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LETTER

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Variable brane tension and dark energy

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Abstract – In this letter, we show that in a particular braneworld scenario with variable brane tension, we obtain matter acting as dark energy while the gravitational constant G promoted to a scalar field on the brane plays the role of matter (both in the sense that they have an "effective" equation of state equivalent to that of dark energy and matter, respectively). This result is interpreted from the Friedmann equation obtained from our model that exactly matches the standard Friedmann equation of general relativity with a cosmological constant Λ in terms of the aforementioned quantities. The universe is assumed to consist of only matter and dark energy in this model which is a good approximation for our universe.

Introduction. – The problem of dark energy is one of the most challenging problems in cosmology. The need for dark energy emerged out of necessity to explain the observation that the universe is expanding in an accelerated manner [1]. It has remained a mystery since and is yet to be solved. In the framework of general relativity, a small and positive cosmological constant Λ [2] is considered as dark energy and since it is proportional to the metric $g_{\mu\nu}$, it is interpreted as vacuum energy. However, its true nature remains elusive since it has an equation of state $\omega = -1$ and also violates the strong energy condition $(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})u^\mu u^\nu < 0$. A number of alternative approaches have been proposed such as quintessence field [3], modified gravity theories (see [4] for a discussion), and braneworld cosmology (see [5] for a review). In the braneworld scenario, for instance, it was shown in [6] by considering the brane in 5D Minkowski bulk, that the Newtonian potential is that of 5D theory at large distances explaining dark energy [7]. This theory effectively has a 4D massless graviton plus a scalar field as its tensorial structure. Recently, it was shown in [8] that a variable brane tension can explain dark energy where the brane tension was proposed to depend upon the scale factor. There has also been a growing theoretical effort to apply the holographic principle to the dark-energy problem called the holographic dark-energy paradigm (see [9] for a review). It has led to Kaniadakis holographic dark energy [10] (which is based on Kaniadakis entropy and is a generalization of Boltzmann-Gibbs entropy), Barrow holographic dark energy [11] (which uses Barrow entropy instead of the standard Bekenstein-Hawking entropy) and power-law holographic dark energy [12].

In this paper, we use the braneworld model with a variable brane tension to explain the problem of dark energy. It is shown that matter effectively acts as the dark energy and the 4D Newton constant G promoted to a scalar field on the brane plays the role of matter. The idea is elaborated in the next section.

Emergence of dark energy. – For a constant brane tension, Einstein's equation on the brane (with no cosmological constant) reads

$$G_{\mu\nu} = \kappa^2_{(4)} T^m_{\mu\nu} + \frac{\kappa^2_{(4)}}{\lambda} S_{\mu\nu} - \xi_{\mu\nu} + \sqrt{\frac{\kappa^2_{(4)}}{\lambda}} F_{\mu\nu}. \quad (1)$$

Here, $T^m_{\mu\nu}$ is the matter energy-momentum tensor, $S_{\mu\nu}$ is the quadratic energy-momentum tensor, $\xi_{\mu\nu}$ is the non-local Weyl tensor and $F_{\mu\nu}$ is the matter field in the bulk. $\kappa^2_{(4)} = 8\pi G_N = \kappa^4_{(5)}\lambda$ is the constant with λ being the constant brane tension. Let us assume that the bulk is empty so that $F_{\mu\nu} = 0$ and, therefore, the pullback on the brane is also zero, which is related to the non-standard model fields [13] as well as the bulk is AdS_5 such that the Weyl term $\xi_{\mu\nu}$ vanishes. This gives

$$G_{\mu\nu} = \kappa^2_{(4)} T^m_{\mu\nu} + \frac{\kappa^2_{(4)}}{\lambda} S_{\mu\nu}. \quad (2)$$

Let us now consider the case where the gravitational constant is promoted to a scalar field G such that $\kappa^2_{(4)} = 8\pi G$ and the five-dimensional gravitational constant is related as usual as $k^4_{(5)}\lambda = 8\pi G$. In this case, the field equation

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becomes

$$G_{\mu\nu} = k_{(5)}^4 \lambda (T_{\mu\nu}^M + T_{\mu\nu}^G) + k_{(5)}^4 S_{\mu\nu}, \tag{3}$$

where $T_G^{\mu\nu}$ is the energy-momentum tensor of the scalar field G on the brane given as

$$T_{\mu\nu}^G = (\partial_\mu G)(\partial_\nu G) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\sigma G)(\partial^\sigma G) - V(G)\right]. \tag{4}$$

