and V are no longer independent, and it is elementary and straightforward to show, using just the first law of thermodynamics, the equation of state for an ideal gas, and adiabaticity, that the equation describing the expansion is

$$VT^{\beta}=\text{const}, \tag{1}$$

where β is a positive constant. From this equation it is self evident that the universe cools down as it expands.

Alternatively, one can relate the total number of photons N of the CBR (using properties of black-body radiation) with the temperature and volume. This relation is[6]

$$N \propto \left(\frac{kT}{\hbar c}\right)^3 V. \tag{2}$$

since N is constant for adiabatic processes, it is again clear that the universe cools down as it expands.

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[1]J. Richard Christman, Am. J. Phys. **63** (1), 13 (1995).
[2]Edward W. Kolb and Michael S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
[3]Ralph A. Alpher and Robert Herman, "Evolution of the Universe," Nature **162**, 774–775 (1948).
[4]A. A. Penzias and R. W. Wilson, "A measurement of excess antenna temperature at 4080 Mc/s," Astrophys. J. **142** (1), 419–421 (1965).
[5]This is only approximately correct. Here we neglect the heat produced by the Universe, e.g., by the stars.
[6]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980), 3rd ed.

Answers to Question #10 ["Cooling and expansion of the universe," J. Richard Christman, Am. J. Phys. 63 (1), 13 (1995)] and Question #17 ["What happens to energy in the cosmic expansion?," Frank Munley, Am. J. Phys. 63 (5), 394 (1995)]

J. Richard Christman notes[1] that the universe cools as it expands and wishes to know why this should be so. After all, he says, ideal gases do not cool as they expand into a vacuum. When an ideal gas expands into a vacuum its entropy increases; in particular, the process is not quasistatic. On the other hand, in the standard cosmology, one takes the expansion of the universe to be so lethargic that the expansion process is essentially reversible. Then since the universe has no external environment with which it can exchange heat, the universal expansion is a constant entropy process. More precisely, one imagines that the expansion of the universe is slow enough that a particle in its *n*th excited state remains in this state as the cosmos expands. The entropy of a gas of photons is proportional to VT^3, so as the gas expands isentropically, its temperature falls.[2] The energy of a gas of photons is proportional to VT^4, so during the expansion the energy also decreases.[3] And that brings us to Frank Munley's question: essentially, is there any meaning to the notion of "conservation of energy" in curved spacetimes?[4]

In a general curved spacetime, in particular in the Friedman–Robertson–Walker spacetimes most often considered in cosmology, there is no useful way to define a constant energy (however in asymptotically flat spacetimes there is; please see Ref. 5). Nonetheless, cosmologists do speak of a type of "conservation," the covariant conservation of the energy-momentum tensor. This conservation is consistent with an energy density proportional to T^4 and a cosmic expansion with constant VT^3 in the radiation dominated epoch. That is, that the universal expansion is isentropic, yet violates the naive conservation of energy, is just what the covariant conservation of energy requires.

The covariant conservation of energy is a statement about the energy momentum tensor, which summaries the energy, momentum, pressures, and stresses of the stuff of the universe and which acts as the source of spacetime curvature in the general theory of relativity according to the Einstein equations,[6]

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-8\pi G T_{\mu\nu}.$$

Here $R_{\mu\nu}$ is the Ricci tensor, and R is its trace. The metric is $g_{\mu\nu}$. Everything on the left-hand side of Einstein's equation is geometric. On the right-hand side, G is Newton's constant and $T_{\mu\nu}$ is the energy-momentum tensor. It follows from the definition of the Ricci tensor in terms of various derivatives of the metric that the left-hand side of Einstein's equation is covariently divergenceless. So then is the left-hand side. That is

$$T_{\mu}{}^{\nu}{}_{;\,\nu}=0,$$

the so-called covariant conservation of the energy-momentum tensor.

The Friedman–Robertson–Walker cosmologies describe spacetimes that are homogeneous and isotropic. In cosmic coordinates the energy momentum tensor is diagonal and has the simple form

$$T_{\mu}{}^{\nu}=\text{diag}(\rho,p,p,p),$$

with ρ the energy density and p the pressure. For a homogeneous and isotropic spacetime (with, for simplicity, flat spatial sections) the line element must have the Friedman–Robertson–Walker form,

$$ds^2=-dt^2+R^2(t)(dx^2+dy^2+dz^2),$$

with $R(t)$ called the scale factor. Given an equation of state $p=p(\rho)$, the Einstein equations give the time dependence of the scale factor. Imposing the covariant conservation of energy gives a useful way to interpret the results. One finds[7]

$$d/dt(\rho R^3)=-p\,d/dt(R^3).$$

The left-hand side describes the rate of change of what might reasonably be called the energy. You can see that this energy is not constant for matter with nonzero pressure. For radiation, $p=1/3\rho$, and we have already noted that the energy density is proportional to T^4. Thus, the covariant conservation of energy is satisfied with RT=const. In the radiation dominated era, the universe expands with constant entropy, but the energy ρR^3 is not conserved. In the current matter dominated era, the pressure is zero. The photon temperature is still given by RT=const because the cosmic expansion is isentropic, but in the matter dominated era, radiation makes a negligible contribution to the cosmic energy.

