

# Modified equation of state, scalar field, and bulk viscosity in Friedmann universe

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## Abstract

A generalized dynamical equation for the scale factor of the universe is proposed to describe the cosmological evolution, of which the  $\Lambda$ CDM model is a special case. It also provides a general example to show the equivalence of the modified equation of state (EOS) and a scalar field model. In the mathematical aspect, the EOS, the scalar field potential  $V(\phi)$ , and the scale factor  $a(t)$  all have possessed analytical solutions. Such features are due to a simple form invariance of the equation inherited which determines the Hubble parameter. From the physical point of view, this dynamical equation can be regarded as the  $\Lambda$ CDM model with bulk viscosity, an existence content in the universe. We employ the SNe Ia data with the parameter  $\mathcal{A}$  measured from the SDSS data and the shift parameter  $\mathcal{R}$  measured from WMAP data to constrain the parameters in our model. The result is that the contribution of the bulk viscosity, accumulated as an effective dark energy responsible for the current cosmic accelerating expansion, is made approximately ten percent to that of the cosmological constant.

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## 1. Introduction

The cosmological observations have provided increasingly convincing evidence that our universe is undergoing a late-time cosmic acceleration expansion [1,2]. In order to explain the acceleration expansion, physicists have introduced a new fluid, which possesses a negative enough pressure, called dark energy. According to the observational evidence, especially from the type Ia supernovae [3,4] and WMAP satellite missions [5], we live in a favored spatially flat universe consisting approximately of 30% dark matter and 70% dark energy. The simplest candidate for dark energy is the cosmological constant, but it has got the (in-)famous and serious fine-tuning problem, while the also elusive dark matter candidate might be a lightest and neutral supersymmetry particle with only gravity interaction.

Recently, a great variety of models are proposed to describe the universe, partly such as scalar fields, exotic equation of state, modified gravity, inhomogeneous cosmology model and super horizon backreaction [6]. It is also instructive to investigate by concrete models the possibility that the both mysterious dark components, dark matter and dark energy, may be two facets of a single complicated fluid, that is, they originate from a unified dark fluid in the universe. From Einstein equation we know that the right-hand side is intimately related to the geometry or gravity, and on the other side the energy–momentum tensor also involves a quantum field vacuum. According to Ref. [7], it is tempting to investigate the properties of cosmological models starting from the equation of state (hereafter EOS) of dark energy directly and by testing whether a given EOS is able to give rise to cosmological models reproducing the available dataset. We hope the situation will be improved with the new generation of more precise observational data upcoming.

The observational constraints indicate that the current EOS parameter  $w = p/\rho$  is around  $-1$  [3], quite probably below  $-1$ , which is called the phantom region and even more myster-

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rious in the cosmological evolution processes. In the standard model of cosmology, if the  $w < -1$ , the universe shows to possess the future finite singularity called Big Rip [8,9]. Several ideas are proposed to prevent the big rip singularity, like by introducing quantum effects terms in the action [10,11]. Based on the motivations of time-dependent viscosity and modified gravity, the Hubble parameter dependent EOS is considered in Refs. [7,12]. The Hubble parameter dependent term in this EOS can drive the phantom barrier being crossed in an easier way [12,13]. Different choices of the parameters may lead to several fates to the cosmological evolution [14]. Recently, the equivalence between the modified EOS, the scalar field model, and the modified gravity is demonstrated in Ref. [15], with a general method to calculate the potential of the scalar field for a given EOS presented.

In Refs. [16–18], the bulk viscosity in cosmology has been studied in various aspects. Dissipative processes are thought to be present in any realistic theory of the evolution of the universe. In the early universe, the thermodynamics is far from equilibrium, the viscosity should be concerned in the studies of the cosmological evolution. Even in the later cosmic evolution stage, for example, the temperature for the intergalactic medium (IGM), the baryonic gas, generally is about  $10^4$  K to  $10^6$  K and the complicated IGM is rather non-trivial. The sound speed  $c_s$  in the baryonic gas is only a few km/s to a few tens km/s and the Jeans length  $\lambda$  yields a term as an effective viscosity  $c_s \lambda$ . On the other hand, the bulk velocity of the baryonic gas is of the order of hundreds km/s [19]. So it is helpful to consider the viscosity element in the later cosmic evolution. It is well known that in the framework of Friedmann–Robertson–Walker (FRW) metric, the shear viscosity [20] has no contribution in the energy–momentum tensor, and the bulk viscosity behaves like an effective pressure. Moreover, the cosmic viscosity here can also be regarded as an effective quantity as caused mainly by the non-perfect cosmic contents interactions and may play a role as a dark energy candidate [21].

