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Higgs inflation in complex geometrical scalar-tensor theory of gravity

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ABSTRACT

We derive a Higgs inflationary model in the context of a complex geometrical scalar-tensor theory of gravity. In this model the Higgs inflaton scalar field has geometrical origin playing the role of the Weyl scalar field in the original non-riemannian background geometry. The energy scale enough to generate inflation from the Higgs energy scale is achieved due to the compatibility of the theory with its background complex Weyl-integrable geometry. We found that for a number of e-foldings $N = 63$, a nearly scale invariant spectrum for the inflaton is obtained with an spectral index $n_s \simeq 0.9735$ and a scalar to tensor ratio $r \simeq 0.01$, which are in agreement with Planck observational data.

1. Introduction

Inflationary models represent a cornerstone of modern cosmology. By postulating the existence of the inflaton scalar field, inflation solves the old problems of the big bang cosmology and also provides a mechanism to explain the formation of cosmological structure. In this theory the inflaton must be capable to generate the enough vacuum energy density to have a suitable model compatible with CMB observational data and the matter distribution in the universe. In the literature we can find different inflationary models that use more than one scalar field, as for example the hybrid inflation models [1–4].

However, until now, the only scalar particle that has experimental evidence of his existence is the Higgs boson [5,6]. The idea that the inflaton field might be the same as the Higgs scalar field has already been considered [7]. The main problem of this idea relies in the fact that the energy scale of the Higgs field is too small to generate the enough quantity of inflation required to solve the problems of the big bang cosmology. In particular, to have the enough inflation to solve the big bang problems, the inflaton is estimated to have a mass $\sim 10^{13}$ GeV, and in some models it prefers a small-interacting quartic coupling constant $\lambda \leq 10^{-9}$ [7–9]. However, all the parameters associated with the Higgs field are determined at TeV scale, such as the dimensionless Higgs quartic coupling $0.11 < \lambda < 0.27$ [9,10]. Models attempting to solve this problem have already appeared in the literature, which in much are non-minimal coupling models [11–14]. In addition, recently there has been a lot of interest in the Higgs inflationary models in the Palatini formulation, where a Palatini variational principle is implemented on an Einstein–Hilbert form of the action. More about these models can be found for example in [15–19].

On the other hand, scalar-tensor theories incorporate a scalar field in the action. However, for some researchers it is not so clear if the scalar field describes gravity or matter [20]. This happens in the so called Jordan frame. By means of a conformal transformation of the metric appears the Einstein frame. In the Jordan frame gravity exhibits a non-minimal coupling with the scalar field while in the Einstein frame it is obtained a minimal coupling [21]. The main controversy relies in determine which of the both frames is the physical one. In the literature we can find opinions in favour of one or the other [20]. However, on the other hand, it is a well-known fact that a geometry is characterized by the compatibility condition between the connection and the metric: $\nabla_{\mu} g_{\alpha\beta} = N_{\alpha\beta\mu}$. However, in general the compatibility condition does not remain invariant only under conformal transformations of the metric. Therefore, the usual manner in which we can pass from the Jordan to Einstein frame in standard scalar-tensor theories, changes the background geometry, and this is why the physics in one or another frame can be different. In particular geodesic observers in one frame are not in the other [20,22,23].

This controversy can be alleviated if the background geometry is not fixed *apriori* as Riemannian. This is the main idea in a recently introduced new kind of scalar-tensor theories known as geometrical scalar-tensor theories of gravity [22,23]. In this theories the background geometry is obtained via the Palatini variational principle. The resulting geometry is one of the Weyl-integrable type [22,23]. As a consequence, the scalar field that

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appears in scalar-tensor theories becomes part of the affine structure of the space–time and in this sense can be considered as geometrical in origin. Hence, the background geometry is essentially the same for both the Weyl and the Riemann frames, which are the analogous for the Jordan and Einstein frames in usual scalar-tensor theories. Hence, the ambiguity about the nature of the scalar field that usually arises in standard scalar-tensor theories and the controversy between the two frames is not present in this new approach [20,22,23]. In the framework of this theory topics like $(2+1)$ gravity models, inflationary cosmology and cosmic magnetic fields, quintessence and some cosmological models have been studied [24–27].

In this letter we extend the formalism of previous geometrical scalar-tensor theories to construct a geometrical Higgs inflationary model. The letter is organized as follows. Section 1 is left for a little motivation and introduction. In Section 2 is developed the general formalism in the Weyl frame. In Section 3 it is obtained the effective field action in the Riemann frame. In Section 4 we present a Higgs inflationary model. Finally, Section 5 is devoted to some final comments.

2. Basic formalism in the Weyl frame

Let us start considering an action for a complex scalar-tensor theory of gravity, which in vacuum is given by

$$\mathcal{S}=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\tilde{\Phi}\tilde{\Phi}^{\dagger}\mathcal{R}+\frac{\tilde{W}(\tilde{\Phi}\tilde{\Phi}^{\dagger})}{\tilde{\Phi}\tilde{\Phi}^{\dagger}}g^{\mu\nu}\tilde{\Phi}_{,\mu}\tilde{\Phi}_{,\nu}-\tilde{U}(\tilde{\Phi}\tilde{\Phi}^{\dagger})\right] \tag{1}$$

