edited by C. R. Smith and W. T. Grandy, Jr. (Reidel, Dordrecht and Boston, 1985).
[5]D. E. Neuenschwander, Am. J. Phys. 63, 108 (1995).

Answer to Question #15 ["What space scales participate in cosmic expansion?," Frank Munley, Am. J. Phys. 63(4), 297 (1995)]

To answer the question being asked here, it is necessary to know what the questioner means by expansion, cosmic or otherwise. Presumably, he is referring to the fact that, in a big-bang cosmology, the so-called proper distance ds between neighboring co-moving points (ct,x,y,z) and $(ct,x+dx,y+dy,z+dz)$, given by

$$ds^2=R(t)^2(dx^2+dy^2+dz^2),$$

is increasing with cosmic time t as long as the scale factor $R(t)$ is an increasing function of t. (For simplicity, I take the form of ds to be that of a spatially flat Robertson–Walker (RW) space-time.) Take this to be the case. Then the conventional answer is that only cosmic-sized objects participate in the cosmic expansion. In particular, measuring rods and clocks are immune to it. Otherwise, as the questioner asserts, "the expansion would be invisible and hence meaningless." In their encyclopedia work, Misner, Thorne and Wheeler[1] (MTW) argue that the situation is analogous to pennies attached to the surface of a balloon. As the balloon is blown up the distance between the pennies increases while their size does not change. But what if, instead of pennies, one paints dots on the balloon? And one can ask, if small scales are immune to the expansion, at what scale does the expansion begin? Are clusters of galaxies immune? Are superclusters? If measuring rods and clocks are immune, what are the expressions for the proper distance and time that they do measure? Does this mean that clocks and rods exist in a space-time different from cosmic space-time and if so, how are these space-times joined? I am going to argue that there is only one space-time and that the dynamics of all extended objects from atoms to superclusters and beyond plays out its role in this space-time. It does not follow, however, that all extended objects expand and that the expansion is therefore invisible. Since the major evidence for the cosmic expansion comes from red-shift observations, the question to ask is, if the expansion takes place on all scales, would one observe a cosmological red-shift?

To answer this latter question, one must ask yet another question, namely, what time do clocks measure. It is sometimes asserted that, in relativity, clocks measure proper time. What kind of clocks? Ideal clocks. Okay, but then, what is an ideal clock? At this point in the discussion it is usually asserted that "atomic" clocks, e.g., hydrogen atoms, are ideal clocks. But what justifies this assertion? Certainly, if my digital watch, which is a kind of atomic clock, is dropped on the floor, it may not measure proper time along its trajectory at all because it has ceased functioning.

In an attempt to answer these questions, I have considered two simple clock models, one an electromagnetic clock consisting of two charges rotating about their center of mass—one a kind of classical hydrogen atom and the other a gravitational clock consisting of two masses so rotating, e.g., a double star system such as the Hulse–Taylor binary pulsar PSR1913-16. To determine what kind of time these model clocks measure, one, of course, needs to know their equations of motion. However, simply to postulate such equations is no help since to do so is only slightly better than asserting that atomic clocks measure proper time. We seem to be caught in a kind of vicious circle.

There is, however, a way out. The equations of motion for a classical hydrogen atom or a double star system can be derived from the Einstein–Maxwell field equations using the Einstein–Infeld–Hoffmann (EIH) surface integral method.[2] In deriving these equations, the only assumption I have made is that these clocks "feel" the effect of the cosmological field so that one perturbs off a RW rather than the "flat" Minkowski field used by Einstein and his co-workers. The resulting equations of motion are, of course, only approximate and are restricted to clocks whose characteristic periods are large compared to the light travel time across them, but small compared to the expansion time (the inverse of the Hubble parameter $\dot{R}/R$) of the universe as a whole.[3] Although the details of the calculation are complicated, the results, at least in the lowest order of approximation—the so-called Newtonian approximation—can be easily stated. There are two effects of the cosmic expansion. One is a kind of cosmic viscosity that is proportional to the product of the Hubble parameter and the particle velocity. (This term appears in the geodesic equations for a test body moving in a RW field.) The other effect is to modify the Newtonian or Coulomb interaction between the components of the clock by a factor $1/R^3$. The resulting equations of motion thus have the form

$$m_A\ddot{\mathbf{x}}_A=-\frac{1}{R^3}\sum_{B\neq A}\frac{\alpha_A\alpha_B}{{r_{AB}}^3}\mathbf{r}_{AB}-2m_A\frac{\dot{R}}{R}\dot{\mathbf{x}}_A,$$

where $\mathbf{r}_{AB}=\mathbf{r}_A-\mathbf{r}_B$ and $\alpha_A=m_A$ for a gravitational clock and $|q_A|$ for an electromagnetic clock.

If R is changing slowly compared to the particle coordinates $\mathbf{x}_A$, then one can solve this set of equations of motion using the adiabatic approximation. The result is that for both the gravitational and electromagnetic clock

$$(Rr)^3\omega^2=A \quad \text{and} \quad Rr\omega=B,$$

where ω is the frequency of the clock, r is its coordinate radius, and A and B are constants. It follows that the frequencies of our clocks are independent of the cosmic time t and hence that such clocks will measure cosmic time. Furthermore, one sees that the quantity Rr, which is usually taken to be the physical separation between the components of the clocks, is also a constant. If one such clock were sitting on a distant galaxy and emitting a wave, another one here on earth would measure the usual cosmological red-shift, even though both clocks are experiencing the effect of the cosmic expansion. Thus one sees that, to the extent that the approximations used in obtaining our results are applicable, our model clocks behave like the pennies on the MTW balloon, even though they live in the space-time of the expanding universe. On the other hand, if the clock period is comparable to the Hubble time, then it will behave like a dot painted on the balloon and will not measure cosmic time. In either case, one expects that the expansion of the universe will make itself felt in the higher orders of approximation, that the effects will be different for gravitational and electromagnetic clocks, and that neither kind of clock will measure cosmic time exactly.

