Gravitation in 4D Euclidean Space-Time Geometry

Franz-Günter Winkler

Stumpergasse 63/11
A-1060 Vienna, Austria
fg.winkler@aon.at

Abstract

The Euclidean interpretation of special relativity provides an intuitive way to understand and derive the Lorentz transformations in the framework of a "natural" 4D Euclidean space-time geometry. In this article the conceptual basis for a purely metric generalization of the Euclidean view is laid. It consists of *i)* the assumption of spatial and directional variations of the speed of light (VSL), *ii)* a formulation of the principle of general covariance in 4D Euclidean geometry, and *iii)* a generally covariant motion law for point particles. For the gravitation model, which is developed on this basis, three out of four effects of the Schwarzschild solution are derived (shift of spectral lines, deflection of light, precession of perihelia of planetary orbits). The explanation of the Shapiro radar echo delay requires modifications of the space-time geometry of the sun's environment. The additional effects brought forth by the respective model entail a possible account of the coronal heating problem and thus make the physics of the sun's environment a test bed for the suggested Euclidean general relativity.

Keywords: gravitation, general relativity, Euclidean space-time geometry, VSL (varying speed of light), coronal heating problem

1 Introduction

The Euclidean interpretation of special relativity [1-3] is an intuitive formulation of special relativity in a "natural" 4D Euclidean geometry, which is nothing but an extension of 3D spatial Euclidean geometry by a fully integrated real time dimension. Being just a different view of one-and-the-same theory, the Euclidean interpretation has no empirical implications. Things change, however, when the Euclidean viewpoint of special relativity is generalized towards what will be called the Euclidean general relativity (EGR).

So far, the envisaged EGR is a standalone approach which cannot be directly linked to known gravitation theories. It differs from standard general relativity by the choice of the basic geometry, by its avoidance of singularities and by its potential extensibility to other fields in addition to gravitation. It also differs from VSL (varying speed of light) models[1] implementing Lorentz-Poincaré type interpretations of relativity by fully subscribing to general covariance and by being purely metric.[2]

In this article we lay EGR's conceptual basis and develop an account of the gravitational field of a spherical massive body. The first model in our three step approach is described by the same equations of motion as Newtonian gravitation, but already explains gravitational red shift. The second model (the "reference model") is built upon the first and allows us to derive the phenomena of light deflection and perihelion precession of planetary orbits known from the Schwarzschild solution. It does, however, not explain the Shapiro radar echo delay. The third model deals with the necessary geometric deviations from the reference model which have to be postulated for the sun's environment. The additional effects following from the altered geometry suggest an account of the coronal heating problem. By this, the physics of the sun's

[1] For an overview, see Magueijo's VSL survey article [4].
[2] More specifically, EGR involves only one metric, while e.g. in Broekaert's gravitation model [5] an „energy-momentum metric" is postulated in addition to the VSL-based space-time metric.

environment becomes a theoretical and empirical test bed for the suggested Euclidean general relativity.

2 Euclidean Special Relativity

The geometric part of the Euclidean interpretation of special relativity can be summarized by a short statement:

> *Special relativity can be fully understood and derived in 4D Euclidean space-time geometry, i.e. a geometry whose metric is defined by the line element*

$$d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + c^2 \Delta t^2 . \tag{1}$$

In the light of Minkowski's non-Euclidean representation of the Lorentz transformations, this may look truly absurd at first sight. In the following, we will illustrate the (geometrically trivial) validity of the above statement.

2.1 Light Clocks

The Euclidean view of special relativity is best introduced by the concept of a light clock. It serves us as a model for both the observer performing space and time measurements and for the object that is being measured.

In *figure 1* the light clock is introduced from the viewpoint of an observer, for whom the invariance of the speed of light is assumed. In addition to Einstein's light clock a second photon is reflected between two mirrors, which are fastened at the end points of a stick. The photons are synchronized such that they always meet in the middle point *M* of the stick. By this, Einstein's definition of synchrony holds for events *A* and *B* as well as for events *C* and *D*.

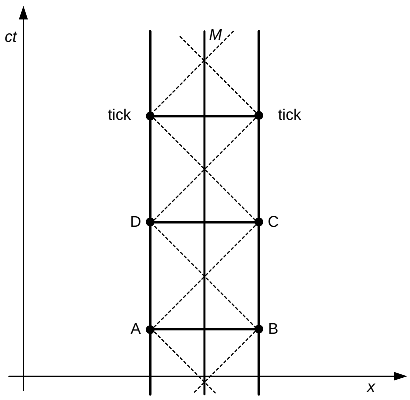

Fig. 1. A resting light clock consisting of two mirrors between which two photons are reflected. The light rays meet in the middle point M. Events A and B respectively C and D are simultaneous.

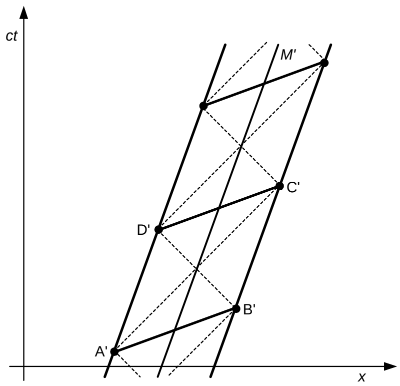

Fig. 2. A moving light clock. Events *A'* and *B'* respectively events *C'* and *D'* are simultaneous.

While *figure 1* shows a resting light clock, the light clock of *figure 2* is in constant motion along the *x*-axis. For the moving clock, events *A'* and *B'* as well as events *C'* and *D'* are simultaneous.

The clocks in *figure 1* and *figure 2* represent inertial observers holding their respective space-time frames. The rectangle *ABCD* respectively the parallelogram *A'B'C'D'* can be regarded as the elementary cells of these frames. This construction can, of course, be extended to more than one space dimension (c.f. *figure 9*).