where $\mathcal{R}$ denotes the Ricci scalar, $\tilde{W}(\tilde{\Phi}\tilde{\Phi}^{\dagger})$ is a well-behaved differentiable function of $\tilde{\Phi}\tilde{\Phi}^{\dagger}$, the dagger $\dagger$ denotes transposed complex conjugate and $\tilde{U}(\tilde{\Phi}\tilde{\Phi}^{\dagger})$ is a scalar potential. With the help of the transformation $\tilde{\Phi}=\frac{1}{\sqrt{G}}\Phi$ the action (1) can be written in the form

$$\mathcal{S}=\int d^4x\sqrt{-g}\left[\frac{\Phi\Phi^{\dagger}\mathcal{R}}{16\pi G}+\frac{\tilde{\omega}(\Phi\Phi^{\dagger})}{\Phi\Phi^{\dagger}}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}-\tilde{V}(\Phi\Phi^{\dagger})\right], \tag{2}$$

where $\tilde{\omega}(\Phi\Phi^{\dagger})=\tilde{W}(\tilde{\Phi}\tilde{\Phi}^{\dagger})/(16\pi)$ and the redefined potential is $\tilde{V}(\Phi\Phi^{\dagger})=\tilde{U}(\tilde{\Phi}\tilde{\Phi}^{\dagger})/(16\pi)$. A Palatini variation of the action (2) with respect to the affine connection leaves to the compatibility condition

$$\nabla_{\mu}g_{\alpha\beta}=-[\ln(\Phi\Phi^{\dagger})]_{,\mu}g_{\alpha\beta}. \tag{3}$$

Hence, the natural background geometry associated to (1) is a non-Riemannian geometry with a quadratic in Φ non-metricity and null torsion. However, through the field transformation $\Phi=e^{-\varphi}$, the non-metricity in (3) can be linearized and written in the form

$$\nabla_{\mu}g_{\alpha\beta}=(\varphi+\varphi^{\dagger})_{,\mu}g_{\alpha\beta}. \tag{4}$$

Notice that this compatibility condition is of the Weyl-Integrable type. Thus, in terms of the new field φ the action (2) reads

$$\mathcal{S}=\int d^4x\sqrt{-g}e^{-(\varphi+\varphi^{\dagger})}\left[\frac{\mathcal{R}}{16\pi G}+\hat{\omega}(\varphi+\varphi^{\dagger})g^{\mu\nu}\varphi_{,\mu}\varphi^{\dagger}_{,\nu}-\hat{V}(\varphi+\varphi^{\dagger})\right], \tag{5}$$

where we have made the identifications $\hat{\omega}(\varphi+\varphi^{\dagger})=\tilde{\omega}(\varphi+\varphi^{\dagger})e^{\varphi+\varphi^{\dagger}}$ and $\hat{V}(\varphi+\varphi^{\dagger})=\tilde{V}(\varphi+\varphi^{\dagger})e^{\varphi+\varphi^{\dagger}}$. Now, we must note that the compatibility condition (4) remains invariant when we apply, at the same time, the transformations

$$\bar{g}_{\mu\nu}=e^{f+f^{\dagger}}g_{\mu\nu}, \tag{6}$$

$$\bar{\varphi}=\varphi+f, \tag{7}$$

$$\bar{\varphi}^{\dagger}=\varphi^{\dagger}+f^{\dagger}, \tag{8}$$

where $f=f(x^{\alpha})$ is a well defined complex function of the space–time coordinates. Thus for the action (5) to be an scalar under the group of transformations of the background geometry, it must be an invariant under the diffeomorphism group and the transformations (6)–(8). However, under (6)–(8) the kinetic term in (5) transforms as

$$\sqrt{-\bar{g}}\,\bar{\hat{\omega}}(\bar{\varphi}+\bar{\varphi}^{\dagger})\bar{g}^{\mu\nu}\bar{\varphi}_{,\mu}\varphi^{\dagger}_{,\nu}= e^{2(f+f^{\dagger})}\sqrt{-g}\,\hat{\omega}(\varphi+f+\varphi^{\dagger}+f^{\dagger})g^{\mu\nu}(\varphi_{,\mu}+f_{,\mu})(\varphi^{\dagger}_{,\nu}+f^{\dagger}_{,\nu}), \tag{9}$$

which indicates that the kinetic term in (5) results to be not invariant and consequently the action (5) is not either. In order to solve this problem we propose the new action

$$\mathcal{S}=\int d^4x\sqrt{-g}\,e^{-(\varphi+\varphi^{\dagger})}\left[\frac{\mathcal{R}}{16\pi G}+\widehat{\omega}(\varphi+\varphi^{\dagger})g^{\mu\nu}\varphi_{:\mu}\varphi^{\dagger}_{:\nu}-e^{-(\varphi+\varphi\dagger)}\widehat{V}(\varphi+\varphi^{\dagger})\right], \tag{10}$$

where we have introduced a gauge covariant derivative defined by $\varphi_{:\mu}={}^{(w)}\nabla_{\mu}\varphi+\gamma B_{\mu}\varphi$, with B_{μ} being a gauge vector field, ${}^{(w)}\nabla_{\mu}$ being the Weyl covariant derivative determined by (4) and γ is a pure imaginary coupling constant introduced to have the correct physical units. Thus, it is not difficult to verify that the invariance under (6) to (8) of (10) is achieved when the vector field B_{μ}, the function $\hat{\omega}$ and the scalar potential $\hat{V}(\varphi)$, obey respectively the transformation rules

