

# Consistency analysis of a Dark Matter velocity dependent force as an alternative to the Cosmological Constant

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## ABSTRACT

A range of cosmological observations demonstrate an accelerated expansion of the Universe, and the most likely explanation of this phenomenon is a cosmological constant. Given the importance of understanding the underlying physics, it is relevant to investigate alternative models. This article uses numerical simulations to test the consistency of one such alternative model. Specifically, this model has no cosmological constant, instead the dark matter particles have an extra force proportional to velocity squared, somewhat reminiscent of the magnetic force in electrodynamics. The constant strength of the force is the only free parameter. Since bottom-up structure formation creates cosmological structures whose internal velocity dispersions increase in time, this model may mimic the temporal evolution of the effect from a cosmological constant. It is shown that models with force linearly proportional to internal velocities, or models proportional to velocity to power three or more cannot mimic the accelerated expansion induced by a cosmological constant. However, models proportional to velocity squared are still consistent with the temporal evolution of a Universe with a cosmological model.

*Keywords:* dark matter – acceleration of particles

## 1. INTRODUCTION

Electromagnetism, dark matter and dark energy vary both in years since each were discovered, and in the establishment of their theoretical foundations and interpretations. Electromagnetism was discovered by Coulomb, Ørsted and many others more than 200 years ago, and finally described definitively by Maxwell 150 years ago. And possibly the only remaining question is why Nature chose at small energies to have a gauge group which involves the  $U(1)$  of the photon.

Dark matter (DM), on the other hand, was discovered gravitationally by Lundmark, Oort and Zwicky about 80-90 years ago, and today we still have very little understanding of the particle properties of the dark matter, except for upper bounds on various parameters. Dark matter is likely an essentially cold particle (Kopp et al. 2018), which is very easy to envisage from a particle physics point of view. The number of dark matter candidates, which have been proposed, is enormous, and these models cover a very wide range of masses, charges and gauge groups (Bertone et al. 2005).

The accelerated expansion of the Universe was established through the observations of supernovae (Perlmutter et al. 1999; Riess et al. 1998), and subsequently these observations have been confirmed through cosmic microwave background and baryonic acoustic oscillations (Komatsu et al. 2011; Percival et al. 2010; Blake et al. 2011). A range of analyses have demonstrated that the interpretation, that the acceleration is induced by a cosmological constant, is in good agreement with data (Larson et al. 2011; Percival et al. 2010; Hicken et al. 2009; Blake et al. 2012).

This article investigates an alternative to the cosmological constant. Instead of the standard model with a cosmological constant and dark matter which only has gravitational interaction, this model has no cosmological constant. Instead the dark matter in this model has, in addition to the gravitational attraction, a repulsive force which depends on the dark matter velocity dispersion. This is somewhat similar to the Lorentz force of electromagnetism which depends on velocity squared, in the sense that the one moving particle creates a magnetic field, and the other particle feels a force proportional to its own velocity times the magnetic field. Since structure formation proceeds bottom-up, implying that later structures have larger internal velocity dispersions, this model may mimic the temporal evolution of the effect

of a cosmological constant. Numerical simulations are used to show that the observational data are still not sufficiently detailed to reject models where the repulsive force depends on the square of the DM velocities.

## 2. THE ACCELERATION EQUATION

A particle at the edge of a homogeneous sphere with radius  $R(t)$  feels the gravitational acceleration,  $GM/R^2$ , where  $M$  is the mass inside the sphere. Here the physical radius  $R(t)$  is related to the comoving distance,  $r$ , through  $R(t) = a(t)r$ , where  $a(t)$  is the time-dependent scale factor.

In this description (Harrison 1965; Peebles 1980) one can express the acceleration of that particle in the real expanding Universe through

$$\frac{\ddot{R}}{R} = \frac{-H_0^2}{2} \left( \Omega_{m,0} \frac{r^3}{R^3} - 2\Omega_{\Lambda,0} \right). \quad (1)$$

Here and below the contribution from relativistic particles is ignored, and  $\Omega_{m,0}$  and  $\Omega_{\Lambda,0}$  represent the values of the densities today.

