# Comments on "Note on varying speed of light theories"

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(Dated: February 1, 2008)

In a recent note Ellis criticizes varying speed of light theories on the grounds of a number of foundational issues. His reflections provide us with an opportunity to clarify some fundamental matters pertaining to these theories.

## I. GENERALITIES

In a recent publication [1], varying speed of light (VSL) theories were criticized for failing to address a number of foundational issues [2]. The criticism allegedly applies to the entirety of the extensive literature in the field, although the author says that he "will not comment on any specific such papers". Although an extensive review is cited [3], it is clear that perusal of the references cited therein was deemed unnecessary, including early pioneering papers [4, 5, 6, 7, 8, 9]. We feel, nonetheless, that Ellis's effort provides an opportunity to clarify a few foundational matters related to these theories.

Before embarking on an examination of the five points he has raised, we stress that VSL means by now a number of very distinct classes of theories, and one cannot generalize. Broadly speaking, these theories fall into two categories: those where c varies in space-time, and those where it varies with the energy scale. The former were motivated by cosmology, the latter by phenomenological quantum gravity. One may further categorize VSL theories according to the fashion in which they contradict (or adapt) Lorentz invariance. As an illustration we shall consider the following:

- Theories with soft breaking of Lorentz invariance (e.g. [4, 6]);
- Theories with hard breaking of Lorentz invariance (e.g. [7, 8]);
- Bimetric theories, for which there is a metric for matter and another for gravity (e.g. [9, 10, 11, 12, 14]);
- Non-linear or deformed special relativity, "DSR" (e.g. [16, 17, 18]).

This classification is far from extensive (for more examples, see [3]), but has the merit of avoiding the perils of sweeping generalizations.

One should then avoid getting tangled up in matters of terminology [1, 2]: of course VSL is an umbrella term for what could be variously called varying maximal attainable speed, varying speed of massless particles, varying speed of particles that follow the null cone, varying c as in the c that appears in many equations in modern physics, varying speed of photons, etc. People who use the term VSL are well aware of these distinctions, but they believe in the benefits of a simple terminology. Which c is it? One should examine the question for VSL theories on a case by case basis. Pedantic terminology can hardly be a substitute.

# II. THE SPEED OF LIGHT AND MEASUREMENT

Following Ellis [1], let us first consider c as the speed of the photon. Can c vary? Could such a variation be measured? As correctly pointed out by Ellis, within the current protocol for measuring time and space the answer is no. The unit of time is defined by an oscillating system or the frequency of an atomic transition, and the unit of space is defined in terms of the distance travelled by light in the unit of time. We therefore have a situation akin to saying that the speed of light is "one light-year per year", i.e. its constancy has become a tautology or a definition.

But then, within such a framework, neither can the constancy of the speed of light be falsified, thus losing its status as a scientific statement. The constancy of c can only be a scientifically sound concept if its variability is a possibility. The physical meaning of a constant c has to lie elsewhere, beyond the convenient but not necessary definitions of units. Instead of embarking on that search, Ellis goes on to say that only the variability of dimensionless constants can be discussed [39]. This is a misleading statement: what about varying G [23] and e theories, where G denotes Newton's gravitational "constant" and e [24, 25] the charge of an electron? Can the Hubble "constant" H not vary?

An historical analogy may be of use here. Consider the acceleration of gravity, little g. This was thought to be a constant in Galileo's time. One can almost hear the Ellis of the day stating that g cannot vary, because "it has units and can always be defined to be constant". The analogy to the present day relativity postulate that c is an absolute constant is applicable, for the most common method for measuring time in use in those days did place the constancy of g on the same footing as c nowadays. If one insists on defining the unit of time from the tick of a given pendulum clock, then the acceleration of gravity is indeed a constant by definition. Just like the modern speed of light c. And yet the Newtonian picture is that the acceleration of gravity varies, the status of constant being shifted to G, determining the proportionality between g and  $M/r^2$ . Why do we do it? Although this would never be phrased in this way in Newton's time, it is because we insist on naturally defining the unit of time by means of electromagnetic clocks. The dynamics tells us that it is a bad idea to define the unit of time by means of a pendulum clock, for the acceleration of gravity varies and that is a dimensional statement. Ultimately, the dynamics is rendered simpler in units for which g varies. Of course we could still define g to be a constant, even within the context of Newtonian (as opposed to Galilean) dynamics but the end result would be needlessly complicated.

