Detecting Vanishing Dimensions via Primordial Gravitational Wave Astronomy

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Lower dimensionality at higher energies has manifold theoretical advantages as recently pointed out by Anchordoqui *et al.* [arXiv:1003.5914]. Moreover, it appears that experimental evidence may already exist for it: A statistically significant planar alignment of events with energies higher than TeV has been observed in some earlier cosmic ray experiments. We propose a robust and independent test for this new paradigm. Since (2 + 1)-dimensional spacetimes have no gravitational degrees of freedom, gravity waves cannot be produced in that epoch. This places a universal maximum frequency at which primordial waves can propagate, marked by the transition between dimensions. We show that this cutoff frequency may be accessible to future gravitational wave detectors such as the Laser Interferometer Space Antenna.

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There is growing theoretical evidence to suggest that the short-distance spatial dimensionality is *less* than the macroscopically observed three. Causal dynamical triangulations [1] demonstrate that the four-dimensional spacetime can emerge from two-dimensional simplicial complexes. It has also been shown that a noncommutative quantum spacetime with minimal length scale will exhibit the properties of a two-dimensional manifold [2]. Reducing the number of dimensions in the far UV limit offers a completely new approach to gauge couplings unification [3]. In a similar vein, the cascading Dvali-Gabadadze-Porrati model [4] provides a mechanism for the emergence of an extra spatial dimension only at Hubble scales, in order to solve the cosmological constant problem. It was even argued that evidence of higher dimensionality at cosmological scales is already present in the current observational data [5].

Combining the essence of both extremes, a framework was recently proposed in which the structure of spacetime is a fundamentally (1 + 1)-dimensional universe but is "wrapped up" in such a way that it appears higher-dimensional at larger distances [6]. The structure of space may be envisioned as an n-dimensional ordered lattice on which dynamics are confined to (presumably) $n = 1$–4, defined by fundamental scales $L_1 < L_2 < L_3 < L_4$. Physics with $\Lambda < \Lambda_3$ on length scales $L > L_3 = \Lambda_3^{-1}$ will appear three-dimensional. When the energy (length) scale becomes of the order of $\Lambda_2 > \Lambda_3$ ($L_2 < L_3$), the manifold transitions from $(3+1)$ to $(2+1)$. If $\Lambda_2 \sim 1$ TeV, planar events and other interesting effects could be observed at the LHC for collisions with $\sqrt{s} \geq \Lambda_2$ [7], in addition to unique signatures of lower-dimensional quantum black hole production [8]. For random orientation of lower-dimensional planes or lines, violations of Lorentz invariance induced by a lattice become nonsystematic and thus evade strong limits put on theories with systematic violation of Lorentz invariance [9].

Beyond this novelty, however, the framework effectively cures all divergences that plague the (3 + 1)-dimensional aspects of the current field theory. For example, the fine-tuning problem is alleviated. The radiative corrections to the Higgs boson mass in d spacetime dimensions are obtained for some cutoff energy Λ from the top, W, and Higgs self-coupling loop diagram contributions:

$$\Delta m_H^2 \sim \sum_i \int^{\Lambda} \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_i^2} = F_d(\Lambda), \qquad (1)$$

where the index i refers to the diagram and the function $F_d(\Lambda)$ denotes the total divergence behavior after full evaluation of the contributing Feynman integrals. While quadratically divergent for $d = 4$ [i.e., $F_4(\Lambda) \sim \Lambda^2$], the one-loop corrections to the Higgs boson mass in (2 + 1) dimensions are linearly divergent in the cutoff scale, while in the (1 + 1)-dimensional case they are only logarithmically divergent.

Furthermore, the problems plaguing (3 + 1)-dimensional quantum gravity quantization programs are solved by virtue of the fact that spacetime is dimensionally reduced. Indeed, effective models of quantum gravity are plentiful in (2 + 1) and even (1 + 1) dimensions [10–12]. Similarly, the cosmological constant problem may be explained as a Casimir-type energy between two adjacent "foliations" of three-dimensional space as the scale size $L > L_4$ opens up a fourth space dimension.

