Chapter 1

THE HYPOTHESIS OF LOCALITY AND ITS LIMITATIONS

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Abstract   The hypothesis of locality, its origin and consequences are discussed. This supposition is necessary for establishing the local spacetime frame of accelerated observers; in this connection, the measurement of length in a rotating system is considered in detail. Various limitations of the hypothesis of locality are examined.

1.   Introduction

The basic laws of microphysics have been formulated with respect to ideal inertial observers. However, all actual observers are accelerated. To interpret the results of experiments, it is therefore necessary to establish a connection between actual and inertial observers. This is achieved in the standard theory of relativity by means of the hypothesis of locality, namely, the assumption that an accelerated observer at each instant along its worldline is physically equivalent to an otherwise identical momentarily comoving inertial observer. In this way a noninertial observer passes through a continuous infinity of hypothetical momentarily comoving inertial observers [1].

The hypothesis of locality stems from Newtonian mechanics, where the state of a particle is given at each instant of time by its position and velocity. Thus the accelerated observer and the hypothetical inertial observer share the same state and are therefore equivalent. Hence, the treatment of accelerated systems in Newtonian mechanics requires no new assumption. More generally, if all physical phenomena could be reduced to pointlike coincidences of classical point particles and electromagnetic *rays*, then the hypothesis of locality would be exactly valid.

However, an electromagnetic *wave* has intrinsic scales of length and time characterized by its wavelength λ and period λ/c. For instance, the measurement of the frequency of the wave necessitates observation of a few oscillations before a reasonable determination can become possible. If the state of the observer does not change appreciably over this period of time, then the hypothesis of locality would be essentially valid. This criterion may be expressed as $\lambda/\mathcal{L} << 1$, where $\mathcal{L}$ is the relevant acceleration length of the observer. That is, the observer has intrinsic scales of length $\mathcal{L}$ and time $\mathcal{L}/c$ that characterize the degree of variation of its state. For instance, $\mathcal{L} = c^2/a$ for an observer with translational acceleration a, while $\mathcal{L} = c/\Omega$ for an observer rotating with frequency Ω [1, 2].

The consistency of these ideas can be seen in the case of an accelerating charged particle. Imagine a particle of mass m and charge q moving under the influence of an external force $\mathbf{F}_{ext}$. The particle radiates electromagnetic waves that have a characteristic wavelength $\lambda \sim \mathcal{L}$, where $\mathcal{L}$ is the acceleration length of the particle. Thus the interaction of the particle with the electromagnetic field violates the hypothesis of locality since $\lambda/\mathcal{L} \sim 1$. The radiating charged particle is therefore not momentarily equivalent to an otherwise identical comoving inertial particle. This agrees with the fact that in the nonrelativistic approximation the Abraham-Lorentz equation of motion of the particle is

$$m\frac{d\mathbf{v}}{dt} - \frac{2}{3}\frac{q^2}{c^3}\frac{d^2\mathbf{v}}{dt^2} + ... = \mathbf{F}_{ext} \ \ , \quad\quad (1.1)$$

so that the state of a radiating particle is not determined by its position and velocity alone.

Imagine an accelerated measuring device in Minkowski spacetime. The internal dynamics of the device is then subject to inertial effects that consist of the inertial forces of classical mechanics together with their generalizations to electromagnetic and quantum domains. If the net influence of these inertial effects integrates — over the relevant length and time scales of a measurement — to perturbations that do not appreciably disturb the result of the measurement and can therefore be neglected, then the hypothesis of locality is valid and the device can be considered standard (or ideal). Consider, for instance, the measurement of time dilation in terms of muon lifetime by observing the decay of muons in a storage ring. It follows from the hypothesis of locality that $\tau_\mu = \gamma\tau_\mu^0$, where γ is the Lorentz factor and τ_μ^0 is the lifetime of the muon at rest in the background inertial frame. On the other hand, the lifetime of such a muon has been calculated on the basis of quantum theory by assuming that the muon occupies a high-energy Landau level

in a constant magnetic field [3]. One can show that the result of [3] can be expressed as [4]

$$\tau_\mu \simeq \gamma \tau_\mu^0 \left[1 + \frac{2}{3} \left(\frac{\lambda}{\mathcal{L}} \right)^2 \right] \quad . \qquad (1.2)$$

Here $\lambda = \hbar/(mc)$ is the Compton wavelength of the muon, m is the muon mass and $\mathcal{L} = c^2/a$, where $a = \gamma^2 v^2/r$ is the effective centripetal acceleration of the muons in the storage ring. The hypothesis of locality is completely adequate for such experiments since $\lambda/\mathcal{L}$ is extremely small. In fact, the hypothesis of locality is clearly valid in many Earth-bound experimental situations since $c^2/g_\oplus \simeq 1$ lyr and $c/\Omega_\oplus \simeq 28$ AU.

