



Analytical shear viscosity in hyperscaling violating black brane



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ABSTRACT

In this letter, with the use of matching method, we investigate the shear viscosity in a non-relativistic boundary field theory without hyperscaling symmetry, which is dual to a bulk charged hyperscaling violating black brane. By matching the solutions to the inner region and outer region at the matching region, we analytically obtain that the ratio of shear viscosity and the entropy density is always $1/4\pi$ at zero temperature and finite temperatures. Our results satisfy the Kovtun–Starinets–Son (KSS) bound.

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1. Introduction

Gauge/gravity duality is a beautiful approach to study the physics of strongly coupled sectors, because it connects a bulk gravitational theory and quantum field theory that lives in one less dimensions [1–3]. This allows us to explore the strongly coupled phenomena with the use of dual gravitational systems with weak coupling. In order to capture physics in a wider class of field theories, the duality has been generalized to the sectors beyond relativistic conformal symmetry. A remarkable generalization proposed in [4–11] is to consider the dual gravity with the metric

$$ds_{d+1}^2 = u^{\frac{2\theta}{d-1}} \left(-\frac{1}{u^{2z}} dt^2 + \frac{1}{u^2} du^2 + \frac{1}{u^2} d\vec{x}^2 \right), \quad (1)$$

which presents both a Lifshitz dynamical critical exponent z ($z \geq 1$) and a hyperscaling violating (HV) exponent θ . Under the scale-transformation $t \rightarrow \lambda^z t$, $x_i \rightarrow \lambda x_i$, $u \rightarrow \lambda u$, the metric transforms as $ds \rightarrow \lambda^{\theta/(d-1)} ds$, which breaks the scale-invariance. When $\theta = 0$, the above analysis recovers the known geometry with Lifshitz symmetry and it goes back to the pure AdS geometry when $z = 1$ and $\theta = 0$. Lots of extensive holographic study based on HV background have been present in [12–21] and references therein.

As the simplest implementation of holography, AdS/CFT correspondence has been widely used in the study of hydrodynamic properties, such as transport coefficients of strongly coupled systems. Specially, it is found that the ratio of the shear viscosity (η) over

the entropy density (s) has a universal value $1/4\pi$ in dual theories described by Einstein gravity [22,23], which has been extended into more general theories, see [24–27] and therein. It is then addressed in [28,29] that the value of ratio $1/4\pi$ gives a universal lower bound, namely the KSS bound, which should be satisfied by all sectors in nature. However, the violation of the viscosity bound has been studied in the presence of higher-derivative gravity corrections [30–40] and in anisotropic gauge/gravity dualities [41,42]. All the descriptions above are focused on finite temperature. Later, by borrowing the matching method proposed in [43] to study the holographic (non-)Fermi liquid, the transport coefficients including shear viscosity and electric conductivity were investigated at extremal AdS RN black hole with finite charge density [44,45]. At zero temperature, the ratio η/s of the boundary field theory is the same as that at the finite temperature boundary field theory. With the same method, the transport coefficients of field theory dual to AdS charged Gauss–Bonnet is performed in [46].

It will be interesting to explore the transport coefficients of hydrodynamic modes in wider boundary geometries like Eq. (1) accompanying with finite charge density, because it is more general than that discussed in AdS gravity. This means that one requires a charged black hole solution with the asymptotical behavior of Eq. (1) in the bulk theory. This kind of solution was firstly proposed in [7], which will be present in the next section.

The aim of this work is to disclose the shear viscosity of the field theory with finite charge density at any temperatures, dual to the bulk theory with hyperscaling violation found in [7]. We will use the matching method via comparing the solution of inner region and outer region at the matching region. Before computing the shear viscosity, we calculate the asymptotical solutions of the

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perturbation modes by matching the solutions. The ratio of shear viscosity to entropy density keeps $1/4\pi$ both at zero and finite temperature, though the HV exponent θ explicitly contributes to the entropy density and shear viscosity. This means the hyperscaling violating in the system does not violate the KSS bound. Note that the shear viscosity via matching method in Lifshitz black hole without hyperscaling violation has been addressed in [47]. And more study of the shear viscosity of non-relativistic effective field theory in neutral case via various methods can be seen [48–52].

