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MASSIVE GAUGE-FLATION

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Gauge-flation model at zeroth order in cosmological perturbation theory offers an interesting scenario for realizing inflation within a particle physics context, allowing us to investigate interesting possible connections between inflation and the subsequent evolution of the Universe. Difficulties, however, arise at the perturbative level, thus motivating a modification of the original model. In order to agree with the latest Planck observations, we modify the model such that the new dynamics can produce a relation between the spectral index n_s and the tensor-to-scalar ratio r allowed by the data. By including an identical mass term for each of the fields of the system, we find interesting dynamics leading to slow-roll inflation of the right length. The presence of the mass term has the potential to modify the n_s vs. r relation so as to agree with the data. As a first step, we study the model at zeroth order in cosmological perturbation theory, finding the conditions required for slow-roll inflation and the number of e-foldings of inflation. Numerical solutions are used to explore the impact of the mass term. We conclude that the massive version of Gauge-flation offers a viable inflationary model.

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1. Introduction

The anomalies in the cosmic microwave background (CMB) map reported by Planck at low statistical significance^{1,2} suggest modifications to the simplest scalar field inflation models since they cannot explain, for instance, the possible preferred direction in the Universe.²⁻⁵ Among the different anomalies involving a preferred direction, the dipolar power asymmetry in the CMB sky on large angular scales has the largest statistical significance (around 3σ) with an amplitude of about 0.07.^{1,2} These anomalies could be explained by field theories involving vector or gauge fields, which make it possible to construct inflationary models that generate statistical anisotropy.⁶⁻⁸ Among the proposed models using vector fields, the proposal based on a gauge invariant theory is particularly appealing. This model, called *Gauge-flation*⁸⁻¹⁰ consists of an $SU(2)$ invariant Lagrangian with a Yang-Mills term as well as a contribution from the one-loop effective action called the κ -term. The latter term modifying the Yang-Mills action is crucial to generating the accelerated expansion. The $SU(2)$ invariance introduces three gauge fields, which can be arranged, naturally, in a way that produces the spatial homogeneity and isotropy (via the homomorphism between the $SU(2)$ and the $SO(3)$ groups).⁹ Although the configuration may appear finely tuned, a dynamical system study performed by a collaboration involving the same authors¹¹ demonstrated that a simple anisotropic initial configuration quickly evolves toward an isotropic configuration. This model exhibits interesting dynamics resulting from combining the Yang-Mills term and the κ -term. The solution to the field equations shows that the combined behaviour of these terms generates slow-roll inflation. Moreover, this inflationary epoch lasts long enough to solve the standard cosmological classic problems.^a

Using cosmological perturbation theory, these authors found the range of parameters of the model and initial conditions for which the values of the spectral index n_s and tensor-to-scalar ratio r agree with the 7-year WMAP results.¹⁴ However, later studies showed that, according to the Planck 2013 results,¹⁵ the conclusions in ref. 9 are not correct since no value of γ (a quantity relating the magnitude of the gauge fields, the gauge coupling g , and the Hubble parameter H) gives r and n_s in the allowed region for these parameters.¹⁶ Moreover, a tachyonic instability not reported in the original papers was found for the region $\gamma < 2$. More concretely, the model is ruled out in the stable region $\gamma \geq 2$ since either n_s is too low (when γ is small) or r is too high (when γ is large).

In order to retain the interesting properties of the Gauge-flation model at the background level, we modify the original proposal to alter the predictions for n_s and r as well to avoid the tachyonic instability. By introducing a mass term in the Lagrangian, we induce a longitudinal mode in the field perturbations which can be

^aOther very interesting models that make use of vector fields in gauge field theories, but this time with spontaneous symmetry breaking, are those presented in refs. 12 and 13. Each of these models is able to reproduce the late-time behaviour of the Universe as well as the primordial inflationary period or the other stages of its thermal history.

expected to modify the allowed region for r and n_s . This modification is beneficial since the spectral index acquires a new contribution from the slow-roll parameter η (i.e., from the mass term). This paper studies the new model at the background level whereas the study of the perturbations is deferred to a subsequent publication. We want to know whether including the mass term can give the right amount of inflationary expansion and to determine the conditions required for accelerated expansion in the new model.

There exists concern regarding quantum and classical instabilities generated in massive vector field models of inflation.¹⁷ However, refs. 18 and 19 show that a Maxwell kinetic term for an Abelian gauge field is one of the three cases that avoid such instabilities. (See for example the successful vector curvaton scenario²⁰). In Gauge-flation, we are not dealing with an Abelian vector field, but we expect that an analysis similar to that carried out in refs. 18 and 19 will demonstrate the absence of such instabilities. It is worth remarking that models with Maxwell-type kinetic term, a mass term, and a κ term lead to second-order field equations, which is a necessary condition for avoiding the famous Ostrogradski instability²¹ in the Abelian^{22–26} and non-Abelian cases.^{27,28}

This paper is organized as follows. Section 2 describes the model setup highlighting differences with the original model. In Section 3, we derive the gauge field and Einstein equations in order to establish the conditions for accelerated expansion. We also discuss gauge fixing. In Section 4, we obtain the conditions for slow-roll inflation and some important relations such as the amount of inflation. In Section 5, we explore numerical solutions to the field and Einstein equations. We show that it is possible to get accelerated expansion with the desired properties and analyse the impact of the mass term in the model. Section 6 presents some conclusions and indicates directions for future work.

