The Dirac-Kerr electron.

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We discuss the relation of the Kerr spinning particle to the Dirac electron and show that the Dirac equation may naturally be incorporated into Kerr-Schild formalism as a master equation controlling the Kerr geometry. As a result, the Dirac electron acquires an extended space-time structure of the Kerr geometry - singular ring of the Compton size and twistorial polarization of the gravitational and electromagnetic fields.

Behavior of this Dirac-Kerr system in the weak and slowly changed electromagnetic fields is determined by the wave function of the Dirac equation, and is indistinguishable from behavior of the Dirac electron.

The model is based on the relation between the Kerr theorem and a 'point-like' *complex* representation of the Kerr geometry. The wave function of the Dirac equation plays in this model the role of an "order parameter" which controls dynamics, spin-polarization and twistorial structure of space-time.

Analyzing the regularization of the Kerr-Newman source and the obtained recently multi-particle Kerr-Schild solutions (hep-th/0506006,hep-th/0510246), we argue that the Dirac-Kerr electron takes an intermediate position between the one-particle Dirac description and multi-particle description of electron in QED.

I. INTRODUCTION

The Kerr solution has a fundamental meaning, penetrating all the regions of theoretical physics from cosmology to superstring theory. Interest to the Kerr solution in the physics of elementary particles has been raised recently by the conjecture that black holes may be created in the laboratory conditions.

On the other hand, the treatment of the Kerr-Newman solution as a model of electron have been considered many times [1–13] after the well known Carter remark [1] that the Kerr-Newman solution has gyromagnetic ratio $g = 2$ as that of the Dirac electron. If this coincidence is not occasional, there appears a fundamental question - what is the relation of the Dirac equation to the structure of Kerr solution?

The Dirac equation is also the discovery of fundamental importance. Practically, most of the theoretical and experimental results in the particle physics are based on the Dirac equation and its consequences. Nevertheless, the Dirac description of electron can not be considered as complete for two reasons:

-it does not take into account gravitational field of electron,

- the Dirac wave function caries a very obscure information on the space-time position and, especially, on the structure of electron.

Forming the wave packets one can only show that electron cannot be localized inside the Compton region. Meanwhile, the multi-particle QED description showed that the "naked" electron is point-like and the "dressed" one is smeared on the the Compton region. The point-like structure of electron is also confirmed by the deep inelastic scattering. There were a lot of attempts to describe the extended structure of spinning particle on the base of field models, and some of them were related to gravity.

The Kerr solution gives us a natural description of spinning particle with gravity, and moreover, it hints us at the relation to electron by the double gyromagnetic ratio [1,3], by the extension in the Compton region [2,4,8], by the wave properties [4], and by the reach spinor and twistorial structures [5,6,10–13].

The aim of this paper is to consider a model which set a relation between spinor solutions of the Dirac equation and the spinor (twistorial) structure of the Kerr solution.

Such a relation is exhibited the most naturally in the supergeneralization of the Kerr geometry [14]. However, the resulting Dirac equation turns out to be massless. So, there appears the problem of the origin of the mass term. There are expectations that the mass term will be created by a nonlinear realization of supersymmetry [15], however this program is rather complicate and has not been studied for the Kerr geometry so far. In the paper [16] authors considered in the whole the twofold Kerr geometry as a quantum state which satisfies the Dirac equation. This point of view contains a rational core, however the relation between the Dirac spinors and the spinor structure of the Kerr geometry does not appear in this approach. In the series of paper [12,13,17], solutions of the Dirac equation were considered as zero modes of some axial strings which appear by excitations of the Kerr geometry and are similar to the Witten superconducting strings. These semi-infinite strings are considered as carriers of the Dirac wave function, and there appears a problem of physical interpretation of the plane Dirac wave, the basic solution for a free electron. It is clear, that the Dirac plane wave cannot dives a distribution of probability which has the support on a strings, and it cannot be cured by the formation of a wave packet. Similar difficulties with physical interpretation of the Dirac wave function are discussed in the modern relativistic quantum theory [18–20] and lead to the conclusion that the one-particle quantum theory may not be consistent.

In this paper we suggest a new approach to the relation of the Dirac and Kerr models which is based on the *assumption that the Dirac equation is complimentary to the Kerr spinning particle and plays the role of a master equation controlling the motion and orientation of the Kerr twistorial space-time structure.* In this model the Dirac spinors are matched to the spinor (twistorial) structure of the Kerr spinning particle, and the Dirac wave function plays the role of an order parameter controlling this system.

As a result, the behavior of this combined Dirac-Kerr system turns out to be determined by the Dirac wave function, and will be undistinguishable from the behavior of the Dirac electron, at least in the frame of one-particle quantum theory.

Plane of the work is following. To realize our aim we have to find a bridge between the Dirac theory and the Kerr structure, i.e. to obtain some objects which are common for both these structures.

In sec.II we consider the real and complex structures of the Kerr geometry and obtain that the motion and polarization of the Kerr spinning particle are controlled by two null vectors k_L and k_R which are related to a "point-like" *complex* representation of the Kerr geometry.

In sec.III, analyzing the structure of Dirac equation in the Weyl basis, we obtain two similar null vectors k_L and k_R which are determined by the Dirac wave function Ψ and control the dynamics and polarization of the Dirac electron.

It allows as to set a link between the Dirac and Kerr spinning particle, which is performed in sec.IV. As a result, the the Dirac-Kerr electron acquires a definite extended space-time structure of the Kerr geometry, and the Dirac wave function takes the role of an "order parameter" which controls polarization and dynamics of this structure. In particular, the Kerr twistorial structure acquires a dependence on the vector potential of the external electromagnetic field via the Dirac equation.

In the following sections we discuss the properties and consequences of the Dirac-Kerr model in comparison with the properties of electron in Dirac theory and QED.

In particular, in sec.V we show that renormalization of the self-energy of the Dirac-Kerr electron may be performed by gravity for different distributions of the mechanical mass and charge, and we describe the approach which allows one to get the regular "dressed" solutions for different values of charge, mass and spin.

In sec.VI, following to the known approach to quantum fields in curved spaces [21,22], we consider regularization of the stress-energy tensor, which has a specific realization in the Kerr geometry.

In sec.VII, treating the obtained recently multiparticle Kerr-Schild solutions [24], we show that the Dirac-Kerr electron model takes an intermediate position between the one-particle Dirac electron and the multi-particle structure of "dressed" electron in QED.

Sec.VIII contains a discussion on the space-time structure of electron and the role of wave function in the Dirac theory, QED and in the Dirac-Kerr model.

In Conclusion we return to the origin of mass term in the Dirac equation and discuss some ways for modification of this model.

We use the spinor notations of the book [15] (see Appendix A). However our spinors are commuting, which changes some relations, and we give them in the Appendix B. Matching notations to the paper [3], we use signature $(-+++)$, Cartesian coordinates $x^0 = t, \quad x^\mu = (t, x, y, z)$ and $E = T^0_0 = p_0 = -p^0 > 0$.