This paper is organized as follows. We briefly review the black brane solution at any temperature in HV theory in section 2. Using the matching method, we study the shear viscosity at zero and finite temperature in section 3. In both cases, the KSS bound is satisfied. The last section is the conclusion and discussion.

2. The charged HV black branes from Einstein–Dilaton–Maxwell theory

We start from Einstein–Dilaton–Maxwell (EDM) action in $3+1$ spacetime dimensions [53]

$$S_g = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{4}(e^{\lambda_1\phi} F^{\mu\nu} F_{\mu\nu} + e^{\lambda_2\phi} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}) \right]. \quad (2)$$

The action contains two $U(1)$ gauge fields coupled to a dilaton field ϕ . The $U(1)$ field A with field strength $F_{\mu\nu}$ is required to have a charged solution, while the other gauge field \mathcal{A} with field strength $\mathcal{F}_{\mu\nu}$ and the dilaton field are necessary to generate an anisotropic scaling. Here λ_1 and λ_2 are free parameters, which will be determined later. We can deduce the equations of motion for all the fields from the above action. The Einstein equation of motion for the metric is

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{V(\phi)}{2}g_{\mu\nu} \\ &+ \frac{1}{2} \left[e^{\lambda_1\phi} (F_{\mu\rho}F_\nu^\rho - \frac{g_{\mu\nu}}{4}F^{\rho\sigma}F_{\rho\sigma}) \right. \\ &\left. + e^{\lambda_2\phi} (\mathcal{F}_{\mu\rho}\mathcal{F}_\nu^\rho - \frac{g_{\mu\nu}}{4}\mathcal{F}^{\rho\sigma}\mathcal{F}_{\rho\sigma}) \right]. \end{aligned} \quad (3)$$

The equation of motion for the dilaton field is

$$\nabla^2\phi = -\frac{dV(\phi)}{d\phi} + \frac{1}{4}(\lambda_1 e^{\lambda_1\phi} F^{\mu\nu} F_{\mu\nu} + \lambda_2 e^{\lambda_2\phi} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}). \quad (4)$$

The Maxwell equations for the gauge fields are

$$\nabla_\mu(\sqrt{-g}e^{\lambda_2\phi} F^{\mu\nu}) = 0, \quad (5)$$

$$\nabla_\mu(\sqrt{-g}e^{\lambda_1\phi} F^{\mu\nu}) = 0. \quad (6)$$

The potential $V(\phi)$ plays a very important role in obtaining a charged HV black brane. Following [7], we set $V(\phi) = V_0 e^{\gamma\phi}$ with γ and V_0 being free parameters. Then the analytical charged HV black brane solution is [7]

$$ds_4^2 = r^{-\theta} \left(-r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2(dx^2 + dy^2) \right), \quad (7)$$

$$f = 1 - \left(\frac{r_h}{r} \right)^{2+z-\theta} + \frac{Q^2}{r^{2(z-\theta+1)}} \left[1 - \left(\frac{r_h}{r} \right)^{\theta-z} \right], \quad (8)$$

$$\mathcal{F}_{rt} = \sqrt{2(z-1)(2+z-\theta)} e^{\frac{2-\theta/2}{2(2-\theta)(z-1-\theta/2)}} \phi_0 r^{1+z-\theta}, \quad (9)$$

$$F_{rt} = Q \sqrt{2(2-\theta)(z-\theta)} e^{-\sqrt{\frac{z-1+\theta/2}{2(2-\theta)}} \phi_0} r^{-(z-\theta+1)}, \quad (10)$$

$$e^\phi = e^{\phi_0} r^{\sqrt{2(2-\theta)(z-1-\theta/2)}}. \quad (11)$$

Here, r_h is the radius of horizon satisfying $f(r_h) = 0$ and $Q = \frac{1}{16\pi G} \int e^{\lambda_1\phi} F_{rt}$ is the total charge of the black brane. All the parameters in the action can be determined by Lifshitz scaling exponent z and HV exponent θ

$$\begin{aligned} \lambda_1 &= \sqrt{\frac{2(z-1-\theta/2)}{2-\theta}}, \quad \lambda_2 = -\frac{2(2-\theta/2)}{\sqrt{2(2-\theta)(z-\theta/2-1)}}, \\ \gamma &= \frac{\theta}{\sqrt{2(2-\theta)(z-1-\theta/2)}}, \\ V_0 &= e^{\frac{-\theta\phi_0}{\sqrt{2(2-\theta)(z-1-\theta/2)}}} (z-\theta+1)(z-\theta+2). \end{aligned} \quad (12)$$