In this Letter, we show that in the framework of Friedmann universe, the general EOS

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H},$$

corresponds to a scalar field model whose potential has got the form

$$V(\varphi) = V_0(e^{-\beta\varphi} + C_1 e^{-\beta\varphi/2} + C_2).$$

We will present analytically that the equation for the scale factor derived from the above EOS or scalar field model is more general than the  $\Lambda$ CDM model can show, and it has possessed an exact solution. The natural interpretation of this model is involved to the bulk viscosity. Concerning on the different forms of the bulk viscosity coefficient, we propose three parameterized  $H(z)$  relations and use the observational data to constrain the parameters.

This Letter is organized as follows: In the next section we give the generalized dynamical equation for the scale factor and show a transformation to reduce the dynamical equation of the scale factor  $a(t)$  into a linear differential equation. In Section 3 we demonstrate that the EOS is corresponding to a scalar

field whose potential can be exactly solved. In Section 4 we find that there exists a form invariance related to the variable cosmological constant which satisfies a renormalization equation. In Section 5 we use the SNe Ia data with the parameters  $\mathcal{A}$  from large scale survey and shift  $\mathcal{R}$  from cosmic background radiation data to constrain our model. Finally, we present our conclusions and discussions in the last section.

## 2. Viscous dark fluid described by an effective EOS

We consider the FRW metric in the flat space geometry ( $k = 0$ ) as the case favored by WMAP data

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2), \quad (1)$$

and assume that the cosmic fluid possesses a bulk viscosity  $\zeta$ . The energy–momentum tensor can be written as

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p - \zeta\theta)H_{\mu\nu}, \quad (2)$$

where in comoving coordinates  $U^\mu = (1, 0, 0, 0)$ , and  $H_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$  [22]. By defining the effective pressure as  $\tilde{p} = p - \zeta\theta$  and from the Einstein equation  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$ , we obtain the Friedmann equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho, \quad (3a)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3\tilde{p}). \quad (3b)$$

The conservation equation for energy,  $T_{;\nu}^{0\nu}$ , yields

$$\dot{\rho} + (\rho + \tilde{p})\theta = 0, \quad (4)$$

where  $\theta = U^\mu_{;\mu} = 3\dot{a}/a$ .

In our previous work [14], we have considered the following EOS form with the same notations

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H}, \quad (5)$$

where  $H$  is the Hubble parameter and  $w_x$  are their corresponding coefficients. We have obtained the exact solution to the scale factor and showed that

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{\dot{a}} \quad (6)$$

is equivalent to the form by using the above EOS. By defining

$$\tilde{\gamma} = \frac{\gamma + (\kappa^2/3)w_{H2}}{1 + (\kappa^2/2)w_{dH}}, \quad (7)$$

$$\frac{1}{T_1} = \frac{-(\kappa^2/2)w_H}{1 + (\kappa^2/2)w_{dH}}, \quad (8)$$

$$\frac{1}{T_2^2} = \frac{-(\kappa^2/2)p_0}{1 + (\kappa^2/2)w_{dH}}, \quad (9)$$

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{6\tilde{\gamma}}{T_2^2}, \quad (10)$$

the dynamical equation of the scale factor  $a(t)$  can be written as

$$\frac{\ddot{a}}{a} = -\frac{3\tilde{\gamma} - 2}{2} \frac{\dot{a}^2}{a^2} + \frac{1}{T_1} \frac{\dot{a}}{a} + \frac{1}{T_2^2}. \quad (11)$$

The analytical solution for  $\tilde{\gamma} \neq 0$  is given out as

$$a(t) = a_0 \left\{ \frac{1}{2} \left( 1 + \tilde{\gamma} \theta_0 T - \frac{T}{T_1} \right) \exp \left[ \frac{t-t_0}{2} \left( \frac{1}{T} + \frac{1}{T_1} \right) \right] + \frac{1}{2} \left( 1 - \tilde{\gamma} \theta_0 T + \frac{T}{T_1} \right) \exp \left[ -\frac{t-t_0}{2} \left( \frac{1}{T} - \frac{1}{T_1} \right) \right] \right\}^{2/3\tilde{\gamma}}. \quad (12)$$

For the case  $\tilde{\gamma} = 0$ , the solution is

$$a(t) = a_0 \exp \left[ \left( \frac{1}{3} \theta_0 T_1 + \frac{T_1^2}{T_2^2} \right) (e^{(t-t_0)/T_1} - 1) - \frac{T_1(t-t_0)}{T_2^2} \right]. \quad (13)$$

The five parameters in Eq. (5) are condensed to three free parameters in Eq. (11) or its solution, later the best fit analyses in Section 5 enable us to obtain the physical evolution of the universe.

