Quantum cosmology and eternal inflation

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This contribution consists of two parts. In the first part, I review the tunneling approach to quantum cosmology and comment on the alternative approaches. In the second part, I discuss the relation between quantum cosmology and eternal inflation. In particular, I discuss whether or not we need quantum cosmology in the light of eternal inflation, and whether or not quantum cosmology makes any testable predictions.

I. INTRODUCTION

Stephen and I at times disagreed about minor things, like the sign in the wave function of the universe, $\psi \sim e^{\pm S}$. But for me Stephen has always been a major source of inspiration. I will mention just one episode, when I talked to Stephen at a conference and was telling him why I thought my wave function was better than his. To which Stephen replied: "You do not have a wave function."

Talking to Stephen is a little like talking to the Oracle of Delphi: you are likely to hear something as profound as it is difficult to decipher. But in that particular case I knew immediately what he meant. He was pointing to the lack of a general definition for the tunneling wave function, which at the time was defined only in a simple model. My talk was scheduled for the next morning, so I spent the night working out a general definition and rewriting my transparencies. I ended up giving a very different talk from what was initially intended. I can thus say with some justification that Stephen contributed to the development of the tunneling approach, although he may not be very pleased with the result.

My contribution here will consist of two parts. In the first part, I will review the tunneling approach to quantum cosmology and will briefly comment on the alternative approaches. In the second part, I will discuss the relation between quantum cosmology and eternal inflation. After a brief review of eternal inflation, I will discuss whether or not we need quantum cosmology in the light of eternal inflation, and then whether or not quantum cosmology makes any testable predictions.

II. QUANTUM COSMOLOGY

If the cosmological evolution is followed back in time, we are driven to the initial singularity where the classical equations of general relativity break down. There was initially some hope that the singularity was a pathological feature of the highly symmetric Friedmann solutions, but this hope evaporated when Stephen and Roger Penrose proved their famous singularity theorems. There was no escape, and cosmologists had to face the problem of the origin of the universe.

Many people suspected that in order to understand what actually happened in the beginning, we should treat the universe quantum-mechanically and describe it by a wave function rather than by a classical spacetime. This quantum approach to cosmology was initiated by DeWitt [1] and Misner [2], and after a somewhat slow start received wide recognition in the last two decades or so. The picture that has emerged from this line of development [3–9] is that a small closed universe can spontaneously nucleate out of nothing, where by 'nothing' I mean a state with no classical space and time. The cosmological wave function can be used to calculate the probability distribution for the initial configurations of the nucleating universes. Once the universe nucleates, it is expected to go through a period of inflation, driven by the energy of a false vacuum. The vacuum energy is eventually thermalized, inflation ends, and from then on the universe follows the standard hot cosmological scenario. Inflation is a necessary ingredient in this kind of scheme, since it gives the only way to get from the tiny nucleated universe to the large universe we live in today.

The wave function of the universe ψ satisfies the Wheeler-DeWitt equation,

$$\mathcal{H}\psi = 0, \tag{1}$$

which is analogous to the Schrodinger equation in ordinary quantum mechanics. To solve this equation, one has to specify some boundary conditions for ψ. In quantum mechanics, the boundary conditions are determined by the physical setup external to the system. But since there is nothing external to the universe, it appears that boundary conditions for the wave function of the universe should be postulated as an independent physical law. The possible form of this law has been debated for nearly 20 years, and one can hope that it will eventually be derived from the fundamental theory.

Presently, there are at least three proposals on the table: the Hartle-Hawking wave function [5,10], the Linde wave function [6], and the tunneling wave function [8,11].

III. THE TUNNELING WAVE FUNCTION

To introduce the tunneling wave function, let us consider a very simple model of a closed Friedmann-Robertson-Walker universe filled with a vacuum of constant energy density ρ_v and some radiation. The total energy density of the universe is given by

$$\rho = \rho_v + \epsilon/a^4, \tag{2}$$

where a is the scale factor and ϵ is a constant characterizing the amount of radiation. The evolution equation for a can be written as

$$p^2 + a^2 - a^4/a_0^2 = \epsilon. \tag{3}$$

Here, $p = -a\dot{a}$ is the momentum conjugate to a and $a_0 = (3/4)\rho_v^{-1/2}$.
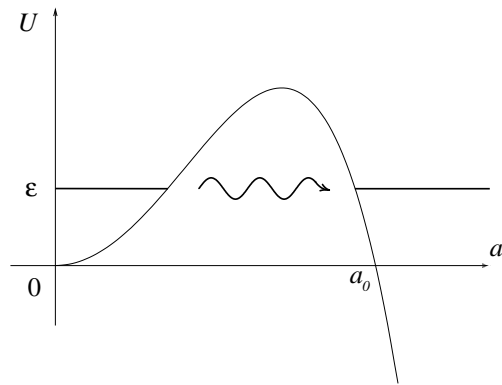
FIG. 1. The potential for the scale factor in Eq.(3). Instead of recollapsing, the universe can tunnel through the potential barrier to the regime of unbounded expansion.