$$\bar{\varphi}\bar{B}_{\mu}=\varphi B_{\mu}-\gamma^{-1}f_{,\mu}, \tag{11}$$

$$\bar{\varphi}^{\dagger}\bar{B}_{\mu}=\varphi B_{\mu}+\gamma^{-1}f^{\dagger}_{,\mu}, \tag{12}$$

$$\bar{\hat{\omega}}(\bar{\varphi}+\bar{\varphi}^{\dagger})\equiv\hat{\omega}(\bar{\varphi}+\bar{\varphi}^{\dagger}-f-f^{\dagger})=\hat{\omega}(\varphi+\varphi^{\dagger}), \tag{13}$$

$$\bar{V}(\varphi+\varphi^{\dagger})\equiv V(\bar{\varphi}+\bar{\varphi}^{\dagger}-f-f^{\dagger})=V(\varphi+\varphi^{\dagger}). \tag{14}$$

Notice that (11) and (12) are transformation rules for the product φB_{μ}. Besides they have the same algebraic form of the algebra of the $U(1)$ group, used to describe the electromagnetic interaction. Thus, we may include a dynamics for φB_{α} extending the action (10) by adding an electromagnetic type term in the form

$$\mathcal{S}=\int d^4x\sqrt{-g}\,e^{-(\varphi+\varphi^{\dagger})}\left[\frac{\mathcal{R}}{16\pi G}+\frac{1}{2}\hat{\omega}(\varphi+\varphi^{\dagger})g^{\alpha\beta}\varphi_{:\alpha}\varphi_{:\beta}-e^{-(\varphi+\varphi^{\dagger})}\hat{V}(\varphi+\varphi^{\dagger})-\frac{1}{4}e^{(\varphi+\varphi^{\dagger})}H_{\alpha\beta}H^{\alpha\beta}\right], \tag{15}$$

where $H_{\alpha\beta}=(\varphi B_{\beta})_{,\alpha}-(\varphi B_{\alpha})_{,\beta}$ is the field strength associated to the gauge boson field B_{μ}. The action (15) is an invariant action compatible with its background geometry and originates a new kind of complex scalar-tensor theory of gravity. Given that its background geometry has a non-metricity of the Weyl-Integrable type, we will refer to $(M,g,\varphi,\varphi^{\dagger},B_{\mu})$ as the Weyl frame. In this frame the dynamics is governed by the field equations derived from the action (15). In addition, the transformations (6) to (8) can be interpreted geometrically as they lead from one frame $(M,g,\varphi,\varphi^{\dagger},B_{\mu})$ to another $(M,\bar{g},\bar{\varphi},\bar{\varphi}^{\dagger},\bar{B}_{\mu})$ sharing the same geometry, the one determined by (4). In this sense all the Weyl frames belong to the same equivalence class. However, there is one element of the class in which by redefining the metric tensor, an effective Riemannian geometry can be obtained. This issue will be the start point of the next section.

3. Field equations in the Riemann frame

As it was mentioned in the previous section, the transformations (6)–(8) lead from one Weyl frame $(M,g,\varphi,\varphi^{\dagger},B_{\alpha})$ to another $(M, \bar{g}, \bar{\varphi}, \bar{\varphi}^{\dagger}, \bar{B}_{\alpha})$. However, for the particular choice $f = -\varphi$, we can define the effective metric $h_{\mu\nu} = \bar{g}_{\mu\nu} = e^{f+f^{\dagger}} g_{\mu\nu}$ such that $\bar{\varphi} = \bar{\varphi}^{\dagger} = 0$. The interesting of this election is that in this case the condition (4) reduces to the effective Riemannian metricity condition: $\nabla_{\lambda} h_{\alpha\beta} = 0$. For this reason we will refer to this frame $(M, \bar{g}, \bar{\varphi} = 0, \bar{\varphi}^{\dagger} = 0, \bar{B}_{\alpha}) = (M, h, \bar{B}_{\alpha})$, as the Riemann frame. We will use this terminology to differentiate it from the traditional Einstein and Jordan frames employed in the scalar tensor theories we can find in the literature. The main reason to differentiate both terminologies is that in the traditional approaches the geodesics are not preserved under conformal transformations, while in the new kind of theories the geodesics are Weyl invariant [24].

In the Weyl frame the scalar field plays the role of a dilatonic geometrical scalar field while in the Riemann frame the Weyl field is no longer part of the affine structure. It means that when we go from the Weyl to the Riemann frame, the Weyl field pass from being geometrical to a physical one. In addition, once we are in the Riemann frame the action needs to be invariant only under the diffeomorphism group, and it implies that the geometrical invariance requirement for the gauge vector field B_{μ} given by (11) is no more valid in this frame. Thus due to the change of geometry, the scalar field φ and the gauge vector field B_{μ} have different properties and interpretations in each frame.

Once we have established some of the physical and geometrical differences between both frames, it is not difficult to verify that the action (15) in the Riemann frame acquires the form

$$\mathcal{S} = \int d^4x\sqrt{-h}\left[\frac{\mathcal{R}}{16\pi G} + \hat{\omega}(\varphi+\varphi^{\dagger})h^{\mu\nu}\mathbb{D}_{\mu}\varphi\mathbb{D}_{\nu}\varphi^{\dagger} - \widehat{V}(\varphi+\varphi^{\dagger}) - \frac{1}{4}H_{\mu\nu}H^{\mu\nu}\right], \tag{16}$$

where now the gauge covariant derivative becomes $\mathbb{D}_{\mu} = {}^{(R)}\nabla_{\mu} + \gamma B_{\mu}$ and the operator ${}^{(R)}\nabla_{\mu}$ denotes the Riemannian covariant derivative.