II. REAL AND COMPLEX STRUCTURES OF THE KERR GEOMETRY

The real structure of the Kerr geometry, Kerr congruence and the Kerr theorem were discussed many times, and we refer readers to our previous papers, for example [11–13,23,24].

Recall, that angular momentum $J = \hbar/2$ for parameters of electron is so high that the black hole horizons disappear and the source of the Kerr spinning particle represents a naked singular ring.

It was suggested [4] that the Kerr singular ring represents a string which may have some excitations generating the spin and mass of the extended particle-like object - "microgeon". The Kerr singular ring is a focal line of the Kerr principal null congruence which is a bundle of the lightlike rays – twistors. The null vector field $k^\mu(x)$, which is tangent to these rays, determines the form of metric

$$g^{\mu\nu} = \eta^{\mu\nu} + 2H k^\mu k^n \tag{1}$$

(where $\eta^{\mu\nu}$ is the auxiliary Minkowski metric) and the form of vector potential

$$A_\mu = \mathcal{A}(x) k_\mu \tag{2}$$

for the charged Kerr-Newman solution.

The Kerr congruence is twisting (see figures in our previous works [11–13,23,24]). It is determined by the Kerr theorem [25,26,23] which constructs the Kerr-Schild ansatz (1), skeleton of the Kerr geometry, as a bundle of twistors [12,13,23,24].

Point-like complex representation of the Kerr geometry.

Complex representation of the Kerr geometry is important for our treatment here since the complex Kerr source is "point-like", which corresponds to a "naked" electron in our treatment.

Applying the complex shift $(x, y, z) \to (x, y, z + ia)$ to the source $(x_0, y_0, z_0) = (0, 0, 0)$ of the Coulomb solution q/r, Appel in 1887(!) considered the resulting solution

$$\phi(x, y, z) = \Re e \; q/\tilde{r}, \tag{3}$$

where $\tilde{r} = \sqrt{x^2 + y^2 + (z - ia)^2}$ turns out to be complex. On the real slice (x, y, z), this solution acquires a singular ring corresponding to $\tilde{r} = 0$. It has radius a and lies in the plane $z=0$. The solution is conveniently described in the oblate spheroidal coordinate system $r,\ \theta$, where $\tilde{r}=r+ia\cos\theta$, and one can see that the space is twofolded having the ring-like singularity as the branch line. Therefore, for the each real point $(t,x,y,z)\in \mathbf{M^4}$ we have two points, one of them is lying on the positive sheet, corresponding to $r>0$, and another one lies on the negative sheet, where $r<0$.

It was obtained that Appel potential corresponds exactly to electromagnetic field of the Kerr-Newman solution written in the Kerr-Schild form [4]. The vector of complex shift $\vec{a}=(a_x,a_y,a_z)$ corresponds to angular momentum of the Kerr solution.

Newman and Lind [27] suggested a description of the Kerr-Newman geometry in the form of a retarded-time construction, where it is generated by a complex source, propagating along a *complex world line* $\overset{\circ}{X}{}^{\mu}(\tau)$ in a complexified Minkowski space-time $\mathbf{CM^4}$. The rigorous substantiation of this representation is possible in the Kerr-Schild approach [3] which is based on the Kerr theorem and the Kerr-Schild form of metric (1) which are related to the auxiliary $\mathbf{CM^4}$ [23,28,29]. In the rest frame of the considered Kerr particle, one can form two null 4-vectors $k_L=(1,0,0,1)$ and $k_R=(1,0,0,-1)$, and represent the 3-vector of complex shift $i\vec{a}=i\Im m\,\overset{\circ}{X}{}^{\mu}$ as the difference $i\vec{a}=\frac{ia}{2}\{k_L-k_R\}$. The straight complex world line corresponding to a free particle may be decomposed to the form

$$\overset{\circ}{X}{}^{\mu}(\tau)=\overset{\circ}{X}{}^{\mu}(0)+\tau u^{\mu}+\frac{ia}{2}\{k_L-k_R\}, \tag{4}$$

where the time-like 4-vector of velocity $u^\mu=(1,0,0,0)$ can also be represented via vectors k_L and k_R

$$u^{\mu}=\partial_t \Re e\,\overset{\circ}{X}{}^{\mu}(\tau)=\frac{1}{2}\{k_L+k_R\}. \tag{5}$$

It allows one to describe the complex shift in the Lorentz covariant form.

One can form two complex world lines related to the complex Kerr source, $\overset{\circ}{X}{}^{\mu}_{+}(t+ia)=\Re e\,\overset{\circ}{X}{}^{\mu}(\tau)+iak_L^{\mu}$ and $\overset{\circ}{X}{}^{\mu}_{-}(t-ia)=\Re e\,\overset{\circ}{X}{}^{\mu}(\tau)-iak_R^{\mu}$. This representation will allow us to match the Kerr geometry to the solutions of the Dirac equation.

Indeed, since the complex world line $\overset{\circ}{X}{}^{\mu}(\tau)$ is parametrized by the complex time parameter $\tau=t+i\sigma$, it represents a stringy world sheet. However, this string is very specific, it is extended along the imaginary time parameter σ. The world lines $\overset{\circ}{X}{}^{\mu}_{+}=\overset{\circ}{X}{}^{\mu}(t+ia)$ and $\overset{\circ}{X}{}^{\mu}_{-}=\overset{\circ}{X}{}^{\mu}(t-ia)$ are the end points of this open complex twistor-string [10–13], see fig.1. By analogue with the real strings where the end points are attached to quarks, one can conventionally call these complex point-like sources as 'quarks'.

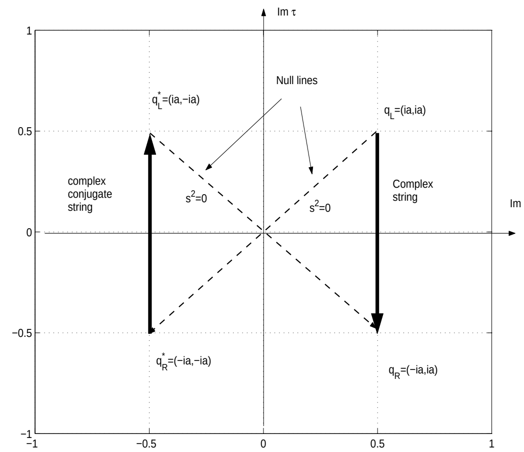

FIG. 1. Positions of the complex point-like sources (conventionally "quarks") at the ends of the open complex Kerr string. The "quarks" form two pairs which are related by the light-like intervals $s^2=0$. Two complex conjugated strings are to be joined forming orientifold, i.e. a folded closed string.