The hypothesis of locality plays a crucial role in Einstein's theory of gravitation: Einstein's principle of equivalence together with the hypothesis of locality implies that an observer in a gravitational field is locally inertial. Indeed, the equivalence between an observer in a gravitational field and an accelerated observer in Minkowski spacetime is useless operationally unless one specifies what an accelerated observer measures.

The hypothesis of locality was formally introduced in [1] and its limitations were pointed out. To clarify the origin of this conception, some background information is provided in section 2. The implications of this assumption for length determination in rotating systems are pointed out in section 3. Section 4 contains a discussion.

2.   Background

Maxwell's considerations regarding optical phenomena in moving systems implicitly contained the hypothesis of locality [5]. The fundamental form of Maxwell's theory of electromagnetism, derived from Maxwell's original electrodynamics of media, is essentially due to Lorentz's development of the theory of electrons.

Lorentz conceived of an electron as an extremely small charged particle with a certain smooth volume charge density. A free electron at rest was regarded as a spherical material system with certain internal forces that ensured the constancy of its size and form. An electron in translational motion would then be a flattened ellipsoid according to Lorentz, since it would be deformed from its original spherical shape by the Lorentz-FitzGerald contraction in the direction of its motion. The internal dynamics of electrons therefore became a subject of scientific inquiry and in 1906 Poincaré postulated the existence of a particular type of internal stress that could balance the electrostatic repulsion even in a moving (and hence flattened) electron. These issues are discussed in

detail in the fifth chapter (on optical phenomena in moving bodies) of Lorentz's book [6] on the theory of electrons.

In extending the Lorentz transformations in a pointwise manner to accelerating electrons, Lorentz encountered a problem regarding the dynamical equilibrium of the internal state of the electron. To avoid this problem, Lorentz introduced a basic assumption that is discussed in section 183 of his book [6]:

> "... it has been presupposed that in a curvilinear motion the electron constantly has its short axis along the tangent to the path, and that, while the velocity changes, the ratio between the axes of the ellipsoid is changing at the same time."

To elucidate this assumption, Lorentz explained its approximate validity as follows (§183 of [6]):

> "... If the form and the orientation of the electron are determined by forces, we cannot be certain that there exists at every instant a state of equilibrium. Even while the translation is constant, there may be small oscillations of the corpuscle, both in shape and in orientation, and under variable circumstances, i.e. when the velocity of translation is changing either in direction or in magnitude, the lagging behind of which we have just spoken cannot be entirely avoided. The case is similar to that of a pendulum bob acted on by a variable force, whose changes, as is well known, it does not instantaneously follow. The pendulum may, however, approximately be said to do so when the variations of the force are very slow in comparison with its own free vibrations. Similarly, the electron may be regarded as being, at every instant, in the state of equilibrium corresponding to its velocity, provided that the time in which the velocity changes perceptibly be very much longer than the period of the oscillations that can be performed under the influence of the regulating forces."

It is therefore clear that the hypothesis of locality and its limitation were discussed by Lorentz for the case of the motion of electrons.

Einstein, in conformity with his general approach of formulating symmetry-like principles that would be independent of the specific nature of matter, simply adopted the same general assumption for rods and clocks. In fact, in discussing the rotating disk problem, Einstein stated in a footnote on page 60 of [7] that:

> "These considerations assume that the behavior of rods and clocks depends only upon velocities, and not upon accelerations, or, at least, that the influence of acceleration does not counteract that of velocity."

The modern experimental foundation of Einstein's theory of gravitation necessitates that this assumption be extended to all (standard) measuring devices; therefore, the hypothesis of locality supersedes the clock hypothesis, etc.