It is interesting that there exists a transformation

$$y = a^{3\tilde{\gamma}/2} \quad (14)$$

to reduce Eq. (11) to a linear differential equation of  $y(t)$

$$\ddot{y} - \frac{1}{T_1} \dot{y} - \frac{3\tilde{\gamma}}{2T_2^2} y = 0, \quad (15)$$

which can be solved easily. The equation of  $a(t)$  can be written as a more general form

$$\frac{\ddot{a}}{a} = -\frac{3\tilde{\gamma} - 2}{2} \frac{\dot{a}^2}{a^2} + \frac{1}{T_1} \frac{\dot{a}}{a} + \frac{1}{T_2^2} + \frac{C}{a^{3\tilde{\gamma}/2}}, \quad (16)$$

and we can also use the transformation of Eq. (14) to obtain the linear differential equation of the function  $y(t)$

$$\ddot{y} - \frac{1}{T_1} \dot{y} - \frac{3\tilde{\gamma}}{2T_2^2} y - \frac{3\tilde{\gamma}}{2} C = 0. \quad (17)$$

We can check directly that the following transformation

$$y = a^{3\tilde{\gamma}/2} + CT_2^2 \quad (18)$$

can reduce the nonlinear Eq. (16) to its corresponding linear Eq. (15). If  $\tilde{\gamma} = 4/3$ , i.e.,  $w = 1/3$ , Eq. (16) becomes

$$\frac{\ddot{a}}{a} = -\frac{\dot{a}^2}{a^2} + \frac{1}{T_1} \frac{\dot{a}}{a} + \frac{1}{T_2^2} + \frac{C}{a^2}, \quad (19)$$

which can be interpreted as the radiation dominated universe when the curvature of the universe is concerned.

### 3. Scalar field and modified gravity

Starting from the action for the gravitational with the matter fields, we can show that an EOS for the universe contents corresponds to a scalar field model. Generally, for a given EOS, the potential  $V(\varphi)$  often has got no analytical solution. Here we demonstrate that the correspondent scalar field model for the EOS of Eq. (5) luckily has possessed an exact solution. First, we revisit the general procedure proposed in Ref. [15] to relate a scalar field model for a given EOS.

Starting from the action of the scalar–tensor theory

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right), \quad (20)$$

the energy density and the pressure are

$$\rho = \frac{1}{2} \omega(\phi) \dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2} \omega(\phi) \dot{\phi}^2 - V(\phi). \quad (21)$$

Combining the above equations and the Friedmann equations, one obtains

$$\omega(\phi) \dot{\phi}^2 = -\frac{2}{\kappa^2} \dot{H}, \quad V(\phi) = \frac{1}{\kappa^2} (3H^2 + \dot{H}). \quad (22)$$

The interesting case is that  $\omega(\phi)$  and  $V(\phi)$  are determined in terms of a single function  $f(\phi)$  as

$$\omega(\phi) = -\frac{2}{\kappa^2} f'(\phi), \quad V(\phi) = \frac{1}{\kappa^2} (3f(\phi)^2 + f'(\phi)). \quad (23)$$

One can check that the special solution  $\phi = t$ ,  $H = f(t)$  satisfies the scalar-field equation. The following relations is obtained in order to solve the function  $f(\phi)$  for a given EOS,

$$\rho = \frac{3}{\kappa^2} f(\phi)^2, \quad p = -\frac{3}{\kappa^2} f(\phi)^2 - \frac{2}{\kappa^2} f'(\phi). \quad (24)$$

By defining a new field  $\varphi$  as

$$\varphi = \int d\phi \sqrt{|\omega(\phi)|}, \quad (25)$$

the action can be rewritten as

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R \mp \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \tilde{V}(\varphi) \right). \quad (26)$$

The energy density and the pressure are now given by

$$\rho = \pm \frac{1}{2} \dot{\varphi}^2 + \tilde{V}(\varphi), \quad p = \pm \frac{1}{2} \dot{\varphi}^2 - \tilde{V}(\varphi). \quad (27)$$

Now we summarize the procedure. For a given EOS, by using Eq. (24), one can obtain an equation for  $f(\phi)$ ; then solving the equation gives  $f(\phi)$ . Using Eq. (23), one obtains  $\omega(\phi)$  and  $V(\phi)$ . And by employing Eq. (25) to transform the variable  $\phi$  to  $\varphi$ , finally, one obtains the  $\tilde{V}(\varphi)$ . In Ref. [15], several examples are presented such as  $p = w\rho$ , and we show in this section that a more general form of Eq. (5) can also have had an analytical solution of the potential.