Thus, in order to restore the quadratic dependence in the scalar field, lost when we linearized (3) to obtain (4), we introduce the field transformations

$$\zeta = \sqrt{\xi}\, e^{-\varphi}, \tag{17}$$

$$A_{\mu} = B_{\mu}\ln(\zeta/\sqrt{\xi}), \tag{18}$$

where ξ is a constant introduced so that the field ζ has the correct physical units. Hence, the action (16) can be written as

$$\mathcal{S} = \int d^4x\sqrt{-h}\left[\frac{\mathcal{R}}{16\pi G} + \frac{1}{2}\omega(\zeta\zeta^{\dagger})h^{\mu\nu}D_{\mu}\zeta(D_{\nu}\zeta)^{\dagger} - V(\zeta\zeta^{\dagger}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{19}$$

being $\mathcal{D}_{\mu}\zeta \equiv \zeta\mathbb{D}_{\mu}(\ln\frac{\zeta}{\sqrt{\xi}}) = {}^{(R)}\nabla_{\mu}\zeta + \gamma A_{\mu}\zeta$ the effective covariant derivative, $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} = -H_{\mu\nu}$ is the Faraday tensor and where we have made the following identifications

$$\frac{\omega(\zeta\zeta^{\dagger})}{2} \equiv \frac{\widehat{\omega}(\ln\frac{\zeta\zeta^{\dagger}}{\xi})}{\zeta\zeta^{\dagger}}, \tag{20}$$

$$V(\zeta\zeta^{\dagger}) \equiv \widehat{V}\left(\ln\frac{\zeta\zeta^{\dagger}}{\xi}\right). \tag{21}$$

Notice that the action (19) is invariant under the gauge transformations

$$\bar{\zeta} = \zeta e^{\gamma\theta(x)} \tag{22}$$

$$\bar{A}_{\mu} = A_{\mu} - \theta_{,\mu}, \tag{23}$$

where $\theta(x)$ is a well-behaved function. Hence, due to the presence of the last term in (19) and the transformations (22) and (23), we can interpret that A_{μ} can play the role of an electromagnetic potential.

The action (19) corresponds to an action of a complex scalar field minimally coupled to gravity in the presence of a free electromagnetic field where the scalar field has $U(1)$ symmetry. Thus, due to the fact that the electromagnetic potential A_{μ} enters in the covariant derivative $\mathcal{D}_{\mu}$, the theory derived from (19) can be interpreted as a gravitoelectromagnetic theory. Notice that in our formalism, the part of (19) that we relate with electromagnetism has its origin in the Weyl invariance of the action (10), which is not the case when electromagnetism is introduced in traditional approaches of scalar-tensor theories of gravity.

4. A Higgs inflation model

In this section we formulate a Higgs inflationary model from the gravitoelectromagnetic theory developed in the previous sections. With this idea in mind let us consider the Higgs potential in the Weyl frame in the form

$$\tilde{V}(\Phi\Phi^{\dagger}) = \frac{\lambda}{4}\left(\Phi\Phi^{\dagger} - \sigma^2\right)^2, \tag{24}$$

where $\lambda = 0.129$ and the vacuum expectation value for electroweak interaction $\sigma = 246$ GeV. These values according to the best-fit experimental data [28,29]. Thus, the Higgs potential in terms of the field ζ in the Riemann frame reads

$$V(\zeta\zeta^{\dagger}) = \frac{\lambda}{4}\left(\frac{\zeta\zeta^{\dagger}}{\xi} - \sigma^2\right)^2. \tag{25}$$

The minimum of the potential $\|\zeta\zeta^{\dagger}\| = \sqrt{\xi}\sigma$ results to be also invariant under (22). However, if we propose $\zeta = \zeta^{\dagger}$ we get $\|\bar{\zeta}^2\| \neq \|\zeta^2\|$, breaking in this manner the symmetry. Thus, excitations about the ground state of (25) can be written in the form

$$\zeta(x^{\mu}) = \sqrt{\xi}\,\sigma + \mathcal{Q}(x^{\mu}), \tag{26}$$

where $\mathcal{Q}(x)$ is the Higgs scalar field. It can be verified with the help of (26) that the kinetic term in (19) gives

$$\frac{\omega(\zeta)}{2}\mathcal{D}^{\nu}\zeta\mathcal{D}_{\nu}\zeta = \frac{\omega_{eff}(\mathcal{Q})}{2}\left(\partial^{\nu}\mathcal{Q}\partial_{\nu}\mathcal{Q} - \gamma^2\xi\sigma^2A^{\nu}A_{\nu} - 2\gamma^2\sqrt{\xi}\,\sigma\,\mathcal{Q}A^{\nu}A_{\nu} - \gamma^2\mathcal{Q}^2A^{\nu}A_{\nu}\right), \tag{27}$$

where $\omega_{eff}(\mathcal{Q}) = \omega(\sqrt{\xi}\sigma + \mathcal{Q})$. However, in order to develop a Higgs inflationary model, the cosmological principle restricts the existence of the field A_{μ} on large cosmological scales. Thus, it results convenient the gauge election: $\theta_{,\mu} = A_{\mu}$ or equivalently $\bar{A}_{\mu} = 0$. Under this gauge election, the terms in (27) that depend of the electromagnetic field A_{μ} become null and thus the action (19) leads to

$$\mathcal{S} = \int d^4x\sqrt{-h}\left[\frac{R}{16\pi G} + \frac{1}{2}\omega_{eff}(\mathcal{Q})h^{\mu\nu}\mathcal{Q}_{,\mu}\mathcal{Q}_{\nu} - V_{eff}(\mathcal{Q})\right], \tag{28}$$

where $V_{eff}(\mathcal{Q}) = V(\sqrt{\xi}\sigma + \mathcal{Q})$. Now, in order to have a scalar field with a canonical kinetic term we use the field transformation

$$\phi(x^{\sigma}) = \int\sqrt{\omega_{eff}(\mathcal{Q})}\,d\mathcal{Q}. \tag{29}$$