2.2 Normalized Light Clocks and Measurement

We call light clocks in arbitrary constant motion states *normalized*, if the Euclidean space-time volumes of their elementary cells are identical and if the respective observers agree to call the length of their own light clock *1 meter* and the time span between two ticks *1 second/c*. By this, the speed of light is *c* for all normalized light clocks.[3]

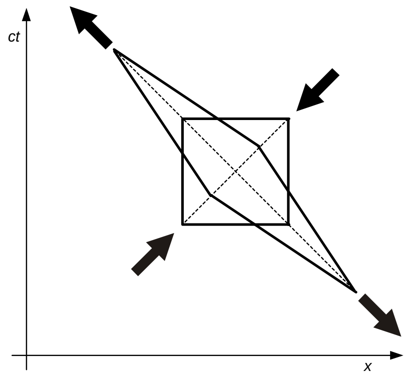

Fig. 3. Illustration of the Lorentz transformation as a combined stretching and compressing operation.

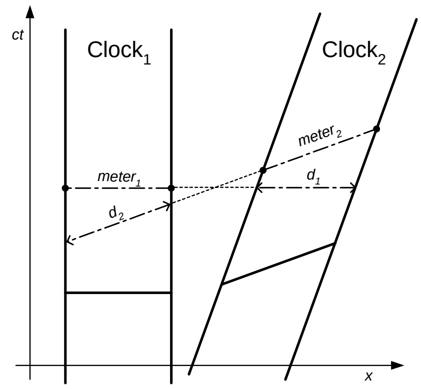

Fig. 4. Two observers measure each other's lengths simultaneously from their own point of view.

Figure 3 shows the relation between the space-time cells of two normalized light clocks. Any combined stretching by some factor *S* along one light diagonal and compressing by the same factor *S* along the other light diagonal produces another normalized space-time cell.
As can be shown very easily [1], the Lorentz transformations are valid for the measurements performed by normalized light clocks, if the results of space and time measurements are understood as *ratios of Euclidean space-time distances* as illustrated below.

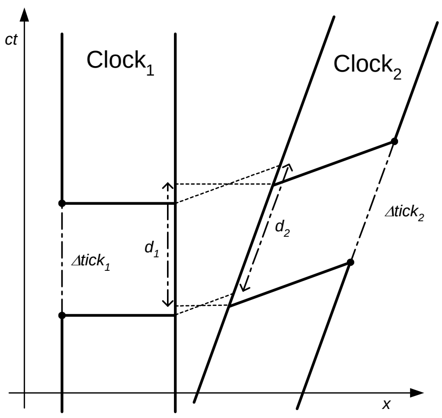

Fig. 5. Two observers measure the time between two ticks of the other observer's light clock.

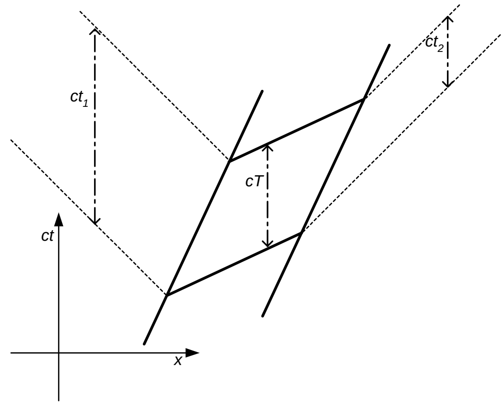

Fig. 6. The time intervals of photons emitted from a moving light clock obey the relativistic Doppler effect.

The normalized light clocks in *figure 4* measure each other's lengths to the same value, for the following relations between Euclidean space-time distances are identical.

$$d_1 : meter_1 = d_2 : meter_2 \tag{2}$$

The normalized light clocks in *figure 5* measure each other's time interval to the same value, for the following ratios of Euclidean space-time distances are identical.

$$d_1 : \Delta tick_1 = d_2 : \Delta tick_2 \tag{3}$$

As a consequence of the geometrical properties (5), the statements (2) and (3) are still valid, if the whole scenario undergoes a Lorentz transformation.

[3] Note that the invariance of the speed of light is mere convention at this stage (c.f. the assumption on "normalization" in section 3).

2.3 Doppler Effect and Energy

The Euclidean view of special relativity allows us to introduce relativistic energy relations on a mere geometric basis. *Figure 6* shows a moving light clock that emits two photons in each direction. The time intervals reflect the relativistic Doppler effect when being measured by a resting observer. In a thought experiment, we let the whole energy of a physical body be emitted by radiation. The energy is now contained in the sum of the radiation energies which are proportional to their frequencies. It can be shown easily that this sum behaves like the total relativistic energy under Lorentz transformations. As a geometric measure for it we can take the time extension of a space-time cell.

$$E \sim \frac{1}{cT} = \frac{1}{2ct_1} + \frac{1}{2ct_2} \tag{4}$$

This intuitive approach to relativistic energy will be sufficient for the mathematical treatment of energy conservation in the gravitation models in sections 5 and 6.

2.4 Euclidean Geometry and the Lorentz Transformations

Behind the suggested explanation of the relativity of space and time measurements stands a class of geometric properties, all of which are easy to show.

After Lorentz transformations (5)

- *parallel lines are still parallel,*
- *sections of a line show the same ratios as before,*
- *sections of an area show the same ratios as before,*
- *sections of a 3D volume show the same ratios as before,*
- *4D volumes show the same ratios as before.*

The listed geometrical properties may either be accepted as mere by-products of the Lorentz transformations or may motivate different derivations and interpretations of Einstein's special relativity. The latter has been suggested in [1-3]. An appropriate minimal set of assumptions to derive the Lorentz transformations on the basis of the conservation of space-time volumes is the following.[4]

- *The speed of light is invariant for some inertial observer.*[5]
- *Changing the speed of a physical object (in a non-destructive way) does not change the Euclidean space-time volume of its space-time cells.*

The relativity principle, though being technically replaced by the second assumption, is fully supported by this derivation and the connected interpretation. This will hold also for the generalization of the Euclidean approach.

2.5 Arbitrary constant light speeds

After introducing the Euclidean view of special relativity, there is just one more conceptual ingredient to EGR, namely the assumption of spatial and directional light speed variations. We get started by showing that the Euclidean perspective can easily handle scenarios where the speed of light is assumed to be different in opposite directions.

---

[4] To be more precise, the assumption of the homogeneity of space and time should be added.

[5] As will become clear in the following, the presented approach is far away from "Lorentz-Poincaré type" interpretations of relativity theory involving an absolute rest frame.

