Review Article

Bulk Viscous Cosmology: Unified Dark Matter

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The bulk viscosity is introduced to model the current observational cosmos and the unified dark matter (dark matter and dark energy). The viscous unified model assumes that the universe is filled with a single nonperfect fluid with a bulk viscosity. We review the general framework for the viscous cosmology model building finding that the Hubble parameter has possessed a direct connection with the bulk viscosity coefficient. For the concrete form of the bulk viscosity, the Hubble parameter which has the scaling relation with the redshift can be obtained generally. We discuss two viscosity models and the cosmological evolution to which they lead in detail. Using SNe Ia data, CMB shift, as well as BAO observations, the viscosity model parameters can be fitted very well. We also briefly review the fitting method in this paper.

1. Introduction

Both the dark matter problem and the cosmic acceleration problem challenge physicists' understanding of the universe. In the standard ΛCDM model, two mixed fluids dark matter and dark energy fluid, are assumed. These two fluids influence the cosmic evolution separately. However, present gravitational probe does not have the ability to differentiate these two fluids. This is the dark degeneracy problem [1, 2]. It is reasonable to model dark matter and dark energy with single fluid or single-field assumption. Some unified models have been proposed to detect the possibility of this unified assumption, like unified dark fluid model [3–7], which assumes the single-fluid equation of state; Chaplygin gas model and generalized Chaplygin gas model [8–12], which discuss the cosmology consequences of an exotic equation of state; scalar field method [13–15].

The introduction of viscosity into cosmology has been investigated from different viewpoints [16, 17]. There are some recent developments like dark energy model [18–21], the cosmic singularity [22]. In this paper, we give a brief introduction to unify dark matter and dark energy with viscosity medium. In such models, the universe is assumed to be filled with viscous single fluid [23–29]. The cosmic density is not separated as dark energy part and dark matter part.

The bulk viscosity contributes to the cosmic pressure and plays the role of accelerating the universe. After considering the bulk viscosity, the cosmic pressure can be written as

$$p = (\gamma - 1)\rho - 3\zeta H, \qquad (1)$$

where γ parameterizes the equation of state. Generally the form of bulk viscosity is chosen as a time-dependent function. In [30–35], a density-dependent viscosity $\zeta = \alpha\rho^m$ coefficient is investigated extensively. For modeling the unified dark matter and dark energy, it is often assumed that the parameter $\gamma = 1$, that the pressure of the viscosity fluid is zero, and the viscosity term contributes an effective pressure. There arise some problems here. From the observational results [36], the cosmic density is nearly equals to the cosmic pressure. In the viscosity model, the viscosity term dominates the cosmic pressure and surpasses the pressure contributions from other cosmic matter constitutions, which contradicts the traditional fluid theory. The works in [37, 38] propose nonstandard interaction mechanism to solve this problem. Obviously, it is important to build solid foundation for the research of the viscous cosmology.

Equation of state $w < -1$ lies in the phantom region. It is shown that cosmology models with such equation of state possess the so-called the future singularity that is called

the Big Rip [39]. The larger viscosity model parameter space can help to solve the cosmic singularity problem and produce different kinds of evolution modes of the future universe; for more details [28].

The rest of this paper is organized as follows. In Section 2, general framework of the viscosity model will be reviewed. In Section 3, we discuss the modeling of the unified model with viscosity. In this section, two concrete models are analyzed. In Section 4, data fitting method is introduced briefly.

2. General Framework

We consider the standard Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \tag{2}$$

For the sake of simplicity, we choose the flat geometry $k = 0$, which is also favored by the update result of the cosmic background radiation measurement.

The general stress-energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} - \zeta \theta h_{\mu\nu}, \tag{3}$$

where ζ is the bulk viscosity. The expansion factor θ is defined by $\theta = U_{;\mu}^\mu = 3(\dot{a}/a)$, and the projection tensor $h_{\mu\nu} \equiv g_{\mu\nu} + U_\mu U_\nu$. In the comoving coordinates, the four velocity $U^\mu = (1, 0, 0, 0)$. We do not specify the concrete form of ζ in this section. Generally speaking, ζ is a quantity evolving with time t or the scale factor $a(t)$. We will see below that nontrivial and more complicated ζ can produce different results especially useful for the late universe modeling.

From the usual Einstein equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{4}$$

we obtain the two equations which we call the modified Friedmann equations:

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{8\pi G}{3} \rho, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3} (\rho + 3\tilde{p}), \end{aligned} \tag{5}$$

where $\tilde{p}$ is an effective pressure and $\tilde{p} = p - \zeta\theta$.