By using the Friedmann equations, Eq. (5) can be rewritten as

$$p = (\tilde{\gamma} - 1)\rho - \frac{2}{\sqrt{3}\kappa T_1} \sqrt{\rho} - \frac{2}{\kappa^2 T_2^2}. \quad (28)$$

The corresponding equation for  $f(\phi)$  is

$$f'(\phi) = \frac{3\tilde{\gamma}}{2} f(\phi)^2 - \frac{1}{T_1} f(\phi) - \frac{1}{T_2}. \quad (29)$$

The solution of this equation is

$$f(\phi) = \alpha \coth \left( \frac{3\tilde{\gamma}}{2} \alpha \phi \right) + \frac{1}{3\tilde{\gamma} T_1}, \quad (30)$$

where

$$\alpha = -\sqrt{\frac{1}{9\tilde{\gamma}^2 T_1^2} + \frac{2}{3\tilde{\gamma} T_2^2}}. \quad (31)$$

Then we obtain the  $\omega(\phi)$  and  $V(\phi)$ . The integration of Eq. (25) gives the relation between  $\phi$  and  $\varphi$  as

$$\varphi = \frac{2}{\kappa} \sqrt{3|\tilde{\gamma}|} \ln \left| \frac{\tanh(3\tilde{\gamma}\alpha\phi/2)}{\tanh(3\tilde{\gamma}\alpha\phi_0/2)} \right|, \quad (32)$$

which gives

$$\coth\left(\frac{3\tilde{\gamma}}{2}\alpha\phi\right) = \coth\left(\frac{3\tilde{\gamma}}{2}\alpha\phi_0\right) \exp\left(-\frac{\kappa}{2}\sqrt{3|\tilde{\gamma}|}|\varphi\right). \quad (33)$$

So substituting Eq. (30) to Eq. (23) and using the above equation to transform the variable  $\phi$  to  $\varphi$ , we obtain

$$\tilde{V}(\varphi) = \frac{\alpha^2}{\kappa^2} \coth^2\left(\frac{3\tilde{\gamma}}{2}\alpha\phi_0\right) \left[ \frac{3(2-\tilde{\gamma})}{2} \exp(-\kappa\sqrt{3|\tilde{\gamma}|}|\varphi) - \frac{2}{\tilde{\gamma}T_1\alpha} \exp\left(-\frac{\kappa}{2}\sqrt{3|\tilde{\gamma}|}|\varphi\right) + \frac{1}{3\tilde{\gamma}^2 T_1^2 \alpha^2} + \frac{3\tilde{\gamma}}{2} \right]. \quad (34)$$

This potential has got the form

$$\tilde{V}(\varphi) = V_0(e^{-\beta\varphi} + C_1 e^{-\beta\varphi/2} + C_2). \quad (35)$$

As a special case,  $p = (\gamma - 1)\rho$  is given out in Ref. [15]. Taking the limit of  $T_1 \rightarrow \infty$  and  $\alpha \rightarrow 0$  of Eq. (34), and omitting the constant term, we obtain

$$\tilde{V}(\varphi) = V_0 e^{-\kappa\sqrt{|\tilde{\gamma}|}|\varphi}, \quad (36)$$

where

$$V_0 = \frac{2(2-\gamma)}{3\gamma^2 \kappa^2 \phi_0^2}. \quad (37)$$

To see the dynamical effects of the scalar field, by defining the EOS parameter  $w = (\pm\dot{\varphi}^2 - \tilde{V})/(\pm\dot{\varphi}^2 + \tilde{V})$ , the evolution of  $w$  with respect to the time  $t$  is illustrated in Fig. 1.

The action for a modified gravity in Einstein frame

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R + \mathcal{L}_{\text{matter}} + f(R) \right) \quad (38)$$

is shown to be related to a modified EOS [15]. In Ref. [12], the following two equations are derived in the framework of

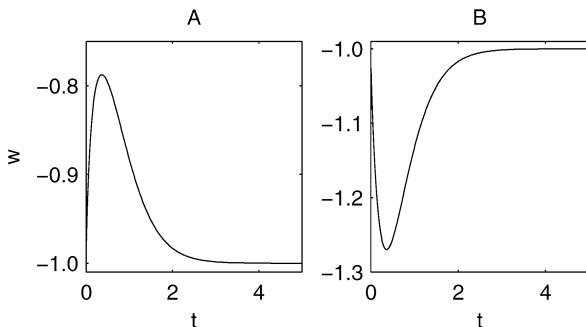


Fig. 1. The  $w-t$  relation of the quintessence case (panel A) and the phantom case (panel B).