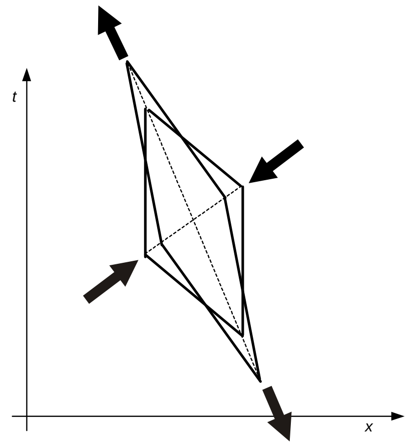

Fig. 7. Combined stretching and compressing of a space-time cell in an environment with different light speeds in opposite directions.

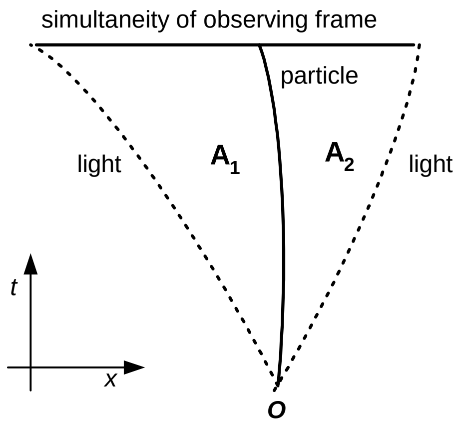

Fig. 8. The relation between areas A_1 and A_2 stays identical for infinitesimal time steps during acceleration.

Figure 7 shows the space-time cells of two normalized observers located in an environment where the speed of light is faster in the positive *x*-direction and slower in the negative *x*-direction. All the geometrical relations from (5) still hold, including the metaphor for the Lorentz transformations as a combined stretching and compressing operation.

An intuitive proof for this statement goes as follows. Considering the role of the constant *c* in the Lorentz transformations, it is clear that special relativity still works if the speed of light takes a different constant value. This means for the light clock model that the space-time cell of the resting light clock may be stretched or compressed along the time (or space) axis. The statements (2) and (3), which express specific relations between Euclidean space-time distances, remain untouched by this operation. They are also still valid after a Euclidean rotation of a space-time diagram.

In fact, any normalized space-time cell can be constructed from a given space-time cell by a combination of the following operations:

- *combined stretching and compressing along space and time axes,*
- *Lorentz transformation (combined stretching and compressing along light diagonals),*
- *Euclidean rotation.*

All three operations do not only conserve the space-time volume, they keep the relations from (5). The Euclidean rotation not only changes the light speeds, but makes them also depend on direction. However, all normalized observers measure their local light speed in all directions to the same value *c*.

3 The Principles of the Euclidean General Relativity

Generalizing the Euclidean view of special relativity is conceptually simple. Instead of confining to straight light trajectories, continuous and several times differentiable light trajectories are considered which are embedded in 4D Euclidean space-time geometry. According to the above arguments, special relativity works locally for small space-time regions.

It should be noted, though, that EGR does not provide the concept of an accelerating observer in the first place, as only observers frames with straight coordinate axes are considered.[6] These observer frames are abstracted from the measurements of local observers. As is more or less explicitly done in standard GRT and other approaches to gravitation, we will

[6] The views of such observers have to be reconstructed by regarding their measurements as physical processes and describing them from the viewpoint of a normalized observer.

take the perspective of a hypothetical distant observer who is unaffected by gravitation and resting relative to the field for the analysis of the suggested field models.

3.1 Observer Concept and General Covariance

Before formulating the Euclidean general covariance principle the concept of a space-time observer and the basic geometric properties are comprised.

Space-time observer

> *Space-time observers are represented by local light clocks. All space-time observers define space and time scales according to the extensions of their own light clock. By this convention, the speed of light is locally invariant for all space-time observers.*

Assumption on the construction of Euclidean views

> *On the basis of their local measurements[7], space-time observer can construct Euclidean views of physical processes taking place in different space-time regions in consistent and coherent ways.*

Conservation laws for coordinate transformations between space-time observers

Coordinate transformations between space-time observers conserve

- *ratios of sections of lines,*
- *ratios of sections of areas,*
- *ratios of sections of 3D volumes, and*
- *ratios of 4D volumes.*

The formulation of the Euclidean general covariance principle on this basis is straightforward.

The Euclidean general covariance principle (EGCP)

> *The laws of physics can be expressed in terms of geometrical relations in a 4D Euclidean space-time geometry with signature (++++). They are covariant, if and only if these geometrical relations are conserved under coordinate transformations between any two space-time observers.*

The first application of this principle is the generalization of the concept of normalization of space and time scales used by different space-time observers.

Normalized Space-time observers

> *Two observers in the same space-time location are normalized, if their space-time cells exhibit identical space-time volumes, independent of their speeds. More generally, two observers located in arbitrary space-time regions are called normalized, if the relation between the volumes of their space-time cells equals the relation between the normalization parameters[8] attributed to these regions.*

The physical meaning of normalization is given by the following assumption:

> *If a physical object undergoes a non-destructive change of its speed or location, it will still be measured to the same space and time extensions in its new rest frame.*

The second application of the relativity principle is the formulation of a law of inertial motion (free fall) for point particles.

[7] The measurement of radar echo times allows „local“ measurements of distant objects.
[8] C.f. eqs. (10) and (25)

3.2 The Law of Inertial Motion

In EGR the curvature of time-like lines is defined by a differential law, which expresses the conservation of a relation between certain space-time areas. In any space-time event, a point particle has a speed along each of the spatial axes, which can be written as a combination of the opposing light speeds along the respective spatial axis.

$$v_{x_i} = K\, c_{x_i+} + (1-K)\, c_{x_i-} \qquad 0 \le K \le 1 \tag{6}$$

The differential motion law describes how the speed of the particle changes according to the changes of the two opposing local light speeds.

$$\frac{dv_{x_i}}{dt} = K \frac{dc_{x_i+}}{dt} + (1-K) \frac{dc_{x_i-}}{dt} \tag{7}$$

The law expresses the conservation of a relation between two space-time areas located on the same space-time plane. While the areas may be different for different normalized observers, their relation is the same for all normalized observers.[9] According to EPGC, the law is covariant, therefore.

