $x = 1$ and we will focus on $x \geq 1$, hence the universe approaches de Sitter geometry for larger \hat{a} . The dimensionless function is defined by

$$\hat{G}(\hat{a}) := \sqrt{\frac{2(n-2)}{(n-1)} \frac{\hat{q}\hat{A}_T(\hat{a})}{(\hat{a})^{n-2}\hat{f}(\hat{a})}} \quad (29)$$

where

$$\begin{aligned} \hat{f}(\hat{a}) = & K - \frac{\hat{m}^2}{(\hat{a})^{n-2}} + \left(\frac{4\hat{\beta}^2}{n(n-1)} + 1 \right) (\hat{a})^2 \\ & - \frac{2\sqrt{2}\hat{\beta}}{n(n-1)} \sqrt{2(\hat{\beta})^2(\hat{a})^{2n-2} + (n-1)(n-2)\hat{q}^2} \\ & + \frac{2(n-1)(\hat{q})^2}{n\hat{a}^{2n-4}} \\ & \times {}_2F_1 \left[\frac{n-2}{2(n-1)}, \frac{1}{2}, \frac{3n-4}{2(n-1)}, -\frac{(n-1)(n-2)\hat{q}^2}{2\hat{\beta}^2(\hat{a})^{2n-2}} \right], \end{aligned} \quad (30)$$

and

$$\begin{aligned} \hat{A}_T = & \sqrt{\frac{n-1}{2(n-2)} \frac{\hat{q}}{\hat{a}^{n-2}}} \\ & \times {}_2F_1 \left[\frac{2(n-2)}{(n-1)}, \frac{1}{2}, \frac{3n-4}{2(n-1)}, -\frac{(n-1)(n-2)\hat{q}^2}{2\hat{\beta}^2\hat{a}^{2n-2}} \right]. \end{aligned}$$

In Fig. 1, the behavior of \hat{f} is shown as the function of \hat{a} in the case of $K = 0$ and for $n = 4$, $\beta = 10$ and $\hat{q} = 3.0$. (We also set $\rho_0 = 0$ for simplicity) We consider the case of no horizon ($m^2 < m_{\text{ext}}^2$), the extremal case ($m^2 = m_{\text{ext}}^2$), that of two horizons ($A > m^2 > m_{\text{ext}}^2$) and that of a single horizon ($A < m^2$). In Fig. 1, each case is described by the solid, thick, dashed and dotted curves, respectively.

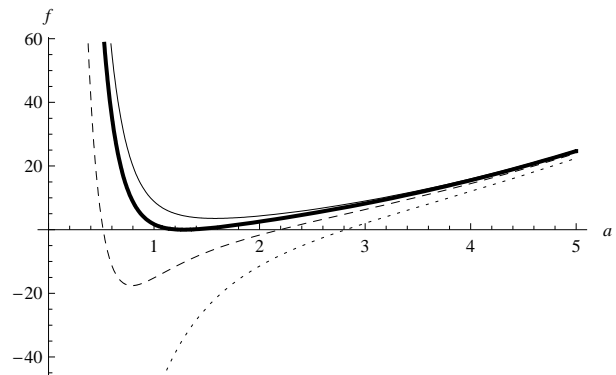


FIG. 1: Typical behaviors of the metric function \hat{f} are shown as functions of \hat{a} . The solid, thick, dashed and dotted curves correspond to the cases of no-horizon ($m^2 < m_{\text{ext}}^2$), the extremal ($m^2 = m_{\text{ext}}^2$), two-horizon ($A > m^2 > m_{\text{ext}}^2$), single-horizon ($A < m^2$), respectively. We have chosen $n = 4$, $\hat{\beta} = 10$ and $\hat{q} = 3$.

In Fig. 2, for the same set of parameters and $x = 1.1$, the behavior of the potential is shown as a function of \hat{a} . From the top to the bottom, the panels correspond to the cases of no horizon, extremal, two-horizon and single-horizon, respectively. In the first two cases, the potential is regular everywhere and the domain wall universe can undergo a bounce, if it is outside the horizon. In the last two cases, the potential negatively diverges at some places inside the horizon. But this divergence does not imply the presence of any singularity of the motion of the wall if it is inside the outer horizon, because the velocity of the brane diverges just at this instance and then becomes finite again. In these cases, in order to investigate its behavior just outside the horizon, we have to enlarge our plots.