III. DIRAC EQUATION IN THE WEYL BASIS.

In the Weyl basis the Dirac equation

$$(\gamma^{\mu}\hat{\Pi}_{\mu}+m)\Psi=0, \tag{6}$$

where $\Psi=\begin{pmatrix}\phi_{\alpha}\\ \chi^{\dot{\alpha}}\end{pmatrix}$, and $\hat{\Pi}_{\mu}=-i\partial_{\mu}-eA_{\mu}$,

splits into the system [18,30]

$$\sigma^{\mu}_{\alpha\dot{\alpha}}(i\partial_{\mu}+eA_{\mu})\chi^{\dot{\alpha}}=m\phi_{\alpha},\quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}(i\partial_{\mu}+eA_{\mu})\phi_{\alpha}=m\chi^{\dot{\alpha}}, \tag{7}$$

The Dirac current

$$J_{\mu}=e(\bar{\Psi}\gamma_{\mu}\Psi)=e(\bar{\chi}\sigma_{\mu}\chi+\bar{\phi}\bar{\sigma}_{\mu}\phi), \tag{8}$$

where $\bar{\Psi}=(\chi^{+},\phi^{+})$, can be represented as a sum of two lightlike components of opposite chirality

$$J_L^{\mu}=e\bar{\chi}\sigma^{\mu}\chi\ ,\qquad J_R^{\mu}=e\bar{\phi}\bar{\sigma}^{\mu}\phi. \tag{9}$$

Forming the null vectors $k_L^{\mu}=\bar{\chi}\sigma^{\mu}\chi$, and $k_R^{\mu}=\bar{\phi}\bar{\sigma}^{\mu}\phi$, one can obtain for their product[1]

$$k_L^{\mu}k_{R\mu}=(\bar{\chi}\sigma^{\mu}\chi)(\bar{\phi}\bar{\sigma}^{\mu}\phi)=-2(\phi\bar{\chi})(\chi\bar{\phi})=2(\bar{\chi}\phi)(\bar{\chi}\phi)^{+}\ . \tag{10}$$

One can also form two more vector combinations from the Dirac spinors, $m^{\mu}=\phi\sigma^{\mu}\chi$, and $\bar{m}^{\mu}=(\phi\sigma^{\mu}\chi)^{+}=(\bar{\chi}\sigma^{\mu}\bar{\phi})$, which are complex conjugated and have also the scalar product

[1] We use the relations (64),(66) and (67) for commuting spinors.

$$m^{\mu}\bar{m}_{\mu}=2(\bar{\chi}\phi)(\bar{\chi}\phi)^{+}. \quad (11)$$

All the other products of the vectors $k_L,\ k_R, m, \bar{m}$ are null.

The normalized four null vectors $n^a = \frac{1}{\sqrt{2(\bar{\chi}\phi)(\bar{\chi}\phi)^{+}}}(m, \bar{m}, k_L, k_R), \quad a = 1,2,3,4$, form *a natural quasi orthogonal null tetrad which is determined by the given solution* Ψ *of the Dirac equation.*

Notice, that the complex vectors m and $\bar{m}$ turn out to be modulated by the phase factor $\exp\{2ip_\mu x^\mu\}$ coming from the Dirac spinors and carry oscillations and also de Broglie periodicity for the moving particle. Meanwhile, this phase factor cancels for the real null vectors k_L and k_R.

If Ψ is a plane wave

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \breve{\phi}_\alpha \\ \breve{\chi}^{\dot{\alpha}} \end{pmatrix} \exp\{ip_\mu x^\mu\}, \quad (12)$$

the Dirac equations take the form

$$m\phi_\alpha = -\Pi_\mu \sigma^\mu_{\alpha\dot{\alpha}}\chi^{\dot{\alpha}}, \qquad m\chi^{\dot{\alpha}} = -\Pi_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha}\phi_\alpha, \quad (13)$$

where

$$\Pi_\mu = p_\mu - eA_\mu. \quad (14)$$

Multiplying these equation by the spinors $\phi, \chi, \bar{\phi}, \bar{\chi}$ to the left, one obtains the null tetrad components of Π_μ

$$\Pi_\mu m^\mu = -m\phi\phi = 0, \qquad \Pi_\mu \bar{m}^\mu = -m\chi\chi = 0. \quad (15)$$

$$\Pi_\mu k^\mu_L = -m\bar{\chi}\phi, \qquad \Pi_\mu k^\mu_R = -m\bar{\phi}\chi. \quad (16)$$

It shows that the vector Π_μ is spanned by the real vectors k_L and k_R, i.e. $\Pi^\mu = ak^\mu_L + bk^\mu_R$, where $a = b = -\frac{m}{2(\phi\bar{\chi})}$. Therefore,

$$\Pi^\mu = -\frac{m}{2\bar{\phi}\chi}(k^m_L + k^m_R) \ . \quad (17)$$

In the rest frame

$$k^0_L = k^0_R, \quad k^i_L = -k^i_R \ , \quad (18)$$

and the space components of Π, $(i = 1,2,3)$ are cancelled.

The spin of electron is determined by the polarization vector which has the form [30]

$$S^\mu = i\overline{\Psi}\gamma^\mu\gamma^5\Psi = k^\mu_L - k^\mu_R \ , \quad (19)$$

and therefore, in the rest frame $S^0 = 0$ and $S = (0, S^i)$.

For normalized spinors

$$|\bar{\phi}\chi| = |\phi_1|^2 + |\phi_2|^2 = 1. \quad (20)$$

For $E > 0$

$$\bar{\phi}\chi = -1, \quad (21)$$

which yields $\Pi^0 > 0$. For $E < 0$

$$\bar{\phi}\chi = 1, \quad (22)$$

and $\Pi^0 < 0$.

IV. DIRAC EQUATION AS A MASTER EQUATION FOR THE KERR TWISTORIAL STRUCTURE

In previous treatment we have seen that the complex Kerr geometry is related to two null vectors k_L and k_R which determine the momentum and angular momentum of Kerr particle. We have also seen that the momentum and spin of the Dirac electron in the Weyl basis are also expressed via two null vectors k_L and k_R. It allows us to connect the solutions of the Dirac equation to twistorial structure of the Kerr spinning particle by setting an equivalence for these null vectors.

The Kerr-Schild ansatz for metric(1) is fixed by the null vector field $k_\mu(x)$, which is tangent to the Kerr principal null congruence (PNC) and is determined by a complex function $Y(x)$

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \quad (23)$$

where $P(Y)$ is a normalizing factor and $\{u, v, \zeta, \bar{\zeta}\}$ are the null Cartesian coordinates

$$2^{\frac{1}{2}}\zeta = x + iy, \qquad 2^{\frac{1}{2}}\bar{\zeta} = x - iy,$$
$$2^{\frac{1}{2}}u = z + t, \qquad 2^{\frac{1}{2}}v = z - t. \quad (24)$$

The Kerr-Schild formalism is based on the null congruences which are geodesic and shear-free.

The Kerr Theorem [25,26] claims that all the geodesic and shear-free congruences are determined by the function $Y(x)$ which is a solution of the algebraic equation

$$F = 0, \quad (25)$$

where the generating function F is arbitrary holomorphic function of the projective twistor variables

$$Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar{\zeta}. \quad (26)$$

Recall, that twistor is the pair $Z^a = \{\psi_\alpha, \mu^{\dot{\alpha}}\}$, where $\mu^{\dot{\alpha}} = x^\mu\bar{\sigma}_\mu\psi_\alpha$, and projective twistor is

$$Z^a/\psi_1 = \{1, Y, \lambda_1, \lambda_2\}. \quad (27)$$

Therefore, the target function $Y(x)$ is a projective spinor coordinate $Y = \psi_2/\psi_1$, and function F may be chosen as homogenous function of Z^a.