What makes this proposal of evolving dimensions very attractive is that some evidence of the lower-dimensional structure of our spacetime at a TeV scale may already exist. Namely, alignment of the main energy fluxes in a target (transverse) plane has been observed in families of cosmic ray particles [13–15]. The fraction of events with alignment is statistically significant for families with energies higher than TeV and a large number of hadrons. This can be interpreted as evidence for coplanar scattering of

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secondary hadrons produced in the early stages of the atmospheric cascade development.

An interesting side effect of such a dimensional reduction scheme is the distinct nature of gravity in lower dimensions. It is well known that, in a (2 + 1)-dimensional universe, there are no local gravitational degrees of freedom, and hence there are no gravitational waves (or gravitons). If the Universe was indeed three-dimensional at some earlier epoch, it is reasonable to deduce that no primordial gravitational waves (PGWs) of this era exist today. There is thus a maximum frequency for PGWs, implicitly related to the dimensional transition scale Λ_2, beyond which no waves can exist. This indicates that gravitational wave astronomy can be used as a tool for probing the novel ''vanishing dimensions'' framework.

We note that the idea of using PGWs and their frequency spectrum to determine dimensional characteristics of spacetime is not new. It has been shown, for example, that phase shifts can be introduced from PGW interactions with extra dimensions [16]. Similarly, Ref. [17] demonstrates the thermalization of PGWs via propagation through extra dimensions. Alternatively, PGWs can reveal the existence of topological Chern-Simons terms in the modified Einstein-Hilbert action [18].

In order to determine an approximate value for the cutoff frequency, we revisit the current state of PGW detection. The standard cosmological theory predicts that gravitational waves will be generated in the pre- or postinflationary regime due to quantum fluctuations of the spacetime manifold. A standard Friedmann-Robertson-Walker cosmology is assumed, with the usual radiation- and matter-dominated eras. Gravity waves can be produced at different times $t_* < t_0 = H^{-1}$, when the temperature of the Universe was T_*. The comoving entropy per volume of the Universe at temperature T_* can be expressed as a function of the scale factor $a(t)$ as

$$S \sim g_S(T) a^3(t) T^3, \tag{2}$$

where the factor g_S represents the effective number of degrees of freedom at temperature T in terms of entropy,

$$g_S(T) = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{j=\text{fermions}} g_j \left(\frac{T_j}{T}\right)^3. \tag{3}$$

The parameters i and j run over all particle species. In the standard model, this assumes a constant value for $T \simeq 300$ GeV, with $g_S(T) = 106.75$ due to the fact that all species were thermalized to a common temperature.

Assuming that entropy is generally conserved over the evolution of the Universe, one can write

$$g_S(T_*) a^3(t_*) T_*^3 = g_S(T_0) a^3(t_0) T_0^3 \tag{4}$$

with $T_0 = 2.728$ K.

The characteristic frequency of a gravitational wave produced at some time t_* in the past is thus redshifted to its present-day value $f_0 = f_* \frac{a(t_*)}{a(t_0)}$ by the factor [19]

$$f_0 = \simeq 9.37 \times 10^{-5}\ \text{Hz} (H \times 1\ \text{mm})^{1/2} g_*^{-1/12} (g_*/g_{*S})^{1/3} T_{2.728}, \tag{5}$$

where the original production frequency f_* is bounded by the horizon size of the Universe at time t_*, i.e., $f_* \sim \lambda_*^{-1} \sim H_*^{-1}$. Note that this is an upper bound, and the actual value may be smaller by a factor $\lambda_* \sim \epsilon H_*^{-1}$, although the final result is weakly sensitive to the value $\epsilon \leq 1$ [19]. This quantity can be related to the temperature T_* by noting that, during the radiation-dominated phase, the scale is