The first term on the right hand side of eq. (1) corresponds to the force from the gravitational attraction of all the matter inside a sphere of radius  $R$ , and the second term corresponds to a gravitational repulsion from a cosmological constant inside a sphere of radius  $R$  (the factor  $-2$  arises from the  $(1+3\omega_\Lambda)$  in the acceleration equation, since  $\omega_\Lambda = -1$ ). Knowing the present values of the total matter,  $\Omega_{m,0} \approx 0.3$ , cosmological constant,  $\Omega_{\Lambda,0} \approx 0.7$ , and Hubble parameter,  $H_0 \approx 70 \text{ km}/(\text{s Mpc})$ , allow us to calculate the expansion of the Universe as a function of time.

In order to see the detailed evolution we normalize with the corresponding acceleration from a matter-only Universe,

$$\frac{\ddot{R}_{\text{m only}}}{R} = \frac{-H_0^2}{2} \left( \Omega_{m,0} \frac{r^3}{R^3} \right), \quad (2)$$

which gives us

$$\frac{\ddot{R}}{\ddot{R}_{\text{m only}}} = 1 - 2 \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} a^3(t). \quad (3)$$

This ratio evolves from unity in the early matter dominated universe, crosses zero a few billion years ago, and is negative today where  $a_{\text{today}} = 1$ .

## 3. AN ALTERNATIVE MODEL

One could in principle consider an expression similar to eq. (3) where the time dependent part could arise from something different from a cosmological constant

$$\frac{\ddot{R}_{\text{alternative}}}{\ddot{R}_{\text{m only}}} = 1 - K(t). \quad (4)$$

If the DM for instance has a charge which was zero at early times and increases as the universe evolves, then a term,  $K(t)$ , could be constructed such that the temporal evolution would follow that of the normal cosmological constant term. The corresponding force would be of the  $1/r^2$  type, just like a Coulomb force.

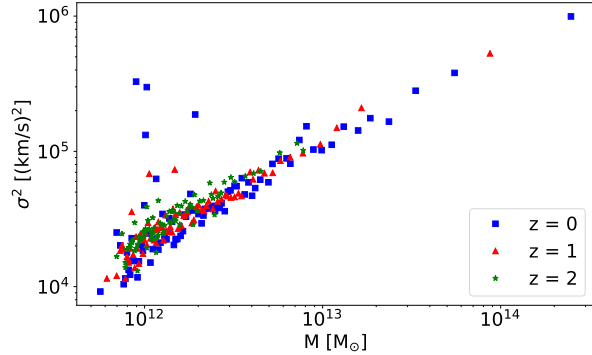
This article considers a new  $1/r^2$  force between DM particles. For simplicity this force is taken to be proportional to the velocity dispersion inside a galaxy: Imagine two galaxies at a large distance. One DM particle in galaxy A will feel the gravitational pull from all particles in galaxy B. And in addition, in this model, particle A will also feel a new force proportional to  $\sigma_B^2$  from all the particles in galaxy B. The form of this force is rather arbitrary, but it is naturally inspired by the Lorentz force which is proportional to the (cross product) of the velocities of particles in galaxies A and B. The only free parameter is now the constant strength of this new force, and by choosing the sign to be negative, this force may be repulsive. The normalized acceleration in this universe, with no cosmological constant but including the new force between DM particles, now reads

$$\frac{\ddot{R}_{DM}}{\ddot{R}_{\text{m only}}} = 1 + \kappa \sum_i \left( \frac{\sigma_i}{c} \right)^2, \quad (5)$$

where the individual  $\sigma_i$ s have been normalized to the speed of light in order to have  $\kappa$  dimensionless. It is clear, that if a typical  $\sigma$  is about 200km/s (see figure 1) and one want this new term to be of the order  $-2 \times 0.7/0.3$  (see eq. (3)), then  $\kappa$  must be of the order  $-10^6$ . If we compare with the forces of electromagnetism, then this model does not have any normal charge, and hence no Coulomb force. In figure 1 we also see an indication of the temporal evolution, where typical velocity dispersions (along with the masses of the structures) increase as the universe evolves. The interesting question is now, what is the temporal evolution of this term in eq. (5). In order to address this question we turn to numerical simulations.