Likewise, a VSL theory is one for which the dynamics is rendered simpler by choosing units for which c varies. One can always define units such that c = 1 (in the same way that one can always define units for which q = 1). but the dynamics naturally picks a set of simpler units, for which c varies. This can be shown by explicitly taking any of these theories and performing a change of units leaving c constant (this was done in one case [3]). The exercise is similar to changing units, so that the Hubble "constant" H is indeed a constant (the Friedmann equations get badly messed up), or the universe is not expanding and instead we are all contracting (we lose minimal coupling to gravity). A crucial aspect in VSL theories is the breaking of Lorentz invariance, as discussed further in the next Section. With such a glaring fundamental dynamical postulate, it would be unacceptable to define units of space and time the way they are currently defined. Ultimately, in theories in which some of the postulated constants of physics are assumed to vary, it is to be understood that these physical quantities were never *truly constant* throughout the age of the universe — they only appear to be constant to observers during specific epochs. So it is perfectly acceptable to consider them as variable quantities, such as the temperature of air, even though they are not dimensionless quantities.

How will then a hypothetical future physicist define units, if VSL were correct? Firstly, in some VSL theories the fine-structure constant  $\alpha$  varies in space and time. Concomitantly all the atomic transitions used to define the unit of time also vary, so that blindly using atomic clocks would be as impractical as using a pendulum clock on a mission to Mars. We could of course still use the same procedure, based on atomic transitions, but only as long as we used the varying- $\alpha$  theory itself to correct the clock, just like we could use a pendulum clock on the Moon as long as we used Newton's laws to correct for the lower acceleration. In other words, you can keep using electromagnetic clocks, but you will need to use the adopted theory to correct them.

The same will happen for the definition of the unit of space. It can be defined from the unit of time and the speed of the graviton (bimetric theories [9, 10, 11, 12, 13, 14, 15]) or the speed of low energy photons (DSR) [16, 17, 18]. In theories with explicit breaking of Lorentz

invariance, one can use a reference  $c_0$ , inferred from the physical (variable) c and possible effects proportional to  $\dot{c}/c$  (such as violations of energy conservation) predicted by a particular choice of theory. In other words, one could correct the physical c in order to infer a constant  $c_0$  precisely using the dynamics of the theory. The  $c_0$  could then be used to define the unit of length.

This is just one possible strategy for defining the units of space and time in a VSL world. It is based on adapting existing protocols and we suspect there might be more direct alternatives.

#### III. THE SPEED OF LIGHT AND THE LORENTZ GROUP

A crucial point complementing the discussion of the previous Section is the issue of the fate of Lorentz symmetry under VSL, issue 3 in [1]. Let c now represent what in special relativity is the invariant speed when transforming between inertial observers. Only this invariance of special relativity justifies the definitions of units currently in use. Before 1905 no one would have thought of defining units in the modern way. All theories under the name of VSL break Lorentz invariance in some way. For the ether theories in vogue before 1905, it was possible to define an absolute frame of reference with respect to the ether in which the speed of light is constant; in all other inertial frames of reference c would vary.

The question is then: If Lorentz invariance is broken, what happens to the speed of light? Given that Lorentz invariance follows from two postulates — (1) relativity of observers in inertial frames of reference and (2) constancy of the speed of light—it is clear that either or both of those principles must be violated. Thus VSL appears almost inevitably associated with the breaking of Lorentz invariance.

It is pointed out correctly by Ellis, that all VSL theories should then make concrete proposals for what happens to Lorentz symmetry. This is in fact at the core of the definition of any such theory. Bimetric theories, for example, replace Lorentz symmetry by two copies of the group SO(3,1) [9, 10, 11, 12, 13, 14, 15]. In the most general case, the dynamics is in the SO(3,1) dynamical matrix relating the two tetrads (*vierbeins*) representing the group.