Thus, the action for the Higgs field (28) yields

$$\mathcal{S} = \int d^4x\sqrt{-h}\left[\frac{\mathcal{R}}{16\pi G} + \frac{1}{2}h^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - U(\phi)\right], \tag{30}$$

where

$$U(\phi) = V_{eff}[\mathcal{Q}(\phi)] = \frac{\lambda}{4}\left[\frac{(\sqrt{\xi}\sigma + \mathcal{Q}(\phi))^2}{\xi} - \sigma^2\right]^2, \tag{31}$$

is the potential written in term of the new field ϕ. Straightforward calculations show that the action (30) leads to the field equations

$$G_{\alpha\beta} = -8\pi G[\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}h_{\alpha\beta}\left(\phi^{,\mu}\phi_{,\mu} + 2U(\phi)\right)], \tag{32}$$

$$\Box\phi + U'(\phi) = 0, \tag{33}$$

with $\Box$ denoting the D'Alambertian operator and the prime representing derivative with respect to ϕ. Now, we consider a 3D-spatially flat Friedmann–Robertson–Walker metric in the form

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{34}$$

with $a(t)$ being the usual cosmological scale factor. As it is usually done in inflationary frameworks, the cosmological principle allow us to assume that the inflaton scalar field ϕ, given by (29), can be written in the form

$$\phi(x^\lambda) = \phi_c(t) + \delta\phi(x^\lambda), \tag{35}$$

where $\phi_c(t) = \left\langle\phi(x^\lambda)\right\rangle$, $\langle\delta\phi\rangle = \left\langle\delta\dot{\phi}\right\rangle = 0$. Here $\delta\phi$ denotes the quantum fluctuations of the inflaton scalar field and $\langle\rangle$ represents expectation value. The fluctuations of the inflaton field correspond in the Weyl frame to fluctuations of the affine connection. It follows from the Eqs. (33) and (34) that the classical and quantum parts for the inflaton field can be written respectively as

$$\ddot{\phi}_c + 3H\dot{\phi}_c + U'(\phi_c) = 0, \tag{36}$$

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{1}{a^2}\nabla\delta\phi + U''(\phi_c)\delta\phi = 0, \tag{37}$$

where $H = \dot{a}/a$ is the Hubble parameter. Now, considering that the universe is filled with a perfect fluid, the classical part of (32) leads to the Friedmann equations

$$H^2 = \frac{\rho}{3M_p^2}, \tag{38}$$

$$\dot{H} = -\frac{1}{2M_p^2}(\rho + p), \tag{39}$$

where $M_p = (8\pi G)^{-1/2} = 2.45 \cdot 10^{18}$ GeV is our planckian mass convention, $\rho = \frac{1}{2}\dot{\phi}_c^2 + U(\phi_c)$ is the energy density and $p = \frac{1}{2}\dot{\phi}_c^2 - U(\phi_c)$ is the pressure, all measured respect to a class of comoving observers. Under the slow-roll condition $|\dot{\phi}_c^2/2| \ll |U(\phi_c)|$, the equation of state parameter become $\eta_\phi = p/\rho \simeq -1$, which is a necessary condition to have inflation. In this manner, the classical part of the inflaton field is given by the Eqs. (36), (38) and (39), whereas their quantum fluctuations are governed by the expression (37).

Now, by means of (36) and (38), the classical part of the inflaton field ϕ_c is determined by

$$\dot{\phi}_c = -\frac{M_p}{\sqrt{3}}\frac{U'(\phi_c)}{\sqrt{U(\phi_c)}}. \tag{40}$$

Thus, in order to illustrate how the formalism works let us consider the anzats

$$\omega_{eff}(Q) = \frac{1}{\left[1 - \beta^2(\sqrt{\xi}\sigma + Q)^4\right]^{5/2}}, \tag{41}$$

where β is a constant parameter with units of M_p^{-2}. Thus Eq. (29) yields

$$\phi = \frac{\sqrt{\xi}\sigma + Q}{\left[1 - \beta^2(\sqrt{\xi}\sigma + Q)^4\right]^{1/4}}. \tag{42}$$

The expression (41) is free of pole singularities when $1 - \beta(\sqrt{\xi}\sigma + Q)^4 > 0$, i.e. when $\frac{\phi^4}{1+\beta\phi^4} < \frac{1}{\beta^2}$ holds, which is fulfilled during inflation. Therefore the potential (31) reads

$$U(\phi) = \frac{\lambda}{4\xi^2}\left(\frac{\phi^4}{1 + \beta^2\phi^4}\right). \tag{43}$$