Eq.(3) is identical to that for a "particle" of energy ϵ moving in a potential $U(a) = a^2 - a^4/a_0^2$. For sufficiently small ϵ, there are two types of classical trajectories. The universe can start at $a = 0$, expand to a maximum radius a_1 and then recollapse. Alternatively, it can contract from infinite size, bounce at a minimum radius a_2 and then re-expand (see Fig. 1). But in quantum cosmology there is yet another possibility. Instead of recollapsing, the universe can tunnel through the potential barrier to the regime of unbounded expansion. The semiclassical tunneling probability can be estimated as

$$\mathcal{P} \sim \exp\left(-2\int_{a_1}^{a_2} |p(a)|da\right).\tag{4}$$

It is interesting that this probability does not vanish in the limit of $\epsilon \to 0$, when there is no radiation and the size of the initial universe shrinks to zero. We then have tunneling from *nothing* to a closed universe of a finite radius a_0; the corresponding probability is

$$\mathcal{P} \sim \exp\left(-2\int_{0}^{a_0} |p(a)|da\right) = \exp\left(-\frac{3}{8\rho_v}\right).\tag{5}$$

The tunneling approach to quantum cosmology assumes that our universe originated in a tunneling event of this kind. Once it nucleates, the universe immediately begins a de Sitter inflationary expansion.

The Wheeler-DeWitt equation for our simple model can be obtained by replacing the momentum p in (3) by a differential operator[1], $p \to -id/da$,

$$\left(\frac{d^2}{da^2} - a^2 + \frac{a^4}{a_0^2}\right)\psi(a) = 0.\tag{6}$$

This equation has outgoing and ingoing wave solutions corresponding to expanding and contracting universes in the classically allowed range $a > a_0$ and exponentially growing and

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[1]Here and below I disregard the ambiguity associated with the ordering of non-commuting factors a and d/da. This ambiguity is unimportant in the semiclassical domain, which we shall be mainly concerned with in this paper.

decaying solutions in the classically forbidden range $0 < a < a_0$. The boundary condition that selects the tunneling wave function requires that ψ should include only an outgouing wave at $a \to \infty$. The under-barrier wave function is then a linear combination of the growing and decaying solutions. The two solutions have comparable magnitudes near the classical turning point, $a = a_0$, but the decaying solution dominates in the rest of the under-barrier region.

The tunneling probability can also be expressed in the language of instantons. The nucleated universe after tunneling is described by de Sitter space, and the under-barrier evolution can be semiclassically represented by the Euclideanized de Sitter space. This de Sitter instanton has the geometry of a four-sphere. By matching it with the Lorentzian de Sitter at $a = a_0$ we can symbolically represent [4] the origin of the universe as in Fig.2. For 'normal' quantum tunneling (without gravity), the tunneling probability $\mathcal{P}$ is proportional to $\exp(-S_E)$, where S_E is the instanton action. In our case,

$$S_E = \int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} + \rho_v \right) = -2\rho_v \Omega_4 = -3/8G^2\rho_v, \qquad (7)$$

where $R = 32\pi G\rho_v$ is the scalar curvature, and $\Omega_4 = (4\pi^2/3)a_0^4$ is the volume of the four-sphere. Hence, I concluded in Ref. [4] that $\mathcal{P} \propto \exp(3/8G^2\rho_v)$.
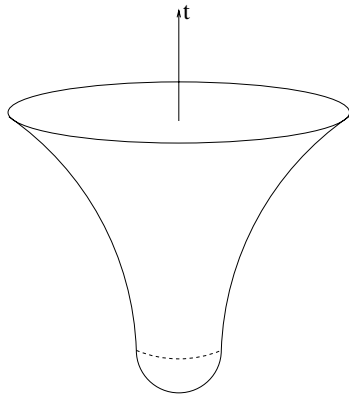

FIG. 2. A schematic representation of the birth of inflationary universe.

I resist the temptation to call this 'the greatest mistake of my life', but I did change my mind on this issue [8]. I now believe that the correct answer is given by Eq.(5). In theories with gravity, the Euclidean action is not positive-definite and the naive answer no longer applies. Similar conclusions were reached independently by Linde [6], Rubakov [7] and by Zeldovich and Starobinsky [9]. But the story does not end there. Not everybody believes that my first answer was a mistake. In fact, Stephen and his collaborators believe that I got it right the first time around and that it is now that I am making a mistake. I shall return to this ongoing debate later in this paper.