In Fig. 3, for the same set of parameters as in the Fig. 1 and 2, the near-horizon behaviors of the metric function and potential are shown. In each panel, the solid and dashed lines represent the behavior of \hat{V} and \hat{f} , respectively. The top panel corresponds to the extremal case, and the remaining three panels to the case of two-horizon (the values of the mass chosen become larger, from the top to the bottom). One can see that in any case a bouncing can happen just in front of the outer horizon, although the height of the potential barrier is being smaller, leading to longer bouncing time for larger values of mass.

As the reference, in Fig. 4, we show the behavior of potential in neglecting the electrostatic energy (namely setting $\hat{G} = 0$ by hand). These results have been essentially shown in Refs. [12, 13, 14, 20]. Except for the final single-horizon case ($m^2 > A$), the domain wall could undergo a bounce inside the (outer) horizon. Our results showed that the inclusion of the electrostatic energy brings the bouncing point outside the horizon.

The feature of the potential is unchanged for difference choices of x as long as $x \geq 1$. Also for $K = \pm 1$, the

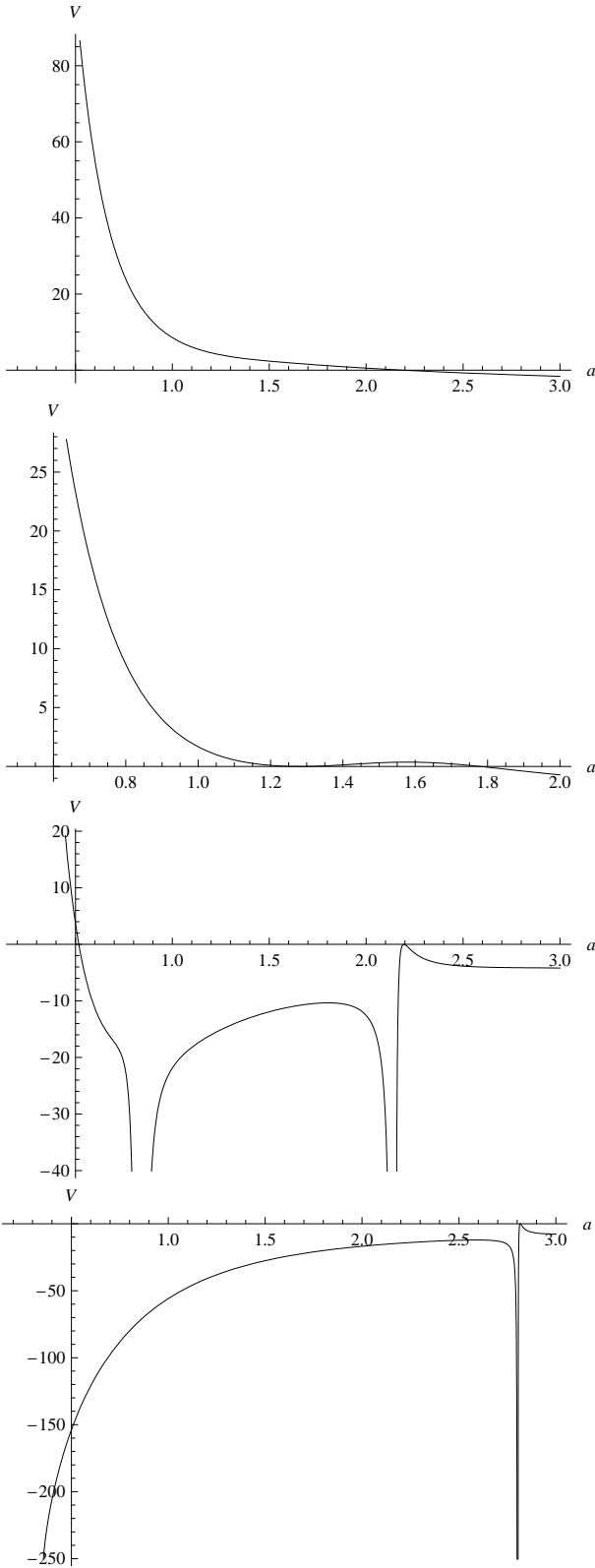


FIG. 2: Typical behaviors of the potential \hat{V} are shown as the function of \hat{a} . The first, second, third and 4th panels show the cases of no horizon ($m^2 < m_{\text{ext}}^2$), the extremal ($m^2 = m_{\text{ext}}^2$), two horizons ($A > m^2 > m_{\text{ext}}^2$) and single horizon ($A < m^2$), respectively. We have chosen $n = 4$, $\hat{\beta} = 10$, $\hat{q} = 3$ and $x = 1.1$.

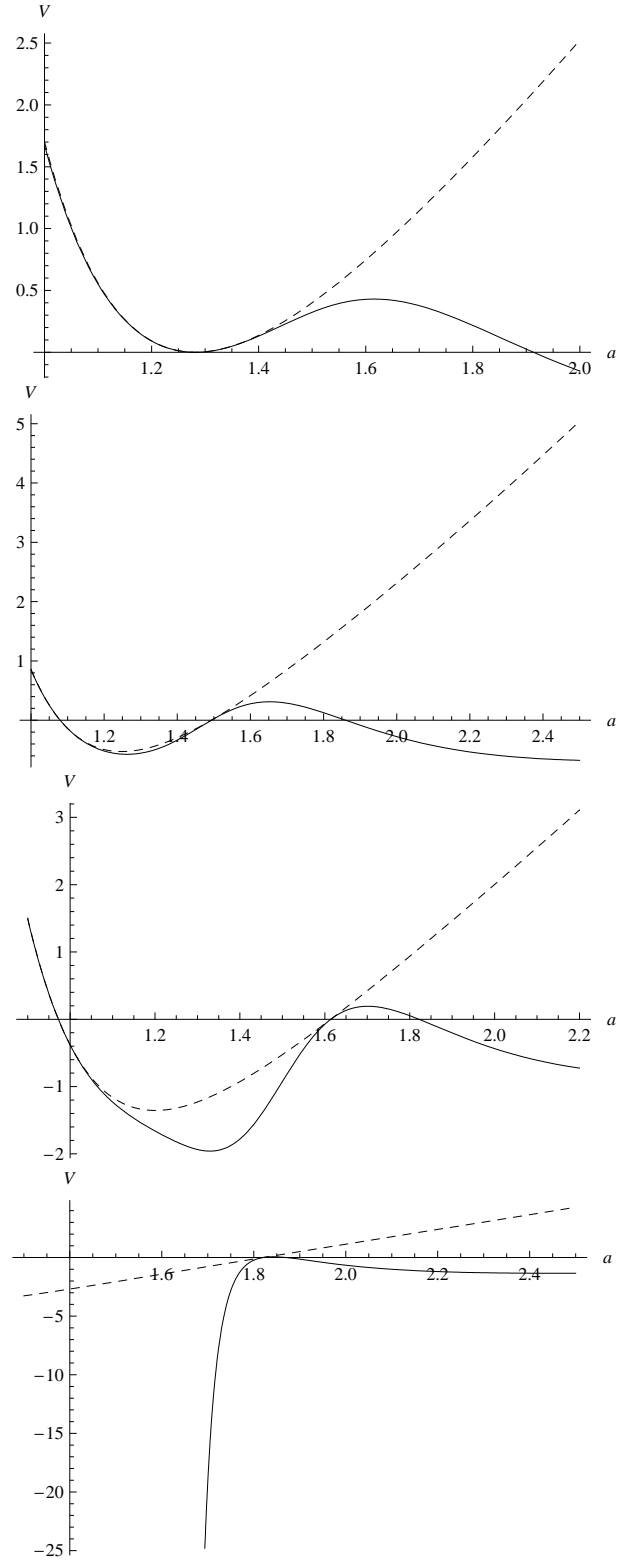


FIG. 3: For the same set of parameters as in the Fig. 1 and 2, the plots on the near-horizon behaviors of the metric function and potential are shown. On the top panel, the plot for the extremal case is shown. In the remaining three plots, from the top to the bottom panels, the values of the black hole mass which we choose become larger. In each panel, the solid and dashed lines represent the behavior of \hat{V} and \hat{f} , respectively.