modified gravity,

$$0 = -\frac{3}{\kappa^2} H^2 + \rho - f(R) + 6 \left( \dot{H} + H^2 - H \frac{d}{dt} \right) f'(R), \quad (39)$$

$$0 = \frac{1}{\kappa^2} (2\dot{H} + 3H^2) + p + f(R) + 2 \left( -\dot{H} - 3H^2 + \frac{d^2}{dt^2} + 2H \frac{d}{dt} \right) f'(R), \quad (40)$$

where  $R = 6\dot{H} + 12H^2$ . If we adopt the approximation  $\dot{H} \ll H^2$  [12], and assume  $f(R) = f_0\sqrt{R}$ , by combining Eqs. (39) and (40), we can obtain

$$p = (\gamma - 1)\rho - \frac{3\sqrt{3}}{2} f_0 \gamma H - \frac{3}{\kappa^2} \gamma H^2, \quad (41)$$

which is a special case of Eq. (5). Additionally, the scalar-tensor theory in Einstein frame is mathematically equivalent to the modified gravity in Jordan frame [15,23]. The conformal transformation  $g_{\mu\nu} \rightarrow e^{\pm\kappa\varphi\sqrt{\frac{2}{3}}} g_{\mu\nu}$  makes the kinetic term in the action of scalar-tensor theory vanish, then one obtains the Jordan frame action

$$S = \int d^4x \sqrt{-g} \left( \frac{e^{\pm\kappa\varphi\sqrt{\frac{2}{3}}}}{2\kappa^2} R - e^{\pm 2\kappa\varphi\sqrt{\frac{3}{2}}} \tilde{V}(\varphi) \right). \quad (42)$$

If the equation of motion of  $\varphi$  can be solved as  $\varphi = \varphi(R)$ , one obtains the action of the modified gravity in Jordan frame. Thus, the Hilbert–Einstein action with an additional term  $f(R)$  has made effective contributions similar to those caused by a scalar field, as well as a modified EOS.

#### 4. Variable cosmological constant and renormalization group equation

The model of variable cosmological constant is another alternative to explain the cosmic evolution, in order to overcome the serious fine-tuning problem. In the framework of variable cosmological constant model, the evolution of the scale factor is determined by both the Friedmann equations and the renormalization group equation (RGE) for the cosmological constant [24,25],

$$\frac{\ddot{a}}{a} = -\frac{3\tilde{\gamma} - 2}{2} \frac{\dot{a}^2}{a} + \frac{3\tilde{\gamma} + 1}{6} \Lambda, \quad (43)$$

$$\frac{d\Lambda}{d \ln \mu} = \frac{1}{4\pi^2} \sigma \mu^2 M^2 + \dots \quad (44)$$

Here we have already used the EOS  $p = (\gamma - 1)\rho$  to eliminate the  $\rho$  and  $p$  in Friedmann equations. With the choice of the renormalization scale  $\mu = H$  [24], the variable  $\Lambda$  is determined by

$$\frac{d\Lambda}{d \ln H} = \frac{1}{4\pi^2} \sigma H^2 M^2. \quad (45)$$

The solution is

$$\Lambda(t) = \Lambda_0 + \xi [H(t)^2 - H(t_0)^2] M_p^2, \quad (46)$$

which has got the form of  $\Lambda = C_0 + C_2 H^2$ . Compared Eq. (43) with Eq. (11), we find that if Eq. (43) is formally invariant under

the transformation

$$\mu \rightarrow \mu + \delta\mu, \tag{47}$$

the solution of RGE becomes

$$\Lambda = C_0 + C_1 H + C_2 H^2. \tag{48}$$

Substitute this result to Eq. (43), we obtain a equation which has the form of Eq. (11). Especially, if  $\delta\mu = 0$ , then  $C_1 = 0$ . It is very interesting that concerning on the bulk viscosity, modified EOS, scalar field model, modified gravity, and the variable cosmological constant can be described in one generally dynamical equation which determines the scale factor.

Actually, the evolution equation of the Hubble parameter,

$$\dot{H} = -\frac{3\tilde{\gamma}}{2}H^2 + \frac{1}{T_1}H + \frac{1}{T_2^2}, \tag{49}$$

has possessed a form invariance for  $H \rightarrow H + H_0$ , i.e.

$$\dot{H} = -\frac{3\tilde{\gamma}}{2}H^2 + \left(\frac{1}{T_1} - 2H_0\right)H + \frac{1}{T_2^2} + H_0^2. \tag{50}$$

It is this form invariance that gives several interesting features of the model, such as both the  $a(t)$  and  $V(\varphi)$  have analytical solutions for the general EOS of Eq. (5).

### 5. A new content of the universe and data fitting

In Eq. (11),  $1/T_2^2$  plays the role of the effective cosmological constant. If  $T_2 \rightarrow \infty$ , the  $H-z$  relation is

$$H(z) = H_0[\Omega_m(1+z)^{3/2} + (1 - \Omega_m)]. \tag{51}$$

We proposed a parameterized  $H-z$  relation as the following form

$$H^2 = H_0^2[\Omega_m(1+z)^{3\tilde{\gamma}} + \Omega_v(1+z)^{3/2} + 1 - \Omega_m - \Omega_v], \tag{52}$$

