Cosmology with a time-varying speed of light [1]

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Abstract

Cosmic inflation is the only known mechanism with the potential to explain the very special initial conditions which are required at the early stages of the evolution of our universe. This article outlines my work with Joao Magueijo which attempts to construct an alternative mechanism based on a time-varying speed of light.

Introduction

As we reconstruct the past history of the universe, we learn that the universe as we see it today must have evolved from a special state that was extremely flat and homogeneous. Such a state is highly unstable toward both the formation of inhomogeneities and evolution away from flatness, because the influence of gravity tends to drive matter away from a flat homogeneous state. Until the advent of inflation, cosmologists had no way of explaining how the universe could have started out so precisely balanced in a state of such high instability.

Inflation addresses this issue by changing the story at very early times. During an early period of inflation, the matter is placed in a peculiar potential dominated state where the effects of gravity are very different. During inflation, flatness is an attractor, and the deviations (or perturbations) from homogeneity destined for our observable universe can be calculated. With a suitable tuning of model parameters, the perturbations can be given a sufficiently small amplitude and even naturally acquire a spectrum which gives good agreement with observations.

The theory of cosmic perturbations has for some time now benefited from the existence of alternative models. The presence of alternatives has made

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it possible to systematically evaluate each alternative, and has even helped us to discover the most fundamental way in which inflationary theory could be falsified[1]. So far, things are looking very good for an inflationary origin for the cosmic perturbations.

But inflation offers us much more than a theory of perturbations. It also is supposed to explain the origin of the flatness and overall smoothness of the universe. In this role, inflation theory has faced no serious competition. While this fact in itself might be seen as a success, it would certainly be much more gratifying if the significant place in the theoretical landscape currently occupied by inflation could be earned by doing better than some serious contenders. After all, inflation is just the first idea we have had to explain these cosmic puzzles.

The above reasoning has motivated me to search for alternatives for some time now. So far everything that I have tried has ended up being just another version of standard inflation, once it was forced into a workable form. This experience encourages the view that inflation might be *the* unique mechanism by which the initial condition problems can be addressed, and also explains the extreme nature of the idea I outline below.

The idea I present here starts with a very simple observation. A statement of the unusual nature of the initial conditions in standard cosmology usually carries with it a description of the "horizon problem". Basically, in the standard big bang model any mechanism which operates in the early universe and attempts to "set up" the correct initial conditions would have to act acausally, because what we observe today is composed of many causally disconnected regions in the early universe (see Figure 1). Inflation solves this problem because a period of superluminal expansion radically changes the causality structure of the universe. Another way of changing the causality structure is to have light travel faster in the early universe (Figure 2).

Joao Magueijo and I have pursued this idea, to the extent of setting up a phenomenological model of how physics might look with a time varying speed of light (VSL)[2]. Interestingly, we have found that our model exhibits energy non-conservation of just the sort that can fill in energy deficits and pull down energy peaks to produce a flat homogeneous universe from a wide range of initial conditions.

Of course our model also breaks Lorentz invariance, which may seem unreasonable to may physicists. In defense of choosing this radical direction, let me comment that many theorists are prejudiced *against* the idea that the spacetime continuum is really a continuum down to arbitrarily small scales. Any deviation from a true continuum would necessarily break Lorentz invariance. In particular many ideas about our 3+1 dimensional world that are coming from superstring theory (and its offspring) suggest that properties of this (3+1D) manifold are emergent as a low-energy limit of something quite different. If the VSL picture really takes root, the picture I describe below could well be a phenomenology of the dynamics of the universe as the 3+1 manifold we inhabit emerges from very different physics governing high energies. In this picture the speed of light varies in the early universe and then holds constant, so that the standard cosmology can proceed. This much is in the same spirit as inflation, where a period of unusual physics is placed in the early universe, without changing the standard physical picture which does an excellent job of explaining many aspects of the universe.