In DSR theories the Lorentz group is replaced by a non-linear representation [16, 17, 18]. One example is the set of transformations between 4-momenta:

$$p'_{0} = \frac{\gamma (p_{0} - vp_{z})}{1 + l_{P}(\gamma - 1)p_{0} - l_{P}\gamma vp_{z}}$$
(1)

$$p'_{z} = \frac{\gamma (p_{z} - vp_{0})}{1 + l_{P}(\gamma - 1)p_{0} - l_{P}\gamma vp_{z}}$$
(2)

$$p'_{x} = \frac{p_{x}}{1 + l_{P}(\gamma - 1)p_{0} - l_{P}\gamma v p_{z}}$$
(3)

$$p'_{y} = \frac{p_{y}}{1 + l_{P}(\gamma - 1)p_{0} - l_{P}\gamma v p_{z}}$$
(4)

The choice of such non-linear representations renders the definition of duals highly non-trivial [18]. This has an impact upon the definition of the space-time metric (to be discussed in the next Section).

In the theories discussed in [6] a distinction is made between the coordinate  $x^0$  (with dimensions of space) and time (as measured by a minimally coupled clock). The distinction allows for a varying-*c* (controlled by a dynamical field), yet it is possible to preserve most of the features of Lorentz invariance which are operationally meaningful (such as those associated with the outcome of the Michelson-Morley experiment). Here the couplings to the matter action are essential in pegging down as the simplest a system of units for which *c* does vary. Local Lorentz invariance can also be violated either spontaneously [4, 5, 22] or dynamically (hard breaking) [7]: these are the only theories where both postulates of relativity are violated.

#### IV. THE SPEED OF LIGHT AND THE METRIC

Next comes the issue of general covariance. How does a varying c enter into the metric of a general space-time and how does it change under a coordinate transformation? It is claimed that a varying c is necessarily something that can be undone by a coordinate transformation. However this is not true: The c in VSL theories is never a coordinate speed of light. It is the physical speed of light measured by free-falling observers and cannot be undone by a coordinate transformation.

Nonetheless Ellis raises an important question. Each VSL theory must specify how diffeomorphism invariance or lack of it works, so that the statement that the varying c is invariant under coordinate transformations is true. Again a generalization is not possible, for each type of VSL theory has a different answer to this question.

For example in bimetric theories the diffeomorphism transformations of frames occur without changing the ratio,  $c_{\gamma}/c_g$ , between the speed of the photon and gravity. In the tetrad formalism of spontaneous violation of Lorentz invariance, the non-vanishing vacuum expectation value of a massive vector field  $\langle A_{\mu} \rangle$  also breaks diffeomorphism invariance as well as local Lorentz invariance [4, 5, 22]. Therefore, in this theory the initial speed of light in the early universe  $c_i$  and the currently measured speed of light  $c_0$ , resulting, say, from a first order phase transition in c(t) (where c is treated as an order parameter) cannot be undone by a transformation from t' to t during the period when the spontaneous violation of the Lorentz group SO(3, 1) and the group of diffeomorphism transformations takes place.

DSR theories, on the contrary, replace the usual metric by the so-called rainbow metric [17], i.e. the metric runs, and we have a different metric for each energy scale. This does not contradict the principle of relativity (and indeed it is implied by this principle) because a non-linear representation for the Lorentz group has been adopted. Diffeomorphism transformations change the metrics without changing the ratio between the speeds of photons with different frequencies. An explanation of the structure of these theories, their Einstein equations, and the impact on solutions such as black hole and cosmological solutions in presented in [17].

Yet another approach is that adopted in the theories of the type described in [6]. Here a  $x^0$  coordinate, with the units of length, should be used in the definition of metric and diffeomorphisms, but it is not to be converted into time in the usual way, as explained in [6]. The physical cappears in the relation between this coordinate and the physical time (measured by clocks defined by minimal coupling of a scalar field to the matter action) and is left invariant by diffeomorphisms (after the concept is suitably modified).

Finally there are VSL theories which crudely violate diffeomorphism invariance [7, 8], for which there is therefore never a question of a varying c being a coordinate artifact.

# V. THE SPEED OF LIGHT AND MAXWELL'S EQUATIONS

As pointed out by Ellis, any VSL theory purporting to represent a varying speed of the photon should contain a derivation of this feature from Maxwell's equations, or whatever equations replace them.