2. Modified Gauge-flation

Gauge-flation was proposed as a model of inflation based on an $SU(2)$ symmetry and presents interesting features in the unperturbed regime.^{8–10} For several initial conditions Gauge-flation produces a sufficient amount of inflation as well as generating homogeneity and isotropy using an interesting field configuration. Modifications to gravity are not needed. Because of the important role that the non-Abelian gauge field theories play at describing the known fundamental interactions, it is clear that Gauge-flation represents a step ahead in the aim of merging inflationary cosmology and particle physics. Indeed, the presence of non-Abelian gauge fields allows us to easily and adequately connect inflation with the physics of the subsequent evolution of the Universe (e.g., preheating²⁹ and reheating³⁰). The model, however, exhibits some negative features at the perturbative level.¹⁶ More concretely, no set of initial conditions generates combinations of n_s and r compatible with the Planck results. We propose here a modification of the model by including a mass term that might arise from some Higgs-like mechanism in a more sophisticated setup (see, for ex-

ample, ref. 27). We expect this mass term to introduce a new longitudinal mode to modify the predictions for n_s and r . The theoretical prediction for n_s would move to the region allowed by observations as the slow-roll parameter η , which contributes to n_s , gets a contribution from the mass term. In this paper, we only analyse the impact of the mass term on the inflationary dynamics. We also check whether it remains possible to produce a successful inflation and under what conditions.

The Lagrangian describing the system is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \frac{\kappa}{384}(\varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^a F_{\lambda\sigma}^a)^2 - \frac{1}{2}m^2 A_\mu^a A_a^\mu, \quad (1)$$

where $\varepsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric Levi-Civita tensor and $\kappa/384$ is a positive dimensionful parameter related to the cutoff of the theory. The mass is the same for all three fields, the only choice which is consistent with the needed symmetries: homogeneity and isotropy. On the other hand, this Lagrangian breaks the gauge symmetry explicitly, and consequently we cannot apply a gauge fixing condition as was done for the original model. Nevertheless, we find a way to use the same field configuration as in the original massless Gauge-flation model as shown in the following Section. Using the flat Friedmann-Robertson-Walker (FRW) metric for the spacetime

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2)$$

we use the orthonormal tetrad

$$\begin{aligned} \hat{e}^{(0)}_\mu &= (1, 0, 0, 0), & \hat{e}^{(1)}_\mu &= (0, a(t), 0, 0), \\ \hat{e}^{(2)}_\mu &= (0, 0, a(t), 0), & \hat{e}^{(3)}_\mu &= (0, 0, 0, a(t)), \end{aligned} \quad (3)$$

to write the spatial components of the fields in the form $A^a_i = \hat{e}^{(j)}_i A^a_{(j)}$, where $A^a_{(j)}$ are the components of each gauge field A^a_i along the orthonormal basis $\hat{e}^{(j)}_i$. We now use as an Ansatz a field configuration with the vectors $A^a_{(j)}$ oriented along each basis vector of the tetrad, in other words

$$A^a_{(j)} = \psi(t)\delta^a_{(j)}. \quad (4)$$

This implies that the three spatial vectors A^a_i are orthonormal, namely $A^a_i = a(t)\psi(t)\delta^a_i = \phi(t)\delta^a_i$. As explained in ref. 11, this Ansatz is an attractor solution of an anisotropic inflationary model described by the Bianchi type I metric. This configuration is, therefore, not a random choice. We expect the same to happen in the case of massive fields since, as we will see later, it is possible to get inflation with similar properties to the original model. With the Ansatz in eq. (4), we see that the system contains four variables coming from the three fields: $\psi(t)$ and the three temporal components of the fields A^a_0 . In the following Section, we study the dynamics of the system where we see, in particular, how to get a constraint equation for A^a_0 .

3. Field and Einstein Equations

We obtain the field and Einstein equations in order to study the inflationary dynamics. As we will see in the following, the field equation for A^a_μ imposes an important constraint on the field configuration. The Euler-Lagrange equations

$$\frac{\partial(\sqrt{-\tilde{g}}\mathcal{L})}{\partial A^a_\mu} - \partial_\nu \left[\frac{\partial(\sqrt{-\tilde{g}}\mathcal{L})}{\partial(\partial_\nu A^a_\mu)} \right] = 0, \quad (5)$$

where \tilde{g} is the determinant of the metric tensor $g_{\mu\nu}$ with the Lagrangian in eq. (1), can be rewritten as

$$D_\mu \frac{\partial\sqrt{-\tilde{g}}\mathcal{L}}{\partial F^a_{\mu\nu}} + m^2 \sqrt{-\tilde{g}} A^a_\mu g^{\mu\nu} = 0, \quad (6)$$

where D_μ is the $SU(2)$ covariant derivative. For $\nu = 0$, we obtain the equation