I should mention that the idea of using a varying speed of light to explain initial conditions first appears in print in a paper by Moffat[3]. His paper

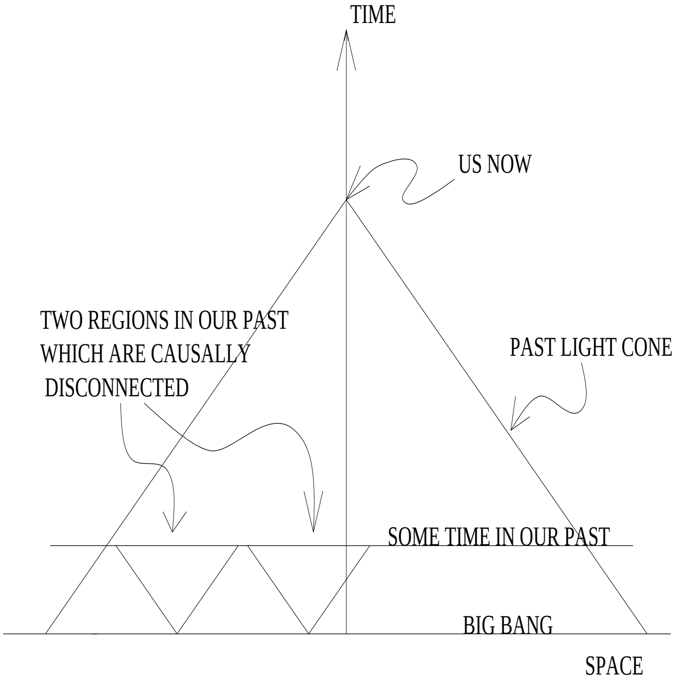

Figure 1: The horizon problem in standard cosmology.

takes the idea in a somewhat different direction than we have. Also, subsequent work by Barrow and Magueijo has taken VSL in a variety of different directions. I will mention this briefly in the final section.

Our prescription

To pursue the idea of VSL, Magueijo and I have used the following simple prescription: VSL models necessarily have preferred frame, because Lorentz invariance is broken. We assume that in that special frame, the Lagrangian is the same as usual, with the substitution $c \rightarrow c(t)$. We also assume that the dynamics of c do not affect the curvature, so that the Riemann tensor and the Ricci scalar are to be computed (in the preferred frame) with c held fixed.

This scheme is spelled out in [2]. There we carefully discuss the question of why this scheme is *not* simply ordinary physics under a strange reparame-

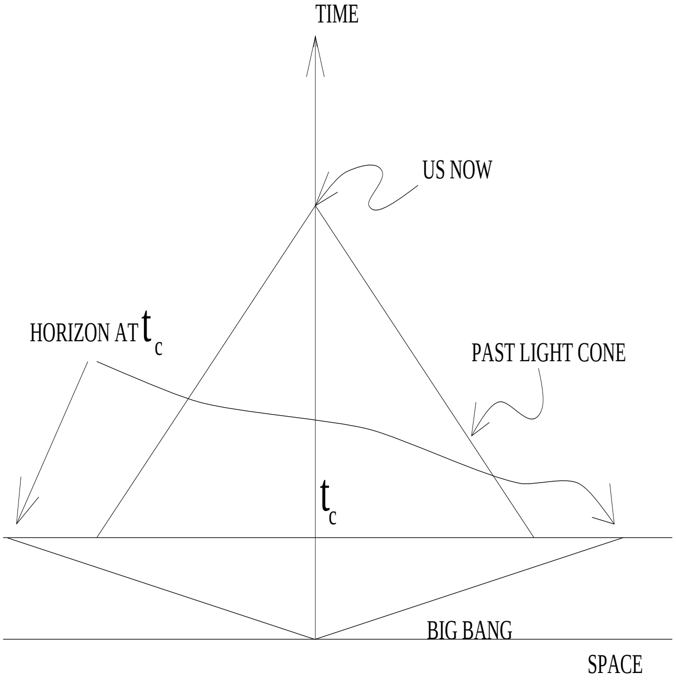

Figure 2: The horizon problem solved by a time-varying speed of light.

terization, and thus why what I describe below can not be viewed as simply an unusual way of describing inflation.

Cosmological Equations

Under our VSL scheme energy is not conserved when c is varying. In a cosmology which is Robertson-Walker in the preferred frame we get

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = -\rho\frac{\dot{G}}{G} + \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c}. \tag{1}$$

This equation is the usual equation for energy conservation when c and G are constant, and I have included the term in $\dot{G}$ for future reference. To observe the effect on the flatness of the universe, we look at the evolution of $\epsilon \equiv \Omega - 1$ ($\Omega \equiv \rho/\rho_c$):

$$\dot{\epsilon} = (1+\epsilon)\epsilon\frac{\dot{a}}{a}(1+3w) + 2\frac{\dot{c}}{c}\epsilon \tag{2}$$

where we have taken $p = w\rho c^2$ with constant w. Here we can see how in standard big bang cosmology ($w > -1/3$, $\dot{c} = 0$) $\epsilon = 0$ is an unstable fixed point, leading to the need to tune ϵ to extremely small values at the beginning, in order to match a value of ϵ which is not large even today. Eqn. 2 also shows how inflation makes $\epsilon = 0$ an attractor, and how $\dot{G}$ drops out of the equation, making at least this version of a varying G ineffective at producing the desired effect. One can also see how negative values of $\dot{c}/c$ will make $\epsilon = 0$ an attractor.