In sum, for an isentropic cosmic expansion, photon temperature is inversely proportional to the cosmic scale factor. The behavior of energy with time is determined by the Einstein field equations. As a consequence of these equations one realizes the covariant conservation of the energy-momentum tensor. In the Friedman–Robertson–Walker spacetimes, the conservation has a ready interpretation in terms of the time rate of change of energy. The energy is not conserved unless the matter of the universe has zero pressure.

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[1]J. Richard Chritman, "Question #10. Cooling and expansion of the universe," Am. J. Phys. **63** (1), 13 (1995).
[2]Charles Kittel and Herbert Kroemer, *Thermal Physics* (Freeman, San Francisco, 1980), p. 95.
[3]See Ref. 2, p. 94.
[4]Frank Munley, "Question #17. What happens to energy in the cosmic expansion?" Am. J. Phys. **63** (5) 394 (1995).
[5]Steven Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972), pp. 165 ff.
[6]See for example, Ref. 5, pp. 165 ff.
[7]S. K. Blau and A. H. Guth in "Inflationary cosmology," edited by S. W. Hawking and W. Israel, *Three Hundred Years of Gravitation* (Cambridge U. P., Cambridge, 1987), p. 527.

Answer to Question #10 ["Cooling and expansion of the universe," J. Richard Christman, Am. J. Phys. 63(1), 13 (1995)]

While it is true that the temperature of an ideal gas remains constant if the gas expands without doing work upon its surroundings, the expanding universe must be compared to an expanding *nonideal* gas. By definition, separated molecules of an ideal gas exert no forces upon one another. Therefore, the potential energy of a system of receding ideal gas molecules remains constant. This implies constant kinetic energy and thus constant temperature for the system. On the other hand, because of attractive forces acting upon receding particles, the universe, like a nonideal gas, gains potential energy during expansion. In both cases, system cooling arises from the resulting decrease in the kinetic energy of the system.

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Answer to Question #17 ["What happens to energy in the cosmic expansion?," Frank Munley, Am. J. Phys. 63(5), 394 (1995)]

This is really a "response," not an "*answer*," because a complete answer to the *four* questions raised by Frank Munley in Question #17 (Ref. 1) would undoubtedly require the full machinery of general relativity. Moreover, it would seem that the answers are not even known.[2] However, within the context of special relativity and Newtonian cosmology,[3,4] an answer can be given for the simpler question which seems to be at the root of Question #17.

Suppose there is a source of electromagnetic radiation, e.g., a galaxy, out there somewhere, which, as far as we are concerned, can be characterized by a positive redshift parameter, z.[5] Then, if an observer in the rest frame of the source (the *source observer*) notes the emission of photons with energy E, an observer back here (the *detector observer*) would eventually detect some of the same photons, but with energy $E/(1+z)$. That is, the photon energies measured by the detector observer are reduced by a factor of $1+z$ from what the emitter observer measures. *What has happened to the lost photon energy?*

The answer to this simpler question, which perhaps is the question that led to Question #17, is that *there is not any lost photon energy*. The two observers, the emitter observer and the detector observer, have different rest frames. These two rest frames are, at least in some approximation, inertial frames, but for $z\neq 0$ they are *different* inertial frames. Since photon energy is not a Lorentz invariant, one should not expect the measurements of the photon energies by the two different observers in their two different inertial rest frames to give the same results; the results should in fact differ by a factor of $1+z$. Thus, the photons in question have not lost energy and so there is no need to look for the lost energy.

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[1]Frank Munley, "Question #17. What happens to energy in the cosmic expansion?" Am. J. Phys. **63**(5), 394 (1995).
[2]P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, NJ, 1993), p. 139.
[3]Joseph Silk, *The Big Bang–Revised and Updated Edition* (W. H. Freeman and Company, New York, 1989), pp. 96–108.
[4]Vernon D. Barger and Martin G. Olsson, *Classical Mechanics: A Modern Perspective–Second Edition* (McGraw-Hill, New York, 1995), Chap. 9.
[5]Steven Weinberg, *The First Three Minutes: A Modern View of the Origin of the Universe–Updated Edition* (Basic Books, New York, 1993), p. 165.

Answer to Question #17 ["What happens to energy in the cosmic expansion?," Frank Munley, Am. J. Phys. 63 (5), 394 (1995)]

In cosmology it is a good approximation to neglect photons from stars, deal only with matter and cosmic background photons, and assume that matter and radiation are decoupled and can be treated separately. In the following R denotes the scale factor [usually written $R(t)$] of cosmological models which measures the expansion as a function of time. For matter we treat the universe as a finite spherical mass distribution with an expansion according to Hubble's law. On Newtonian theory the kinetic energy of the expanding distribution can be calculated and is proportional to $\dot{R}^2$ [$\dot{R}=dR(t)/dt$]. The potential energy can be calculated, and is proportional to $-1/R$. The energy equation can then be written down. The same equation is found to hold in general relativistic cosmology. As the universe expands and R increases, the potential energy of the universe increases (becomes less negative). Thus $\dot{R}$ decreases, and the rate of expansion of the universe slows down. This result holds for positive, zero, or negative total energy, i.e., for hyperbolic, flat, and elliptical geometries.

Now consider the photons. In the early universe when they interacted with matter they acquired a black body energy distribution. It is an important result in cosmology[1,2] that as the universe expands, a black body distribution remains a black body distribution (it is not diluted to a grey body) but with a temperature which gets lower and is given by $T = T_0R_0/R$, where the subscripts denote present values. Thus,