From Eqs. (9) and (10), we can obtain the gauge fields

$$\begin{aligned} \mathcal{A}_t &= -\mu \left[1 - \left(\frac{r}{r_h} \right)^{2+z-\theta} \right] \\ \text{with } \mu &= \frac{\sqrt{2(z-1)(2+z-\theta)}}{2+z-\theta} e^{\frac{2-\theta/2}{\sqrt{2(2-\theta)(z-1-\theta/2)}} \phi_0} r_h^{2+z-\theta}, \end{aligned} \quad (13)$$

$$\begin{aligned} A_t &= \mu \left[1 - \left(\frac{r_h}{r} \right)^{z-\theta} \right] \\ \text{with } \mu &= Q \sqrt{\frac{2(2-\theta)}{z-\theta}} e^{-\sqrt{\frac{z-1+\theta/2}{2(2-\theta)}} \phi_0} r_h^{\theta-z}. \end{aligned} \quad (14)$$

In addition, the Hawking temperature and entropy density are respectively

$$T = \frac{(2+z-\theta)r_h^z}{4\pi} \left[1 - \frac{(z-\theta)Q^2}{2+z-\theta} r_h^{2(\theta-z-1)} \right], \quad (15)$$

$$s = \frac{r_h^{2-\theta}}{4G}. \quad (16)$$

Before proceeding, we have to fix the valid region of the parameters z and θ . First, the background solution (7)–(11) are valid only if $z \geq 1$ and $\theta \geq 0$. Second, the condition $z - \theta \geq 0$ should be satisfied to make sure a well-defined chemical potential in the dual field theory. Third, it is obvious from equation (13) that $\theta < 2$. The null energy condition $(-\frac{\theta}{2} + 1)(-\frac{\theta}{2} + z - 1) \geq 0$ [7] gives us $\theta \leq 2(z-1)$. Thus, in this charged background, the range of the parameters is

$$\begin{cases} 0 \leq \theta \leq 2(z-1) & \text{for } 1 \leq z < 2, \\ 0 \leq \theta < 2 & \text{for } z \geq 2. \end{cases} \quad (17)$$

Furthermore, if we set $Q = \sqrt{\frac{2+z-\theta}{z-\theta}} r_h^{z-\theta+1}$, i.e., $\mu = \sqrt{\frac{2(2-\theta)(2+z-\theta)}{z-\theta}} r_h$, one gets the zero-temperature limit. Then the redshift factor $f(r)$ becomes

$$\begin{aligned} f(r)|_{T=0} &= 1 - \frac{2(z-\theta+1)}{z-\theta} \left(\frac{r_h}{r} \right)^{z-\theta+2} \\ &+ \frac{z-\theta+2}{z-\theta} \left(\frac{r_h}{r} \right)^{2(z-\theta+1)}. \end{aligned} \quad (18)$$

Obviously, in the near horizon limit $r \rightarrow r_h$,

$$f(r)|_{T=0, r \rightarrow r_h} \simeq \frac{(z-\theta+1)(z-\theta+2)}{r_h^2} (r - r_h)^2. \quad (19)$$

Therefore, at the zero temperature, there exists the same near horizon geometry, $AdS_2 \times \mathbb{R}^2$, as that for RN-AdS background. Specially, we can define $u = r/r_h$ and change the coordinate via

$$u - 1 = \frac{\alpha}{\zeta}, \quad (20)$$

with $\alpha = \frac{1}{(z-\theta+1)(z-\theta+2)r_h^2}$. Consequently, near the horizon, the metric is given by

$$ds^2 = r_h^{-\theta} \left[\frac{-dt^2 + d\zeta^2}{(z-\theta+1)(z-\theta+2)\zeta^2} + r_h^2(dx^2 + dy^2) \right], \quad (21)$$

with the curvature radius of AdS_2 $L_2 \equiv 1/\sqrt{(z-\theta+1)(z-\theta+2)}$, while the gauge fields are $A_\tau = \frac{\mu(z-\theta)\alpha}{\zeta}$ and $\mathcal{A}_\tau = \frac{\mu(2+z-\theta)\alpha}{\zeta}$, respectively.