The covariant conservation equation $T_{;\mu}^{0\mu} = 0$ yields

$$\dot{\rho} + (\rho + \tilde{p}) \theta = 0. \tag{6}$$

The existence of a bulk viscosity contributes a modification to the pressure p, thus we see that the Friedmann equation and the covariant conservation equation are invariant under the transformation

$$p \longrightarrow \tilde{p} = p - \zeta\theta. \tag{7}$$

The covariant energy conservation equation becomes

$$\dot{\rho} + (\rho - \zeta\theta) \theta = 0. \tag{8}$$

We define the dimensionless Hubble parameter here as

$$h^2 = \frac{H^2}{H_0^2} = \frac{\rho}{\rho_{cr}}, \tag{9}$$

where $\rho_{cr} = 3H_0^2/8\pi G$ is the critical density now. Using the dimensionless Hubble parameter, (8) can be transformed as

$$\frac{1}{H_0} \frac{d(h^2)}{dt} + 3h^3 = 9\lambda h^2, \tag{10}$$

where the bulk viscosity is redefined as $\lambda = H_0 \zeta / \rho_{cr}$. Through the simple relation between scale factor $a(t)$ and the redshift z

$$dt = \frac{1}{aH} da, \tag{11}$$

we transform (10) into a differential equation with respect to the scale factor $a(t)$ as follows:

$$\frac{dH}{da} + \frac{3}{2a} H = \frac{3\zeta}{2a}. \tag{12}$$

Solving this equation, we obtain an integral form of $H(a)$ as

$$H(a) = C_1 a^{-3/2} + \left[\int \frac{3\zeta}{2a} \exp\left(\int \frac{3}{2a} da \right) da \right] \exp\left(- \int \frac{3}{2a} da \right). \tag{13}$$

Different forms of viscosity can be used here to make this integral calculable, numerically or exactly.

3. Unified Single Fluid

3.1. Redshift-Dependent Model. In [40], the authors assume that the bulk viscosity takes the form of a Hubble parameter-dependent function. A redshift-dependent viscosity is proposed in [27]. This bulk viscosity is a combination of a constant and a scaling relation term

$$9\lambda = \lambda_0 + \lambda_1 (1 + z)^n, \tag{14}$$

where n is an integer and λ_0 and λ_1 are two constants, which will be fitted from the observational datasets.

After taking account of this ansatz, the integration is easy to work out. We get

$$\begin{aligned} h^2(z) = \lambda_2^2 (1 + z)^3 + \frac{2}{3} \lambda_0 \lambda_2 (1 + z)^{1.5} \\ - \frac{2\lambda_0 \lambda_1}{3(2n - 3)} (1 + z)^n + \frac{\lambda_1^2}{(2n - 3)^2} (1 + z)^{2n} \\ - \frac{2\lambda_1 \lambda_2}{2n - 3} (1 + z)^{n+1.5} + \frac{\lambda_0^2}{9}. \end{aligned} \tag{15}$$

Since we have assumed the spatial flatness of the universe, the consistency condition requires that $h(0) = 1$. Thus this sets a constraint on the model parameters as

$$\frac{\lambda_0}{3} = 1 - \lambda_2 + \frac{\lambda_1}{2n - 3}. \tag{16}$$

We remind the readers that we make the single-fluid assumption above, and we do not concretely specify the constitutions of the cosmic density ρ. In this single-fluid model, values of model parameters λ_0, λ_1, and λ_2 will be fitted, and their meanings are not explained. But when we compare it with two-fluid model, having in mind that the universe is filled with dark matter and dark energy fluid, more constraints can be added. The solution consists of terms with different scaling relation. The first term has the form of $C(1+z)^3$, which has the same evolution behavior as that of the cold dark matter. Their simplicity leads us to parameter λ_2 correspond to dark matter ratio Ω_m

$$\lambda_2^2 = \Omega_m. \tag{17}$$

This identity can help us utilize more data to constrain the viscosity model. The result is also consistent with that obtained from the standard model (ΛCDM). The shift parameter $\mathcal{R}$ [41, 42] and the distance parameter $\mathcal{A}$ are defined as

$$\mathcal{R} \equiv \sqrt{\Omega_m} \int_0^{z_*} \frac{\mathrm{d}\, z'}{h(z')}, \tag{18}$$

$$\mathcal{A} \equiv \sqrt{\Omega_m}\, h(z_b)^{-1/3} \left(\frac{1}{z_b} \int_0^{z_b} \frac{\mathrm{d}z'}{h(z')} \right)^{2/3}, \tag{19}$$

respectively. Both of them are dependent on dark matter ratio Ω_m, and in the joint statistical analysis, they provide strong constraint on Ω_m.