The complex world line $\overset{\circ}{X}{}^\mu_+$ can be used as a complex Kerr's source for generating function F of the Kerr theorem.

$$\overset{\circ}{X}{}^\mu_+ = \Re e \ \overset{\circ}{X}{}^\mu_+ + iak^\mu_L = \Re e \ \overset{\circ}{X}{}^\mu_+ + ia\bar{\chi}\sigma^\mu\chi. \quad (28)$$

Indeed, as it was shown in [5,6,28,29,23], the complex time parameter τ is cancelled in the Kerr generating function F. Because of that, the world line $\overset{\circ}{X}{}^\mu_-$, having the same complex shift in the space-like direction and the same 4-velocity, may also be used on the equal reason and yields the same result.

The Kerr generating function F may be represented in the form [5,6,28,29,23]

$$F = (\lambda_{\dot\alpha} - \overset{\circ}{\lambda}_{\dot\alpha})\check{K}\lambda^{\dot\alpha} \ , \tag{29}$$

where

$$\overset{\circ}{\lambda}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta} \overset{\circ}{X}{}^{\mu}_{+} \bar\sigma^{\dot\beta\alpha}_{\mu}\psi_\alpha \ . \tag{30}$$

are the values of twistor parameters at the complex world line of the Kerr source $\overset{\circ}{X}{}^{\mu}_{+}$. Operator $\check{K} = u^\mu\partial_\mu$ is related to momentum of the complex particle and expressed, in accordance with (5), via vectors k_L and k_R. Setting the equivalence of these vectors for the Kerr and Dirac particles, one matches their spin and momentum.

Without loss of generality one can set $\Re e \ \overset{\circ}{X}{}^{\mu}_{+} = 0$. Taking into account the relations (64) and (66) one obtains for the commuting spinors

$$\overset{\circ}{\lambda}{}^{\dot\alpha} = -2ia(\bar\chi\psi)\chi^{\dot\alpha}, \tag{31}$$

and

$$K\lambda^{\dot\alpha} = \Pi^\mu\bar\sigma_\mu\psi = -\frac{m}{(\bar\phi\chi)}[(\phi\psi)\bar\phi^{\dot\alpha} + (\bar\chi\psi)\chi^{\dot\alpha}]. \tag{32}$$

As a result, function F acquires the form (up to the nonessential factor m)

$$F(\psi, x^\mu) = x^\mu[(\phi\psi)(\bar\phi\bar\sigma_\mu\psi) + (\bar\chi\psi)(\chi\bar\sigma_\mu\psi)]/(\bar\phi\chi) \\ -2ia(\phi\psi)(\bar\chi\psi). \tag{33}$$

One sees that it is the function which possesses all the necessary for generating function properties. It is holomorphic in ψ in accord with the conditions of the Kerr theorem, and it is homogenous in ψ, which allows one to transform it to the standard Kerr-Schild form by the replacement $Y = \psi_2/\psi_1$. Finally, it is quadratic in ψ, which corresponds to the studied before case yielding the Kerr congruence up to the shifts and Lorentz transformations [5,31,29,23].

The Dirac spinor solutions ϕ, χ depend via (14) from the vector potential A_μ which is an external electromagnetic field. Therefore, the Dirac wave function $\Psi = (\phi, \chi)$ plays in this model the role of order parameters which control dynamics of the Dirac-Kerr particle, spin-polarization, momentum and deformation of the Kerr congruence caused by external electromagnetic field.

Behavior of the Dirac-Kerr spinning particle turns out to be fully determined by the wave function and will be indistinguishable from behavior of the Dirac electron, at least in the weak and slowly changed on the Compton lengths electromagnetic field.

V. RENORMALIZATION BY GRAVITY AND REGULARIZATION OF SELF-ENERGY

The mass renormalization is the most complicate and the most vulnerable procedure in QED. Gravitational field is ignored in QED, relying on the argument that its local action is negligible. However, gravity has a strong non-local action which automatically provides the self-energy renormalization for island sources.

Indeed, mass of an isolated source is determined by only asymptotic gravitational field, and therefore, it depends only on the mass parameter m which survives in the asymptotic expansion for the metric. On the other hand, the total mass can be calculated as a volume integral which takes into account densities of the electromagnetic energy ρ_{em}, material (mechanical mass) sources ρ_m and energy of gravitational field ρ_g. The last term is not taken into account in QED, but namely this term provides perfect renormalization. For a spherically symmetric system, the expression may be reduced to an integral over radial distance r[2]

$$m = 4\pi\int_0^\infty \rho_{em}dr + 4\pi\int_0^\infty \rho_m dr + 4\pi\int_0^\infty \rho_g dr. \tag{34}$$

Some of these terms may be divergent, but the total result will not be changed, since divergences always will be compensated by contribution from gravitational term. Therefore, *gravity performs perfectly the mass-energy renormalization by arbitrary distributions of the charges and matter.*

It shows that, due to the strong non-local action, gravity turns out to be essential for elementary particles, on the distances which are very far from the Planck scale.

The Kerr-Schild form of metric allows one to consider a broad class of regularized solutions which remove the Kerr singular ring, covering it by a matter source. There is a long-term story of the attempts to find some interior regular solution for the Kerr or Kerr-Newman solutions [2,8,9,34,35]. [3] Usually, the regularized solutions have to retain the Kerr-Schild form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_n \tag{35}$$

and the form of Kerr principal null congruence $k_\mu(x)$, as well as its property to be geodesic and shear-free. The space part $\vec n$ of the Kerr congruence $k_\mu = (1, \vec n)$ has the form of a spinning hedgehog. In the case $a = 0$, it takes the form of the spherically symmetric hedgehog which is usually considered as an ansatz for the solitonic models

[2]It looks like the expressions in a flat space-time. However, in the Kerr-Schild background, the exact Tollman relations taking into account gravitational field and rotation give just the same result [35].

[3]Extra references may be found in [9,34,35].

of elementary particles and quarks. It suggests that this model may also have relation to the other elementary particles. Indeed, the Kerr-Schild class of metric has a remarkable property, allowing us to consider a broad class of the charged and uncharged, the spinning and spinless solutions from an unified point of view.

Our treatment will be based on the approach given in [34,35], where the *smooth* regularized sources were obtained for the rotating and non-rotating solutions of the Kerr-Schild class. These smooth and regular solutions have the scalar function H of the general form

$$H = f(r)/(r^2 + a^2 \cos^2 \theta). \tag{36}$$

For the Kerr-Newman solution function $f(r)$ has the form

$$f(r) \equiv f_{KN} = mr - e^2/2. \tag{37}$$

Regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = f_{KN}$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) a narrow intermediate region $r \in [r_0 - \delta, r_0]$ which allows one to get a smooth solution interpolating between regions i) and ii).