$$\begin{aligned} \partial_i \left[F^0_i - \frac{\kappa}{48} \varepsilon^{0i\lambda\sigma} F^a_{\lambda\sigma} (\varepsilon^{\rho\omega\gamma\kappa} F^b_{\rho\omega} F^b_{\gamma\kappa}) \right] - m^2 A^a_0 \\ - g \varepsilon^a_{dc} A^d_i \left[F^0_c - \frac{\kappa}{48} \varepsilon^{0i\lambda\sigma} F^c_{\lambda\sigma} (\varepsilon^{\rho\omega\gamma\kappa} F^b_{\rho\omega} F^b_{\gamma\kappa}) \right] = 0, \end{aligned} \quad (7)$$

where ε^a_{dc} (i.e., the totally antisymmetric three-dimensional Levi-Civita tensor) represents the structure constants of the $SU(2)$ group and g is the respective coupling constant. The homogeneity property of the background sets all the derivatives $\partial_i A^a_\mu$ to zero, giving a constraint on the temporal components of the fields

$$A^a_0 \left(m^2 + \frac{2g^2\phi^2}{a^2} \right) = 0. \quad (8)$$

This implies that all the zero components of the fields vanish, and we thus recover the initial configuration of the fields given in ref. 9:

$$A^a_0 = 0, \quad A^a_i = \phi(t) \delta^a_i. \quad (9)$$

We have just seen, therefore, that the explicit symmetry breaking of the $SU(2)$ invariance does not alter the configuration in eq. (9) provided that we assume homogeneity and isotropy. Consequently, the spatial components of the Euler-Lagrange equations reduce to

$$\left(1 + \kappa \frac{g^4\phi^4}{a^4} \right) \frac{\ddot{\phi}}{a} + \left(1 + \kappa \frac{\dot{\phi}^2}{a^2} \right) \frac{2g^2\phi^3}{a^3} + \left(1 - 3\kappa \frac{g^2\phi^4}{a^4} \right) \frac{H\dot{\phi}}{a} + m^2 \frac{\phi}{a} = 0, \quad (10)$$

where the dot indicates a derivative with respect to cosmic time.

On the other hand, the energy-momentum tensor

$$T_{\mu\nu} = -2 \frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}, \quad (11)$$

for our model is

$$T_{\mu\nu} = F^a_{\mu\beta} F^a_{\nu\alpha} g^{\beta\alpha} - \frac{\kappa}{192} \frac{g_{\mu\nu}}{(\sqrt{-\tilde{g}})^2} (\varepsilon^{\alpha\beta\lambda\sigma} F^a_{\alpha\beta} F^a_{\lambda\sigma})^2 + m^2 A^a_\mu A^a_\nu + g_{\mu\nu} \mathcal{L}_{red}, \quad (12)$$

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where \mathcal{L}_{red} is the reduced Lagrangian of the model. This reduced Lagrangian is derived by replacing the field configuration in eq. (9) in the original Lagrangian in eq. (1):

$$\mathcal{L}_{red} = \frac{3}{2} \left[\frac{\dot{\phi}^2}{a^2} - \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} - \frac{m^2 \phi^2}{a^2} \right]. \quad (13)$$

The non-vanishing components of the energy-momentum tensor are

$$T_{00} = \frac{3}{2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} + \frac{m^2 \phi^2}{a^2} \right], \quad (14)$$

$$T_{ij} = \frac{1}{2} \left[\dot{\phi}^2 + \frac{g^2 \phi^4}{a^2} - 3\kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^4} - m^2 \phi^2 \right] \delta_{ij}. \quad (15)$$

As expected, the Universe is represented by a perfect fluid with an energy-momentum tensor of the form $T_{\mu\nu} = (\rho + P)U_\mu U_\nu + P g_{\mu\nu}$, ρ being the energy density, P the isotropic pressure, and U_μ the fluid 4-velocity. For a comoving observer, the expressions for the energy density and pressure are

$$\rho = \frac{3}{2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} + \frac{m^2 \phi^2}{a^2} \right], \quad (16)$$

$$P = \frac{1}{2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} - 3\kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} - \frac{m^2 \phi^2}{a^2} \right]. \quad (17)$$

With these formulae, we obtain the Einstein equations for the model

$$H^2 = \frac{1}{2m_{Pl}^2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} + \frac{m^2 \phi^2}{a^2} \right], \quad (18)$$

$$\dot{H} = -\frac{1}{m_{Pl}^2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \frac{1}{2} \frac{m^2 \phi^2}{a^2} \right], \quad (19)$$

where m_{Pl} is the reduced Planck mass.

We now must solve eqs. (10), (18), and (19) to check whether the conditions for inflation are satisfied. However, we can simplify the Einstein equations and gain more intuition by expressing the energy density as the sum of three contributions coming from the different terms in the Lagrangian in eq. (1):

$$\rho = \rho_{YM} + \rho_\kappa + \rho_{mass}, \quad (20)$$

where

$$\rho_{YM} = \frac{3}{2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} \right], \quad \rho_\kappa = \frac{3}{2} \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6}, \quad \rho_{mass} = \frac{3}{2} \frac{m^2 \phi^2}{a^2}. \quad (21)$$

The isotropic pressure takes the form

$$P = \frac{1}{3} \rho_{YM} - \rho_\kappa - \frac{1}{3} \rho_{mass}, \quad (22)$$

and the condition for a period of accelerated expansion, given by $\rho + 3P < 0$, implies that

$$\rho_\kappa > \rho_{YM}. \quad (23)$$

The mass term *does not* affect the existence of an accelerated expansion, which requires only that $\rho_\kappa > \rho_{YM}$. The inequality inverts as inflation comes to an end. An interesting dynamics results from the interplay of the contributions of the Yang-Mills term and the κ -term to the energy density. The latter starts inflation and the former ends inflation. Although the mass term seems irrelevant to the overall behaviour during inflation, we will see in the following section that it strongly influences its length.