The space-time areas A_1 and A_2 in *figure 8* are limited by two parabolic[10] curves (one representing the particle trajectory, the other one a light trajectory) and a horizontal line connecting events that are simultaneous from the perspective of the observing frame. In event O the two photons have speeds c_+ respectively c_- while the particle's speed v is a combination of both according to eq. (6).[11]

4 The Construction of Field Models in EGR

In EGR the space-time geometry of static fields is defined by the extensions and shapes of normalized space-time cells resting in different regions of the field. By doing so, the local light speeds in all directions and the normalization parameters are determined. The motion of point particles follows exclusively from this geometric definition of a field and the motion law.

An important criterion to discriminate types of fields is the question whether inward and outward radial light speeds are equal for the field's rest frame.[12] In the first case, i.e. for our first two gravitation models, flat hyper-planes connecting simultaneous events can be defined for the whole field. In the second case, simultaneity hyper-planes are curved.

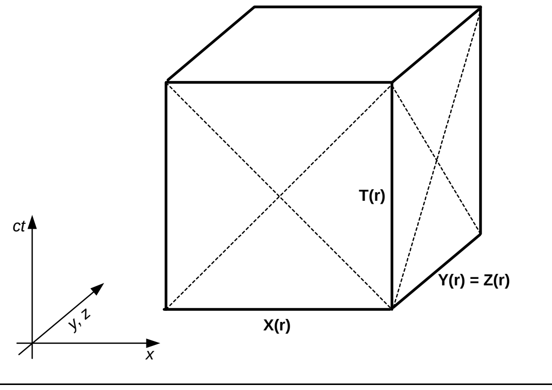

Fig. 9. A normalized space-time cell resting in the gravitational field. The x-axis is taken as the radial dimension.

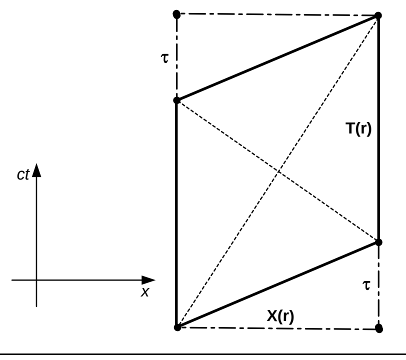

Fig. 10. A time shifted space-time cell in one space dimension.

---

[9] To be precise, observers do not even have to be normalized to make this statement true.

[10] In section 3 we postulated that light curves are continuous and several times differentiable. For infinitesimally small time intervals, they can be modelled as parabolic curves therefore, i.e. curves with constant second time derivatives.

[11] It is a simple geometrical task to show that the relation between the areas A_1 and A_2 is conserved as long as the light trajectories are parabolic and the (as well parabolic) particle trajectory obeys the motion law.

[12] An asymmetry of tangent light speeds could be the geometric representation of spin.

Illustrating the first case, *Figure 9* shows a normalized space-time cell of a light clock that rests in some field from the perspective of a distant normalized observer who is unaffected by the field. The *x*-axis is taken as the radial dimension. The constant *c* stands for the speed of light in free space.

While radial light speeds in *figure 9* are symmetric, *figure 10* shows the radial component of a space-time cell representing a field that involves asymmetric radial speeds. Such a field will be discussed in *model C*. As will become clear in the following, symmetry of radial light speeds makes radial acceleration of point particles independent of particle speeds while asymmetry causes speed dependence of acceleration.

Before developing the three field models we shortly summarize their respective purposes in the present context and their properties.

Model A - Reconstruction of Newtonian gravitation

The first model which assumes equal effects on radial and tangent light speeds is a reconstruction of Newtonian gravitation including an explanation of spectral shift. It shows how the geometric approach to relativistic energy introduced in section 2.3 connects to the conservation of Newtonian energy. The differential equations which are (in compliance with the motion law) derived from the conservation of relativistic energy are identical to the classical equations.

Model B – The reference model

While *model A* assumes identical effects on radial and tangent light speeds, *model B* leaves tangent light speeds untouched. The phenomena of perihelion precession and light deflection follow from this difference. Due to its simplicity, we call this model the "reference model" describing pure gravitation. It does not explain the Shapiro radar echo delay, though. In EGR, the decrease of light travel time in the sun's environment is not a gravitational effect and requires variations of the space-time geometry.

Model C – Speed dependence of acceleration

Model C which involves asymmetric radial light speeds explains the Shapiro delay without destroying the effect of light deflection from the *reference model*. It will be suggested to regard the actual field of the sun as a combination of *models B* and *C*.

The asymmetry of radial light speeds implies an outward acceleration of particles emitted by the sun and an asymmetric acceleration behavior of particles with different speeds. It will be argued that *model C* might thus provide an account of the coronal heating problem.

5 Model A: Reconstruction of Newtonian Gravitation

The space-time geometry of *model A* can be defined by the use of *figure 9*. Depending on the distance *r* to the center of gravity and on the Schwarzschild radius *s* the space-time cells of the resting normalized space-time observers have the following extensions.

$$X(r) = Y(r) = Z(r) = \left(1+\frac{s}{r}\right) \quad , \; T(r) = c^{-1}\left(1+\frac{s}{r}\right)^{1/2} \tag{8}$$

Radial and tangent light speeds are equal.

$$c_{rad} = \frac{X(r)}{T(r)} = c_{tan} = \frac{Y(r)}{T(r)} = \frac{Z(r)}{T(r)} = \pm c\left(1+\frac{s}{r}\right)^{1/2} \tag{9}$$

The normalization parameter *N(r)* holds the volume of a space-time cell in the respective location in comparison to the volume of an identical[13] cell located far away from the center of gravity.

$$N(r) = c\; X(r)\,Y(r)\,Z(r)\,T(r) = \left(1+\frac{s}{r}\right)^{7/2} \tag{10}$$

[13] C.f. section 3.1.