It is, however, the case that in most VSL constructions Maxwell's theory is derivative, the core being in the proposal of a new structure, typically dealing with the fate of Lorentz and diffeomorphism invariance (see the last two sections). Once this structure is defined, it is usually trivial to set up the matter action (for all spins, namely 1). The situation has parallels with setting up Maxwell's theory in General Relativity, where once minimal coupling is introduced there is not much to say about Maxwell's theory except that "derivatives become covariant derivatives". There are exceptions to this rule, however. In some early VSL theories, such as those of Drummond, Hathrell and Shore [19, 20] changes to the Maxwell action are in fact at the core of VSL, without any extra structure being introduced. Here non-minimal couplings to the metric leave the speed of the photon polarization dependent, a physical prediction that is derived from first principles.

However, this is not always the case. Take for example bimetric VSL theories. Once we define the double metric structure, Maxwell's action is just the same as usual, referred to the matter metric (as opposed to gravity's metric.) The usual derivations of the photon phase and group speeds can be made. The standard Maxwell electromagnetic field action is kept, while c emerges as a variable physical quantity, when referred to the speed of the graviton.

This is recognized in [1] with regards to bimetric theories, but it is also true in all other theories, for example in [6]. Here a  $x^0$  coordinate is used, but it is not to be converted into time in the usual way, as explained in [6]. Again the standard derivations follow, but when the speed of the photon is identified a new element crucial to the VSL theory makes this speed variable. The new structure derives precisely from consideration of the issues of the last 3 sections, namely the concept of minimally coupled clock as the simplest unit of time for that particular theory. Another possibility is that the Lorentz symmetry of Maxwell's action is spontaneously violated or violated by some form of dynamical symmetry breaking.

Other times, still, the new fundamental structure embodying VSL requires that Maxwell's action is modified. For example a field theory realization of DSR necessarily changes the action, either by invoking higher order derivatives or a non-commutative space-time. The speed of the photon is then color dependent, something that is derived from first principles from the new action. The new action, however, is not the fundamental element of the new theory; rather it follows from a deformation of the Lorentz group, in this case based on the choice of a non-linear representation (see [16, 17, 18, 35]).

Once again generalization is not possible; the issue raised must be addressed within the framework of each theory proposed.

# VI. THE SPEED OF LIGHT AND DYNAMICAL EQUATIONS

More generally Ellis correctly points out that in view of the fact that c enters into many modern equations of physics, one needs to consider the effects of a varying c on the whole of physics. It is not advisable to simply insert a varying speed of light, c = c(x), into equations before constructing a consistent dynamical theory of VSL.

But like with Maxwell's theory, this is usually dealt with by the definition of the new structure embodying a varying c and explaining the fate of diffeomorphism and Lorentz invariance. For example, in bimetric models [9, 10, 11, 12] there is never any ambiguity in what to do with any component of the matter action, electromagnetic or otherwise. Simply write the theory, as before, making sure you use the matter rather than the gravity metric. The same applies to the Einstein action, this time derived from a Ricci scalar made from the gravity metric. A derivation of Einstein's equations from first principles can then be achieved.

This is equally true in theories of the type defined in [6], where all one needs to do is make sure the connection between  $x^0$  and t is suitably modified. Implications for all sorts of laws (including second quantization) were discussed in [6]. In Einstein's field equations:

$$G_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{5}$$

where  $\kappa = 8\pi G/c^4$  a varying c could equally be interpreted as a varying G with c kept constant, either choice leading to different physical conclusions. This is explained carefully in [6], where the issue of minimal coupling to the matter action pegs down which system of units one should be using, and therefore whether we have a varying G or varying c. The implications for the standard tests of general relativity were discussed in [26].

The issue raised is indeed a very important one, as in many cases a varying c does not appear in all kinds of laws. Take for example the c that occurs in the fine structure constant,  $\alpha = e^2/\hbar c$ . This does not vary in bimetric theories but does so in theories of the type defined in [6].

Finally, according to our long experience with constructing consistent dynamical theories, it is as argued by Ellis advisable to construct a Lagrangian-Hamiltonian formulation of the theory at the outset and admit only those solutions to the equations following the application of a least action principle. This was the route followed in early VSL papers [4, 5] and in bimetric theories [9, 10, 11, 12, 13, 14, 15]. But it is not true that a theory has to be defined by a Lagrangian or a Hamiltonian.