## 4. NUMERICALLY SIMULATED UNIVERSE

The initial conditions have been produced with MUSIC (Hahn & Abel 2011), using cosmological parameters as observed by Planck (Aghanim et al 2011). The numerical code RAMSES (Teyssier 2002) is used to perform a set of pure DM simulations with different initial conditions. With a boxlength of 96Mpc and  $2.1 \times 10^6$  particles, the individual particle masses are  $1.6 \times 10^{10} M_\odot$ . The HOP Halofinder from the yt Project (Turk et al 2010; Eisenstein & Hut 2010) is used to identify structures.



**Figure 1.** The connection between halo mass and velocity dispersion. Structure formation proceeds bottom-up, which implies that at later times the structures have larger masses and hence higher velocity dispersions.

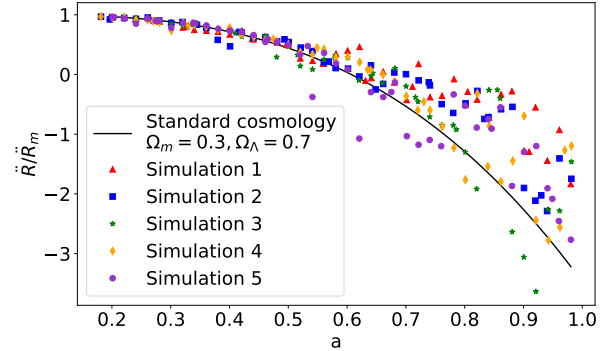
For each of the 50 snapshots between redshift 20 and zero, 64 independent spheres of 9.6 Mpc comoving radii are selected. For each sphere the acceleration felt by a particle at a random position on that sphere is averaged, from each of the DM haloes inside the sphere, according to eq. (5). This includes both the gravitational attraction and the repulsive effect of acceleration due to the new DM force.

In order to determine the value of  $\kappa$  the acceleration equation, eq. (5) is fit to the analytical behaviour in a standard  $\Lambda$ -dominated universe, eq. (1), between redshift 20 and  $z = 0.6$ . The latter value is chosen since this represents the transition between a matter dominated and a cosmological dominated universe. If the model described above should be able to mimic the observed acceleration of the Universe, then the entire curve, from redshift 20 to  $z = 0$  should have the same shape as the analytical shape from a Lambda-dominated universe. In figure 2 is shown the temporal evolution of 5 numerical simulations with different random seeds for the initial condition. As is clear from this figure, the model has a temporal evolution which mimics a cosmological constant fairly well.

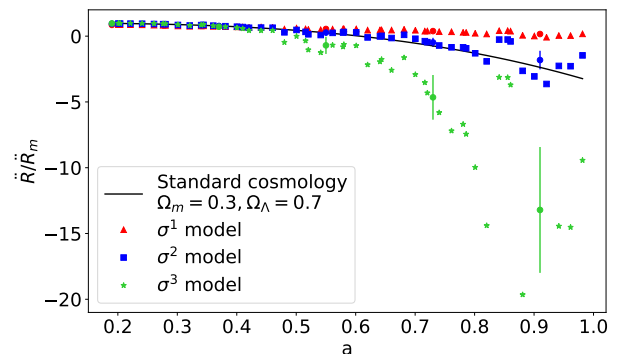
If one instead had postulated a force which is linearly proportional to  $|\sigma|$ , or to the third power,  $|\sigma|^3$ , then the temporal evolution would be significantly different from that of a cosmological constant. This is seen in figure 3, where the plotted error-bars represent a spread calculated from scatter between simulations with different initial conditions.

## 5. LIMITATIONS OF THE MODEL

The numerical simulations in this article have been performed using a standard cosmological model. As was shown in figure 2, the expansion of the universe with



**Figure 2.** The acceleration normalized to that of a matter dominated universe. The solid line is the analytical result of a standard cosmology including DM and a cosmological constant. The calculated symbols show the acceleration from the new force induced by the velocity dispersions of DM in halos. The 5 numerical simulations have different initial conditions, and hence the scatter reflects the cosmic variance. For each simulation the magnitude of the force is fitted in the range  $0.2 < a < 0.6$ .