It is not difficult to see that the choice of the anzats (41) allows the effective Higgs potential (31) to exhibit a plateau for enough large field values making possible a suitable slow-roll inflation. Something similar is used for example in [30]. After inflation begins when the condition $\beta^2\phi^4 \ll 1$ holds, the potential (43) becomes

$$U(\phi) \simeq \frac{\lambda}{4\xi^2}\phi^4. \tag{44}$$

Thus, it follows from (43) and (40) that ϕ_c is given implicitly by

$$t - t_0 + \frac{\beta^2}{6\alpha}\left(\phi_c^4\sqrt{1 + \beta^2\phi_c^4} - \phi_0^4\sqrt{1 + \beta^2\phi_0^4}\right) + \frac{2}{3\alpha}\left(\sqrt{1 + \beta^2\phi_c^4} - \sqrt{1 + \beta^2\phi_0^4}\right) + \frac{1}{2\alpha}tanh^{-1}\left(\frac{1}{\sqrt{1 + \beta^2\phi_c^4}}\right) - \frac{1}{2\alpha}tanh^{-1}\left(\frac{1}{\sqrt{1 + \beta^2\phi_0^4}}\right) = 0, \tag{45}$$

where $\phi_0 = \phi(t_0)$ with t_0 being the time when inflation begins. Thus, in view that the number of e-foldings is given by

$$N(\phi) = M_p^{-2}\int_{\phi_e}^{\phi}\frac{U(\phi)}{U'(\phi)}d\phi, \tag{46}$$

being ϕ_e the value of the inflaton field at the end of inflation, the classical scalar field ϕ_c in terms of N has the form

$$\phi_c(N) = \frac{\sqrt{\beta\Delta^{1/3}\left(\Delta^{2/3} - 1\right)}}{\beta\Delta^{1/3}}, \tag{47}$$

where

$$\Delta = 12\beta NM_p^2 + \sqrt{1 + 144N^2\beta^2M_p^4}. \tag{48}$$

For the potential (44) the expression (45) reduces to

$$\phi_c(t) = \phi_e e^{2M_p\sqrt{\frac{\lambda}{3\xi^2}}(t_e - t)}, \tag{49}$$

which near to the end of inflation can be approximated by

$$\phi_c(t) \simeq \phi_e\left[1 + 2M_p\sqrt{\frac{\lambda}{3\xi^2}}(t_e - t)\right], \tag{50}$$

with t_e denoting the time when inflation ends. Thus, employing (38), (44) and (49) it is obtained an approximated scale factor of the form

$$a = a_e\exp\left[\frac{\phi_e^2}{8M_p^2}\left(1 - \exp\left(4M_p\sqrt{\frac{\lambda}{3\xi^2}}(t_e - t)\right)\right)\right], \tag{51}$$

where $a_e = a(t_e)$. For $t \simeq t_e$ (51) can be approximated by

$$a(t) \simeq \tilde{a}_e\exp\left(\frac{\phi_e^2}{2M_p}\sqrt{\frac{\lambda}{3\xi^2}}\,t\right) \tag{52}$$

where $\tilde{a}_e = a_e\exp\left(-\frac{\phi_e^2}{2M_p}\sqrt{\frac{\lambda}{3\xi^2}}\,t_e\right)$. Thus the Hubble parameter associated with (51) is then

$$H(t) = \frac{1}{\sqrt{3}M_p}\sqrt{\frac{\lambda}{4\xi^2}}\,\phi_e^2\exp\left(4M_p\sqrt{\frac{\lambda}{3\xi^2}}(t_e - t)\right). \tag{53}$$

Therefore, near to the end of inflation (53) can be approximated by

$$H(t) \simeq \frac{\phi_e^2}{\sqrt{3}M_p}\sqrt{\frac{\lambda}{4\xi^2}}\left[1+4M_p\sqrt{\frac{\lambda}{3\xi^2}}\,(t_e-t)\right]. \tag{54}$$

On the other hand, in order to have agreement with PLANCK data, Higgs inflation requires an energy scale corresponding to an initial Hubble parameter of the order $H_0 \simeq 10^{11}$–10^{12} GeV, for an average Higgs mass of the order $M_h \simeq 125.7$ GeV [31,32]. Therefore we obtain

$$H_0 \simeq \frac{\lambda}{2\sqrt{3}}\frac{1}{\beta\xi M_p} \simeq 10^{11} - 10^{12}\ \text{GeV}. \tag{55}$$

Using $\lambda = 0.13$ and $M_p = 1.22 \cdot 10^{19}$ GeV [32], we obtain that ξ must vary in the interval: $[3.7528 \cdot 10^{-14}, 3.7528 \cdot 10^{-13}](\beta M_p)^{-1}\ (\text{GeV})^{-1}$.