4. Slow-Roll Inflation

We derive the conditions required for slow-roll inflation. As stated before, this type of inflation can easily lead to a sufficient length of accelerated expansion which is needed to solve the classic problems of the standard cosmology.^{31–34} Moreover, slow-roll inflation guarantees a spectral index for the power spectrum of the primordial curvature perturbation very close to one, as needed to satisfy the observational restrictions presented in ref. 35.

The slow roll parameters

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv -\frac{\ddot{H}}{2\dot{H}H}, \quad \delta \equiv -\frac{\dot{\psi}}{\psi H}, \quad (24)$$

characterize the evolution of the Hubble parameter H and the field $\psi = \phi/a$. It follows that

$$\eta = \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}. \quad (25)$$

Before giving the analytic expressions for ε , η and ψ in terms of the model variables, we find the conditions on the energy density needed for slow-roll inflation, which requires that $\varepsilon, |\eta|, \delta \ll 1$.^{31–34} From eqs. (18), (19), and (24), we obtain

$$\varepsilon = \frac{2\rho_{YM} + \rho_{mass}}{\rho_\kappa + \rho_{YM} + \rho_{mass}}. \quad (26)$$

The condition on ε for slow-roll inflation is satisfied if $\rho_\kappa \gg \rho_{YM}$ and $\rho_\kappa \gg \rho_{mass}$. We can see how much the mass term affects slow-roll inflation: ρ_{mass} must be much smaller than ρ_κ . Although ρ_{mass} is not relevant for generating inflation, *it is* relevant if we want inflation to be slow-roll, which in turn implies a significant influence of the mass on the length of inflation. Numerical simulations demonstrating this fact will be presented in the following Section.

We now study the conditions to ensure that $|\eta|, \delta \ll 1$, which is expressed as conditions on ε and its time derivative. From eq. (25), it follows that $|\eta| \sim \varepsilon \ll 1$

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only if $\dot{\varepsilon}/H\varepsilon \sim \varepsilon$, which we shall assume, i.e., if the relative variation of ε is very small, then $|\eta| \ll 1$. Using the Friedmann equation and eqs. (21) and (26), we obtain

$$\varepsilon = \frac{1}{H^2 m_{Pl}^2} \left[\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \frac{m^2 \phi^2}{2a^2} \right], \quad (27)$$

or equivalently

$$\varepsilon = \frac{\psi^2}{m_{Pl}^2} \left((1 - \delta)^2 + \gamma + \omega/2 \right), \quad (28)$$

where

$$\gamma \equiv \frac{g^2 \psi^2}{H^2}, \quad \omega \equiv \frac{m^2}{H^2}. \quad (29)$$

ε also follows from the Friedmann equation and eqs. (21) and (26), so that

$$\varepsilon = 2 - \frac{\kappa g^2 \psi^6 (1 - \delta)^2}{m_{Pl}^2} - \frac{m^2 \psi^2}{2H^2 m_{Pl}^2}. \quad (30)$$

If we derive the previous equation with respect to the cosmic time and using eqs. (25) and (28), we find

$$\begin{aligned} \eta = \varepsilon - \left[\frac{3\delta}{\varepsilon} + \frac{\dot{\delta}}{(1 - \delta)H\varepsilon} \right] \left[2 - \varepsilon - \frac{\varepsilon\omega/2}{((1 - \delta)^2 + \gamma + \omega/2)} \right] \\ + \frac{(\varepsilon - \delta)\omega/2}{((1 - \delta)^2 + \gamma + \omega/2)}. \end{aligned} \quad (31)$$

Therefore, from $|\eta| \sim \varepsilon$ and $\frac{\omega/2}{((1 - \delta)^2 + \gamma + \omega/2)} < 1$, we conclude that $\delta \sim \varepsilon^2$ must be satisfied in order to have slow-roll inflation. Moreover, from this result and the condition on the time variation of ε (i.e., $\dot{\varepsilon} \sim H\varepsilon^2$), we obtain that δ also varies slowly in time (i.e., $\dot{\delta}/H\delta \sim \varepsilon$).

Given the conditions for slow-roll inflation, we can now obtain expressions for the parameters $\varepsilon, \eta, \delta$ in the slow-roll approximation. Eq. (28) gives

$$\varepsilon \approx \frac{\psi^2}{m_{Pl}^2} (1 + \gamma + \omega/2). \quad (32)$$

Combining this result with eqs. (25) and (29), we obtain

$$\eta \approx \frac{\psi^2}{m_{Pl}^2}. \quad (33)$$

Finally, using $(\omega/2)/((1 - \delta)^2 + \gamma + \omega/2) < 1$, we rewrite eq. (31) as

$$\eta \approx \varepsilon - 2 \left[\frac{3\delta}{\varepsilon} + \frac{\dot{\delta}}{H\varepsilon} \right] + \frac{\varepsilon\omega/2}{(1 + \gamma + \omega/2)}, \quad (34)$$

which helps us to find an expression for δ using the previous results for ε and η :

$$\delta \approx \frac{(\gamma + \omega/2)}{6(1 + \gamma + \omega/2)} \varepsilon^2. \quad (35)$$

These expressions for ε , η and δ coincide with the behaviour described above. For example, we see that $\delta \sim \varepsilon^2$.