In the general case, the wave function of the universe is defined on superspace, which is the space of all 3-dimensional geometries and matter field configurations, $\psi[g_{ij}(\mathbf{x}), \varphi(\mathbf{x})]$, where g_{ij} is the 3-metric, and matter fields are represented by a single field φ. The tunneling boundary condition can be extended to full superspace by requiring that ψ should include only outgoing waves at the boundary of superspace, except the part of the boundary corresponding to vanishing 3-geometries (see [11,12] for more details).

Alternatively, the tunneling wave function can be defined as a path integral

$$\psi_T(g, \varphi) = \int_{\emptyset}^{(g,\varphi)} e^{iS}, \tag{8}$$

where the integration is over paths interpolating between a vanishing 3-geometry $\emptyset$ ('nothing') and (g, φ). I argued in [12] that this definition is equivalent to the tunneling boundary condition in a wide class of models.

Note that according to the definition (8), ψ_T is strictly not a wave function, but rather a propagator

$$\psi_T(g, \varphi) = K(g, \varphi|\emptyset), \tag{9}$$

where $K(g, \varphi|g', \varphi')$ is given by the path integral (8) taken over Lorentzian histories interpolating between (g', φ') and (g, φ). One expects, therefore, that ψ_T should generally be singular at $g = \emptyset$. In simple minisuperspace models this singularity was noted in my paper [11] and also by Kontoleon and Wiltshire [13] (who regarded it as an undesirable feature of ψ_T).

I should also mention an interesting issue raised by Rubakov [7,14] who argues that cosmic tunneling of the type illustrated in Fig. 1 results in a catastrophic particle production and in a breakdown of the semiclassical approximation. This conclusion is reached using a Euclidean formalism in which particles are defined by an instantaneous diagonalization of the Hamiltonian. This is a rather unconventional approach and I am not convinced that 'particles' defined in this way are the same particles that observers will detect when the universe emerges from under the barrier. If Rubakov is right, then the tunneling wave function cannot be obtained as a limit of tunneling from a small initial universe in a generic quantum state when the size of that universe goes to zero. (There is no dispute that for a particular quantum state, corresponding to the de Sitter invariant Bunch-Davis vacuum, there is no catastrophic particle production and the semiclassical approximation is well justified [15,14].) This issue requires further study.

At present, the general definitions of the tunneling wave function remain largely formal, since we do not know how to solve the Wheeler-DeWitt equation[2] or how to calculate the path integral (8), except for simple models (and small perturbations about them), or in the semiclassical limit. A promising recent development is the work by Ambjorn, Jurkiewicz and Loll [16] who developed a Lorentzian path integral approach to quantum gravity. It would be interesting to see an application of this approach to the tunneling problem.

IV. ALTERNATIVE PROPOSALS FOR THE WAVE FUNCTION

I shall now comment on the other proposals for the wave function of the universe.

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[2]Note that the Wheeler-DeWitt equation applies assuming that the topology of the universe is fixed. Possible modification of this equation accounting for topology change have been discussed in [12]. Here, I have disregarded topology change, assuming the simplest S_3 topology of the universe. For a discussion of topology-changing processes in quantum gravity, see Fay Dowker's contribution to this volume.

A. The DeWitt wave function

I should first mention what I believe was the first such proposal, made by DeWitt in his 1967 paper [1]. DeWitt suggested that the wave function should vanish for the vanishing scale factor,

$$\psi_{DW}(a = 0) = 0. \tag{10}$$

The motivation for this is that $a = 0$ corresponds to the cosmological singularity, so (10) says that the probability for the singularity to occur is zero.

The boundary condition (10) is easy to satisfy in a minisuperspace model with a single degree of freedom, but in more general models it tends to give an identically vanishing wave function. No generalizations of the DeWitt boundary condition (10) have yet been proposed.

B. The Hartle-Hawking wave function

The Hartle-Hawking wave function is expressed as a path integral over compact Euclidean geometries bounded by a given 3-geometry g,

$$\psi_{HH}(g, \varphi) = \int^{(g,\varphi)} e^{-S_E}. \tag{11}$$

The Euclidean rotation of the time axis, $t \to i\tau$, is often used in quantum field theory because it improves the convergence of the path integrals. However, in quantum gravity the situation is the opposite. The gravitational part of the Euclidean action S_E is unbounded from below, and the integral (11) is badly divergent. One can attempt to fix the problem by additional contour rotations, extending the path integral to complex metrics. However, the space of complex metrics is very large, and no obvious choice of integration contour suggests itself as the preferred one [17].

In practice, one assumes that the dominant contribution to the path integral is given by the stationary points of the action and evaluates ψ_{HH} simply as $\psi_{HH} \sim e^{-S_E}$. For our simple model, $S_E = -3/8\rho_v$ and the nucleation probability is $\mathcal{P} \sim \exp(+3/8\rho_v)$. The wave function $\psi_{HH}(a)$ for this model has only the growing solution under the barrier and a superposition of ingoing and outgoing waves with equal amplitudes in the classically allowed region. This wave function appears to describe a contracting and re-expanding universe.