Now, following a standard quantization procedure, the commutator relation for $\delta\phi$ and its canonical conjugate momentum $\Pi^0_{(\delta\phi)} = \frac{\partial L}{\partial\dot{\delta\phi}}$ is given by

$$\left[\delta\phi(t,\bar{x}), \Pi^0_{(\delta\phi)}(t,\bar{x}')\right] = i\delta^{(3)}(\bar{x}-\bar{x}'). \tag{56}$$

Thus, using $\Pi^0_{(\delta\phi)} = \sqrt{-h}\,[(\dot{\phi}_c + \dot{\delta\phi})]$ the commutator (56) reads

$$\left[\delta\phi(t,\bar{x}), \dot{\delta\phi}(t,\bar{x}')\right] = \frac{i}{\sqrt{-h}}\,\delta^{(3)}(\bar{x}-\bar{x}'). \tag{57}$$

We introduce the auxiliary field $\delta\chi$ as

$$\delta\phi(t,\bar{x}) = \exp\left(-\frac{3}{2}\int H(t)dt\right)\delta\chi(t,\bar{x}). \tag{58}$$

We consider the Fourier expansion

$$\delta\chi(t,\bar{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left[a_k e^{i\bar{k}\cdot\bar{x}}\eta_k(t) + a_k^\dagger e^{-i\bar{k}\cdot\bar{x}}\eta_k^*(t)\right], \tag{59}$$

with the asterisk mark denoting complex conjugate and, a_k and $a_k^\dagger$ being the annihilation and creation operators. These operators satisfy the commutator algebra

$$\left[a_k, a_{k'}^\dagger\right] = i\delta^{(3)}(\bar{k}-\bar{k}'), \qquad [a_k, a_{k'}] = \left[a_k^\dagger, a_{k'}^\dagger\right] = 0. \tag{60}$$

The quantum modes $\eta_k^{(end)}(t)$ at the end of inflation, according to (37), (43), (52), (54) and (58) are given by

$$\ddot{\eta}_k^{(end)} + \left[\frac{k^2}{\tilde{a}_e^2 e^{2H_e t}} - \frac{9}{4}H_e^2 + U''(\phi_e)\right]\eta_k^{(end)} = 0, \tag{61}$$

where $H_e = H(t_e)$ and

$$U''(\phi_e) = -\frac{\lambda\phi_e^2(-3+5\beta^2\phi_e^4)}{\xi^2(1+\beta^2\phi_e^4)}. \tag{62}$$

Selecting the Bunch Davies condition [33], the normalized solution of (61) reads

$$\eta_k^{(end)} = \frac{1}{2\tilde{a}_e}\sqrt{\frac{\pi}{\tilde{a}_e H_e}}\,\mathcal{H}_\nu^{(1)}[z(t)], \tag{63}$$

where $\mathcal{H}_\nu^{(1)}$ is the first kind Hankel function, the parameter $\nu = (1/2)\sqrt{9-(4U''(\phi_e)/H_e^2)}$ and $z(t) = [k/(\tilde{a}_e H_e)]e^{-H_e t}$.

The amplitude of $\delta\phi$ on the infrared sector is given by the expression

$$\left\langle\delta\phi^2\right\rangle_{IR} = \frac{2^{2\nu}\Gamma^2(\nu)}{8\pi^3\tilde{a}_e^2}\frac{e^{-(3-2\nu)H_e t_e}}{(\tilde{a}_e H_e)^{1-2\nu}}\int_0^{\varepsilon k_H}\frac{dk}{k}k^{3-2\nu}, \tag{64}$$

where $\varepsilon = k_{max}^{IR}/k_p \ll 1$ is a dimensionless parameter with $k_{max}^{IR} = k_H(t_r)$ being the wave number related to the Hubble radius at the time t_r, which is the time when the modes re-enter to the horizon and k_p is the Planckian wave number. It is well-known that for a Hubble parameter $H = 0.5 \times 10^{-9} M_p$, the values of ε range between 10^{-5} and 10^{-8}, and this corresponds to a number of e-foldings at the end of inflation $N_e = 63$. Hence the squared $\delta\phi$-fluctuations has a power-spectrum

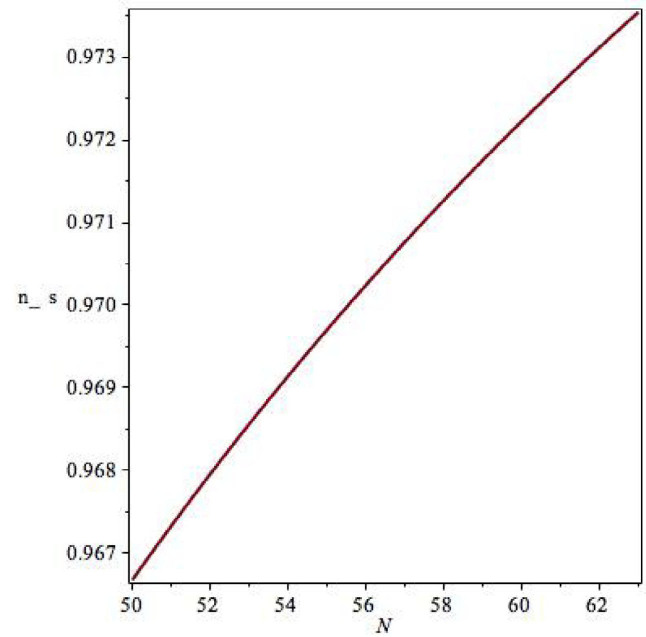

Fig. 1. This plot shows the spectral index n_s versus the number of e-foldings N, which runs from 50 to 63.

$$\mathcal{P}_s(k) = \frac{2^{2\nu}\Gamma^2(\nu)}{8\pi^3\tilde{a}_e^2}\frac{e^{-(3-2\nu)H_e t_e}}{(\tilde{a}_e H_e)^{1-2\nu}}k^{3-2\nu}. \tag{65}$$

On the other hand, the scalar to tensor ratio r and the scalar spectral index n_s are given by $r = 16\epsilon$ and $1 - n_s = 6\epsilon - 2\eta$, being ϵ and η the slow-roll parameters

$$\epsilon = \frac{M_p^2}{2}\left(\frac{U'}{U}\right), \quad \eta = M_p^2\left(\frac{U''}{U}\right). \tag{66}$$