One motivation for slow-roll is to produce enough e -folds of inflation N_e . To calculate N_e , we take into account that $\delta = -\dot{\psi}/H\psi$ and $\delta \sim \varepsilon^2$, so that ψ remains almost constant during the slow-roll inflationary period. This allows us to write, using eq. (32),

$$\frac{\varepsilon}{\varepsilon_i} \approx \frac{1 + \gamma + \omega/2}{1 + \gamma_i + \omega_i/2}, \quad (36)$$

where the subscript i denotes the beginning of slow-roll inflation. Since $\varepsilon = 1$ at the end of inflation,

$$\gamma_f + \omega_f/2 \approx \frac{1 + \gamma_i + \omega_i/2}{\varepsilon_i}, \quad (37)$$

where the subscript f denotes the end of slow-roll inflation. This expression is useful to obtaining N_e . The definition of N_e between times t_i and t_f is

$$N_e = \int_{t_i}^{t_f} H dt, \quad (38)$$

which using the change of variable $dt = dH/\dot{H}$ can be rewritten as

$$N_e = - \int_{H_i}^{H_f} \frac{dH}{H\varepsilon}. \quad (39)$$

To solve the integral, we introduce the variable $\theta = \gamma + \omega/2$, finding that

$$N_e \approx - \frac{m_{Pl}^2}{2\psi^2} \int_{\theta_i}^{\theta_f} \frac{-1/\theta^2}{(1+\theta)/\theta} d\theta. \quad (40)$$

Finally, we use eqs. (32) and (37) to obtain the following formula for N_e in terms of the initial values of ε , γ and ω :

$$N_e \approx \frac{1 + \gamma_i + \omega_i/2}{2\varepsilon_i} \ln \left[\frac{1 + \gamma_i + \omega_i/2}{\gamma_i + \omega_i/2} \right]. \quad (41)$$

The final amount of inflation does not depend on the evolution of ε , γ , or ω , but rather on their initial values. From the initial conditions of the model, we can check whether or not the right amount of inflation is produced. Thus the initial conditions on ψ , $\dot{\psi}$, and H are constrained such that the value of N_e satisfies the required bound ($N_e \gtrsim 60$).³¹⁻³⁴ In the next Section, we solve numerically the field and Einstein equations of the model and check the validity of eq. (41) as well as the impact of ω (the mass term) on N_e .

5. Numerical Solution

We solve numerically the field and Einstein equations of the model, which are eqs. (10), (18), and (19). Only two of these equations are independent. The idea is to check whether introducing the mass term allows the model to produce the

necessary accelerated expansion. By analysing the evolution of ε , we check whether the inequality in eq. (23) is satisfied and also whether the inflationary period is slow-roll. Furthermore, we check the validity of eq. (41) for the number of e -folds of inflation. The model has three parameters (g , κ , and m) and two dynamical degrees of freedom ($\phi(t)$ and $a(t)$). We must therefore specify the initial values of ϕ , a and their first time derivatives. As the physical field is $\psi = \phi/a$, and the measurable quantity is H instead of a , we specify initial conditions in terms of ψ and H . We then find the temporal evolution of ψ , ε , ρ_κ/ρ_{YM} , ρ_κ/ρ_{mass} , ρ_{mass}/ρ_{YM} , and N , as well as $\dot{\phi}/a$ vs. ψ for two sets of initial conditions. These initial conditions are chosen so that the expected properties of the model are obtained, keeping a similarity with the conditions chosen in the original massless Gauge-flation model.⁹

The solution for the first set of initial conditions is plotted in Fig. 1. The time evolution of ψ shows slow variation at the beginning of inflation. This behaviour is consistent with $\delta \ll 1$, and then we infer slow-roll behaviour. This conclusion is also supported by the plot of ε showing a very small value at the beginning that varies slowly until inflation ends. Moreover, we note that the conditions for slow-roll inflation are satisfied since $\rho_\kappa \gg \rho_{YM}$ and $\rho_\kappa \gg \rho_{mass}$ while the field is almost constant. These relations break down when the field starts to move fast and to oscillate. As shown in the plot $\dot{\phi}/a$ vs. ψ , these two quantities oscillate with the same frequency but out of phase by $\pi/2$. Nevertheless, since the quantities ε , ρ_{YM} , ρ_κ , and ρ_{mass} involve powers of $\dot{\phi}/a$ and ψ , as seen in eqs. (18), (19), (21), and (24), their behaviour is oscillatory with a modulated amplitude. This can be seen in the the plot of ε on the right (which is an appropriate zoomed version of the plot of ε on the left) as well as in the zoomed plots for ρ_κ/ρ_{YM} , ρ_κ/ρ_{mass} , and ρ_{mass}/ρ_{YM} . Although not included in the figure, we have numerically checked that \dot{H} exhibits oscillations modulated in amplitude, whereas H does not as one normally expects. Overall, the evolution of ψ , ε and the components of ρ suggests that we have obtained the expected slow-roll inflationary behaviour with a sufficient length to solve the standard cosmological classic problems. In fact, we also exhibit the evolution of the amount of inflation where we see that N_e reaches a value around 60. Moreover, the main contribution to N comes from the slow-roll period. This result is consistent with the value from eq. (41) once we use the initial conditions to obtain γ_i , ε_i , and ω_i . We assumed a small value for m compared to H_i (i.e., ω_i) for the first set of initial conditions. Therefore, the model is not drastically affected by the mass term (the ratio ρ_{mass}/ρ_{YM} always stays negligible despite its pronounced oscillations). We obtain almost the same behaviour as found for the original model. Nevertheless, it is also possible to use a value of m larger than H_i and obtain inflation as shown below.