3.2. Effective Equation of State Model. Another viscosity model reviewed here is proposed in [25], where a general form of time-dependent viscosity is discussed:

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{\dot{a}}. \tag{20}$$

An interesting feature of this model is its effective equivalence to the following equation of state:

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H}, \tag{21}$$

where p_0, w_H, w_{H2}, and w_{dh} are free parameters. The corresponding between two groups of coefficients are

$$w_H = -3\zeta_0,$$
$$w_{H2} = -3(\zeta_1 + \zeta_2),$$
$$w_{dH} = -3\zeta_2. \tag{22}$$

The parameterized bulk viscosity combines terms related to the "velocity" $\dot{a}$ and "acceleration" $\ddot{a}$, which can be seen to describe the dynamics of the cosmic nonperfect fluid. After eliminating p and ρ, a differential equation about the scale factor $a(t)$ can be obtained as

$$\frac{\ddot{a}}{a} = \frac{-(3\gamma - 2)/2 - (\kappa^2/2)w_{H2} + (\kappa^2)w_{dH}}{1 + (\kappa^2)w_{dH}} \left(\frac{\dot{a}}{a} \right)^2$$
$$+ \frac{-(\kappa^2)w_H}{1 + (\kappa^2/2)w_{dH}} \frac{\dot{a}}{a} + \frac{-(\kappa^2/2)p_0}{1 + (\kappa^2/2)w_{dH}}. \tag{23}$$

Another feature of this model is that this differential equation can be solved exactly, and the evolution function of the scale factor $a(t)$ is definite. This evolution function is especially convenient for discussing the cosmic singularity.

With the initial conditions $a(t_0) = a_0$ and $\theta(t_0) = \theta_0$, when $\tilde{\gamma} \neq 0$, the scale factor can be obtained as

$$a(t) = a_0 \left\{ \frac{1}{2} \left(1 + \tilde{\gamma}\theta_0 T - \frac{T}{T_1} \right) \exp\left[\frac{t - t_0}{2} \left(\frac{1}{T} + \frac{1}{T_1} \right) \right] \right.$$
$$\left. + \frac{1}{2} \left(1 - \tilde{\gamma}\theta_0 T + \frac{T}{T_1} \right) \exp\left[-\frac{t - t_0}{2} \left(\frac{1}{T} - \frac{1}{T_1} \right) \right] \right\}^{2/3\tilde{\gamma}}, \tag{24}$$

where the parameters are redefined as

$$\tilde{\gamma} = \frac{\gamma + (\kappa^2/3)w_{H2}}{1 + (\kappa^2/2)w_{dH}},$$

$$\frac{1}{T_1} = \frac{-(\kappa^2/2)w_H}{1 + (\kappa^2/2)w_{dH}},$$

$$\frac{1}{T_2^2} = \frac{-(\kappa^2/2)p_0}{1 + (\kappa^2/2)w_{dH}},$$

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{6\tilde{\gamma}}{T_2^2}. \tag{25}$$

From Friedmann equation, ρ can be written as

$$\rho(t) = \frac{1}{3\kappa^2 \tilde{\gamma}^2} \left[\frac{(1 + \tilde{\gamma}\theta_0 T - T/T_1)(1/T + 1/T_1)\exp((t-t_0)/T) - (1 - \tilde{\gamma}\theta_0 T + T/T_1)(1/T - 1/T_1)}{(1 + \tilde{\gamma}\theta_0 T - T/T_1)\exp((t-t_0)/T) + (1 - \tilde{\gamma}\theta_0 T + T/T_1)} \right]^2. \tag{26}$$

The model parameters leave enough space to produce various evolution behaviors, which can be interpreted in different ways. In this review, we emphasze their power to unify dark energy and dark matter with the single-fluid

assumption. According to the parameters redefined above and the Friedmann equation, the equation of state can be converted to

$$p = (\widetilde{\gamma} - 1)\rho - \frac{2}{\sqrt{3}\kappa T_1}\sqrt{\rho} - \frac{2}{\kappa^2 T_2^2}. \tag{27}$$

The case $\widetilde{\gamma} = 0$ and $T_1 \rightarrow \infty$ corresponds to the ΛCDM. With the aim to unify dark energy and dark matter, the case $\widetilde{\gamma} = 1$ and $T_2 \rightarrow \infty$ is especially considered. This case corresponds to a single fluid with constant viscosity. The relation between p and ρ can be obtained from the general equation of state above as follows:

$$p = -\frac{2}{\sqrt{3}\kappa T_1}\sqrt{\rho}. \tag{28}$$

Therefore, it is straightforward to eliminate p from the covariant energy conservation equation and to work out the solution of ρ. Using Friedmann equation, $H(z)$ can be obtained

$$H(z) = H_0\Big[\Omega_\gamma(1 + z)^{3/2} + \left(1 - \Omega_\gamma\right)\Big], \tag{29}$$

where Ω_γ is the only one model parameter. Its value can be fitted from SNe Ia observational data.