It is advisable to consider first the non-rotating cases, since the rotation can later be taken into account by an easy trick. In this case, taking $n = 4$ and the parameter $\alpha = 8\pi\Lambda/6$, one obtains for the source (interior) a space-time of constant curvature $R = -24\alpha$ which is generated by a source with energy density

$$\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2, \tag{38}$$

and tangential and radial pressures

$$p_{rad} = -\rho, \quad p_{tan} = \rho - \frac{1}{8\pi} f''/\Sigma, \tag{39}$$

where $\Sigma = r^2$. It yields for the interior the stress-energy tensor $T_{\mu\nu} = \frac{3\alpha}{4\pi} diag(1, -1, -1, -1)$, or

$$\rho = -p_{rad} = -p_{tan} = \frac{3\alpha}{4\pi}, \tag{40}$$

which generates a de Sitter interior for $\alpha > 0$, anti de Sitter interior for $\alpha < 0$. If $\alpha = 0$, we have a flat interior which corresponds to some previous classical models of electron, in particular, to the Dirac model of a charged sphere and to the Lopez model in the form of a rotating elliptic shell [8].

The resulting sources may be considered as the bags filled by a special matter with positive $\alpha > 0$ or negative $\alpha < 0$ energy density.[4]

The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from 'true' to 'false' vacuum in a supersymmetric $U(1) \times \tilde{U}(1)$ Higgs model [34].

Assuming that transition region iii) is very thin, one can consider the following graphical representation which turns out to be very useful.

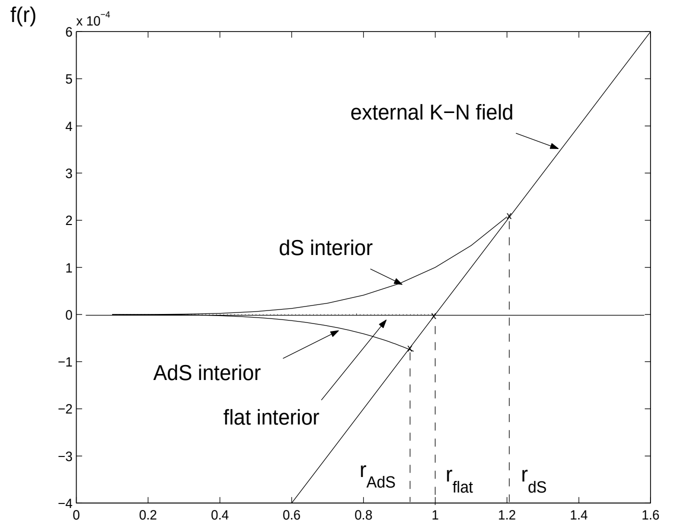

FIG. 2. Regularization of the Kerr spinning particle by matching the external field with dS, flat or AdS interior.

The point of phase transition r_0 is determined by the equation $f_{int}(r_0) = f_{KN}(r_0)$ which yields $\alpha r_0^4 = mr_0 - e^2/2$. From (40), we have $\rho = \frac{3\alpha}{4\pi}$ and obtain the equation

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \tag{41}$$

In the first term on the right side, one can easily recognize the electromagnetic mass of a charged sphere with radius r_0, $M_{em}(r_0) = \frac{e^2}{2r_0}$, while the second term is the mass of this sphere filled by a material with a homogenous density ρ, $M_m = \frac{4}{3}\pi r_0^3 \rho$. Thus, the point of intersection r_0 acquires a deep physical sense, providing an energy balance by the mass formation. In particular, for the classical Dirac model of a charged sphere with radius $r_0 = r_e = \frac{e^2}{2m}$, the balance equation yields the flat internal space with $\rho = 0$. If $r_0 > r_e$, a material mass of positive energy $M_m > 0$ gives a contribution to total mass m. If $r_0 < r_e$, this contribution has to be negative $M_m < 0$, which is accompanied by the formation of an AdS internal space.

[4] It resembles the discussed at present structure of dark matter in Universe. The case $\alpha > 0$ is reminiscent of the old Markov suggestions to consider particle as a semi-closed Universe [36].

All the above treatment retains valid for the rotating cases, and for the passage to a rotating case, one has to set

$$\Sigma = r^2 + a^2 \cos^2 \theta, \tag{42}$$

and consider r and θ as the oblate spheroidal coordinates.

The Kerr-Newman spinning particle with a spin $J = \frac{n}{2}\hbar$, acquires the form of a relativistically rotating disk. The corresponding stress-energy tensor (40) describes in this case the matter of source in a co-rotating with this disk coordinate system. Disk has the form of a highly oblate ellipsoid with thickness r_0 and radius $a = \frac{n}{2}\hbar/m$ which is of order the Compton length. Interior of the disk represents a 'false' vacuum having superconducting properties, so the charges are concentrated on the surface of this disk, at $r = r_0$. Inside the disk the local gravitational field is negligible.

VI. NONSTATIONARITY AND REGULARIZATION OF THE ZERO-POINT FIELD.

Classical models of spinning particle encounter an unavoidable contradiction with quantum theory. With respect to the Kerr spinning particle, this question was raised in [9]. The stationary Kerr space cannot be matched to quantum uncertainty principle.

Metric of the Kerr-Newman solution is only a stationary approximation for the metric which may be related to spinning particle. In the old Kerr's microgeon model [4], the Kerr singular ring acquires electromagnetic wave excitations which were interpreted as excitations of a circular string [6,5,11]. Such excitations break axial symmetry of the Kerr-Newman solution and stationarity. As a result, only an average metric takes the Kerr-Newman form. Integration of the Kerr-Schild equations in [3] leads to the appearance of an electromagnetic field $\gamma(x)$ which describes electromagnetic radiation along the Kerr congruence k_μ and is related to nonstationarity of the solutions. In [3], authors set the restriction $\gamma = 0$ to obtain the final form of the Kerr-Newman solution. The exact Einstein-Maxwell solutions with $\gamma \neq 0$ have not been obtained so far and represent the old hard problem. There are known the Vaidia "shining star" solutions [26,23], where $\gamma \neq 0$, but the Maxwell equation have been switched out there, since a non-coherent radiation is considered. These solutions show that radiation leads also to the nonstationarity via the loss of mass.

Note, that in quantum theory oscillations are stationary and absence of radiation caused by oscillations is postulated. Contrary to quantum physics, in classical theory nonstationarity entails radiation, and here lies a rather sharp boundary between the classical and quantum theories.

It should be mentioned that radiation is present in QED too, since the structure of free particles in QED is related to radiative corrections of the vacuum fields: the field of virtual photons, vacuum zero point field and vacuum polarization.