For the second set of initial conditions and parameters, we take $m > H_i$ (see Fig. 2). In this case, all the quantities evolve with essentially the same shape as in Fig. 1, except for ε and ρ_{mass}/ρ_{YM} . During an initial period, the field ψ varies slowly generating an amount of inflation above the lower bound, taking into account that we have to increase κ in order to have a sufficiently long ρ_κ dominance. The

initial conditions and parameters are suitable for describing a slow-roll inflationary period. As in the previous case, we checked the validity of eq. (41) for calculating the final amount of inflation. We found the value 67, which is in agreement with the plot in Fig. 2. As in the previous figure, the quantities ε , ρ_{YM} , ρ_κ , and ρ_{mass} exhibit oscillations modulated in amplitude. The curious behaviour of ε , with alternating peaks of oscillation, can be explained by the high mass value: the plot for ρ_{mass}/ρ_{YM} reveals a series of oscillations where the energy density is alternately dominated by ρ_{mass} , giving ε values quite near 1, and by ρ_{YM} , giving ε a value equal to 2. More specifically, since ρ_{mass} and ρ_κ vanish periodically while ρ_{YM} never vanishes, according to eq. (21), because of the $\pi/2$ phase difference between the oscillations of ϕ and $\dot{\phi}$, the highest peaks in ε always reach 2 according to eq. (26) ($\phi = 0$ so that $\rho_{mass} = \rho_\kappa = 0$) whereas the lowest peaks reach values near 1 ($\rho_{mass} \gg \rho_{YM} \gg \rho_\kappa$).

Finally, by plotting N_e as a function of $\omega_i = m^2/H_i^2$ (keeping H_i constant) for the two sets of initial conditions (see Fig. 3), we find that N_e decreases substantially provided that m increases above H_i . This shows how the mass term modifies the original massless Gauge-flaton model: it reduces the length of inflation.

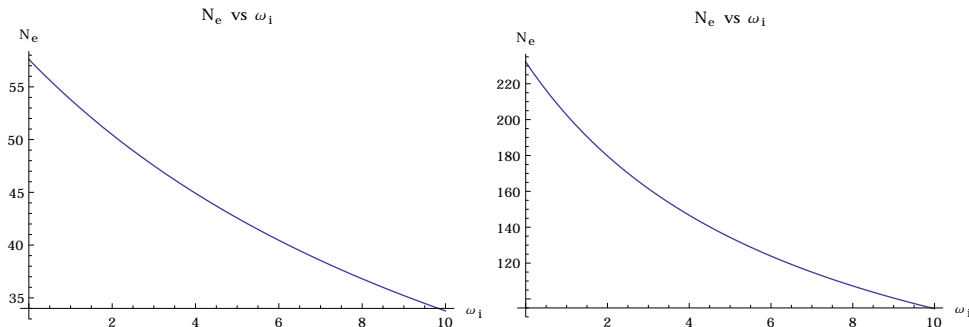


Fig. 3. Change in the final amount of inflation N_e , given by eq. (41), as a function of ω_i for the two sets of initial conditions used in Figs. 1 and 2 (left and right respectively).

6. Conclusions

Including the mass term in the Gauge-flaton model presents interesting features analysed throughout the paper and allows us to obtain a period of accelerated expansion. We have found that the presence of inflationary expansion does not depend on the mass of the fields. The mathematical condition for inflation is $\rho_\kappa > \rho_{YM}$ in which the mass contribution does not appear. However, slow-roll inflation does not occur unless $\rho_\kappa \gg \rho_{mass}$, i.e., the mass term contribution to the energy density ρ must be negligible compared to the κ -term contribution at the beginning of inflation. This is because the κ -term produces the equation of state $P = -\rho$ leading to an era of accelerated expansion. By analysing the numerical solution, we observe that the conditions $\rho_\kappa \gg \rho_{YM}$ and $\rho_\kappa \gg \rho_{mass}$ hold for some initial

