Axion monodromy formalism

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1 Introduction

The formalism of axion monodromy as a theory of inflation has been studied extensively. The possible UV completion as a string theory and apparent simplicity of the effective field theory makes this theory an attractive candidate to explain the dynamics of inflation. In this note, we highlight the mechanism that allows for a duality between a theory of a massive three form with a shift symmetry and a theory of a massive scalar field exhibiting a shift symmetry.

2 A theory of a massive scalar field with a shift symmetry.

We start by considering the Lagrangian of a massless pseudo scalar field with a shift symmetry:

$$\mathcal{L} = f^2 \left(\partial_\mu \phi\right)^2 \tag{1}$$

We want to create a mass gap while maintaining the shift symmetry and without introducing any additional degrees of freedom.

In this note we show that a theory of a massive three form with a shift symmetry is dual to a theory of a massive scalar field with a shift symmetry. The number of degrees of freedom is equal to one in both theories.

Let us start by considering a dynamical two-form $B_{\mu\nu}$. (In this dual picture, the pseudo scalar ϕ is replaced by a two-form $B_{\mu\nu}$).
The Lagrangian for B is:

$$\mathcal{L} = h_{\mu\nu\lambda} h^{\mu\nu\lambda} \tag{2}$$

where $h_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$

This theory exhibits a global shift symmetry:

$$B_{\mu\nu} \to B_{\mu\nu} + \Omega_{\mu\nu} \tag{3}$$

Now we want to promote that global shift symmetry to a gauge symmetry and create a mass gap for B without introducing additional degrees of freedom:

First let us review the dynamics of a theory of a four-form.
The Lagrangian for such a theory is:

$$\mathcal{L} = F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} \tag{4}$$

where,

$$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\rho\lambda]} \tag{5}$$

F is a totally antisymmetric tensor in 4 dimensions.
Hence :

$$F_{\mu\nu\lambda\rho} = q(x^a)\epsilon_{\mu\nu\rho\lambda} \tag{6}$$

The equation of motion for F is:

$$\partial^\mu F_{\mu\nu\lambda\rho} = 0 \implies q(x^a) = cst \tag{7}$$

This is a theory of constant with one degree of freedom.

Now, let us consider the following Lagrangian, where $A_{\mu\nu\lambda}$ couples to the external conserved current $J_{\mu\nu\lambda}$:

$$\mathcal{L} = F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} + A_{\mu\nu\lambda} J^{\mu\nu\lambda} \tag{8}$$

This Lagrangian is invariant under the gauge transformation:

$$A_{\mu\nu\rho} \to A_{\mu\nu\rho} + d_{[\mu}\Omega_{\nu\rho]} \tag{9}$$

It is worth noting that A contains no propagating degrees of freedom because of the gauge freedom.

By coupling the three-form A to the two-form B we effectively gauge the shift symmetry:

$$\mathcal{L} = -\frac{1}{48} F^2_{\mu\nu\lambda\rho} + \frac{m^2}{12}\left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^2 \tag{10}$$

We have successfully created a mass gap for B while maintaining the shift symmetry and without introducing any additional degree of freedom.

We now wish to promote $h_{\nu\lambda\rho}$ as a fundamental three-form by realizing the Bianchi identity:

$$\epsilon^{\mu\nu\lambda\rho}\partial_{[\mu}h_{\nu\lambda\rho]} = 0 \tag{11}$$

The Lagrangian for our theory becomes:

$$\mathcal{L} = -\frac{1}{48}F^2_{\mu\nu\lambda\rho} + \frac{m^2}{12}\left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^2 + \frac{m}{6}\phi\epsilon^{\mu\nu\lambda\rho}\partial_\mu h_{\nu\lambda\rho} \tag{12}$$

where the pseudo scalar ϕ appears as a Lagrange multiplier.

In the next part of this note, we wish to show how this theory is dual to a theory of a massive scalar field with a shift symmetry.

First, let us integrate out h :

$$\mathcal{L} = -\frac{1}{48}F^2_{\mu\nu\lambda\sigma} + \frac{1}{2}\left(\partial\phi\right) + \frac{m}{24}\phi\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho} \tag{13}$$

We then enforce $F_{\mu\nu\lambda\sigma} = 4\partial_{[\mu}A_{\nu\lambda\rho]}$ by introducing the Lagrange multiplier Q:

$$\mathcal{L} = -\frac{1}{48}F^2_{\mu\nu\lambda\sigma} + \frac{1}{2}\left(\partial\phi\right)^2 + \frac{m\phi + Q}{24}\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu\lambda\sigma} - \frac{Q}{6}\epsilon_{\mu\nu\lambda\sigma}\partial^\mu A^{\nu\lambda\rho} \tag{14}$$

The equation of motion for F give us:

$$F_{\mu\nu\lambda\sigma} = \left(m\phi + Q\right)\epsilon_{\mu\nu\lambda\sigma} \tag{15}$$

We can integrate out F and we get:

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}\left(m\phi + Q\right)^2 + \frac{1}{6}\epsilon_{\mu\nu\lambda\sigma}\left(\partial^\mu Q\right)A^{\nu\lambda\sigma} \tag{16}$$

This is a theory of a massive scalar field with a shift symmetry:

$$\phi \to \phi + \phi_0 \tag{17}$$

$$Q \to Q - \frac{\phi_0}{\mu} \tag{18}$$

References:

Gia Dvali (2005). *Three-Form Gauging of axion Symmetries and Gravity.*
N.Kaloper, M.König, A.Lawrence, and J.H.C. Scargill (2020). *On Hybrid Monodromy Inflation — Hic Sunt Dracones —.*
E. Silverstein and A. Westphal (2008). *Monodromy in the CMB: Gravity Waves and String Inflation.*