It is sometimes argued [10,14] that changing expansion to contraction does not do anything, as long as the directions of all other physical processes are also reversed. So if the ingoing and outgoing waves are CPT conjugates of one another, they may both correspond to expanding universes, provided that the internal direction of time is determined as that in which the entropy increases. I would like to note that this issue is clarified in models where the universe is described by a brane propagating in an infinite higher-dimensional bulk space [18–20]. The nucleation of the universe then appears as bubble nucleation from the point of view of the bulk observer. In such models, there is an extrinsic bulk time variable, and the interpretation of incoming and outgoing waves is unambiguous [21]. The tunneling wave function appears to be the only correct choice in this case.

C. The Linde wave function

Linde suggested that the wave function of the universe is given by a Euclidean path integral like (11), but with the Euclidean time rotation performed in the opposite sense, $t \to +i\tau$, yielding

$$\psi_L = \int^{(g,\varphi)} e^{+S_E}. \tag{12}$$

For our simple model, this wave function gives the same nucleation probability (23) as the tunneling wave function.

The problem with this proposal is that the Euclidean action is also unbounded from above and, once again, the path integral is divergent. If one regards Eq.(12) as a general definition that applies beyond the simple model (something that Linde himself never suggested), then the divergence is even more disastrous than in the Hartle-Hawking case, because now all integrations over matter fields and over inhomogeneous modes of the metric are divergent. This problem of the (extended) Linde's wave function makes it an easy target, and I suspect it is for this reason that Stephen likes to confuse ψ_L and ψ_T and refers to both of them as "the tunneling wave function". In fact, the two wave functions are quite different, even in the simplest model [22]. The Linde wave function includes only the decaying solution under the barrier and a superposition of ingoing and outgoing modes with equal amplitudes outside the barrier.

Using Stephen's expression, I think it would be fair to say that at present none of us "has a wave function". All four proposals are well defined only in simple minisuperspace models or in the semiclassical approximation. So they are to be regarded only as prototypes for future work in this area, and not as well defined mathematical objects.

V. SEMICLASSICAL PROBABILITIES

Quantum cosmology is based on quantum gravity and shares all of its problems. In addition, it has some extra problems which arise when one tries to quantize a closed universe. The first problem stems from the fact that ψ is independent of time. This can be understood [1] in the sense that the wave function of the universe should describe everything, including the clocks which show time. In other words, time should be defined intrinsically in terms of the geometric or matter variables. However, no general prescription has yet been found that would give a function $t(g_{ij}, \varphi)$ that would be, in some sense, monotonic.

A related problem is the definition of probability. Given a wave function ψ, how can we calculate probabilities? There was some debate about this in the 1980's, but now it seems that the only reasonable definition that we have is in terms of the conserved current of the Wheeler-DeWitt equation [1,2,23]. The Wheeler-DeWitt equation can be symbolically written in the form

$$(\nabla^2 - U)\psi = 0, \tag{13}$$

which is similar to the Klein-Gordon equation. Here, ∇^2 is the superspace Laplacian and the 'potential' U is a functional of g_{ij} and φ. (We shall not need explicit forms of ∇^2 and U.) This equation has a conserved current

$$J = i(\psi^* \nabla \psi - \psi \nabla \psi^*), \qquad \nabla \cdot J = 0. \tag{14}$$

The conservation is a useful property, since we want probability to be conserved. But one runs into the same problem as with the Klein-Gordon equation: the probability defined using (14) is not positive-definite. Although we do not know how to solve these problems in general, they can both be solved in the semiclassical domain. In fact, it is possible that this is all we need.

Let us consider the situation when some of the variables $\{c\}$ describing the universe behave classically, while the rest of the variables $\{q\}$ must be treated quantum-mechanically. Then the wave function of the universe can be written as a superposition

$$\psi = \sum_k A_k(c) e^{iS_k(c)} \chi_k(c,q) \equiv \sum_k \psi_k^{(c)} \chi_k, \tag{15}$$

where the classical variables are described by the WKB wave functions $\psi_k^{(c)} = A_k e^{iS_k}$. In the semiclassical regime, ∇S is large, and substitution of (15) into the Wheeler-DeWitt equation (13) yields the Hamilton-Jacobi equation for $S(c)$,

$$\nabla S \cdot \nabla S + U = 0. \tag{16}$$

The summation in (15) is over different solutions of this equation. Each solution of (16) is a classical action describing a congruence of classical trajectories (which are essentially the gradient curves of S). Hence, a semiclassical wave function $\psi_c = A e^{iS}$ describes an ensemble of classical universes evolving along the trajectories of $S(c)$. A probability distribution for these trajectories can be obtained using the conserved current (14). Since the variables c behave classically, these probabilities do not change in the course of evolution and can be thought of as probabilities for various initial conditions. The time variable t can be defined as any monotonic parameter along the trajectories, and it can be shown [1,23] that in this case the corresponding component of the current J is non-negative, $J_t \geq 0$. Moreover, one finds [24–26] that the 'quantum' wave function χ satisfies the usual Schrodinger equation,

$$i\partial\chi/\partial t = \mathcal{H}_\chi \chi \tag{17}$$

with an appropriate Hamiltonian $\mathcal{H}_\chi$. Hence, all the familiar physics is recovered in the semiclassical regime.