where  $\Omega_v = 2/(3\tilde{\gamma}T_1H_0)$  and  $1 - \Omega_m - \Omega_v = 2/(3\tilde{\gamma}T_2^2H_0^2)$ . Note that Eq. (11) can have several interpretations [14], such as the ‘‘inflessence’’ model [26] when the first term and third term alternatively dominate. The three terms in the right-hand side of Eq. (11) are proportional to  $H^2$ ,  $H^1$ , and  $H^0$ , respectively. In the early times, the first term is dominant, which may lead to inflation if  $\tilde{\gamma} \sim 0$ . In the medium times, the second term dominates, which leads to deceleration if  $T_1 < 0$ . In the late times as current stage, the third term is dominant, which leads to cosmic acceleration expansion behaving as the de Sitter universe if  $T_2^2 > 0$ . So, a single equation may describe three epoches of the cosmological evolution. In this Letter, however, we only focus on the interpretation of bulk viscosity for this model, as the viscous universe has been discussed for various cosmology evolution stages with very naturally physical motivations. And the viscosity and the dissipative processes in describing physical universe have been studied in various aspects, for example [27–31].

So far, the universe contents and their dynamical contributions are listed in Table 1.

Compared with the  $\Lambda$ CDM model, we use three cosmological scenarios as parameterizations of the  $H-z$  relations, which are listed in Table 3.

Table 1

$w$	Universe content	Contribution to $H(z)^2$
-1	vacuum	$(1+z)^0$
-1/2	(effective) viscosity	$(1+z)^{3/2}$
-1/3	curvature	$(1+z)^2$
0	dust	$(1+z)^3$
1/3	radiation	$(1+z)^4$
1	stiff matter	$(1+z)^6$

- (i) The  $\Lambda$ CDM model, the simplest model to explain the dark energy.
- (ii) The viscosity model without cosmological constant.
- (iii) The bulk viscosity is constant, so that the bulk viscosity has got the dynamical effects of  $(1+z)^{3/2}$ .
- (iv) The bulk viscosity has the form of  $\zeta = \zeta_0 + \zeta_1\dot{a}/a$ , where the constant term has the dynamical effects of  $(1+z)^{3/2}$ , and the term proportional to  $H$  has an effect to change  $\gamma$  to  $\tilde{\gamma}$ . (See Eq. (7) and Ref. [13]).

The observations of the SNe Ia have provided the direct evidence for the cosmic accelerating expansion for our current universe. Any model attempting to explain the acceleration mechanism should be consistent with the SNe Ia data implying results, as a basic requirement. The method of the data fitting is illustrated in Refs. [32,33]. The observations of supernovae measure essentially the apparent magnitude  $m$ , which is related to the luminosity distance  $d_L$  by

$$m(z) = \mathcal{M} + 5 \log_{10} D_L(z), \tag{53}$$

where  $D_L(z) \equiv (H_0/c)d_L(z)$  is the dimensionless luminosity distance and

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{1}{E(z')} dz'. \tag{54}$$

Also,

$$\mathcal{M} = M + 5 \log_{10} \left( \frac{c/H_0}{1 \text{ Mpc}} \right) + 25, \tag{55}$$

where  $M$  is the absolute magnitude which is believed to be constant for all supernovae of type Ia. We use the 157 golden sample of supernovae Ia data compiled by Riess et al. [3] to fit our model. The data points in these samples are given in terms of the distance modulus

$$\mu_{\text{obs}}(z) \equiv m(z) - M_{\text{obs}}(z). \tag{56}$$

The  $\chi^2$  is calculated from

$$\chi^2 = \sum_{i=1}^n \left[ \frac{\mu_{\text{obs}}(z_i) - \mathcal{M}' - 5 \log_{10} D_{\text{Lth}}(z_i; c_\alpha)}{\sigma_{\text{obs}}(z_i)} \right]^2 + \left( \frac{\mathcal{A} - 0.469}{0.017} \right)^2 + \left( \frac{\mathcal{R} - 1.716}{0.062} \right)^2, \tag{57}$$

where  $\mathcal{M}' = \mathcal{M} - M_{\text{obs}}$  is a free parameter and  $D_{\text{Lth}}(z_i; c_\alpha)$  is the theoretical prediction for the dimensionless luminosity distance of a SNe Ia at a particular distance, for a given model

Table 2  
Summary of the best fit parameters

	Models	Best fit of parameters	$\chi^2$
(i)	$H^2 = H_0^2[\Omega_m(1+z)^3 + 1 - \Omega_m]$	$\Omega_m = 0.283$	177.84
(ii)	$H = H_0^2[\Omega_m(1+z)^{3/2} + 1 - \Omega_m]$	$\Omega_m = 0.435$	319.78
(iii)	$H^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_v(1+z)^{3/2} + 1 - \Omega_m - \Omega_v]$	$(\Omega_m, \Omega_v) = (0.281, 0.065)$	177.38
(iv)	$H^2 = H_0^2[\Omega_m(1+z)^{3\tilde{\gamma}} + \Omega_v(1+z)^{3/2} + 1 - \Omega_m - \Omega_v]$	$(\Omega_m, \Omega_v, \tilde{\gamma}) = (0.298, 0.053, 1.004)$	177.32