Though the hypothesis of locality originates from Newtonian mechanics, one should point out that the state of a relativistic point particle differs from that in Newtonian mechanics: the magnitude of velocity is always less than c. Moreover, the hypothesis of locality rests on the possibility of defining instantaneous inertial rest frames along the worldline of an arbitrary point particle. In fact, Minkowski raised this possibility and hence the corresponding hypothesis of locality to the level of a fundamental axiom [8].

Another aspect of Lorentz's presupposition must be mentioned here that involves the extension of the notion of rigid motion to the relativistic domain: the electron moves rigidly as it is always undeformed in its momentary rest frame. The notion of rigid motion in the special and general theories of relativity has been discussed by a number of authors [9–13]. It is important to note that the concept of an infinitesimal rigid rod is indispensable in the theory of relativity (cf. section 3).

In some expositions of relativity theory, such as [10] and [14], the hypothesis of locality is completely implicit. For instance, in Robertson's paper on "Postulate *versus* Observation in the Special Theory of Relativity" [14], attention is simply confined to "the kinematics *im kleinen* of physical spacetime" [14]. However, when interpreting the observational foundations of special relativity, one must recognize that actual observers are all accelerated and that the difference between accelerated and inertial observers must be investigated; in fact, this problem is ignored in [14] by simply asserting that physics is essentially local.

3.   Length measurement

To illustrate the nature of the hypothesis of locality, it is interesting to consider spatial measurements of rotating observers. Imagine observers A and B moving on a circle of radius r about the origin in the (x, y)-plane of a background global inertial frame with coordinates (t, x, y, z). Expressed in terms of the azimuthal angle φ, the location of A and B at $t = 0$ can be chosen such that $\varphi_A = 0$ and $\varphi_B = \Delta$ with no loss in generality. The motion of A and B is then assumed to be such that for $t > 0$ they rotate in *exactly the same way* along the circle with angular frequency $\hat{\Omega}_0(t) > 0$. Thus for $t > 0$ observers A and B can be characterized by the azimuthal angles

$$\varphi_A(t) = \int_0^t \hat{\Omega}_0(t')dt' \ , \ \ \varphi_B(t) = \Delta + \int_0^t \hat{\Omega}_0(t')dt' \ . \qquad (1.3)$$

According to the static inertial observers in the background global frame, the angular separation of A and B is constant at any time $t > 0$

and is given by $\varphi_B(t) - \varphi_A(t) = \Delta$; moreover, the spatial separation of the two observers along the circular arc at time $t > 0$ is $\ell(t) = r\Delta$.

Consider now a class of observers O populating the whole arc from A to B and moving exactly the same way as A and B. At any time $t > 0$, it appears to inertial observers at rest in the background frame that these rotating observers are all at rest in the (x', y', z') system that is obtained from (x, y, z) by a simple rotation about the z-axis with frequency $\hat{\Omega}_0(t)$. What is the length of the arc according to these rotating observers? It follows from an application of the hypothesis of locality that for $t > 0$ the spatial separation between A and B as measured by the rotating observers is $\ell' = \hat{\gamma}\ell(t)$, where $\hat{\gamma}$ is the Lorentz factor corresponding to $\hat{v} = r\hat{\Omega}_0(t)$. Units are chosen here such that $c = 1$ throughout this section. Indeed at any time $t > 0$ in the inertial frame, each observer O is momentarily equivalent to a comoving inertial observer and the corresponding infinitesimal element of the arc $\delta\ell$ has a rest length $\delta\ell'$ in the momentarily comoving inertial frame such that from the Lorentz transformation between this local inertial frame and the global background inertial frame one obtains

$$\sqrt{1 - \hat{v}^2} \ \delta\ell' = \delta\ell \tag{1.4}$$

in accordance with the Lorentz-FitzGerald contraction. Defining

$$\ell' = \Sigma \ \delta\ell' \ , \tag{1.5}$$

where each $\delta\ell'$ is the infinitesimal length at rest in a different local inertial frame, one arrives at $\ell' = \hat{\gamma}\ell$, since $\hat{v}(t)$ is the same for the class of observers O at time t. The same result is obtained if length is measured using light travel time over infinitesimal distances between observers O, since in each local inertial frame the two methods give the same answer. As is well known, the light signals could also be used for the synchronization of standard clocks carried by observers O.