This semiclassical interpretation of the wave function ψ is valid to the extent that the WKB approximation for ψ_c is justified and the interference between different terms in (15) can be neglected. Otherwise, time and probability cannot be defined, suggesting that the wave function has no meaningful interpretation. In a universe where no object behaves classically (that is, predictably), no clocks can be constructed, no measurements can be made, and there is nothing to interpret. It would be interesting, however, to investigate the effects of small corrections to the WKB form of the wave functions and of non-vanishing interference.

VI. COMPARING DIFFERENT WAVE FUNCTIONS

To see what kind of cosmological predictions can be obtained from different wave functions, one needs to consider an extension of the minisuperspace model (6). Instead of a

constant vacuum energy ρ_v, one introduces an inflaton field φ with a potential $V(\varphi)$. The Wheeler-DeWitt equation for this two-dimensional model can be approximately solved assuming that $V(\varphi)$ is a slowly-varying function and is well below the Planck density.

After an appropriate rescaling of the scale factor a and the scalar field φ, the Wheeler-DeWitt equation can be written as

$$\left[\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\frac{\partial^2}{\partial \varphi^2} - U(a, \varphi)\right]\psi(a, \varphi) = 0, \tag{18}$$

where

$$U(a, \varphi) = a^2[1 - a^2 V(\varphi)]. \tag{19}$$

With the above assumptions, one finds [27] that Hartle-Hawking and tunneling solutions of this equation are given essentially by the same expressions as for the simple model (6), but with ρ_v replaced by $V(\varphi)$. The only difference is that the wave function is multiplied by a factor $C(\varphi)$, such that $\psi(a, \varphi)$ becomes φ-independent in the limit $a \to 0$ (with $|\varphi| < \infty$).

The initial state of the nucleating universe in this model is characterized by the value of the scalar field φ, with the initial value of a given by $a_0(\varphi) = V^{-1/2}(\varphi)$. The probability distribution for φ can be found using the conserved current (14). For the tunneling wave function one finds

$$\mathcal{P}_T(\varphi) \propto \exp\left(-\frac{3}{8G^2 V(\varphi)}\right), \tag{20}$$

This is the same as Eq.(5) with ρ_v replaced by $V(\varphi)$.

Eq.(20) can be interpreted as the probability distribution for the initial values of φ in the ensemble of nucleated universes. The highest probability is obtained for the largest values of $V(\varphi)$ (and smallest initial size). Thus, the tunneling wave function 'predicts' that the universe is most likely to nucleate with the largest possible vacuum energy. This is just the right initial condition for inflation. The high vacuum energy drives the inflationary expansion, while the field φ gradually 'rolls down' the potential hill, and ends up at the minimum with $V(\varphi) \approx 0$, where we are now.

The Hartle-Hawking wave function gives a similar distribution, but with a crucial difference in sign,

$$\mathcal{P}_H(\varphi) \propto \exp\left(+\frac{3}{8G^2 V(\varphi)}\right). \tag{21}$$

This is peaked at the smallest values of $V(\varphi)$. Thus the Hartle-Hawking wave function tends to predict initial conditions that disfavor inflation. There has been much discussion of this point in the literature [27–31], but as I will explain in Section 8, the eternal nature of inflation makes this distinction between the wave functions rather irrelevant.

VII. DO WE NEED QUANTUM COSMOLOGY?

The status of quantum cosmology is closely related to that of eternal inflation, and I am going to discuss this relation in the rest of the paper.

A very generic feature of inflation is its future-eternal character [32–34]. The evolution of the inflaton field φ is influenced by quantum fluctuations, and as a result thermalization does not occur simultaneously in different parts of the universe. One finds that, at any time, the universe consists of post-inflationary, thermalized regions embedded in an inflating background. Thermalized regions grow by annexing adjacent inflating regions, and new thermalized regions are constantly formed in the midst of the inflating sea. At the same time, the inflating regions expand and their combined volume grows exponentially with time. It can be shown that the inflating regions forms a self-similar fractal of dimension somewhat smaller than 3 [35,36].

Given this picture, it is natural to ask if the universe could also be past-eternal. If it could, we would have a model of an infinite, eternally inflating universe without a beginning. We would then need no initial or boundary conditions for the universe, and quantum cosmology would arguably be unnecesary.