$$H_* = \frac{8\pi^3 g_* T_*^4}{90 M_{\text{Pl}}^2}. \tag{6}$$

By combining Eqs. (5) and (6), the frequency of PGWs that would be detectable is

$$f_\Lambda = 7.655 \times 10^{-5} (g_*)^{1/6} \left(\frac{T_*}{\text{TeV}}\right) \text{Hz} \approx 1.67 \times 10^{-4} \left(\frac{T_*}{\text{TeV}}\right) \text{Hz}, \tag{7}$$

where the latter equality holds for $g_* \sim 10^2$. When $T_* = 1$ TeV, the frequency is $f_\Lambda \sim 10^{-4}$ Hz. This is well below the seismic limit of $f \sim 40$ Hz on ground-based gravity wave interferometer experiments like LIGO or VIRGO [20] but sits precisely at the threshold of the Laser Interferometer Space Antenna (LISA)'s sensitivity range. Indeed, the latter observatory is expected to probe a variety of early-Universe phenomenology from the 100 GeV–1000 TeV period [21]. Figure 1 demonstrates the threshold PGW frequency as a function of transition energy $\Lambda_2 = T_{2D}$.

At this point, it is instructive to study the physics of expanding (1 + 1) and (2 + 1)-dimensional universes in order to check if some unexpected dynamical features can

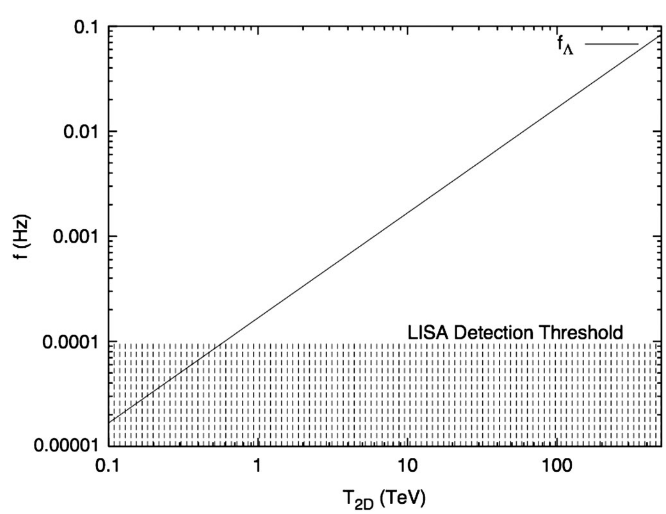

FIG. 1. Frequency threshold for primordial gravitational waves produced when the Universe was at temperature T_*. The hatched region is outside the sensitivity cutoff of LISA.

change the conclusions derived so far. We will show below that this does not happen.

In any spacetime, the curvature tensor $R_{\mu\nu\rho\sigma}$ may be decomposed into a Ricci scalar R, Ricci tensor $R_{\mu\nu}$, and conformally invariant Weyl tensor $C^{\sigma}_{\mu\nu\rho}$. In three dimensions the Weyl tensor vanishes, and $R_{\mu\nu\rho\sigma}$ can be expressed solely through $R_{\mu\nu}$ and R. Explicitly,

$$R_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\alpha}\epsilon_{\rho\sigma\beta}G^{\alpha\beta}. \tag{8}$$

This in turn implies that any solution of the vacuum Einstein's equations is locally flat. Thus, (2 + 1)-dimensional spacetime has no local gravitational degrees of freedom, i.e., no gravitational waves in the classical theory and no gravitons in the quantum theory. Gravity is then uniquely determined by a local distribution of matter. The number of degrees of freedom in such a theory is finite, the quantum field theory reduces to quantum mechanics, and the problem of nonrenormalizability disappears. A (2 + 1)-dimensional Friedmann-Robertson-Walker metric is

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right), \tag{9}$$

where $a(t)$ is the scale factor and $k = -1, 0, +1$. Einstein's equations for this metric are

$$\left(\frac{\dot{a}}{a}\right)^2 = 2\pi G\rho - \frac{k}{a^2}, \qquad \frac{d}{dt}(\rho a^2) + p\frac{d}{dt}a^2 = 0, \tag{10}$$

where G is the (2 + 1)-dimensional gravitational constant, p is the pressure, and ρ is the energy density. In a radiation-dominated universe, $p = \frac{1}{2}\rho$ and $\rho a^3 = \rho_0 a_0^3 = \text{const}$, which gives

$$\dot{a} = \pm\sqrt{\frac{2\pi G\rho_0 a_0^3}{a} - k}, \qquad \ddot{a} = \frac{\pi G\rho_0 a_0^3}{a^2}. \tag{11}$$