There is an important conclusion to be drawn from the above discussion and I will end my reply with it. We saw that it was unnecessary to invoke either the notion of an ideal clock or the notion of proper time to answer the question of what kind of time our model clocks measure. In principle, the question of what kind of time a given clock measures must be answered from a knowledge of its dynamics and, at least in the case of the simple clocks considered here, that knowledge comes directly from the field equations of general relativity. In a very real sense, general relativity has eliminated the need for geometry in physics and replaced it by the gravitational field.[4]

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[1]See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), p. 719.

[2]A. Einstein, L. Infeld, and B. Hoffmann, "Gravitational equations and the problems of motion," Ann. Math., Ser. 2 **39**, 65–100 (1939); A. Einstein and L. Infeld, "Gravitational equations and the problems of motion. II," Ann. Math., Ser. 2 **40**, 455–464 (1940); A. Einstein and L. Infeld, "Motion of particles in general relativity theory," Can. J. Math. **3**, 209–241 (1949).

[3]James L. Anderson, "Multiparticle dynamics in an expanding universe," Phys. Rev. **75** (20), 3602–3604 (1995).

[4]After this answer was submitted, an answer by Steven K. Blau, Am. J. Phys. **63**(9), 779–780 (1995), to the question discussed here appeared that is, I contend, fundamentally wrong and also assumes tactitly what is to be proven. It is not true that the number of meter sticks that will fit between Earth and a galaxy is constant, as Blau contends. As the above discussion demonstrates, this number will increase. Also, in deriving the expression for the cosmic red shift, Blau assumes that the frequency of emission of an atom remains constant over cosmic times. This is true but not obvious and must be demonstrated as I have outlined here.

Answer to Question #22 ["Is there a gravitational force or not?," Barbara S. Andereck, Am. J. Phys. 63(7), 583 (1995)]

How can gravity be unified with the other fundamental forces of physics if, as general relativity teaches us, there is no gravitational force but just the geometry of spacetime, i.e., particles in a gravitational field have geodesic world lines? There are, broadly speaking, two answers, depending on how one understands unification. The first is that one gives a geometrical account of the nongeometrical forces; the second is that one gives up a geometrical account of gravity at a fundamental level.

An example of the first answer is the Kaluza–Klein program. Here, for example, to unify gravity and electromagnetism, we postulate that spacetime has 4+1 dimensions and that Einstein's field equations are satisfied in 5 dimensions, and all particles have geodesic world lines in the 5-dimensional spacetime. Relative to the effective 3+1-dimensional world, however, the electromagnetic four-vector potential $A_u \sim g_{u5}$. "Charged" particles are those which travel curved (relative to the 3+1 world) world lines when the electromagnetic field is nonzero. To generalize the scheme to the weak and strong interactions requires still further spatial dimensions satisfying appropriate isometries.

In general, examples of the first answer are classical, in the sense that general relativity is a classical theory, whereas those of the second answer are quantum mechanical. In fact, it is difficult to discuss the second answer in a straightforward way because there is no successful quantum theory of gravity—whether unified or not. But generally, we can say that if we had such a successful quantum theory of gravity, then we would not expect the equations of general relativity to be valid at a fundamental level. Rather, we would expect them to hold only as expectation values relative to certain semiclassical states, such as a coherent state or a WKB wave function. Gravity as geometry would therefore emerge at a phenomenal macroscopic (i.e., above the Planck scale) level, even though it would not hold at a fundamental level.

An instructive example of a possible unified theory containing gravity is string theory. Here, the quanta of all the forces, including gravity, are different excitations of the fundamental strings (or string field). The gravitational field cannot then be fundamental to the theory, but only a special state of the string (or string field). Since the gravitational field is the metric field (or metric curvature) of spacetime, the metrical geometry of spacetime would not be present at a fundamental level.

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Answer to Question #22. ["Is there a gravitational force or not?," Barbara S. Andereck, Am. J. Phys. 63(7), 583 (1995)]

The beauty of general relativity with its apparent geometrization of gravity motivated a quest for the geometrization of all of the forces of nature. Einstein himself spent the last years of his life attempting to construct a theory of gravity and electromagnetism in which both fields were parts of an underlying unified geometrical object which itself was to be determined by simple geometrical requirements. Sadly, all of these attempts by Einstein and others ultimately failed. Nevertheless, the geometrical interpretation of the gravitational force has become a standard dogma of present day physics. But, as the author of this question points out, it leads to a seeming paradox.

My answer to the question posed here is that general relativity actually *eliminated* geometry from physics. In both Newtonian mechanics and special relativity one made use of geometric objects to formulate the basic laws of these structures. In Newtonian mechanics one introduced planes of absolute simultaneity and straight lines, while in special relativity the planes of absolute simultaneity were replaced by light cones. But, in both cases, these geometrical objects were fixed and independent of whatever else went on. What general relativity succeeded in doing was to replace these absolute geometrical objects by the dynamical gravitational field, obeying field equations in many ways similar to the field equations of electromagnetism. What was not clear in the beginning but by now has been recognized is that one does not need the "geometrical" hypotheses of the theory, namely, the identification of a metric with the gravitational field, the assumption of geodesic motion, and the assumption that "ideal" clocks measure proper time as determined by this metric. Indeed, we now know that both of these latter assumptions follow as approximate results directly from the field equations of the theory without further assumptions.[1] Thus one can, and in my view should, eschew any geometri-