Note that since the brane tension is now a variable and the energy-momentum tensor of matter and scalar field is divergenceless, Einstein's equation requires the following relation to hold:

$$k_{(5)}^4 T^{\mu\nu}\nabla_\mu \lambda + k_{(5)}^4 \nabla_\mu S^{\mu\nu} - \nabla_\mu \xi^{\mu\nu} = 0, \tag{5}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of matter plus the scalar field G. The field equation (3) can be derived from the action $S = S_{bulk} + S_{brane}$ on the brane where

$$S_{bulk} = \frac{1}{2k_{(5)}^2}\int \mathrm{d}^5 x\sqrt{-g^{(5)}}R^{(5)}, \tag{6}$$

$$S_{brane} = \int \mathrm{d}^4 x \sqrt{-g^{(4)}} \times \left[\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu G \partial_\nu G - V(G)\right) + \mathcal{L}_M\right], \tag{7}$$

where $\mathcal{L}_M$ is the matter Lagrangian on the brane. Considering the flat FLRW geometry of the brane, the Friedmann equation on the brane reads

$$3H^2 = k_{(5)}^4 \lambda \sum_i \rho_i \left(1 + \frac{\rho_i}{2\lambda}\right), \tag{8}$$

where $\Sigma\rho_i = \rho_M + \rho_r + \rho_G$ is the total energy density (of matter, radiation, and scalar field G, respectively). Since we are interested in late-time cosmology, the second term in the brackets of (8) drops out in the low-energy limit and we can write the above Friedmann equation in terms of redshift factor z as

$$H^2 = H_0^2[\lambda(\Omega_{M0}(1+z)^3 + \Omega_{G0}(1+z)^{3(1+w)} + \Omega_{r0}(1+z)^4)], \tag{9}$$

where the density parameter $\Omega_i(t)$ is given by

$$\Omega_i(t) = \frac{k_{(5)}^4}{3H^2(t)}\rho_i(t) \tag{10}$$

and is the same as that in general relativity in the sense that they carry the same information. w is the Equation of State (EoS) of the scalar field G. For the scalar field (assuming no spatial variation in G), we have

$$\rho_G = \frac{\dot{G}^2}{2} + V(G), \tag{11}$$

$$p_G = \frac{\dot{G}^2}{2} - V(G). \tag{12}$$

If we now consider the kinetic term to be much greater than the potential term $\dot{G}^2 \gg V(G)$ which can be thought of as the scalar field G rolling down a very steep potential, then we have $w = 1$. Using the divergenceless property of the scalar-field energy-momentum tensor ($\nabla_\mu T_G^{\mu\nu} = 0$) and eqs. (11) and (12), the Equation of Motion (EoM) of the scalar field coupled to gravity is given by (assuming no spatial variation in G)[1]

$$\ddot{G} + 3H\dot{G} + \frac{\mathrm{d}V(G)}{\mathrm{d}G} = 0. \tag{13}$$

Thus, $w = 1$ means that the friction term due to the Hubble parameter H is non-dominant and the scalar field easily rolls down the potential generating a larger kinetic term. The (variable) brane tension λ is taken to be the function of the redshift factor z as

$$\lambda(z) = \lambda_0(1+z)^{-3}. \tag{14}$$

Inserting this value of λ and $w = 1$ in (9) with $\Omega_{r0} = 0$ for our universe, we obtain

$$H^2 = H_0^2[\lambda_0(\Omega_{M0} + \Omega_{G0}(1+z)^3)]. \tag{15}$$

This is equivalent to the standard Friedmann equation of general relativity with cosmological constant Λ (and $\Omega_{r0} = 0$) such that

$$\lambda_0\Omega_{M0} = \Omega_{\Lambda 0} \approx 0.7, \tag{16}$$

$$\lambda_0\Omega_{G0} = \Omega_{mo} \approx 0.3. \tag{17}$$

Therefore, in this braneworld model, the scalar field G effectively acts as the matter in the sense that although its EoS is $w = 1$, the "effective" EoS (*i.e.*, after taking the effect of the variable brane tension) is that of matter. This can be interpreted from the Friedmann equation (15) while the matter field effectively acts as the cosmological constant! This result is quite surprising.

Conclusion. – In this letter, we presented a possible explanation of the dark-energy problem by using the braneworld model with a variable brane tension. The four-dimensional Newton constant is promoted to a scalar field on the brane. Counterintuitively, it turned out that the matter on the brane acts as an effective dark-energy term while the scalar field on the brane effectively acts as matter. Considering the late-time cosmology, the Friedmann equation obtained using this model exactly matches the standard Friedmann model of general relativity with a cosmological constant and $\Omega_{r0} = 0$ for our universe. Therefore, in this particular braneworld scenario, we successfully achieved our aim of explaining dark energy without the need to introduce any extra field.

Data availability statement: No new data were created or analysed in this study.

[1] The EoM of the scalar field can also be obtained by extremizing the action (7).

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