For $k = 0$ the solution to these equations is

$$a(t) = \left(\tfrac{9}{2}\pi G\rho_0 a_0^3\right)^{1/3} t^{2/3}. \tag{12}$$

One can note that three-dimensional solution $a(t) \propto t^{2/3}$ is different from the usual four-dimensional behavior $a(t) \propto t^{1/2}$ in the radiation-dominated era.

We have to note here that Einstein's equations, i.e., $G_{\mu\nu} = \kappa T_{\mu\nu}$, are not the only valid option in (2 + 1)-dimensional space. For example, it has been known that theories with $R = \kappa T$, where R is the Ricci scalar and T is the trace of $T_{\mu\nu}$, are not good (3 + 1)-dimensional theories of gravity since they do not have a good Newtonian limit. However, in the context of evolving dimensions, a good Newtonian limit is not a requirement since the spacetime becomes (3 + 1)-dimensional at distances larger than TeV^{-1}. The solutions of the $R = \kappa T$ theory were discussed, for example, in Ref. [22]. The solution for a radiation-dominated universe for the metric given in (9) is $a(t) = t$.

The crossover from a (2 + 1)- to a (3 + 1)-dimensional universe happened when the temperature of the universe was $T_{2D\to3D} = \Lambda_2 \sim 1$ TeV. Working backwards, we can estimate the size of the Universe at the transition from the ratio of scale sizes at various epochs, specifically between the present day ($t_{\text{today}} \sim 10^{17}$ s), the radiation- or matter-dominated era ($t_{\text{RM}} \sim 10^{10}$ s), and the TeV era ($t_{\text{TeV}} \sim 10^{-12}$ s). The scale factor at the latter epoch is thus

$$a_{\text{TeV}} = a_{\text{today}}\left(\frac{t_{\text{TeV}}}{t_{\text{RM}}}\right)^{1/2}\left(\frac{t_{\text{RM}}}{t_{\text{today}}}\right)^{2/3} = 10^{-15}a_{\text{today}}. \tag{13}$$

This value may also be obtained by noting that conservation of entropy requires the product aT to be constant, and so $a_{\text{TeV}} = 10^{-15}a_{\text{today}}$ (since $T_{\text{today}} \sim 10^{-3}$ eV). Equation (13) implies that the size of the currently visible Universe (10^{28} cm) at $T \sim 1$ TeV was 10^{13} cm. This distance is macroscopic, but it is not in contrast with our assumption that the crossover from a (2 + 1)- to a (3 + 1)-dimensional universe happened when the temperature of the Universe was $T \sim 1$ TeV, since the causally connected Universe today contains many causally connected regions of some earlier time. Finding the exact size of the causally connected Universe at the dimensional crossover is not a unique task, since it would strongly depend on an underlying cosmological model. In particular, it would depend on which scale inflation and reheating happened [23] (if at all). The absolute lower limit in the energy scale of inflation is about 10 MeV (in order not to affect the earliest landmark of the standard cosmology—nucleosynthesis), but inflation may as well happen at any energy above the dimensional crossover scale. Further, the dimensional crossover may perhaps be a violent highly nonadiabatic process with huge entropy production. In that case the standard relation $aT = \text{constant}$ would not be valid anymore at temperatures above TeV [but it would still be valid from $T \sim$ TeV until today, as assumed in Eq. (13)].

Fortunately, our limits on PGW are robust with respect to the underlying cosmological model. The only explicit input we used was that the dimensional crossover scale is $T \sim 1$ TeV, which is the value strongly favored for theoretical reasons [6] and perhaps also indicated by the planar events in cosmic ray experiments [13–15].

