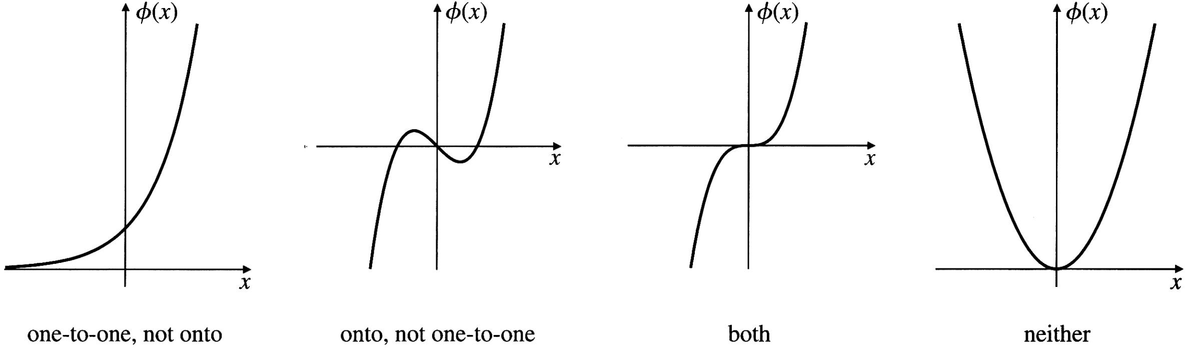


FIGURE 2.10 Types of maps.

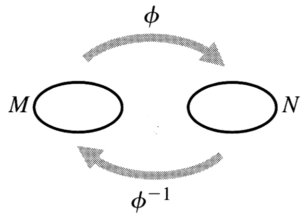
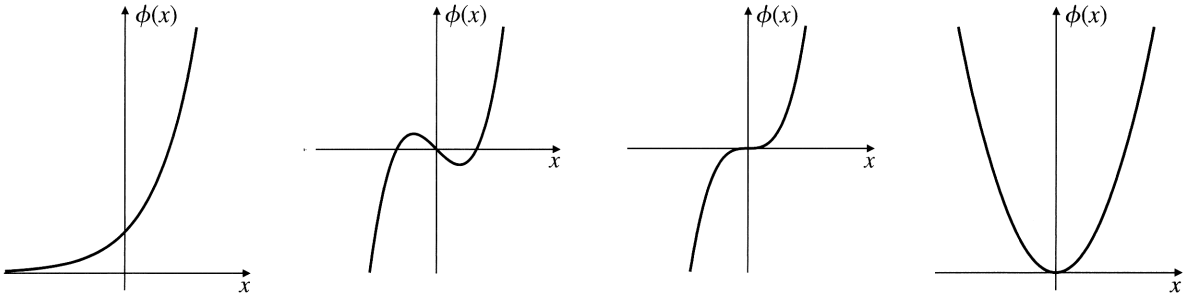


FIGURE 2.11 A map and its inverse.



one-to-one, not onto | onto, not one-to-one | both | neither

FIGURE 2.10 Types of maps.

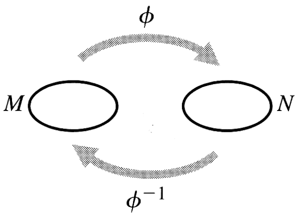


FIGURE 2.11 A map and its inverse.

The set M is known as the **domain** of the map ϕ, and the set of points in N that M gets mapped into is called the **image** of ϕ. For any subset $U \subset N$, the set of elements of M that get mapped to U is called the **preimage** of U under ϕ, or $\phi^{-1}(U)$. A map that is both one-to-one and onto is known as **invertible** (or bijective). In this case we can define the **inverse map** $\phi^{-1} : N \to M$ by $(\phi^{-1} \circ \phi)(a) = a$, as in Figure 2.11. Note that the same symbol ϕ^{-1} is used for both the preimage and the inverse map, even though the former is always defined and the latter is only defined in some special cases.

The notion of **continuity** of a map is actually a very subtle one, the precise formulation of which we won't need. Instead we will assume you understand the concepts of continuity and differentiability as applied to ordinary functions, maps $\phi : \mathbf{R} \to \mathbf{R}$. It will then be useful to extend these notions to maps between more general Euclidean spaces, $\phi : \mathbf{R}^m \to \mathbf{R}^n$. A map from $\mathbf{R}^m$ to $\mathbf{R}^n$ takes an m-tuple $(x^1, x^2, \ldots, x^m)$ to an n-tuple $(y^1, y^2, \ldots, y^n)$, and can therefore be thought of as a collection of n functions ϕ^i of m variables:

$$\begin{aligned} y^1 &= \phi^1(x^1, x^2, \ldots, x^m) \\ y^2 &= \phi^2(x^1, x^2, \ldots, x^m) \\ &\vdots \\ y^n &= \phi^n(x^1, x^2, \ldots, x^m). \end{aligned} \tag{2.8}$$

We will refer to any one of these functions as C^p if its pth derivative exists and is continuous, and refer to the entire map $\phi : \mathbf{R}^m \to \mathbf{R}^n$ as C^p if each of its component functions are at least C^p. Thus a C^0 map is continuous but not necessarily differentiable, while a C^∞ map is continuous and can be differentiated as many times as you like. Consider for example the function of one variable $\phi(x) = |x^3|$. This function is infinitely differentiable everywhere except at $x = 0$, where it is differentiable twice but not three times; we therefore say that it is C^2. C^∞ maps are sometimes called **smooth**.