time after which these inequalities are inverted, causing inflation comes to an end (when $\varepsilon \rightarrow 1$). We found that it is possible to choose sets of initial conditions and parameters of the model so that inflation lasts long enough to solve the classic problems of the standard cosmology. By using the slow-roll approximation, we found an analytic expression for the amount of inflation in terms of γ_i , ε_i and ω_i . The validity of this expression was checked using a numerical solution, which was useful for clarifying the influence of the mass term on the model dynamics. Fig. 3 shows that an increase in m results in a smaller N_e (i.e., the mass term is relevant at determining the length of inflation). We conclude that the new model provides a successful setup for the description of the primordial inflationary evolution of the Universe. A study of the first-order perturbations will be reported in a future paper illustrating the impact of the longitudinal mode of the field perturbations on the graph r vs. n_s and on the tachyonic instability at $\gamma < 2$. We expect an enhancement in the η parameter because of the mass term as happens in single-field inflation, and, consequently, an enhancement in n_s . Thus, when γ is small, but still in the stable region (and we will have to check how it changes when the mass is introduced), n_s would not continue being too low and, therefore, the pair $r - n_s$ would lie in the allowed parameter window.

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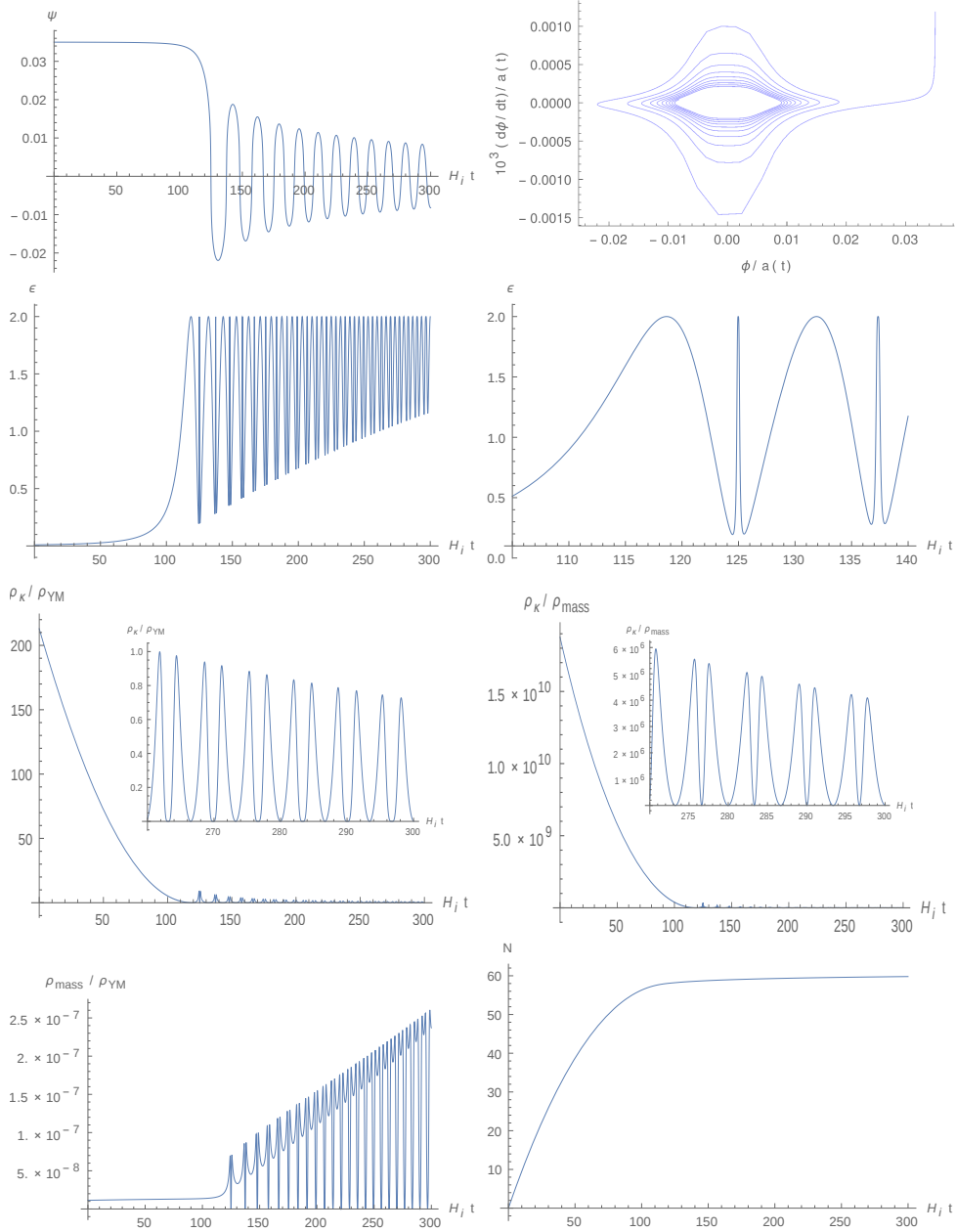
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Fig. 1. Time evolution of ψ , ε , ρ_κ/ρ_{YM} , ρ_κ/ρ_{mass} , ρ_{mass}/ρ_{YM} , and N , as well as $\dot{\phi}/a$ vs. ψ , found with the initial conditions $\psi_i = 0.035$; $\psi = -3.27 \times 10^{-9}$; $\kappa = 1.733 \times 10^{14}$; $g = 2.5 \times 10^{-3}$; $m = 1 \times 10^{-8}$; $H_i = 3.4 \times 10^{-5}$. These conditions produce the parameters $\gamma_i = 6.59$, $\varepsilon_i = 9.3 \times 10^{-3}$ and $\delta_i = 1 \times 10^{-4}$. Note that κ , m , H and ψ are given in m_{pl} units. The plot of ε on the right is actually a zoom of the plot on the left. Several other panels show insets with appropriate zoomed versions.