It is important to remark here that equation (1.5) is far from a proper geometric definition of length and one must question whether it is even physically reasonable, since each $\delta\ell'$ in equation (1.5) refers to a different local Lorentz frame. In any case, in this approach the length of the arc as measured by the accelerated observers is

$$\ell' = \hat{\gamma}(t)r\Delta \ . \tag{1.6}$$

The sum in equation (1.5) involves infinitesimal rest segments each from a separate local inertial frame. Perhaps the situation could be improved by combining these infinite disjoint local inertial rest frames

into one continuous *accelerated* frame of reference. The most natural way to accomplish this would involve choosing one of the noninertial observers on the arc and establishing a geodesic coordinate system along its worldline. In such a system, the measure of separation along the worldline (proper time) and away from it (proper length) would also be determined by the hypothesis of locality. That is, at any instant of proper time the rules of Euclidean geometry are applicable as the accelerated observer is instantaneously inertial. It turns out that the length of the arc determined in this way would in general be different from ℓ' and would depend on which reference observer $O : A \to B$ is chosen for this purpose [15]. To illustrate this state of affairs and for the sake of concreteness, in the rest of this section the length of the arc will be determined in a geodesic coordinate system along the worldline of observer A and the result will be compared with equation (1.6).

In the background inertial frame, the coordinates of observer A are

$$x_A^\mu = (t, r \cos \varphi_A, r \sin \varphi_A, 0) \ , \tag{1.7}$$

and the proper time along the worldline of A is given by

$$\tau = \int_0^t \sqrt{1 - \hat{v}^2(t')} \ dt' \ , \tag{1.8}$$

where $\tau = 0$ at $t = 0$ by assumption. It is further assumed that $\tau = \tau(t)$ has an inverse and the inverse function is denoted by $t = F(\tau)$. Thus $dt/d\tau = dF/d\tau = \gamma(\tau) = (1 - v^2)^{-1/2}$ is the Lorentz factor along the worldline of A, so that $v(\tau) := \hat{v}(t)$ and $\gamma(\tau) := \hat{\gamma}(t)$. Moreover, it is useful to define $\phi(\tau) := \varphi_A(t)$ and $d\phi/d\tau = \gamma \Omega_0(\tau)$, where $\Omega_0(\tau) := \hat{\Omega}_0(t)$. With these definitions, the natural orthonormal tetrad frame along the worldline of A for $\tau > 0$ is given by

$$\lambda^\mu{}_{(0)} = \gamma(1, -v \sin \phi, v \cos \phi, 0) \ , \tag{1.9}$$

$$\lambda^\mu{}_{(1)} = (0, \cos \phi, \sin \phi, 0) \ , \tag{1.10}$$

$$\lambda^\mu{}_{(2)} = \gamma(v, -\sin \phi, \cos \phi, 0) \ , \tag{1.11}$$

$$\lambda^\mu{}_{(3)} = (0 \ , 0 \ , 0 \ , 1) \ , \tag{1.12}$$

where $\lambda^\mu{}_{(0)} = dx_A^\mu/d\tau$ is the temporal axis and the spatial triad corresponds to the natural spatial frame of the rotating observer. To obtain this tetrad in a simple fashion, first note that by setting $r = 0$ and hence $v = 0$ and $\gamma = 1$ in equations (1.9) - (1.12) one has the natural tetrad of the fixed noninertial observer at the spatial origin — as well as the class of noninertial observers at rest in the background inertial frame —

that refers its observations to the axes of the (x', y', z') coordinate system alluded to before; then, boosting this tetrad with speed v along the second spatial axis tangent to the circle of radius r results in equations (1.9) - (1.12).

It follows from the orthonormality of the tetrad system (1.9) - (1.12) that the acceleration tensor $\mathcal{A}_{\alpha\beta}$ defined by

$$\frac{d\lambda^{\mu}{}_{(\alpha)}}{d\tau} = \mathcal{A}_{\alpha}{}^{\beta}\lambda^{\mu}{}_{(\beta)} \tag{1.13}$$

is antisymmetric. The translational acceleration of observer A, which is the "electric" part of the acceleration tensor ($a_i = \mathcal{A}_{0i}$), is given by