This possibility was discussed in the early days of inflation, but it was soon realized [37,38] that the idea could not be implemented in the simplest model in which the inflating universe is described by an exact de Sitter space. The reason is that in the full de Sitter space, exponential expansion is preceded by an exponential contraction. If thermalized regions were allowed to form all the way to the past infinity, they would rapidly fill the space, and the whole universe would be thermalized before the inflationary expansion could begin.

More recently, general theorems were proved [39], using the global techniques of Penrose and Hawking, where it was shown that inflationary spacetimes are geodesically incomplete to the past. However, it is now believed [40,41] that one of the key assumptions made in these theorems, the weak energy condition, is likely to be violated by quantum fluctuations in the inflating parts of the universe. This appears to open the door again to the possibility that inflation, by itself, can eliminate the need for initial conditions. Now I would like to report on a new theorem, proved in collaboration with Arvind Borde and Alan Guth [42], which appears to close that door completely. (For a more detailed discussion of the theorem and an outline of the proof, see Alan Guth's contribution to this volume.)

The theorem assumes that (i) the spacetime is globally expanding and (ii) that the expansion rate is bounded below by a positive constant,

$$H > H_{min} \geq 0. \tag{22}$$

The theorem states that any spacetime with these properties is past geodesically incomplete. Both of the above conditions need to be spelled out.

It is important to realize that expansion and contraction are not local properties of spacetime. Rather, they refer to the relative motion of comoving observers filling the spacetime, with observers being described by a congruence of timelike geodesics. The global expansion condition requires that the spacetime can be filled by an expanding congruence of geodesics.[3]

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[3]This formulation of the global expansion condition may be somewhat too restrictive. Congruences of geodesics tend to develop caustics and often cannot be globally defined. A weaker form of the condition, which is still sufficient for the proof of the theorem, requires that an expanding con-

This condition is meant to exclude spacetimes like de Sitter space (which can be said to have a globally contracting phase).

The Hubble expansion rate H is defined as usual, as the relative velocity divided by the distance, with all quantities measured in the local comoving frame. Since we do not assume any symmetries of spacetime, the expansion rates are generally different at different places and in different directions. The bound (22) is assumed to be satisfied at all points and in all directions. This is a very reasonable requirement in the inflating region of spacetime.

The theorem is straightforwardly extended to higher-dimensional models. For example, in Bucher's model [19], brane worlds are created in collisions of bubbles nucleating in an inflating higher-dimensional bulk spacetime. Our theorem implies that the inflating bulk cannot be past-eternal. Another example is the cyclic brane world model of Steinhardt and Turok [43]. One of the two branes in this model is globally expanding and thus should have a past boundary.

This is good news for quantum cosmology. It follows from the theorem that the inflating region has a boundary in the past, and some new physics (other than inflation) is necessary to determine the conditions at that boundary. Quantum cosmology is the prime candidate for this role. The picture suggested by quantum cosmology is that the universe starts as a small, closed 3-geometry and immediately enters the regime of eternal inflation, with new thermalized regions being constantly formed. In this picture, the universe has a beginning, but it has no end.

VIII. IS QUANTUM COSMOLOGY TESTABLE?

There are also some bad news. In the course of eternal inflation, the universe quickly forgets its initial conditions. Since the number of thermalized regions to be formed in an eternally inflating universe is unbounded, a typical observer is removed arbitrarily far from the beginning, and the memory of the initial state is completely erased. This implies that any predictions that quantum cosmology could make about the initial state of the universe cannot be tested observationally. All three proposals for the wave function of the universe are therefore in equally good agreement with observations, as well as a wide class of other wave functions – as long as they give a non-vanishing probability for eternal inflation to start [44].[4]

The only case that requires special consideration is when there are some constants of nature, α_j, which are constant within individual universes, but can take different values in different universes of the ensemble. (One example is the cosmological constant in models where it is determined by a four-form field.) In this case, the memory of the initial state

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gruence satisfying (22) can be continuously defined along a past-directed timelike or null geodesic. Members of the congruence may cross or focus away from that geodesic.

[4]There is, arguably, a much wider class of wave functions which describe highly excited states of the fields, but these will generally exhibit no quasiclassical behavior and will not, therefore, allow for the existence of observers [45].

is never erased completely, since the values of α_j are always equal to their initial values. One might hope that probabilistic predictions for the values of α_j could be derived from quantum cosmology and could, in principle, be tested observationally. Unfortunately, this prospect does not look very promising either.