**Figure 3.** The acceleration normalized to that of a matter dominated universe. The solid line is the analytical result of a standard cosmology including DM and a cosmological constant. The calculated symbols show the acceleration from the new force induced by the velocity dispersions of DM in halos. The different symbols reflect different possible dependencies of how the new force depends on the velocity dispersion, and each set of symbols is fitted in the range  $0.2 < a < 0.6$ . It is clear from the figure, that only forces proportional to  $\sigma^2$  have a temporal evolution which approximately mimics that of a cosmological constant.

the new DM model would rather accurately follow this behaviour. This means that the model, within the scatter of the numerical simulations, is consistent with the evolution driven by a cosmological constant model. Naturally, it would be very interesting to modify the numerical code to correctly calculate the acceleration from the new force, with no reference to the cosmological con-

stant model. That would also allow for local variations from the uniform acceleration induced by a cosmological constant.

If the model discussed in this article should have any relevance to Nature, then one would want to measure the *difference* between the new DM force and the effect of a standard cosmological constant. That difference will be visible in figure 2, where a departure from the analytical line represents a variation with respect to the cosmological constant. We will leave it to more sophisticated simulations to investigate this. Similarly, one should expect that the temporal evolution in the future should level out at a constant normalized acceleration for this new DM force, because in the far future structure formation effectively ends, leading to no increased acceleration in this model. This is in stark contrast to the evolution from a cosmological constant, which keeps accelerating the expansion rate.

A very important question is, to which degree other cosmological observables already exclude the simple model discussed above, including the results from BAO, CMB and SN1a observations. The simple answer is, that to first degree there is no difference at all. All these observables are derived under the assumption of the universe containing a distribution of photons, baryons, leptons and dark matter, in exactly the same way as the model described above. In addition the standard cosmological model used to derive these observables also include the changed expansion rate of the universe, as induced by the cosmological constant. However, this acceleration is typically included in the calculations by going from proper to comoving coordinates, and at the same time the scale parameter,  $a(t)$ , is calculated from the background cosmological model. To the extent that the data-points in Fig. 2 above, agree with the analytical derivation from the standard cosmological model (solid line in Fig. 2), then the scale parameter is unchanged. Within the scatter from the simulations with different initial conditions, there is perfect consistency between the standard cosmological model and the model discussed above, as seen in Fig. 2. Specifically will all the linear properties derived in the standard cosmological model, including the CMB power spectra, be identical to the ones calculated in the model we discuss in this paper. The only potentially serious problem with this model we discuss it at small scales. Using a magnitude of the new DM force, which is sufficiently big to explain the accelerated expansion of the Universe, probably leads to inconsistencies with the internal stability of dwarf galaxies, galaxies and galaxy clusters, since the individual DM particles are moving with such high veloci-

ties, that the repulsive force locally may be much larger than the gravitational attraction today. This problem could be circumvented by some screening-mechanism, which should render this new force negligible on smaller scales than few Mpc. This would have negligible influence on the accumulated effect on large scales. In addition, infalling and merging processes will continue as in the standard picture.

The simulations presented above are limited in number of particles and boxsize, and both affect the halo distribution. To test the effect of the highest mass structures we perform a simulation with  $16.8 \times 10^6$  particles and boxlength of 192Mpc. When calculating the normalized acceleration we find that there is virtually no difference from the smaller boxsize simulation because of the very sharp drop in the halo-mass-function at high masses. Similarly we perform a simulation with  $16.8 \times 10^6$  particles and boxlengths of 96 Mpc, in order to see the effect of the many smaller structures. Again we see that there is a very small effect, because the  $M - \sigma^2$  variation is more important than the increase in number of smaller haloes. Finally we have assumed that the new force is proportional to the velocity squared, summed over all haloes. To this end we are implicitly assuming that first identifying the haloes (including all particles which have departed from the general expansion of the universe), and then subsequently summing over all haloes, is the correct approach. We wish to consider alternatives to this approach in a future study.

## 6. CONCLUSION

With a purely phenomenological approach, we allow the dark matter particles to have a new repulsive force proportional to the squared internal velocity dispersions of cosmological halos. Since structure-formation proceeds bottom-up, this implies that this force grows as a function of time. We use numerical cosmological simulations to show that this force may mimic the temporal evolution of a cosmological constant. We also find that similar forces linearly proportional to the velocity dispersion, or to power three, are inconsistent with the temporal evolution of our Universe.

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