Gravitational time dilation

In the Schwarzschild solution a gravitational time dilation and consequently a red shift of spectral lines follows from the time component of the line element. For *models A, B,* and *C,* the time dilation can be read from the time extension of a normalized space-time cell. The difference between GRT and EGR results can be neglected for small values of *s*, as in this case the following two expressions yield almost identical values. The first expression describes the time dilation for the Schwarzschild solution, the second one the time dilation for the three EGR models.

$$\left(1-\frac{s}{r}\right)^{-1/2} \quad , \quad \left(1+\frac{s}{r}\right)^{1/2} \tag{11}$$

The first order approximations for frequency relations in two locations are identical.

$$\frac{\nu_2}{\nu_1}(GRT)=\sqrt{\frac{1-\frac{s}{r_1}}{1-\frac{s}{r_2}}}\approx 1+\frac{s}{2}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\approx\sqrt{\frac{1+\frac{s}{r_2}}{1+\frac{s}{r_1}}}=\frac{\nu_2}{\nu_1}(EGR) \tag{12}$$

Radial acceleration

Applying the motion law (eq. (7)) implies the equality of the radial acceleration of particles and the radial acceleration of light. Radial particle acceleration does not depend on the factor *K* and thus on the particle's speed.

$$v_{rad}=K\,c_{rad+}+(1-K)\,c_{rad-} \tag{13}$$

$$\frac{\partial v_{rad}}{\partial r}\frac{dr}{dt}=K\frac{dc_{rad+}}{dt}+(1-K)\frac{dc_{rad-}}{dt}=\frac{dc_{rad}}{dt}=-\frac{s\,c^2}{2\,r^2}$$

This leads to the first part of the differential equation system for particle motion in a polar coordinate system:

$$\ddot{r}=r\,\dot{\varphi}^2-\frac{s\,c^2}{2\,r^2} \tag{14}$$

Tangent acceleration

A useful statement about the evolution of the tangent particle speed can be derived from the motion law.

$$v_{tan}=K\,c_{tan+}+(1-K)\,c_{tan-}=(2K-1)\,c_{tan} \tag{15}$$

$$\frac{v_{tan}}{c_{tan}}=2K-1$$

Inserting this to eq. (7) yields an expression for the change of the tangent particle speed in terms of the change of the tangent light speed and the relation between particle and light speeds.

$$\frac{\partial v_{tan}}{\partial r}\frac{dr}{dt}=K\frac{dc_{tan+}}{dr}\frac{dr}{dt}+(1-K)\frac{dc_{tan-}}{dr}\frac{dr}{dt}=\frac{v_{tan}}{c_{tan}}\frac{dc_{tan}}{dt} \tag{16}$$

From this it follows that the tangent particle acceleration is proportional to the tangent particle speed. We can write

$$\ddot{\varphi}=f(r,\dot{r})\dot{\varphi}\,. \tag{17}$$

Instead of deriving the missing function *f* directly from the motion law, we derive the respective tangent differential equation from the radial differential equation (eq. (14)) and the conservation of relativistic energy. As the result satisfies eq. (17) and holds for light (i.e. is in accordance with the geometry), it complies with the motion law.

Thought experiment: Energy conservation during free fall

Relating to the considerations on relativistic energy from section 2.3, the conservation of the total relativistic energy of a free falling body in a gravitational field can be illustrated by a thought experiment.

We assume that resting observers who are represented by normalized space-time cells are located at different distances to the center of gravity. These observers communicate by exchanging light of certain frequencies. After locally measuring the total relativistic energy of a falling body, an observer codes the result into the frequency of the light waves he sends to the other observers. In this scenario, conservation of energy means that all observers always receive light of the same frequency for one falling body.

The mathematical formulation of energy conservation takes into account *i)* the *r*-dependence of the time extension of the normalized space-time cells and *ii)* the fact that local speed measurements put the speed of the moving body in relation to the local light speeds.[14]

$$cT = \left(1+\frac{s}{r}\right)^{1/2}\sqrt{1-\frac{v_{rad}^2}{c_{rad}^2}-\frac{v_{tan}^2}{c_{tan}^2}} = const \tag{18}$$

Going to polar coordinates, we get:

$$c^2T^2 = \left(1+\frac{s}{r}\right)\left(1-\frac{\dot{r}^2}{c^2\left(1+\frac{s}{r}\right)}-\frac{r^2\dot{\varphi}^2}{c^2\left(1+\frac{s}{r}\right)}\right) = const \tag{19}$$

The substitution for $\ddot{r}$ in the time derivative of eq. (19) according to eq. (14) allows us to complete the differential equation system for point particles:

$$\ddot{r} = r\,\dot{\varphi}^2 - \frac{s\,c^2}{2\,r^2} \quad , \quad \ddot{\varphi} = -2\frac{\dot{r}\,\dot{\varphi}}{r} \tag{20}$$

This complies with the tangent motion law, because tangent acceleration is proportional to tangent speed (eq. (17)) and because the law perfectly describes photon trajectories that are directly defined by the geometry. This can be shown by deriving the same result from light speed conservation instead of from energy conservation.

$$\sqrt{\frac{v_{rad}^2}{c_{rad}^2}+\frac{v_{tan}^2}{c_{tan}^2}} = 1 \tag{21}$$

For later comparison with the reference model we list the expressions for the conserved classical quantities L (angular momentum per mass) and E (energy per mass) that follow from the differential equations.

$$L = \dot{\varphi}\,r^2 \quad , \quad E = \frac{\dot{r}^2}{2}+\frac{L^2}{2\,r^2}-\frac{s\,c^2}{2\,r} \tag{22}$$

6 Model B: The Reference Model

The space-time geometry of *model B* differs from *model A's* geometry only by the non-effect on tangent light speed.

$$X(r) = \left(1+\frac{s}{r}\right) \;,\; Y(r) = Z(r) = \left(1+\frac{s}{r}\right)^{1/2} \;,\; T(r) = c^{-1}\left(1+\frac{s}{r}\right)^{1/2} \tag{23}$$

$$c_{rad} = \frac{X(r)}{T(r)} = \pm c\left(1+\frac{s}{r}\right)^{-1} \;,\; c_{tan} = \frac{Y(r)}{T(r)} = \frac{Z(r)}{T(r)} = \pm c \tag{24}$$

The normalization parameter calculates to

$$N(r) = c\,X(r)\,Y(r)\,Z(r)\,T(r) = \left(1+\frac{s}{r}\right)^{5/2}. \tag{25}$$

The differential equations are derived by the same procedure as for *model A*. While the radial component is the same, the energy expression differs from that of *model A*:

[14] C.f. the illustration of measurements in *figs. 4* and *5*.