$$\mathbf{a} = \left(-\gamma^2 v\Omega_0 \;\; , \;\; \gamma^2\frac{dv}{d\tau} \;\; , \;\; 0\right) \tag{1.14}$$

with respect to the tetrad frame and similarly the rotational frequency of A, which is the "magnetic" part of the acceleration tensor ($\Omega_i = \frac{1}{2}\epsilon_{ijk}\mathcal{A}^{jk}$), is given by

$$\mathbf{\Omega} = (0, 0, \gamma^2\Omega_0) \;\; . \tag{1.15}$$

Moreover, in close analogy with electrodynamics, one can define the invariants of the acceleration tensor as

$$I = -a^2 + \Omega^2 = \gamma^2\Omega_0^2 - \gamma^4\left(\frac{dv}{d\tau}\right)^2 \tag{1.16}$$

and $I^* = -\mathbf{a}\cdot\mathbf{\Omega} = 0$. The analogue of a null electromagnetic field is in this case a null acceleration tensor; that is, an acceleration tensor is null if both I and I^* vanish. A rotating observer with a null acceleration tensor is discussed in the appendix.

The translational acceleration $\mathbf{a}$ consists of the well-known centripetal acceleration $\gamma^2 v^2/r$ and the tangential acceleration $\gamma^2 dv/d\tau$. The latter formula is consistent with the corresponding result in the case of linear acceleration along a fixed direction. To interpret equation (1.15) as the frequency of rotation of the spatial frame with respect to a local nonrotating frame, it is necessary to construct a nonrotating, i.e. Fermi-Walker transported, orthonormal tetrad frame $\tilde{\lambda}^{\mu}{}_{(\alpha)}$ along the worldline of observer A. Let $\tilde{\lambda}^{\mu}{}_{(0)} = \lambda^{\mu}{}_{(0)} \;\; , \;\; \tilde{\lambda}^{\mu}{}_{(3)} = \lambda^{\mu}{}_{(3)}$ and

$$\tilde{\lambda}^{\mu}{}_{(1)} = \cos\Phi \; \lambda^{\mu}{}_{(1)} - \sin\Phi \; \lambda^{\mu}{}_{(2)} \;\; , \tag{1.17}$$

$$\tilde{\lambda}^{\mu}{}_{(2)} = \sin\Phi \; \lambda^{\mu}{}_{(1)} + \cos\Phi \; \lambda^{\mu}{}_{(2)} \;\; , \tag{1.18}$$

where the angle Φ is defined by

$$\Phi = \int_0^\tau \Omega(\tau')d\tau' \ , \tag{1.19}$$

so that $d\Phi/d\tau = \gamma^2 \Omega_0$. It remains to show that $\tilde{\lambda}^\mu{}_{(i)}$, $i = 1, 2, 3$, correspond to local ideal gyroscope directions. This can be demonstrated explicitly using equations (1.17) - (1.19) and one finds that

$$\frac{d\tilde{\lambda}^\mu{}_{(i)}}{d\tau} = \tilde{a}_i \ \tilde{\lambda}^\mu{}_{(0)} \ , \tag{1.20}$$

where $\tilde{\mathbf{a}}$ is the translational acceleration with respect to the nonrotating frame, as expected. It is straightforward to study the average motion of the spatial frame $\tilde{\lambda}^\mu{}_{(i)}$ with respect to the background inertial axes and illustrate Thomas precession with frequency $(1-\hat{\gamma})\hat{\Omega}_0$ per unit time t. That is, the frame of the accelerated observer rotates with frequency $\hat{\Omega}_0(t)$ about the background inertial axes, while the Fermi-Walker transported frame rotates with frequency $-\hat{\gamma}\hat{\Omega}_0$ per unit time t with respect to the frame of the accelerated observer according to equations (1.17) - (1.19); therefore, the unit gyroscope directions precess with respect to the background inertial frame with frequency $(1-\hat{\gamma})\hat{\Omega}_0$ as measured by the static background inertial observers.