By application of the quantum field theory in curved spaces [22,21], quantum effects are concentrated in the vacuum zero point field which is divergent, and by a transfer to the classical Einstein-Maxwell theory, the quantum vacuum fields have to be subtracted. It means that the classical stress-energy tensor has to be regularized with respect to the vacuum fluctuations [22]

$$T^{(reg)}_{\mu\nu} = T_{\mu\nu} - < 0|T_{\mu\nu}|0 >, \tag{43}$$

under the condition

$$T^{(reg)\ \mu\nu}{}_{,\mu} = 0. \tag{44}$$

Since nonstationarity of the Kerr-Schild solutions is related to the field γ, it was conjectured [6,12,23] that this field has to be subtracted by some regularization. Indeed, the field γ is an electromagnetic radiation propagating along the Kerr congruence k_μ, and it has to involve a loss of mass. However, *twofoldedness* of the Kerr geometry leads to a very specific effect: *the outgoing radiation on the "positive" out-sheet of the metric is compensated by an ingoing radiation on the "negative" in-sheet , and therefore, the solution has to be stationary, as if the loss of mass is absent.* It shows, that the field γ has to be identified with the vacuum zero-point field and it has to be subtracted from the stress-energy tensor by means of a procedure of regularization which has to satisfy the condition (44). Such regularization may be performed, [6,11], and leads to some modified Kerr-Schild equations (see Appendix C). Although the equations are essentially simplified, there is still a remnant of the field γ in the Maxwell equations, which reflects a relation between the real electromagnetic excitations and vacuum fields. By such a regularization, electromagnetic excitations may be interpreted as a resonance of the zero-point fluctuations on the (superconducting) source of the Kerr spinning particle [6,11,13].

Although, the exact nontrivial solutions of the regularized system were not obtained so far, there were obtained corresponding exact solutions of the Maxwell equations which showed that any excitation of the Kerr geometry leads to the appearance of some extra "axial" singular line (string) which is semi-infinite and modulated by de Broglie periodicity [11–13]. [5] The obtained recently multiparticle Kerr-Schild solutions [24] supported this point of view, showing that interaction occurs via pp-strings of this type, which we shall discuss in the next section.

[5] It was argued in [12,13] that these strings may acquire the quark indices and may be responsible for the scattering at high energies. Similar strings appear as the real images of the complex point-like "quarks" which were mentioned in the end of section II.

VII. DIRAC-KERR ELECTRON AND MULTIPARTICLE KERR-SCHILD SOLUTIONS.

Some evidences that the Dirac-Kerr model is related to a multi-particle representation may be extracted from the obtained recently multiparticle Kerr-Schild solutions. It was shown in [24] that taking the generating function of the Kerr theorem F in the form of the product of partial functions for i-th particle

$$F = \prod_i F_i(Y|q_i), \tag{45}$$

where q_i is the set of parameters of motion and orientation of particle i, one can obtain the multi-particle Kerr-Schild solutions of the Einstein-Maxwell system in the assumption that particles are stationarily moving along some different trajectories.

The main equation of the Kerr theorem for twistorial structure (25) is satisfied by any partial solution $F_i(Y) = 0$. It means that the twistorial multi-particle space-time splits on the sheets corresponding to different roots of the equation $F(Y) = 0$, similar to the Riemann surface sheets.

Twistorial structures on the different sheets turn out to be independent and twistorial structure of i-th particle "does not feel" the structure of particle j, forming a sort of its internal space. This property is a direct generalization of the known corresponding property of the Kerr geometry - the twofold structure.

Since function $F(Y)$ for one Kerr particle is quadratic in Y [7,31,28,23], the equation $F_i(Y|q_i) = 0$ has two roots Y_i^+ and Y_i^- corresponding to the positive ('out') and negative ('in') sheets. In terms of these roots one can express F_i in the form [24]

$$F_i(Y) = A_i(x)(Y - Y_i^+)(Y - Y_i^-). \tag{46}$$

One sees that metric of a multi-particle solution will depend on the solution $Y_i(x)$ on the considered sheet of i-th particle. Indeed, substituting the $(+)$ or $(-)$ roots $Y_i^\pm(x)$ in the relation (23), one determines the Kerr congruence $k_\mu^{(i)}(x)$ and corresponding function h_i of the Kerr-Schild ansatz (1) on the i-th sheet

$$H_i = \frac{m}{2}\left(\frac{1}{\mu_i \tilde r_i} + \frac{1}{\mu_i^* \tilde r_i^*}\right) + \frac{(e/\mu_i)^2}{2|\tilde r_i|^2}. \tag{47}$$

Electromagnetic field generated by electric charge is given by the vector potential

$$A_\mu^{(i)} = \Re e\left(\frac{e}{\mu_i \tilde r_i}\right) k_\mu^{(i)}. \tag{48}$$

The complex radial distance $\tilde r_i$ and function $\mu_i(Y)$ are also determined from the extended version of the Kerr theorem [24],

$$\tilde r_i = d_Y F_i, \tag{49}$$

$$\mu_i(Y_i) = \prod_{j\neq i} A_j(x)(Y_i - Y_j^+)(Y_i - Y_j^-). \tag{50}$$

Contrary to independence of twistorial structures for different particles, there is a field interaction, since the function $\mu_i(Y)$ acquires a pole $\mu_i \sim A(x)(Y_i^+ - Y_j^-)$ on the twistor line which is common for the particles i and j. The metric and electromagnetic field will be singular along the common twistor lines. For example, a light-like interaction occurs along the line which connects the *out* - sheet of particle i to the *in* - sheet of particle j , see fig.3.

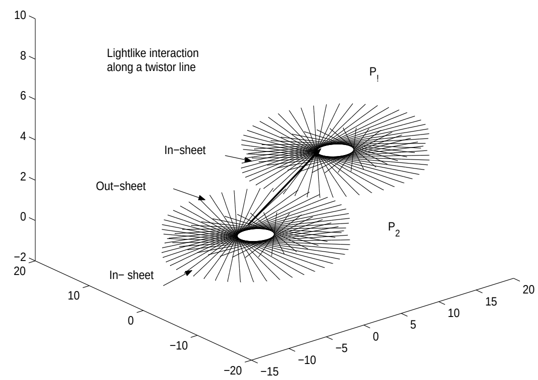

FIG. 3. The light-like interaction via a common twistor line connecting out-sheet of one particle to in-sheet of another.

This singular line is extended to infinity, since it will also be common for the *out* - sheets of the both particles. The field structure of this line is similar to singular pp-wave solutions. Analysis of some simple cases shows that each particle has a pair of semi-infinite singular lines which is caused by interaction with some external particle. [6] As it was discussed in [12,13,17] such a pair of strings turns out to be a carrier of de Broglie periodicity.

[6] The light-like strings contain only the one-way light-like modes. On the other hand, the modes of opposite directions link the positive sheet of the second particle to negative sheet of the first one, which is a stringy analog of the photon exchanges.