Quantum cosmology can give us the probability distribution $P_{nucl}(\alpha)$ for a universe to nucleate with given values of α_j. In other words, this is the probability for a universe arbitrarily picked in the ensemble to have this set of values. To get the probability of observing these values, it should be multiplied by the average number of independent observers, $N(\alpha)$, that will evolve in such a universe [46],

$$P_{obs}(\alpha) \propto P_{nucl}(\alpha)N(\alpha). \tag{23}$$

The number of observers in each eternally inflating universe grows exponentially with time,

$$N(\alpha;t) = B(\alpha)\exp[\chi(\alpha)t]. \tag{24}$$

The prefactor $B(\alpha)$ depends on the details of the biochemical processes, and at present we have no idea how to calculate it. But the rate of growth χ is determined by the growth of thermalized volume and can be found as an eigenvalue of the Fokker-Planck operator, as discussed in Refs. [34,44]. It is independent of biology, but generally depends on α_j. This suggests that the most probable values of α_j should be the ones maximizing the expansion rate $\chi(\alpha)$. As time goes on, the number of observers in universes with this preferred set of α_j gets larger than the competition by an arbitrarily large factor. In the limit $t \to \infty$, this set has a 100% probability, while the probability of any other values is zero,

$$P_{obs}(\alpha) \propto \delta(\alpha - \alpha_*), \tag{25}$$

where $\chi(\alpha_*) = \max$. We thus see that the probability of observing the constants α_j is determined entirely by the physics of eternal inflation and is independent of the nucleation probability $P_{nucl}(\alpha)$ – as long as $P_{nucl}(\alpha_*) \neq 0$.

The situation I have just described is somewhat clouded by the problem of gauge-dependence. The problem is that the expansion rate $\chi(\alpha)$ and the values of α_j maximizing this rate depend on one's choice of the time coordinate t [44]. Time in General Relativity is an arbitrary label, and this gauge dependence casts doubt on the meaningfulness of the probability (25). We now have some proposals on how this problem can be resolved in a single eternally inflating universe [47,48], but comparing the numbers of observers in disconnected eternally inflating universes still remains a challenge.

I can think of two possible responses to this situation. (i) There may be some preferred, on physical grounds, choice of the time variable t, which should be used in this case for the calculation of probabilities. For example, one could choose the proper time along the worldlines of comoving observers. (ii) One can take the point of view that no meaningful definition of probabilities is possible for observations in disconnected, eternally inflating universes. While this issue requires further investigation, the important point for us here is that, in either case, an observational test of quantum cosmology does not seem possible.

Thus, the conclusion is that, sadly, quantum cosmology is not likely to become an observational science. However, without quantum cosmology our picture of the universe is

incomplete. It raises very intriguing questions of principle and will, no doubt, inspire future research. I wish that Stephen continues to lead and challenge us in this adventure.

[1] DeWitt, B.S. (1967). Quantum theory of gravity. I. The canonical theory, *Phys. Rev.* **160**, 1113.

[2] Misner, C.W. (1972). Minisuperspace, in *Magic without Magic* (Freeman, San Francisco).

[3] Grishchuk, L.P. and Zeldovich, Ya.B. (1982). Complete cosmological theories, in *Quantum Structure of Space and Time*, ed. M. Duff and C. Isham (Cambridge University Press, Cambridge).

[4] Vilenkin, A. (1982). Creation of universes from nothing, *Phys. Lett.* **B117**, 25.

[5] Hartle, J.B. and Hawking, S.W. (1983). Wave function of the universe, *Phys. Rev.* **D28**, 2960.

[6] Linde, A.D. (1984). Quantum creation of the inflationary universe, *Lett. Nuovo Cimento* **39**, 401.

[7] Rubakov, V.A. (1984). Quantum mechanics in the tunneling universe, *Phys. Lett.* **148B**, 280.

[8] Vilenkin, A. (1984). Quantum creation of universes, *Phys. Rev.* **D30**, 509.

[9] Zeldovich, Ya.B. and Starobinsky, A.A. (1984). Quantum creation of a universe in an nontrivial topology, *Sov. Astron. Lett.* **10**, 135.

[10] Hawking, S.W. (1986). Quantum cosmology, in *300 Years of Gravitation*, ed S.W. Hawking and W. Israel (Cambridge University Press, Cambridge).

[11] Vilenkin, A. (1986). Boundary conditions in quantum cosmology, *Phys. Rev.* **D33**, 3560.

[12] Vilenkin, A. (1994). Approaches to quantum cosmology, *Phys. Rev.* **50**, 2581.

[13] Kontoleon, N. and Wiltshire, D.L. (1999). Operator ordering and consistency of the wave function of the universe, *Phys. Rev.* **D59**, 063513.

[14] Rubakov, V.A. (1999). Quantum cosmology, gr-qc/9910025.

[15] Vachaspati, T. and Vilenkin, A. (1988). On the uniqueness of the tunneling wave function of the universe, *Phys. Rev.* **D37**, 898.

[16] Ambjorn, J. Jurkiewicz, J. and Loll, R. (2001). Dynamically triangulating Lorentzian quantum gravity, *Nucl. Phys.* **B610**, 347.

[17] Halliwell, J.J. and Hartle, J.B. (1990). Integration contours for the no boundary wave function of the universe, *Phys. Rev.* **D41**, 1815.