Indeed for centuries theories were defined by the equations of motion, the Lagrangian formulation being regarded as a mathematical accessory. We now know that this is far from true. The Lagrangian formulation brings to the front the concept of symmetry and how it relates to conservation laws which are otherwise miraculous accidents. However we must not lose sight of the fact that experiment is at the root of physics more than formalism. At the heart of some VSL theories [7, 27] is the idea that the laws of physics may intrinsically evolve. Usually this is precluded by the fact that we could always define an invariant super-law explaining how the laws are changing. But not if we state that the time translational invariance of physics is broken as a result of a time variability in the speed of light.

This is far from metaphysical and has a very concrete implication: energy conservation is violated, and nothing, like the proposal of a new energy form, can be done to fix it. This is in fact the ingredient behind the solution of the flatness problem proposed in [7]. The Lagrangian formulation may be useful in bringing this to the front but in many cases it just so happens that it becomes very awkward. If energy is not conserved, then the Lagrangian formulation may not be the best way to set up the theory at all (this is a situation reminiscent of the description of friction forces).

Absence of a Lagrangian formulation is far from being a general feature of VSL, but we argue that it may be *the* point of those that attack the philosophical foundations of physics at its most fundamental level, introducing the concept of intrinsic evolution in the laws of physics.

#### VII. CONCLUSIONS

We are glad that reference [1] gave us the opportunity to clarify these important foundational issues. However we should finish by stressing that the success or lack thereof of VSL theories will likely depend on a variety of other issues.

For VSL theories describing early universe cosmology, it is natural to compare them to the widely accepted paradigm of inflation [28, 29, 30, 31]. Already in an early paper [4] and in a more recent work [22], it was shown that a calculation of the spectrum of primordial fluctuations can predict successfully a scale invariant Gaussian spectrum. Such a successful calculation was also performed in a bimetric model [13]. In these calculations the problem of causally connecting the fluctuations outside the horizon was achieved through the superluminal speed of light well beyond the horizon. This superluminal behavior of the speed of light performs the same role as the superluminal expansion of spacetime in inflation models.

We are clearly unable to directly observe whether the universe underwent an inflationary expansion or whether the speed of light was very large during a short period of time in the early universe, due to the opaqueness of the surface of last scattering. However, indirect consequences such as the predicted ratio of the tensor to scalar modes of the fluctuation spectrum may differ in VSL and inflationary model calculations, as well as the predictions for the spectrum of gravitational waves.

On the other hand, the VSL models can be criticized in the same way as many inflationary models, for fundamental theories of VSL and inflation are lacking. The question of whether these models truly solve the initial value problems of the Big Bang model is still debateable. Both VSL and inflationary models must contend with the problem of the Big Bang singularity at t = 0 and possible violations of the second law of thermodynamics. Although the possibility of a violation of Lorentz symmetry is now widely accepted by the physics community, there is as yet no measurable indication that this actually happens in nature. Such a violation of Lorentz invariance and possibly diffeomorphism invariance is part of the VSL paradigm, although it may only occur in the very early universe.

One reason why VSL may be ahead of inflation is that, at least in some of its guises, it may be directly testable (and here it is once again important not to generalize; the following does *not* apply to bimetric and DSR theories). It looks as if the observational evidence for a redshift dependence in the fine structure parameter  $\alpha$  is here to stay [36, 37, 38]. No such direct prediction graces the inflationary literature. Of course a varying alpha may be due to a variety of theories, namely, the more conservative varying *e* theories [32, 33]. But there are always distinct predictions differentiating these theories [34]. We feel that the future of VSL may be not in its confrontation with inflation but rather in the following:

- The execution of further high redshift spectroscopic observations, placing the Webb results on an even firmer footing.
- Atomic clock experiments, capable of direct detection of varying α over the space of a year, as suggested in [3].
- An array of supplementary experiments, required for distinguishing between VSL and other varying  $\alpha$  theories (e.g. [34]).

Herein may lie the future of VSL.

Acknowledgements: We thank George Ellis for providing us with the motivation to write this paper. The Perimeter Institute is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MEDT.

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