Going towards even higher temperatures, the spacetime becomes (1 + 1)-dimensional. To avoid large hierarchy in the standard model, the crossover from an (1 + 1)-dimensional to a (2 + 1)-dimensional universe needs to happen when the temperature of the universe was $T_{1D\to2D} = \Lambda_1 \leq 100$ TeV. Conservation of entropy (if between $T \sim 1$ TeV and $T \sim 100$ TeV nothing nonadiabatic happened) requires $aT = \text{const}$. This implies

$$\frac{a_{2D\to3D}}{a_{1D\to2D}} = \frac{T_{1D\to2D}}{T_{2D\to3D}} \sim 100. \tag{14}$$

Similarly,

$$\frac{a_{1D\to2D}}{a_0} = \frac{T_{1D\to2D}}{T_0}, \tag{15}$$

where a_0 and T_0 are the scale factor and temperature of the universe the first time it appears classically. It is tempting to set $a_0 = M_{\rm Pl}^{-1}$ and $T_0 = M_{\rm Pl}$; however, $M_{\rm Pl} = 10^{19}$ GeV is an inherently (3 + 1)-dimensional quantity whose meaning is not quite clear in the context of evolving dimensions.

A (1 + 1)-dimensional Friedmann-Robertson-Walker metric is [24]

$$ds^2 = dt^2 - a(t)^2 \frac{dx^2}{1 - kx^2}. \tag{16}$$

The denominator in the second term in (9) can be absorbed into a definition of the spatial coordinate x. Moreover, all (1 + 1)-dimensional spaces are conformally flat; i.e., one can always use coordinate transformations (independently of the dynamics) and put the metric in the form $g_{\mu\nu} = e^{\phi}\eta_{\mu\nu}$. Einstein's action in a two-dimensional spacetime is just the Euler characteristics of the manifold in question, so the theory does not have any dynamics, unless the scalar field ϕ is promoted into a dynamical field by adding a kinetic term for it. Even in this case there are no gravitons in the theory, so there are no gravity waves and the threshold of importance remains the 2D → 3D transition.

We finally note that the action for gravity was taken at each step of dimensional reduction to be the dimensionally continued Einstein-Hilbert action. At scales much larger or much smaller than the dimensional crossover, this may be justified. However, exactly at the crossover the description could be very complicated. For example, systems whose effective dimensionality changes with the scale can exhibit fractal behavior, even if they are defined on smooth manifolds. As a good step in that direction, in Ref. [25] a field theory which lives in fractal spacetime and is argued to be Lorentz invariant, power-counting renormalizable, and causal was proposed.

Dimensional crossover would also induce nonrenormalizable operators suppressed by the crossover energy scale. While their explicit form is unknown, one can in fact put constraints on their form. Light coming to us from large cosmological distances would be subject to stochastic fluctuations which would induce uncertainties in the wavelength $\delta\lambda \sim L_3(\lambda/L_3)^{1-\alpha}$, where α measures the suppression. Photons that were initially coherent will lose phase coherence as they propagate. For a propagation distance L, the cumulative phase dispersion is $\Delta\phi \sim 2\pi L_3^{\alpha} L^{1-\alpha}/\lambda$ [26]. PKS1413 + 135, a galaxy at a distance of 1.2 Gpc that shows Airy rings at a wavelength of 1.6 μm, is a typical probe for effects of this sort. For $\Lambda_3 \sim 1$ TeV, the requirement $\Delta\phi \gtrsim 2\pi$ gives $\alpha \gtrsim 0.8$, which is not very restrictive.

In conclusion, we proposed a generic and robust test for the new paradigm of ''vanishing'' or ''evolving'' dimensions where the spacetime we live in is lower-dimensional on higher energies. Since (2 + 1)-dimensional spacetimes have no gravitational degrees of freedom, gravity waves cannot be produced in that epoch. This places a universal maximum frequency at which primordial waves can propagate, marked by the transition between dimensions. We showed that this cutoff frequency may be accessible to future gravitational wave detectors such as LISA.

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