There is also an interesting effect on the cosmological constant. In order to discuss this, we must be careful about which constant we are talking about.

$$S = \int dx^4 \sqrt{-g}\left(\frac{c^4(R + 2\Lambda_1)}{16\pi G} + \mathcal{L}_M + \mathcal{L}_{\Lambda_2}\right) \tag{3}$$

Equation 3 shows the action in the preferred frame, where $\mathcal{L}_M$ is the matter fields Lagrangian. The term in Λ_1 is a geometrical cosmological constant, as first introduced by Einstein. The term in Λ_2 represents the vacuum energy density of the quantum fields [4]. VSL is only able to affect Λ_1, and we simply call this Λ in what follows. If we define $\epsilon_\Lambda = \rho_\Lambda/\rho_m$ where $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$ we find

$$\dot{\epsilon}_\Lambda = \epsilon_\Lambda\left(3\frac{\dot{a}}{a}(1+w) + 2\frac{\dot{c}}{c}\frac{1+\epsilon_\Lambda}{1+\epsilon}\right). \tag{4}$$

Ordinary cosmology has a Λ problem in the sense that Λ rapidly comes to dominate, and must be tuned initially in order not to be super-dominant today. Here we can see that inflation, with $-1 \leq w \leq -1/3$, cannot provide the necessary tuning, while VSL ($\dot{c} < 0$) can have that effect. This, by the way also helps illustrate how VSL is not physically equivalent to inflation.

We have also considered the evolution of perturbations. We have found that for a sudden change in c, the density contrast Δ obeys

$$\frac{\Delta^+}{\Delta^-} = \frac{c^+}{c^-}. \tag{5}$$

For the large variation in c required to produce a flat universe, we have found that the universe has all perturbations reduced to unobservable levels. This leave a blank slate which requires something like the defect models of cosmic structure formation to provide perturbations[5, 6].

Scenario Building

So far I have discussed the machinery of VSL, but how can this be turned into a cosmological scenario? One can start the discussion by considering a sudden transition between c^- and c^+. Let us assume for a moment that before the transition we have a flat FRW universe. We find under these circumstances the temperature obeys

$$T^+/T^- = c^-/c^+. \tag{6}$$

If we want $T^+ \approx T_{Planck}$ and require $c^-/c^+ > 10^{60}$ to solve the flatness problem, on is starting with a very cold T^-. Immediately, one can see that fine tuning is required to have a cold flat universe before the transition, and this scenario does not make much sense. Interestingly, unlike inflation Equation 2 shows that VSL can create energy even in a empty open ($\epsilon = -1$) universe. It might be more interesting to build scenarios based on that starting point. Just as inflation has seen the scenario-building change radially over the years, we feel there is a lot to be learned about how to implement VSL before we understand the best scheme to use. What we have so far is a very interesting mechanism that can move the universe toward a flat homogeneous state.

Discussion and Conclusions

Since our paper there have been a number of other publications on VSL. One new direction pursued by Barrow and Magueijo[7, 8] looks at a possible power-law $c(t)$ that could have c varying even today. They have found interesting attractor solutions which keeps Ω_Λ at a constant fraction of the total Ω, but they have their work cut out for them understanding primordial nucleosynthesis in that model. Also, Moffat has further explored the idea of spontaneous breaking of Lorentz symmetry[9].

Probably the greatest problem with the VSL idea is that we have no fundamental picture of what makes c vary. The phenomenological treatment in [2] does not address this question. What we can say is our work shows that there are interesting cosmological rewards for considering a time varying speed of light, and with that motivation it may be possible to make some interesting discoveries.

Despite this problem, it is already possible to falsify at least the fast-transition version of VSL. Since the perturbations turn out to be infinitesimal after the transition, structure formation must be left to active models which have their own characteristic signatures that differentiate them from inflation[10]. If the microwave background comes out with characteristic inflationary features, these VSL models will be ruled out.

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