[18] Davidson, A., Karasik, D. and Lederer, Y. (1999). Wave function of a brane-like universe, *Class. Quant. Grav.* **16**, 1349.

[19] Bucher, M. (2001). A braneworld universe from colliding bubbles, hep-th/0107148.

[20] Dvali, G., Gabadadze, G. and Porrati, M. (2000). 4D gravity on a brane in 5D Minkowski space, *Phys. Lett.* **B484**, 112.

[21] Cordero, R. and Vilenkin, A. (2001). Stealth branes, *Phys. Rev.* **D65**, 083519.

[22] Vilenkin, A. (1998). Wave function discord, *Phys. Rev.* **D58**, 067301.

[23] Vilenkin, A. (1989). Interpretation of the wave function of the universe, *Phys. Rev.* **D39**, 1116.

[24] Lapchinsky, V. and Rubakov, V.A. (1979). Canonical quantization of gravity and quantum field theory in curved space-time, *Acta Phys. Polon.* **B10**, 1041.

[25] Halliwell, J.J. and Hawking, S.W. (1995). The origin of structure in the universe, *Phys. Rev.* **D31**, 1777.

[26] Banks, T. (1985). TCP, quantum gravity, the cosmological constant and all that ... *Nucl. Phys.* **B249**, 332.

[27] Vilenkin, A. (1988). Quantum cosmology and the initial state of the universe, *Phys. Rev.* **D37**, 888.

[28] Hawking, S.W. and Page, D.N. (1986). Operator ordering and the flatness of the universe, *Nucl. Phys.* **B264**, 185.

[29] Grishchuk, L.P. and Rozhansky, L.V. (1988). On the beginning and the end of classical evolution in quantum cosmology, *Phys. Lett.* **B208**, 369.

[30] Barvinsky, A.O. and Kamenshchik, A.Y. and Mishakov, I.V. (1997). Quantum origin of the early inflationary universe, *Phys. Lett.* **B332**, 270.

[31] Page, D.N. (1997). Space for both no-boundary and tunneling quantum states of the universe, *Phys. Rev.* **D56**, 2065.

[32] Vilenkin, A. (1983). Birth of inflationary universes, *Phys. Rev.* **D27**, 2848.

[33] Linde, A.D. (1986). Eternally existing selfreproducing chaotic inflationary universe, *Phys. Lett.* **B175**, 395.

[34] Starobinsky, A.A. (1986). Stochastic de Sitter (inflationary) stage in the early universe, in *Field Theory, Quantum Gravity, and Strings*, ed. H.J. de Vega and N. Sanchez, Lecture Notes in Physics Vol. 246 (Springer, Heidelberg).

[35] Aryal, M. and Vilenkin, A. (1987). Fractal dimension of inflationary universe, *Phys. Lett.* **B199**, 351.

[36] Winitzki, S. (2002). The eternal fractal in the universe, *Phys. Rev.* **D65**, 083506.

[37] Steinhardt, P.J. (1983). Natural inflation, in *The Very Early Universe*, ed. G.W. Gibbons and S.W. Hawking (Cambridge University Press, Cambridge).

[38] Linde, A.D. (1983). The new inflationary universe scenario, in *The Very Early Universe*, ed. G.W. Gibbons and S.W. Hawking (Cambridge University Press, Cambridge).

[39] Borde, A. and Vilenkin, A. (1994). Eternal inflation and the initial singularity, *Phys. Rev. Lett.* **72**, 3305; (1996). Singularities in inflationary cosmology: a review, *Int. J. Mod. Phys.* **D5**, 813.

[40] Borde, A. and Vilenkin, A. (1997). Violation of the weak energy condition in inflating spacetimes, *Phys. Rev.* **D56**, 717.

[41] Winitzki, S. (2001). Null energy condition violations in eternal inflation, gr-qc/0111109.

[42] Borde, A., Guth, A.H. and Vilenkin, A. (2001). Inflation is past-incomplete, gr-qc/0110012.

[43] Steinhardt, P.J. and Turok, N. (2001). A cyclic model of the universe, hep-th/0111030.

[44] Linde, A.D., Linde D.A. and Mezhlumian, A. (1994). From the big bang to the theory of a stationary universe, *Phys. Rev.* **D49**, 1783.

[45] Hartle, J.B. (1994). Quasiclassical domains in a quantum universe, gr-qc/9404017; (1997). Sources of predictability, in *Complexity*, **3**, 22.

[46] Vilenkin, A. (1995). Predictions from quantum cosmology, *Phys. Rev. Lett.* **74**, 846.

[47] Vilenkin, A. (1998). Unambiguous probabilities in an eternally inflating universe, *Phys. Rev. Lett.* **81**, 5501.

[48] Garriga, J. and Vilenkin, A. (2001). A prescription for probabilities in eternal inflation, *Phys. Rev.* **D64**, 023507.