4. Data Fitting

We review the method to fit the model parameters. More details are illustrated in [43]. The datasets we use are SNe Ia, BAO, and CMB. The 397 Constitution sample [44] combines the Union sample [45] and the low-redshift ($z < 0.08$) sample [46]. The comoving distance d_M in FRW coordinate is

$$d_M = \int_0^z \frac{1}{H(z')}dz'. \tag{30}$$

The apparent magnitude which is measured is

$$m \equiv M + 5\log_{10}D_L(z), \tag{31}$$

where the dimensionless luminosity $D_L \equiv H_0 d_L(z)$ and

$$d_L = (1 + z)d_M(z), \tag{32}$$

where M is the absolute magnitude which is believed to be constant for all SNe Ia. In the SNe Ia samples, data are given in terms of the distance modulus $\mu_{\rm obs} \equiv m(z) - M_{\rm obs}(z)$. The χ^2 for this procedure is written as

$$\chi^2 = \sum_{i=1}^n \left[\frac{\mu_{\rm obs}(z_i) - \mu_{\rm th}(z_i; c_\alpha)}{\sigma_{\rm obs}(z_i)}\right]^2, \tag{33}$$

where $\mu_{\rm th}$ means the distance modulus calculated from model with parameters c_α ($\alpha = 0, 1, 2, \ldots$). Together with the shift parameter $\mathcal{R}$ and the distance $\mathcal{A}$, the total $\chi^2_{\rm total}$ for the joint data analysis is given as

$$\chi^2_{\rm total} = \chi^2 + \left(\frac{\mathcal{R} - \mathcal{R}_{\rm obs}}{\sigma_{\mathcal{R}}}\right)^2 + \left(\frac{\mathcal{A} - \mathcal{A}_{\rm obs}}{\sigma_{\mathcal{A}}}\right)^2. \tag{34}$$

For the redshift-dependent model, the relation between distance modulus and redshift is plotted in Figure 1. The model calculated value and the Constitution data are compared in the figure.
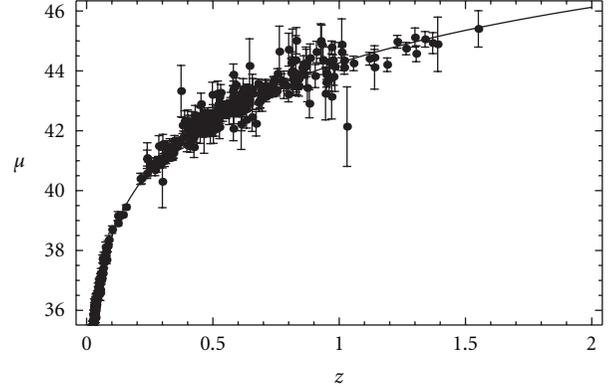
FIGURE 1: Relation between distance modulus and redshift. The solid line corresponds to the theoretical value calculated from the model concerned. The dots with error bar are the data from the 397 Constitution sample.

5. Conclusion

In this review, we discuss three aspects of the viscosity model.

(i) There is general framework for viscosity modeling. General form of Hubble parameter is presented. This general form is convenient for comparing different scale factor- (or redshift-) dependent viscosity models.

(ii) Two kinds of viscosity models are used to model unified models.

(iii) Observation constraint is necessary for model building. We can see that the fitting results are consistent with data. It is prospected that more accurate direct measurements of the Hubble constant will provide a new constraint on cosmological parameters [47].

Especially we focus on its application on modeling the unified dark energy and dark matter.

In the cosmic background level, dynamical analysis can be performed. The statefinder method is useful for discriminating different models [48–52]. Compared with ΛCDM model, evolution of the statefinder of the viscosity model is different and can be discriminated easily; more details can be found in [28, 53]. More plentiful and accurate data will improve the power of the statefinder method, which will give enough constraint on the late universe model.

We review the viscosity model which is on the level of zero order. The perturbation analysis and to confront the observed large-scale structure are especially useful for the model building and distinguishing. The model predictions need to be consistent with CMB and LSS data. Some works have investigated the perturbation aspects of the viscosity cosmology model [35, 54]. After choosing properly the corresponding model parameters, the viscosity cosmology model has been shown to have the connection with the Chaplygin gas model. Though the Chaplygin gas model can fit the SNe Ia data well at the zero order, in the perturbation level it is found that the Chaplygin gas model does not behave

in a satisfactory way. Whether the viscosity cosmology models could behave well needs further investigation.

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