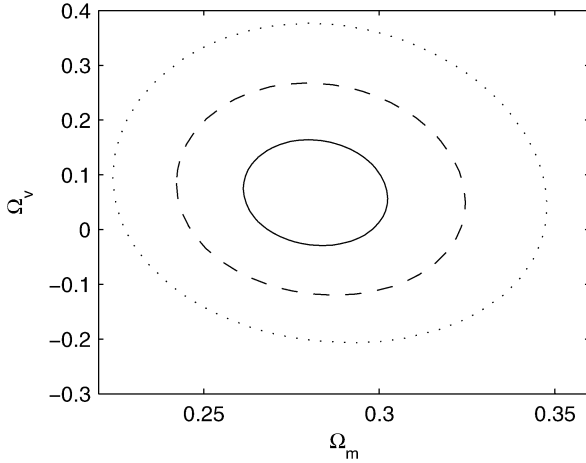


Fig. 2. The  $1\sigma$  (solid line),  $2\sigma$  (dashed line), and  $3\sigma$  (dotted line) contour plots of model (iii).

with parameters  $c_\alpha$ . The parameter  $\mathcal{A}$  is defined as [34]

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{0.35} \left[ \frac{0.35}{E(0.35)} \int_0^{0.35} \frac{dz}{E(z)} \right]^{1/3}, \quad (58)$$

and the shift parameter  $\mathcal{R}$  is [35,36]

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_{ls}} \frac{dz}{E(z)}. \quad (59)$$

We will consider the  $\Lambda$ CDM model for comparison and perform a best-fit analysis with the minimization of the  $\chi^2$ , with respect to  $\mathcal{M}'$ ,  $\Omega_m$ ,  $\Omega_v$ , and  $\tilde{\gamma}$ . The results are summarized in Table 2.

From the results, we see that the bulk viscosity part has made approximately 10% contributions to that of the cosmological constant. In model (ii), the cosmic acceleration expansion is due to the bulk viscosity, without the cosmological constant, and with comparing this model is rather disfavored. For model (iii), Fig. 2 plots the likelihood contour of the parameters  $\Omega_m$  and  $\Omega_v$ . We can see that the bulk viscosity contributes approximately 10% of the cosmological constant. For model (iv), Figs. 3 and 4 plot the likelihood contours of  $\Omega_m - \Omega_v$  and  $\Omega_m - w$  ( $w = \tilde{\gamma} - 1$ ), respectively.

## 6. Conclusion and discussion

In conclusion, we have shown that several different approaches to explain the current accelerating universe expansion can give the same form of a dynamical equation for  $a(t)$ . Also

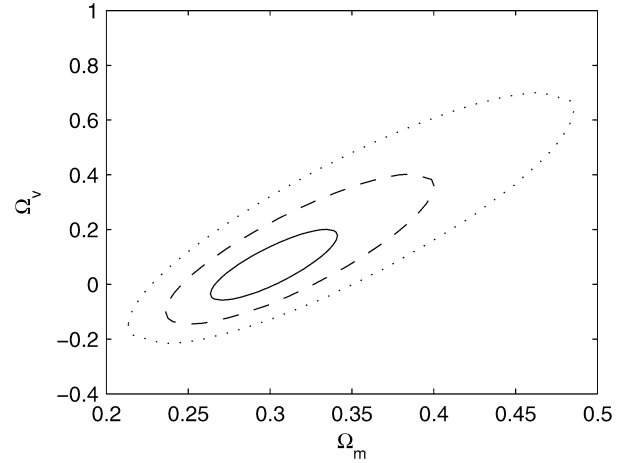


Fig. 3. The  $1\sigma$  (solid line),  $2\sigma$  (dashed line), and  $3\sigma$  (dotted line) contour plots of  $\Omega_m - \Omega_v$  relation in model (iv).