Along the worldline of observer A, the geodesic coordinates can be introduced as follows: At a proper time τ, consider the straight spacelike geodesics that span the hyperplane orthogonal to the worldline. An event $x^\mu = (t, x, y, z)$ on this hyperplane is assigned geodesic coordinates $X^\mu = (T, \mathbf{X})$ such that

$$x^\mu = x_A^\mu(\tau) + X^i \lambda^\mu{}_{(i)}(\tau) \ , \quad \tau = T \ . \tag{1.21}$$

Let $\mathbf{X} = (X, Y, Z)$ and recall that along the worldline of A, $t = F(\tau)$ and $\varphi_A(t) = \phi(\tau)$; then, the transformation to the new coordinates is given by

$$\begin{align}
t &= F(T) + \gamma(T)v(T)Y \ , \tag{1.22}\\
x &= (X + r)\cos\phi(T) - \gamma(T)Y\sin\phi(T) \ , \tag{1.23}\\
y &= (X + r)\sin\phi(T) + \gamma(T)Y\cos\phi(T) \ , \tag{1.24}\\
z &= Z \ . \tag{1.25}
\end{align}$$

For $r = 0$, the geodesic coordinate system reduces to (t', x', y', z'), where $t' = t$; that is, the standard rotating coordinate system is simply the

geodesic coordinate system constructed along the worldline of the non-inertial observer at rest at the origin of spatial coordinates.

The form of the metric tensor in the geodesic coordinate system has been discussed in [1, 15, 16]. It turns out that in the case under consideration here the geodesic coordinates are admissible within a cylindrical region [16]. The boundary of this region is a real elliptic cylinder for $I > 0$, a parabolic cylinder for $I = 0$ or a hyperbolic cylinder for $I < 0$, where the acceleration invariant I is given by equation (1.16).

The class of observers $O : A \to B$ lies on an arc of the circle $x^2 + y^2 = r^2$ in the background coordinate system; therefore, it follows from equations (1.23) and (1.24) that in the geodesic coordinate system the corresponding figure is an ellipse

$$\frac{(X + r)^2}{r^2} + \frac{Y^2}{(r\gamma^{-1})^2} = 1 \tag{1.26}$$

with semimajor axis r, semiminor axis $r\sqrt{1 - v^2}$ and eccentricity v. The latter quantities are in general dependent upon time T, hence at a given time t each observer lies on a different ellipse. It is natural to think of the ellipse (1.26) as a circle of radius r that has suffered Lorentz-FitzGerald contraction along the direction of motion [1, 15].

The measurement of the length from A to B in the new system involves the integration of $dL, dL^2 = dX^2 + dY^2$, along the curve from $A : (T_A, 0, 0, 0)$ to $B : (T_B, X_B, Y_B, 0)$ corresponding to $A : (t, r \cos \varphi_A, r \sin \varphi_A, 0)$ and $B : (t, r \cos \varphi_B, r \sin \varphi_B, 0)$ in the background inertial frame. To clarify the situation, it is useful to introduce — in analogy with the elliptic motion in the Kepler problem — the eccentric anomaly θ by

$$X + r = r \cos \theta \ , \quad Y = r\sqrt{1 - v^2} \sin \theta \ . \tag{1.27}$$

Then, for a typical rotating observer $O : (t, r \cos \varphi, r \sin \varphi, 0)$ on the arc from $A \to B$ with

$$\varphi = \delta + \int_0^t \hat{\Omega}_0(t')dt' \tag{1.28}$$

one has in geodesic coordinates $O : (T, X, Y, 0)$, where X and Y are given by equations (1.27), and equations (1.22) - (1.24) imply that

$$t = F(T) + rv(T) \sin \theta \ , \tag{1.29}$$

$$\varphi = \theta + \phi(T) \ . \tag{1.30}$$

As O ranges from A to B, $\delta : 0 \to \Delta$ in equation (1.28) and hence $\theta : 0 \to \Theta$. For a fixed t, $t = F(T_A)$, equation (1.29) can be solved to give T as a function of θ; then, a detailed calculation involving equations (1.27) - (1.30) shows that

$$L = r \int_0^\Theta \sqrt{1 - v^2 W \cos^2 \theta} \, d\theta \ . \tag{1.31}$$

Here W is defined by

$$W = \gamma^2 \frac{1 - r^2 \dot{v}^2 \sin^2 \theta}{(\gamma + r \dot{v} \sin \theta)^2} \ , \tag{1.32}$$