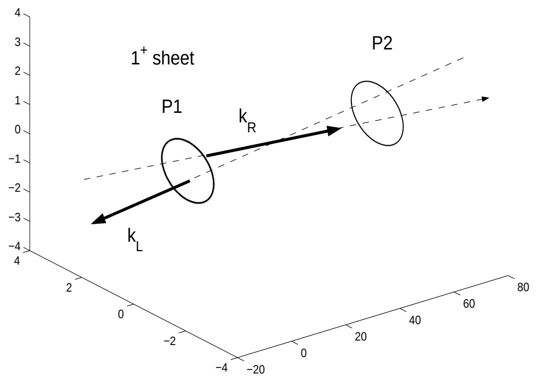

FIG. 4. Two outgoing semi-infinite singular lines of the particle P1, caused by its interaction with external particle P2.

In the limit of infinitely many external particles, the selected Kerr's particle will be connected by singular twistor lines with many other external particle, and the singular twistor lines will have every dense distribution among the twistor lines, covering the principal null congruence of the selected particle.

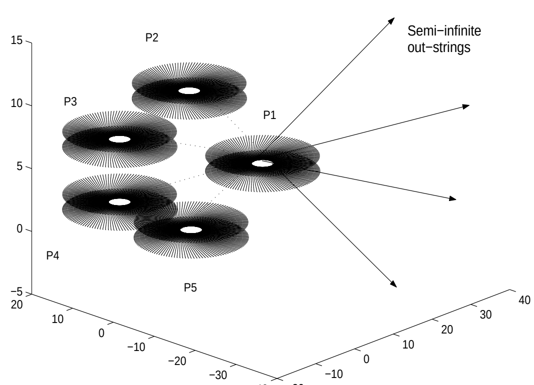

FIG. 5. Formation of the outgoing semi-infinite singular lines by interaction via the common 'out-sheet'-'out-sheet' twistor lines.

The multiparticle Kerr-Schild solutions show us that the usual Kerr-Newman solution is solution for an isolated particle, while there is a series of corresponding solutions, in which the selected Kerr particle is surrounded by other particles, perhaps on the far distances. This new type of the Kerr-Newman solution differs from the initiate one in one respect only - some of the twistor lines of the Kerr principal null congruence turn out to be *singular*. These singular lines are the light-like Schild strings which are described by singular pp-waves [12,17]. It shows, that the Dirac-Kerr model is multiparticle indeed, because of the stringy inter-particle interactions, and also that the singular twistor lines are analogs of the virtual photons which provide radiative corrections by regularization in QED. Therefore, the Dirac-Kerr electron takes an intermediate position between the one-particle Dirac model and multiparticle "dressed" electron of QED.

Returning to the discussed in sec.VII regularization of the the stress-energy tensor in the Kerr-Schild solutions, one sees that the fields propagating along the Kerr congruence have to be identified with the vacuum zero-point fields and have to be removed by regularization. Therefore, we arrive at the conclusion that *the singular pp-strings of the multiparticle Kerr-Schild solutions are elements of the vacuum, suggesting that vacuum has a twistorial texture.*

VIII. DISCUSSION: THE WAVE FUNCTION AND SPACE-TIME STRUCTURE OF ELECTRON

The 'point-like' and 'extended' electron

One of the main consequences of the considered here Dirac-Kerr model is that electron acquires a definite space-time structure of the Kerr geometry. In particular, it acquires a twistorial structure and a source of the Compton size, excitations of which lead to the wave properties [12,13].

On the other hand, the Dirac equation controls the motion of the Kerr spinning particle, and consequently, the behavior of this model does not differ from the corresponding behavior of the Dirac electron. It is known, that there are some restrictions on electromagnetic field for the one-particle Dirac model. It has to be small enough and weakly changing on the Compton distances [18,20,19], but it is exactly the conditions, which allow us to consider the Kerr spinning particle as a united system. It has to be noted that similar restrictions exist by multi-particle treatment in QED, as the conditions for decomposition on the creation and annihilation operators.

In the modern quantum theory, the Dirac wave solutions cannot be considered consequently as the wave functions, since relativistic quantum theory cannot be considered as one-particle theory [18,20,19]. In accordance with QED, the naked electron is point-like [32] and turns out to be smeared over the Compton region by the multi-particle effects of vacuum polarization, which corresponds to a dressed electron. However, there is no description of this structure which shows explicitly origin of spin, at least in coordinate representation. As a consequence of the QED problems, in particular in coordinate description, there appear some extreme points of view that the modern relativistic quantum theory may refuse to deal with the wave function and with the treatment of processes in space-time at all, in the sense that the Feinman graphs may represent itself the final form of the quantum theory.

Meanwhile, the Kerr spinning particle has a definite nontrivial space-time structure which does not contradict to conclusions of the one-particle Dirac theory and to predictions of QED. The considered in QED features of the electron acquire a clear geometrical meaning in the Dirac-Kerr model:

the "point-like" structure of the "naked" electron is related to the point-like complex representation of the Kerr geometry, while the real structure of "dressed" electron is determined by the extended twistorial structure and by the gravitational regularization of the Kerr source in the Compton region.

Wave function as an order parameter

In the considered Dirac-Kerr model, the Dirac wave function controls the Kerr space-time structure. However, one can conjecture that the role of wave function in this model may be more essential, indeed. In sec.III we obtained that there is a natural null tetrad n^a related to the Dirac wave function. This tetrad is a direct generalization of the null tetrad $\{d\zeta,\ d\bar{\zeta},\ du,\ dv\}$ which is determined by the Cartesian null coordinates and plays important role in the Kerr-Schild formalism and twistor theory. The spin-direction of the Kerr's particle in a standard representation is determined by the real direction $du + dv$ which has for the Dirac electron the analog $(k_{L\mu} - k_{R\mu})dx^\mu$. Similar, the complex forms $d\zeta,\ d\bar{\zeta}$ are analogs of the Dirac null vectors $m_\mu dx^\mu$ and $\bar{m}_\mu dx^\mu$. However, these complex vectors of the Dirac tetrad have a very important peculiarity. They are oscillating with the double Compton frequency and are carrying de Broglie periodicity for the moving electron. There is especial interest to use corresponding generalization of the Kerr-Schild formalism to the case of oscillating tetrad. Formally, the choice of the tetrad is only a gauge transformation which will not result to any change in the nature of the corresponding solutions of the Einstein-Maxwell system of equation. However, two factors are important:

i - it may simplify the equation, opening a way for obtaining some new solutions (for example oscillating ones) which were not obtained so far in the Kerr-Schild formalism, and

ii - there may be a topologically non-trivial situation, in which the new tetrad will not be equivalent to initiate one. An important example of this situation is the twist of spin-structure[7] which mixes the internal and orbital degrees of freedom and has found important applications in the solitonic models, topological sigma-models and twistor-string theory [33].

In this case, the Dirac wave function acquires again the role of an order parameter (Goldstone fermion) controlling the broken symmetry related fixation of the tetrad.

Therefore, in the Dirac-Kerr model, following to the authors of the books [18,19], we have to refuse from the naive point of view that the Dirac wave function $\psi(x)$ carries information on a space-time localization and the structure of electron. In the Dirac-Kerr model, the Dirac wave function plays the role of an 'order parameter' which controls the Kerr twistorial structure fixing the broken symmetry of the surrounding vacuum.