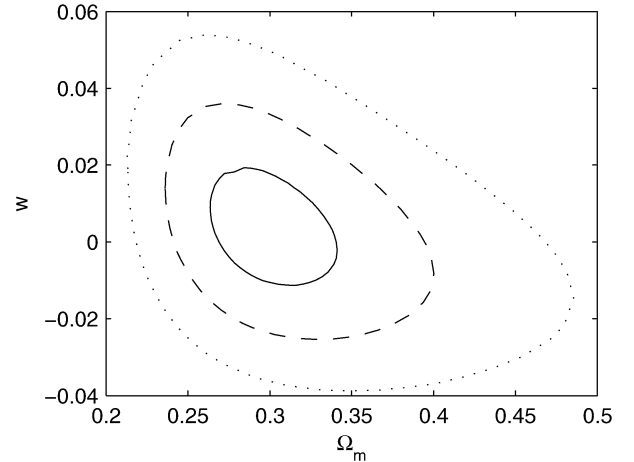


Fig. 4. The  $1\sigma$  (solid line),  $2\sigma$  (dashed line), and  $3\sigma$  (dotted line) contour plots of  $\Omega_m - w$  relation in model (iv).

in the sense that different terms in the right-hand side of the dynamical equation may dominate correspondingly different periods we call the media described by the general EOS a dark fluid. The case of the  $\Lambda$ CDM model can be regarded as a special case of this equation descriptions. In the framework of Friedmann universe, the correspondences of the bulk viscosity, EOS, scalar field, modified gravity, and variable cosmological constant are summarized in Table 3. The scalar field model as a prototype is intensely used to study the dynamical behaviors for the scalar factor in the literature because of its simplicity in formulation and treatment; here we have provided a general and exact solution of the scalar field which can be either quintessence or

Table 3

The mathematical equivalence of different approaches in cosmology ( $C_i$  only denote the coefficients)

Bulk viscosity	$\zeta = \zeta_0 + \zeta_1 \dot{a}/a + \zeta_2 \ddot{a}/\dot{a}$
EOS	$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H}$ (or $p = C_1 \rho + C_2 \sqrt{\rho} + C_3$ )
Scalar field	$V(\varphi) = V_0(e^{-\beta\varphi} + C_1 e^{-\beta\varphi/2} + C_2)$
Modified gravity	$f(R) = f_0 \sqrt{R}$ (by approximation)
Variable CC	$\Lambda = C_0 + C_1 H + C_2 H^2$

Table 4

Fitting results for the model  $H^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_v(1+z)^{3/2} + 1 - \Omega_m - \Omega_v]$  with different data

	$\Omega_m$	$\Omega_v$	$\chi^2$
157	0.330	-0.583	174.53
157 + $\mathcal{A}$ + $\mathcal{R}$	0.281	0.065	177.38
157 + $\mathcal{A}$	0.300	0.063	177.36
157 + $\mathcal{R}$	0.303	0.045	177.34
$\mathcal{A}$ + $\mathcal{R}$	0.316	0.117	$1.88 \times 10^{-7}$
$\mathcal{A}$	0.283	0.031	$1.34 \times 10^{-9}$
$\mathcal{R}$	0.266	0.033	$3.47 \times 10^{-8}$

phantom. Mathematically, this equation enjoys several interesting features, such as the EOS, the potential  $V(\varphi)$  of the scalar field model, and the scale factor  $a(t)$  all have got analytical solutions. This equation can be transformed to a linear equation and has possessed a form invariance related to the Hubble parameter. Physically, the bulk viscosity is a most natural interpretation of this model. We have proposed three cosmological scenarios which contain the contributions from the bulk viscosity and used the 157 golden sample of SNe Ia data with the parameter  $\mathcal{A}$  and  $\mathcal{R}$  to constrain our model. The special model that only the bulk viscosity contributes the cosmic accelerating expansion without the cosmological constant may be excluded. And the (effective) bulk viscosity has made approximately 10% contributions to that of the cosmological constant.

The puzzling cosmic dark components: dark matter and dark energy, responsible mainly for large scale structure formation of universe and cosmic accelerating expansion as well as our universe evolution fate as we now understand in the standard hot big bang and inflation models have challenged our previous intelligence on the physical world. A unification picture description for the two elusive dark composites either from complicated fluid dynamics, modified gravity, inhomogeneous cosmology or quantum field models with introducing more degrees of freedom and supported by more precious experiments like LHC and PLANCK in 2007 is certainly valuable for us to uncover the mysterious dark side of the Universe, which even will bring us new knowledge on fundamental physics.

**Note.** In this brief presentation we find by numerical calculations that parameters  $\mathcal{A}$  and  $\mathcal{R}$  respectively from large scale survey and cosmic background radiation detections significantly affect the fitting results for some models. For example, if we do not use  $\mathcal{A}$  and  $\mathcal{R}$ , the minimum  $\chi^2$  of model (ii) is 179.36, which is acceptable. Furthermore, the best-fit results of model (iii) are listed in Table 4. From this table, it is obvious that the

result by using the 157 golden samples (hereafter 157 as in the below table) with parameters  $\mathcal{A}$  and  $\mathcal{R}$  is very different from that by only using the 157 golden samples, for which we are trying to figure out the physical reasons in late study. It is very likely that we can obtain more accuracy results with high level certainties by next year (2007) PLANCK mission, continuous large scale structure and later large sample of SNe Ia survey, to which we are very confident.

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