$\dot{v} = dv/dT$ and Θ can be found in terms of Δ by solving equations (1.29) and (1.30) at B:

$$t = F(T_B) + rv(T_B) \sin \Theta \ , \tag{1.33}$$

$$\Delta + \int_0^t \hat{\Omega}_0(t') dt' = \Theta + \phi(T_B) \ . \tag{1.34}$$

In practice, the explicit calculation of L can be rather complicated; therefore, for the sake of simplicity only the case of constant v (i.e. uniform rotation) will be considered further here [1, 15]. Then, $W = 1$ and equation (1.31) simply refers to the arc of a constant ellipse for which a Kepler-like equation

$$\theta - v^2 \sin \theta = \delta \tag{1.35}$$

follows from equations (1.29) and (1.30). Furthermore, the proper acceleration length of the uniformly rotating observer A is given by $\mathcal{L} = I^{-1/2} = (\gamma \Omega_0)^{-1}$. The case of uniform rotation, where L and ℓ' are independent of time and $L \neq \ell'$ in general, has been treated in detail in [1, 15] and it is clear that irrespective of the magnitude of Δ, $L/\ell' \to 1$ as $r/\mathcal{L} = v\gamma \to 0$; on the other hand for $\Delta \to 0$, $L/\ell' \to 1$ irrespective of $v < 1$. That is, consistency can be achieved only if the length under consideration is negligibly small compared to the acceleration length of the observer.

4.   Discussion

It is important to recognize that the hypothesis of locality is an essential element of the theories of special and general relativity. In particular, it is indispensable for the measurement of spatial and temporal intervals

by accelerated observers. Therefore, relativistic measurement theory must take this basic assumption and its limitations into account. This has been done for the measurement of time in [17]. In connection with the measurement of distance, it has been shown that there is a lack of uniqueness; however, this problem can be resolved if the distance under consideration is much smaller than the relevant acceleration length of the observer [1, 15]. This means that from a basic standpoint the significance of noninertial reference frames is rather limited [16]. In practice, however, the difference between L and ℓ' (discussed in section 3) is usually rather small; for instance, in the case of the equatorial circumference of the Earth this difference amounts to about 10^{-2} cm [15].

The application of these concepts to standard accelerated measuring devices that are by definition consistent with the hypothesis of locality results in a certain maximal acceleration [18, 19] that is imposed by the quantum theory. For a classical device of mass M, the dimensions of the device must be much larger than $\hbar/(Mc)$ according to the quantum theory of measurement [20, 21]. On the other hand, the dimensions of the device must be much smaller than its acceleration length $\mathcal{L}$. It follows that $\mathcal{L} >> \hbar/(Mc)$ for any standard classical measuring device [2, 4]. Thus for $\mathcal{L} = c^2/a$, the translational acceleration a must be much smaller than $Mc^3/\hbar$, while for $\mathcal{L} = c/\Omega$, the rotational frequency Ω must be much smaller than $Mc^2/\hbar$. Further discussion of the notion of maximal acceleration is contained in [22].

The hypothesis of locality is compatible with wave phenomena only when the latter are considered in the ray limit $(\lambda/\mathcal{L} \to 0)$. To go beyond the basic limitations inherent in the hypothesis of locality regarding the treatment of wave phenomena, a nonlocal theory of accelerated observers has been developed [23–25]. In this theory, the amplitude of a radiation field as measured by an accelerated observer depends on its history, namely, its past worldline in Minkowski spacetime. This acceleration-induced nonlocality constitutes the first step in the program of developing a nonlocal theory of gravitation.

Appendix: Null acceleration

The relativistic theory of an observer in arbitrary circular motion is treated in section 3. In this case, the proper acceleration length of the observer is defined to be $|I|^{-1/2}$, where I is given by equation (1.16). It is interesting to study the circular motion of an observer with a constant prescribed magnitude of I. In fact, equation (1.16) can be written as

$$\left(\frac{d\hat{v}}{dt}\right)^2 = \frac{1}{r^2}\,\hat{v}^2(1-\hat{v}^2)^2 - I(1-\hat{v}^2)^3\;,$$

which for constant I can be simply integrated. For the null acceleration case $I = 0$, the solution is

$$\hat{v}^{-2} = 1 + \eta\,e^{\mp 2\frac{t}{r}}$$

for $\eta > 0$. The upper sign refers to motion that asymptotically $(t \to \infty)$ approaches the speed of light, while the lower sign corresponds to an asymptotic state of rest.

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