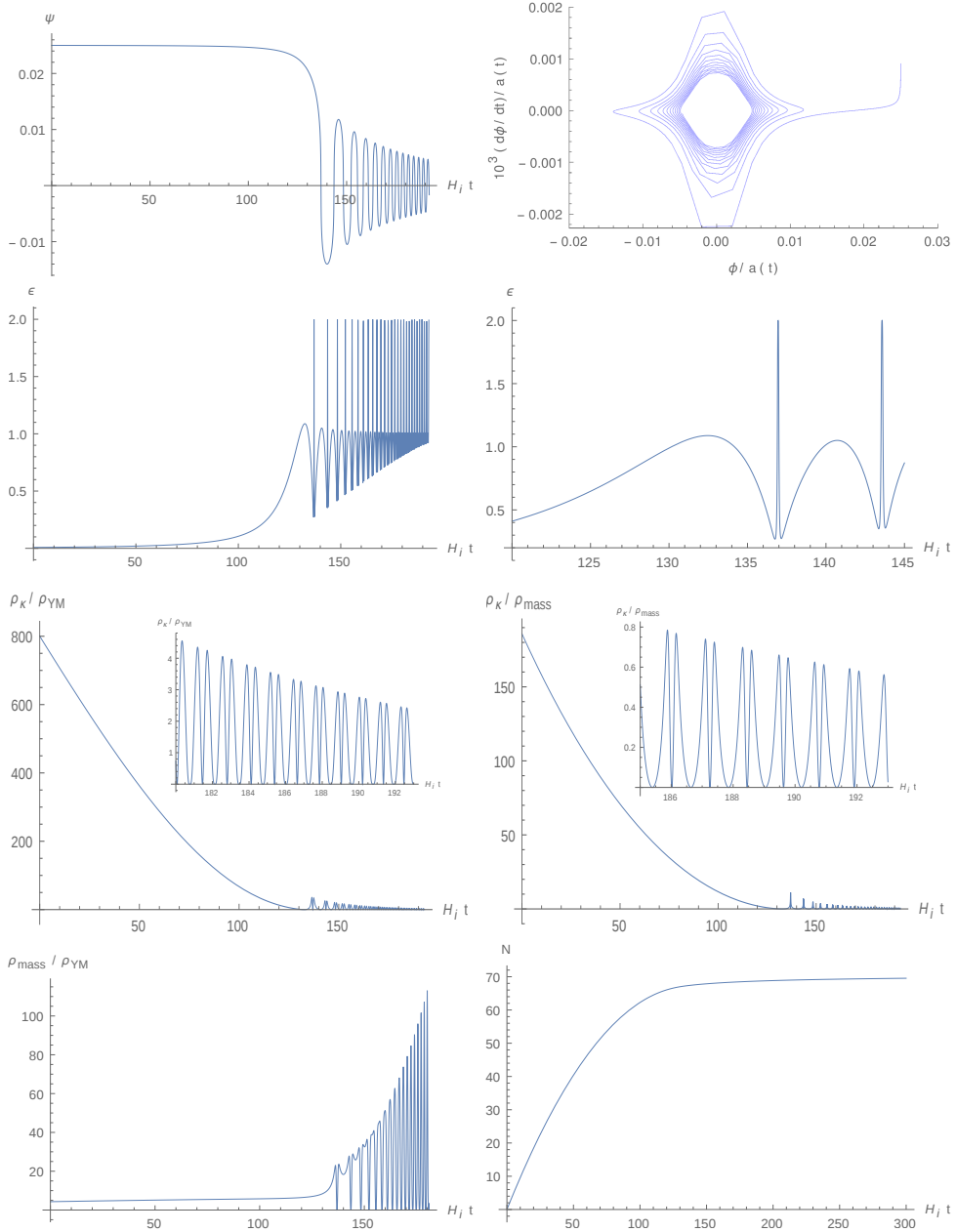


Fig. 2. Time evolution of ψ , ε , ρ_κ/ρ_{YM} , ρ_κ/ρ_{mass} , ρ_{mass}/ρ_{YM} , and N , as well as $\dot{\phi}/a$ vs. ψ , found with the initial conditions $\psi_i = 0.025$; $\dot{\psi} = 9.07 \times 10^{-9}$; $\kappa = 1.3 \times 10^{15}$; $g = 2.5 \times 10^{-3}$; $m = 1.5 \times 10^{-4}$; $H_i = 3.6 \times 10^{-5}$. These conditions produce the parameters $\gamma_i = 2.96$, $\varepsilon_i = 7.8 \times 10^{-3}$ and $\delta_i = -8 \times 10^{-4}$. Note that κ , m , H and ψ are given in m_{pl} units. The plot of ε on the right is actually a zoom of the plot on the left. Several other panels show insets with appropriate zoomed versions.