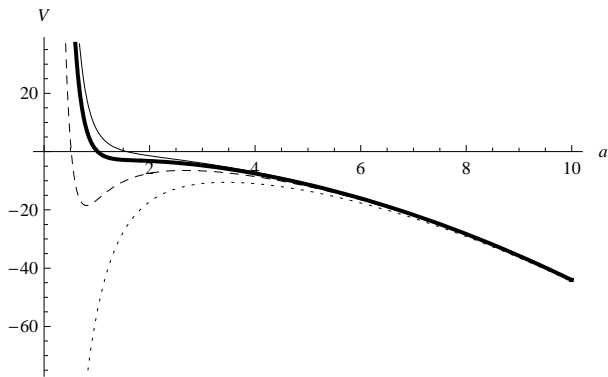


FIG. 4: Typical behaviors of the potential \hat{V} are shown as the function of \hat{a} , by setting $\hat{G} = 0$ by hand. The solid, thick, dashed and dotted curves show no horizon ($m^2 < m_{\text{ext}}^2$), extremal ($m^2 = m_{\text{ext}}^2$), two-horizon ($A > m^2 > m_{\text{ext}}^2$) and single horizon ($A < m^2$) cases. We have chosen $n = 4$, $\hat{\beta} = 10$, $\hat{q} = 3$ and $x = 1.1$ (RS-tuning).

results remaining essentially the same. The bouncing of the domain wall universe always happens just outside the outer horizon and height of the barrier becomes smaller for larger values of the black hole mass, leading to longer time scales for bouncing.

Summary: In this Letter, we discussed the dynamics of a domain wall universe in the Einstein-Born-Infeld (EBI) theory. We assumed that the spacetime is asymptotically anti-de Sitter (AdS). In the previous works, the electrostatic energy of the wall is not taken into account. In this work, we have taken it into consideration, which is determined through the $U(1)$ junction condition.

We obtained the effective Friedmann equation on the wall in the EBI theory. There are four possible spacetime structures, i.e., those with no horizon (naked singularity), the extremal one, those with two horizons (as the Reissner-Nordström black hole), and those with a single horizon (as the Schwarzschild black hole) cases. We find that a cosmological bounce always can happen *outside* the (outer) horizon. The height of the barrier between the bouncing point and horizon in the effective potential is being smaller for larger values of the black hole

mass, which leads to the longer bouncing time. These results are in contrast to the results obtained in previous works, suggesting that regular bounce can happen *inside* the horizon except for $m^2 > A$.

In all the cases discussed in this Letter, at the later times, the contribution of the electrostatic energy falls off very rapidly proportional to a^{-7} and the domain wall cosmology reduces to the one in the neutral black hole background, which include the case of the Randall-Sundrum model.

Finally, we shall mention the stability of the system discussed in this Letter. We speculate that our system is stable for most possible choices of parameters, in terms of the stability of asymptotically AdS, charged and static black hole solutions. To our knowledge, there has been no work which investigated the dynamical stability of the charged, asymptotically AdS black hole solutions in the EBI theory. In the case of the $D \geq 5$ -dimensional Einstein-Maxwell theory, in Ref. [31] the stability of Reissner-Nordström AdS black hole in $D \geq 5$ against the vector and tensor perturbations was shown. The stability against the scalar perturbations in $D = 5, 6, \dots, 11$ -dimensional EM theory was shown for all the parameters of charge and cosmological constant in Ref. [32]. In addition, in terms of the thermodynamical arguments, in Ref. [21], it was shown that asymptotically AdS black holes in EBI theory are thermodynamical stable, always for $K = 0, -1$, and for $K = +1$ if the BI parameter β is larger than some critical value β_c . These facts suggest that the asymptotic AdS black hole solutions in the EBI theory are also dynamically stable for at least most possible choices of parameters. Thus, the system discussed in this Letter is also expected to be stable.

Acknowledgements

This work was supported by the Korea Science and Engineering Foundation(KOSEF) grant funded by the Korea government (MEST) through the Center for Quantum Spacetime (CQUeST). W.L. was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD)(KRF-2007-355-C00014).

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