Thus, with the help of (43) and (47) the scalar spectral index is given by

$$1 - n_s = \frac{8\beta M_p^2\Delta(5\Delta^{4/3} - 7\Delta^{2/3} + 5)}{(\Delta^{2/3}-1)(\Delta^{4/3}-\Delta^{2/3}+1)^2} \simeq \frac{5}{3N}. \tag{67}$$

It is not difficult to see from (67) that for a number of e-foldings $N = 63$ the scalar spectral index is approximately $n_s \simeq 0.9735$, which is in agreement with PLANCK observations: $n = 0.968 \pm 0.006$ [34]. Similarly, the scalar to tensor ratio results to be

$$r = \frac{128M_p^2\beta\Delta^{5/3}}{(\Delta^{2/3}-1)(\Delta^{4/3}-\Delta^{2/3}+1)^2} \simeq \frac{128}{\beta^{2/3}M_p^{4/3}}\frac{1}{(24N)^{5/3}}. \tag{68}$$

Again, it follows from (68) that for $N = 63$ and the parameter $\beta \simeq 0.01629 M_p^{-2}$, that the scalar to tensor ratio is of the order $r \simeq 0.01$, in consistency with the PLANCK data ($r < 0.11$). Hence, for such value of β the parameter ξ must ranges in the interval $(2.81047 \cdot 10^7, 2.81047 \cdot 10^8)$. Thus, taking this value of ξ Eq. (62) indicates that $U''(\phi_e) \ll 1$, and therefore $\nu \simeq 3/2$ which according to (65) corresponds to a nearly scale invariant power spectrum at the end of inflation $\mathcal{P}_s(k) \sim H_e^2$. Fig. 1 exhibits how vary the spectral index n_s versus the number of e-foldings N, while Fig. 2 shows the behaviour of the scalar to tensor index versus N and the parameter β.

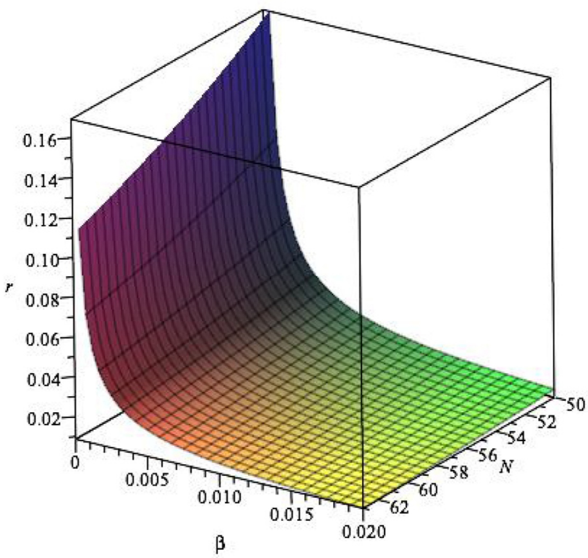

Fig. 2. This plot shows the behaviour of the scalar to tensor ratio r versus the parameter β and the number of e-foldings N. We have used β from 0 to 0.020.

5. Final remarks

In this letter we have derived a Higgs inflationary model on the framework of a new kind of complex scalar-tensor theory of gravity that we called: geometrical complex scalar-tensor theory of gravity. In this approach we consider a complex scalar-tensor theory compatible with it's background geometry. We mean by compatibility the invariance of the action under the symmetry group of its background geometry. The former is obtained here by employing the Palatini's variational principle, resulting that the background geometry for a complex scalar-tensor theory is a kind of complex Weyl-integrable geometry. As has occurred in the case of real geometrical scalar-tensor theories, there are two frames: the Weyl and the Riemann frames. The Riemann frame is obtained by a particular gauge election of the Weyl-transformations: $f = -\varphi$. In the Weyl frame the scalar field is part of the affine structure of the space–time manifold, whereas in the Riemann frame it can be considered as a physical field. This is why general relativity can be recovered in the Riemann frame.

As an application of the formalism we developed a Higgs inflationary model. An interesting feature of our model is that the inflaton and the Higgs fields can be both identified with the Weyl scalar field. Moreover, due to the compatibility of the new complex scalar-tensor theory with its background geometry, the Higgs potential can be rescaled enough to generate the primordial inflation of the universe. We obtain a super-De-Sitter expansion at the beginning of inflation. The infrared power spectrum results nearly scale invariant at the end of inflation for $\beta \simeq 0.01629\, M_p^{-2}$. For $N = 63$ e-foldings we obtain an spectral index $n_s \simeq 0.9735$ and a scalar to tensor ratio $r \simeq 0.01$, which are in agreement with PLANCK observations [34]. The value of the spectral index n_s could fit even better in PLANCK data, if the contribution of neutrinos were considered in our formalism, however, we leave this topic for further investigation. Evidence of this improvement in n_s considering neutrino contributions has already been done for example in [35].

One important feature of Higgs inflation developed in our formalism is that the model is free of the unitarity problem. This is because when we pass from the Weyl to the Riemann frame by means of the Weyl transformations (6)–(8) the Ricci tensor is maintained unaltered, avoiding in this way the appearance of the extra term that originates the unitarity violation. In the Weyl frame we have employed the Palatini principle and in the same manner as it was shown in [36], this consideration avoids the unitarity problem.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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