IX. CONCLUSION

The considered Dirac-Kerr model of electron has a few attractive properties:

- electron acquires an extended space-time structure which does-not contradict to conclusions of the Dirac model and QED,

- the Kerr twistorial structure is controlled by the Dirac equation, so all the consequences of this model have to match to the reach experimental data obtained from the Dirac equation, and since the Feinman rules are also based on the Dirac equation, one can expect that this model will not conflict with the results of QED too,

- the model shows that gravity performs renormalization and regularization of the particle-like models in a very elegant manner,

- the model has a non-trivial geometrical structure which displays a relationship to quantum theory. It opens a way to a geometrical view on the formal quantum prescriptions.

A peculiarity of this model is that the Dirac equation takes an especial role with respect to the other fields - it is considered on the auxiliary Minkowski space-time, while the other fields are considered on the Kerr-Schild background. One could argue that this difference is not essential, since the local gravitational field of electron is extremely small and will not affect on the solutions of the Dirac equation. However, it is not true, indeed. The Kerr solution has the nontrivial twofold topology which is retained even in the limit of the infinitely small mass. Because of that the aligned to the Kerr congruence electromagnetic and spinor solutions, which could lead to the self-consistent solutions, turn out to be essentially different from the plane wave solutions on Minkowski space-time, which is illustrated by the appearance of the extra axial singular filament. So, the plane wave solutions are deformed with concentration of the fields near singular lines.

In the considered Dirac-Kerr model the Dirac wave function plays the especial role of a global 'reference' field (order parameter) which controls the broken symmetry and set a synchronization of tetrad. The models of another sort, based on supergravity, are started from the massless Dirac equation, and the mass term has to be generated by some especial mechanisms, such as the broken symmetry or a nonlinear realization of supersymmetry [15]. This approach may be very prospective, but it needs a special adaptation to the Kerr geometry, and we expect to consider it elsewhere. Meanwhile, the many results of this paper will be invariant with respect to such a modification.

[7] Do not mix with twist of the Kerr congruence.

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APPENDIX A: SPINOR NOTATIONS

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} , \tag{51}$$

where

$$\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}, \tag{52}$$

and

$$\sigma_0 = \bar\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \sigma_1 = -\bar\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$

$$\sigma_2 = -\bar\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = -\bar\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{53}$$

$\epsilon^{12} = 1, \quad \epsilon^{21} = \epsilon_{12} = -1.$

Also,

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{54}$$

$$\gamma_5 = \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}. \tag{55}$$

APPENDIX B: SOME RELATIONS FOR COMMUTING SPINORS

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha{}^\gamma, \tag{56}$$

$$\epsilon^{12} = \epsilon_{21} = 1, \quad \epsilon^{21} = \epsilon_{12} = -1. \tag{57}$$

$$\psi_\alpha\chi^\alpha = \epsilon_{\alpha\beta}\psi^\beta\epsilon^{\alpha\gamma}\chi_\gamma = -\psi^\beta\epsilon_{\beta\alpha}\epsilon^{\alpha\gamma}\chi_\gamma = -\psi^\beta\chi_\beta = -\psi\chi . \tag{58}$$

It yields

$$\psi\chi = -\psi_\alpha\chi^\alpha = -\chi^\alpha\psi_\alpha = -\chi\psi \Rightarrow \psi\psi = 0 , \tag{59}$$

Complex conjugation changes the order of spinors without change of sign, which yields

$$(\chi\psi)^+ = (\chi^\alpha\psi_\alpha)^{*T} = (\bar\chi^{\dot\alpha}\bar\psi_{\dot\alpha})^T = (\bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}), \tag{60}$$

and due to (59)

$$(\chi\psi)^+ = (\chi^\alpha\psi_\alpha)^+ = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = \bar\psi\bar\chi = -\bar\chi\bar\psi. \tag{61}$$

Next, using the relation

$$\sigma_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\bar\sigma^{\dot\beta\beta} \tag{62}$$

and taking into account that $\epsilon\psi = -\psi\epsilon$, one obtains

$$\chi^\alpha\sigma_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} = \chi_\beta\bar\psi_{\dot\beta}\bar\sigma^{\dot\beta\beta} , \tag{63}$$

which yields (assuming that $\chi^\alpha \to \chi_\alpha$and $\bar\psi^{\dot\alpha} \to \bar\psi_{\dot\alpha}$)

$$\chi\sigma\bar\psi = \bar\psi\bar\sigma\chi . \tag{64}$$

Basing on the relation

$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu^{\dot\beta\beta} = -2\delta_\alpha{}^\beta\delta_{\dot\alpha}{}^{\dot\beta}, \tag{65}$$

and taking into account the order of the co- and contra-variant spinors, one obtains

$$(\bar\chi\sigma^\mu\phi)(\bar\sigma_\mu\psi)^{\dot\alpha} = -2(\bar\chi\psi)\phi^{\dot\alpha}, \tag{66}$$

and

$$(\psi\phi)\bar\chi_{\dot\beta} = -\frac{1}{2}(\phi\sigma^\mu\bar\chi)(\psi\sigma_\mu)_{\dot\beta} . \tag{67}$$

We have also

$$(\chi\sigma\bar\psi)^+ = \psi\sigma\bar\chi , \tag{68}$$

and

$$(\chi\sigma^\mu\bar\sigma^\nu\psi)^+ = \bar\psi\bar\sigma^\nu\sigma^\mu\bar\chi . \tag{69}$$

APPENDIX C: REGULARIZED EINSTEIN-MAXWELL EQUATIONS

The field equations for the Einstein-Maxwell system after preliminary integration in [3] are the following. Electromagnetic field is given by tetrad components of the selfdual tensor $\mathcal{F}_{ab}$,

$$\mathcal{F}_{12} = AZ^2, \tag{70}$$

$$\mathcal{F}_{31} = \gamma Z - (AZ),_1 , \tag{71}$$

and the equations relating the functions $A(x)$ and $\gamma(x)$ are

$$A,_2 - 2Z^{-1}\bar Z Y,_3 A = 0, \tag{72}$$

$$\mathcal{D}A + \bar Z^{-1}\gamma,_2 - Z^{-1}Y,_3\gamma = 0. \tag{73}$$

Gravitational equations take the form

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}\, M = A\bar{\gamma}\bar{Z}, \tag{74}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \tag{75}$$

where

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\,\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\,\partial_2\ . \tag{76}$$

Solutions of this system were given in [3] only for the case for $\gamma = 0$. Regularization of the stress-energy tensor

$$T^{\mu\nu}_{reg} = :T^{\mu\nu}: \equiv T^{\mu\nu} - <0|T^{\mu\nu}|0> \tag{77}$$

under the condition $\nabla_\mu T^{\mu\nu}_{reg} = 0$ retains the term γ in the Maxwell equations (70), (71), (72), (73) and in the gravitational equation (74), while the equation (75) takes the simple form

$$\mathcal{D}M = 0, \tag{78}$$

which provides stationarity.

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