$$c^2T^2 = \left(1+\frac{s}{r}\right)\left(1-\frac{\dot{r}^2}{c^2\left(1+\frac{s}{r}\right)}-\frac{\dot{\varphi}^2 r^2}{c^2}\right) = const \tag{26}$$

For the resulting complete differential equation system

$$\ddot{r} = r\,\dot{\varphi}^2 - \frac{s\,c^2}{2\,r^2} \quad , \; \ddot{\varphi} = \frac{\dot{r}\,\dot{\varphi}}{r}\left(-2+\frac{3\,s}{2\,(r+s)}\right) \tag{27}$$

the conservation laws

$$L = \dot{\varphi}\,r^2\left(1+\frac{s}{r}\right)^{3/2} \quad , \quad E = \frac{c^2\left(1-c^2T^2\right)}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2\,(r+s)^2} - \frac{s\,c^2}{2\,r} \tag{28}$$

can be found. For photons the time distance T vanishes, which leads to the energy expression for light:

$$T = 0 \quad , \quad E = \frac{c^2}{2} \tag{29}$$

Orbital characteristics for particles (including photons) will be calculated on the basis of the conservation laws (eqs. (28)) by integrating the following expression, respectively by developing approximations.

$$\frac{d\,\varphi}{d\,r} = \pm\frac{L}{r^2\left(1+\frac{s}{r}\right)^{3/2}\sqrt{2E-\frac{L^2}{(r+s)^2}+\frac{s\,c^2}{r}}} = \pm\frac{L}{r^2\sqrt{\left(2\,E+\frac{s\,c^2}{r}\right)\left(1+\frac{s}{r}\right)^3-\frac{L^2}{r^3}(r+s)}} \tag{30}$$

For weak fields we can use a first order approximation for the power 3 expression:

$$\left(1+\frac{s}{r}\right)^3 \approx 1+\frac{3\,s}{r} \quad , \quad \frac{s}{r} << 1 \tag{31}$$

This leads to:

$$\frac{d\,\varphi}{d\,r} \approx \pm\frac{L}{r^2\sqrt{\left(2\,E+\frac{s\,c^2}{r}\right)\left(1+\frac{3\,s}{r}\right)-\frac{L^2}{r^3}(r+s)}} \tag{32}$$

Perihelion precession of planetary orbits

The calculations in this and the following sections use standard techniques (e.g. [6]).
After setting

$$u_0 = \frac{1}{r_0} \quad , \quad u_1 = \frac{1}{r_1} \tag{33}$$

for the radial distances of the flexion points r_0 (perihelion) and r_1 (aphelion) the angular perihelion shift $\Delta\varphi$ is given by:

$$\Delta\varphi = 2[\varphi(u_1) - \varphi(u_0) - \pi] = 2\int_{r_0}^{r_1}\frac{d\varphi}{dr}dr - 2\pi \tag{34}$$

The substitutions

$$r = \frac{1}{u} \qquad \dot{r} = -\frac{\dot{u}}{u^2} \tag{35}$$

in eq. (32) lead to

$$\frac{d\,\varphi}{d\,u} = \pm\frac{L}{\sqrt{2\,E + s\left(c^2+6\,E\right)u - \left(L^2-3\,c^2\,s^2\right)u^2 - L^2\,s\,u^3}} \tag{36}$$

and consequently (according to eq. (34)) to:

$$\Delta\varphi = 2[\varphi(u_1) - \varphi(u_0) - \pi] = \int_{u_0}^{u_1}\frac{2L\,du}{\sqrt{2\,E + s\left(c^2+6\,E\right)u - \left(L^2-3\,c^2\,s^2\right)u^2 - L^2\,s\,u^3}} - 2\pi \tag{37}$$

The polynomial under the square root has two real roots at u_0 and u_1, therefore it must have a third real root u_2. We can write the polynomial as

$$2\,E + s\left(c^2 + 6\,E\right)u - \left(L^2 - 3\,c^2\,s^2\right)u^2 - L^2\,s\,u^3 = \alpha\left(u-u_0\right)\left(u-u_1\right)\left(u-u_2\right) \tag{38}$$

and get for the parameters:

$$\alpha = -L^2\,s \;\;,\;\; u_2 = \frac{3\,c^2\,s}{L^2} - \frac{1}{s} - \left(u_0 + u_1\right) \tag{39}$$

The substitution

$$\delta = u_0 s = \frac{s}{r_0} \tag{40}$$

and some reordering yield:

$$\frac{d\varphi}{du} = \frac{1}{\sqrt{\left(u-u_0\right)\left(u-u_1\right)\left(1+\frac{\delta}{u_0}\left(u_0+u_1+u-\frac{3\,c^2\,s}{L^2}\right)\right)}} \tag{41}$$

The power series expansion for δ gives us the first order approximation:

$$\frac{d\varphi}{du} \approx \frac{1-\frac{\delta}{2u_0}\left(u_0+u_1+u-\frac{3\,c^2\,s}{L^2}\right)}{\sqrt{\left(u-u_0\right)\left(u_1-u\right)}} \tag{42}$$

After the replacement of δ, the substitution

$$u = \frac{u_0+u_1}{2} + \left(u_1-u_0\right)\sin\psi \tag{43}$$

leads to the new integration interval [$-\pi/2$, $\pi/2$]. We can now integrate:

$$\varphi(u_1)-\varphi(u_0) = \int_{-\pi/2}^{\pi/2}\left(1-\frac{s}{2}\left(\frac{3\left(u_0+u_1\right)}{2}+\frac{u_1-u_0}{2}\sin\psi-\frac{3\,c^2\,s}{L^2}\right)\right)d\psi = \pi + \frac{3\,\pi\,s^2c^2}{2L^2} - \frac{3\,s\left(u_0+u_1\right)}{4} \tag{44}$$

Going back to r_0 and r_1 and substituting from classical theory (ε stands for the eccentricity and a for the length of the semi-major axis of ellipse),

$$u_0 = \frac{1}{r_0} = \frac{1}{a\left(1+\varepsilon\right)} \;\;,\;\; u_1 = \frac{1}{r_1} = \frac{1}{a\left(1-\varepsilon\right)} \;\;,\;\; L^2 = \frac{s\,c^2 a\left(1-\varepsilon^2\right)}{2}, \tag{45}$$

allows us to calculate the GRT result for the perihelion precession from eq. (37):

$$\Delta\varphi = 2\left[\varphi(u_1)-\varphi(u_0)-\pi\right] = \frac{3\,\pi\,s}{a\left(1-\varepsilon^2\right)} \tag{46}$$

Light Deflection

Writing r_0 for the point of closest approach of a photon passing by the sun, the deflection angle $\Delta\varphi$ is given by

$$\Delta\varphi = 2\left[\varphi(\infty)-\varphi(r_0)\right]-\pi \tag{47}$$

We use a reformulation of eq. (32)

$$\frac{d\,\varphi}{d\,r} = \pm\frac{L}{r\sqrt{r^2\left(1+\frac{3s}{r}\right)\left(2E+\frac{sc^2}{r}\right)-L^2\left(1+\frac{s}{r}\right)}} \tag{48}$$

and get

$$\Delta\varphi = 2\left[\varphi(\infty)-\varphi(r_0)\right]-\pi = 2\int_{r_0}^{\infty}\frac{Ldr}{r\sqrt{r^2\left(1+\frac{3s}{r}\right)\left(2E+\frac{sc^2}{r}\right)-L^2\left(1+\frac{s}{r}\right)}}-\pi\,. \tag{49}$$

The photon energy E, which has already been determined (eq. (29)), does not depend on r_0. The calculation of L is done for $r=r_0$ and approximated:

$$v_{\tan}\left(r_0\right) = c \;\;,\;\; L = c\,r_0\left(1+\frac{s}{r_0}\right)^{3/2} \;\;,\;\; L \approx c\,r_0\sqrt{1+\frac{3s}{r_0}} \tag{50}$$

From eq. (48) we get:

$$\frac{d\varphi}{dr}=\frac{1}{r\sqrt{1+\frac{s}{r}}\sqrt{\frac{r(r+3s)}{r_0(r_0+3s)}-1}} \tag{51}$$

After substituting

$$\varepsilon=\frac{s}{2r_0}\ ,\ x=\frac{r}{r_0}\ ,\ dx=\frac{dr}{r_0}\ ,\ r=r_0x\ ,\ dr=r_0dx \tag{52}$$

the new integration interval is [1, ∞]. Eq. (51) writes now as:

$$\frac{d\varphi}{dr}=\frac{1}{xr_0\sqrt{1+\frac{2\varepsilon}{x}}\sqrt{\frac{(x-1)(1+x+6\varepsilon)}{1+6\varepsilon}}} \tag{53}$$

The power series expansion of eq. (53) for ε yields the first order approximation

$$\frac{d\varphi}{dr}=\frac{1}{r_0}\left(\frac{1}{x\sqrt{x^2-1}}-\varepsilon\left(\frac{3(x-1)}{(x^2-1)^{3/2}}-\frac{1}{x^2\sqrt{x^2-1}}\right)\right), \tag{54}$$

which allows us to calculate the GRT result for light deflection:

$$\Delta\varphi\approx 2\int_1^\infty\left[\frac{dx}{x\sqrt{x^2-1}}-\varepsilon\left(\frac{3(x-1)}{(x^2-1)^{3/2}}-\frac{1}{x^2\sqrt{x^2-1}}\right)dx\right]-\pi=4\varepsilon=\frac{2s}{r_0c^2}=\frac{4GM}{r_0c^2} \tag{55}$$

7 Model C: Speed dependence of acceleration

The reference model does not explain the Shapiro radar echo delay. Quite to the contrary, it explicitly assumes higher radial light speeds and leads therefore to shorter radar echo times. The variations of the reference model that are necessary to produce the Shapiro delay involve a certain amount of speed dependence of radial acceleration.

A look at *figure 10* makes clear that the two-way speed of light which is relevant for radar echo experiments depends only on the ratio of *X(r)* and *T(r)*, creating a free choice for the time shift parameter τ. The extensions *Y(r)*, *Z(r)*, and *T(r)* are left the same compared to *model B*. As will be shown, the new expression for *X(r)* leads to the delay term from GRT.

$$X(r)=\left(1+\frac{s}{r}\right)^{-1/2}\ ,\ Y(r)=Z(r)=\left(1+\frac{s}{r}\right)^{1/2}\ ,\ T(r)=c^{-1}\left(1+\frac{s}{r}\right)^{1/2} \tag{56}$$

With c_1 as the speed of outgoing light and c_2 as the speed of ingoing light we get:

$$c_1=\frac{X(r)}{T(r)+\tau}=\frac{\left(1+\frac{s}{r}\right)^{-1/2}}{c^{-1}\left(1+\frac{s}{r}\right)^{1/2}+\tau}\ ,\ c_2=-\frac{X(r)}{T(r)-\tau}=-\frac{\left(1+\frac{s}{r}\right)^{-1/2}}{c^{-1}\left(1+\frac{s}{r}\right)^{1/2}-\tau} \tag{57}$$

A useful choice for τ must result in radial accelerations for low speeds that are comparable to Newtonian gravitation. An appropriate solution follows from assuming

$$c_1c_2=-c^2\left(1+\frac{s}{r}\right)^{-1}\text{, which leads to }\ \tau=c^{-1}\sqrt{\frac{s}{r}}\ . \tag{58}$$

The power series expansion of the time derivatives of the two light speeds makes clear that for *K=1/2* in the motion law (which is the case for a small negative speed) the overall radial acceleration is almost identical to Newton’s law:

$$\frac{dc_1}{dt}=c^2\left[\frac{\sqrt{s}}{2r^{3/2}}-\frac{s}{2r^2}-\frac{3s^{3/2}}{4r^{5/2}}+\frac{s^2}{r^3}+\ldots\right] \tag{59}$$

$$\frac{dc_2}{dt}=c^2\left[-\frac{\sqrt{s}}{2r^{3/2}}-\frac{s}{2r^2}+\frac{3s^{3/2}}{4r^{5/2}}+\frac{s^2}{r^3}+\ldots\right]$$

More generally, it follows from eqs. (59) that

$$\frac{dv}{dt}\approx-\frac{s\,c^2}{2r^2}\ \text{ holds for speeds }\ \frac{v}{c}<<\sqrt{\frac{s}{r}}\ . \tag{60}$$

This speed dependence is, however, strong enough to exclude the validity of *model C* for distances from the sun of 1 AU or more (e.g. the radial speeds of comets near the earth can be higher).

Light travel time

The Shapiro experiment can only measure the two-way travel time of light. For the following approximation of the respective result for *model C* we multiply the radial speed expression from the *reference model* (a reformulation of the energy law from eq. (28)) by the factor

$$\left(1+\frac{s}{r}\right)^{-3/2} \tag{61}$$

which expresses the ratio of the two-way light speeds of *model C* and of the *reference model*. For our purpose *E* and *L* can be taken from the *reference model*, which leads to:

$$\frac{dt}{dr} \approx \frac{\left(1+\frac{s}{r}\right)^{3/2}}{\sqrt{2E-\frac{L^2}{(r+s)^2}+\frac{sc^2}{r}}} \quad , \quad E \approx \frac{c^2}{2} \quad , \quad L \approx cr_0\left(1+\frac{s}{r}\right)^{3/2} \tag{62}$$

The power series expansion for

$$\varepsilon = \frac{s}{r_0} \tag{63}$$

yields a first order approximation and allows us to estimate the average travel time of light on a two-way trip:

$$\Delta t \approx \int_{r_0}^{r} \frac{r+s}{r\sqrt{c^2\left(1-\frac{r_0^2}{r^2}\right)}}dr = \frac{1}{c}\left(\sqrt{r^2-r_0^2} + s\, Log\left[r+\sqrt{r^2-r_0^2}\right]\right) \tag{64}$$

The only difference to the GRT expression is a missing extra term which is experimentally irrelevant, though.

8 Different field types and the coronal heating problem

Neither the *reference model* nor *model C* alone can explain all the experimentally confirmed "gravitational" effects. What is required is a continuous combination of the two models, which has to provide *i)* approximately Newton radial acceleration for small speeds at all distances from the center, *ii)* shrinking or vanishing speed dependence of acceleration at larger distances, and *iii)* shrinking two-way light speeds for smaller distances. As an illustration of the fact that these requirements do not contradict each other the following example for small speed dependence can be taken:

The choice of radial light speeds

$$c_1 = c\left(1+\frac{s}{r}\right)^{-1/2} \quad , \quad c_2 = -c\left(1+\frac{s}{r}\right)^{3/2} \tag{65}$$

leads to the light speed accelerations:

$$\frac{dc_1}{dt} = c^2\left[\frac{s}{2r^2}-\frac{s^2}{r^3}+\ldots\right] \tag{66}$$

$$\frac{dc_2}{dt} = c^2\left[-\frac{3s}{2r^2}-\frac{2s^2}{r^3}+\ldots\right]$$

Although outgoing light is accelerated outwards, the overall acceleration of a particle shows only small speed dependence compared to eq. (60). There would be no acceleration for outgoing particles with speeds around ¾ *c*, if *s/r* is supposed to be much smaller than unity.

The combination of *models B* and *C* might be regarded as a mere pragmatic way of dealing with all the different aspects of gravitation which are in need of explanation. What looks as a drawback in the first place, allows us to locate a first theoretical and empirical test bed for the presented approach to gravitation. As has been shown, the simple geometry of the *reference model* must be supplemented by a different geometry for the sun's environment. *Model C*, which implements this geometric requirement, shows a property in addition to the requested radar echo delay, namely speed dependence of acceleration. More specifically, particles that move away from the field's center with more than a certain speed are accelerated outwards instead of being attracted by the field. A natural candidate for this behavior is solar wind, which is emitted from the sun (respectively somewhere in its closer environment) into the heliosphere at speeds v/c of the same order as $\sqrt{s/r_{sun}}$, which is in good accordance with eqs. (59, 60).

Going one step further, *model C* might provide a solution to the so-called *coronal heating problem*: The transition region and especially the corona of the sun are much hotter (up to several Million° Kelvin) than the sun's photosphere (around 6000° Kelvin). While the ionization of H-atoms (which mostly form the solar wind) is seen as being due to the enormous heat, it remains an open question[15] how to explain the required energy increase. From the perspective of EGR, the high amount of energy is the result of asymmetric acceleration and can be read from the first terms of the two light accelerations in eqs. (59), which cancel out only for low particle speeds.

9 Conclusions

We presented the foundations of a generally covariant geometric generalization of the Euclidean interpretation of special relativity. In the suggested Euclidean general relativity, static fields are defined by a location-depending scaling factor for normalized space-time cells (the normalization parameter) and by the distribution of light speeds, which depend not only on location, but also on direction. The behavior of point particles has been described for three gravitation models. The first model shows the connection to Newtonian gravitation and already explains gravitational red shift. The second model explains the perihelion precession of planetary orbits and the deflection of light, but falls short of producing Shapiro's radar echo delay. The necessary geometric adaptations lead to a third model, whose side effects are reminiscent of phenomena in the sun's environment and suggest an account of the coronal heating problem.

Acknowledgments

The author wishes to thank Yagmur Denizhan and Franz Pichler for mathematical advice.

References

[1] F.-G. Winkler. The Normalization Problem of Special Relativity, Physics Essays, Vol. 15, 2, 199-208, 2002.

[2] F.-G. Winkler. The Euclidean Interpretation of Special Relativity. Proceedings of PIRT 2002 (Physical Interpretations of Relativity Theory), M.C. Duffy (ed), PD Publications, Liverpool, 640-651, 2003.

[3] F.-G. Winkler. *Die Spezielle Relativitätstheorie – anschaulich erklärt, neu interpretiert.* Peter Lang, Frankfurt, 2005.

[4] J. Magueijo. New varying speed of light theories. Rept.Prog.Phys. 66, 2025, 2003.

[5] J. Broekaert. On a modified-Lorentz-transformation based gravity model confirming basic GRT experiments. Found.Phys. 35, 839-864, 2005.

[6] E. Rebhan. Theoretische Physik I. Spektrum Akademischer Verlag GmbH, Heidelberg, Berlin, 1999.

[7] M. J. Aschwanden. *Physics of the Solar Corona. An Introduction*. Praxis Publishing Ltd., 2004.

[15] There are mainly two approaches to the coronal heating problem (wave heating and magnetic reconnection), which are not seen as fully satisfying, though [7].