3. Analytical study of shear viscosity

The shear viscosity of the dual boundary theory is related with the retarded Green function via the Kubo formula

$$\eta = -\lim_{\omega \rightarrow 0} \frac{\text{Im}G_{xy,xy}^R(\omega)}{\omega}. \quad (22)$$

According to the real-time recipe proposed in [54], the formula of the retarded Green function is

$$G_{xy,xy}^R(\omega) = \frac{1}{16\pi G} \sqrt{-g} g^{uu} h_y^{x*}(u) \partial_u h_y^x(u) |_{u \rightarrow \infty}, \quad (23)$$

where $h_y^x(u)e^{-i\omega t}$ is the tensor perturbation, $g_{xy} = \bar{g}_{xy} + h_{xy}$, of the background Eqs. (7)–(11), satisfying the linearized equation of motion

$$u^2 f h_y^{xx''}(u) + (3uf + uzf - u\theta f + u^2 f') h_y^{xx'}(u) + \frac{\omega^2}{u^{2z} r_h^{2z}} h_y^x(u) = 0. \quad (24)$$

In the equation above, the prime denotes to the derivative to u .

3.1. Shear viscosity at zero temperature

We shall calculate the shear viscosity according to the retarded Green function following the matching method proposed in [43]. To this end, we divide the radial axis into inner region with $u-1 = \frac{\alpha\omega}{\epsilon}$ and outer region with $u-1 > \frac{\alpha\omega}{\epsilon}$, and we consider the limit

$$\omega \rightarrow 0, \quad \zeta = \text{finite}, \quad \epsilon \rightarrow 0, \quad \frac{\alpha\omega}{\epsilon} \rightarrow 0. \quad (25)$$

And then, we match the solutions of the inner region and the outer region in the matching region with $\zeta \rightarrow 0$ and $u-1 = \frac{\alpha\omega}{\epsilon} \rightarrow 0$. For convenience of notation, we will set $h_y^x = \psi$. After introducing

$$\varpi = \alpha\omega \quad \text{and} \quad \zeta = \omega\zeta, \quad (26)$$

we have $u = 1 + \frac{\varpi}{\zeta}$ from (20). So the matching region near the horizon means taking a double limit $\zeta \rightarrow 0$ and $\varpi/\zeta \rightarrow 0$. The equation (24) can be rewritten in term of ϖ as

$$u^2 f \psi''(u) + (3uf + uzf - u\theta f + u^2 f') \psi'(u) + \frac{((z-\theta+1)(z-\theta+2))^2 \varpi^2}{u^{2z} f} \psi(u) = 0. \quad (27)$$

3.1.1. Solution of inner region

The perturbation mode near the horizon can be expanded in the low frequency limit as

$$\psi_I(\zeta) = \psi_I^{(0)}(\zeta) + \varpi \psi_I^{(1)}(\zeta) + \varpi^2 \psi_I^{(2)}(\zeta) + \dots \quad (28)$$

Here the leading term attributes to the near horizon $AdS_2 \times \mathbb{R}^2$ geometry. Subsequently, in terms of the coordinate ζ , the leading term of (27) reads as

$$\psi_I^{(0)''}(\zeta) + \psi_I^{(0)}(\zeta) = 0, \quad (29)$$

the general solution of which is

$$\psi_I^{(0)}(\zeta) = a_I^{(0)} \exp(i\zeta) + b_I^{(0)} \exp(-i\zeta). \quad (30)$$

Keeping the regularity in mind, we intend to choose the in-going boundary condition which requires to set $b_I^{(0)} = 0$ to cancel the out-going branch.

Then, near the matching region, i.e., in the limit of $\zeta \rightarrow 0$, the in-going result can be expanded as

$$\psi_I^{(0)}(\zeta) |_{\zeta \rightarrow 0} \simeq a_I^{(0)}(1+i\zeta) = a_I^{(0)} \left[1 + \mathcal{G}_R(\varpi) \frac{1}{u-1} \right] \quad (31)$$

where we have used $\zeta = \frac{\varpi}{u-1}$ to express the second equality in the coordinate u . Meanwhile,

$$\mathcal{G}_R(\varpi) = i\varpi \quad (32)$$

is nothing but the retarded Green function of a zero-charge scalar operator with conformal dimension one in the IR conformal field theory, of which the dual field is $\psi_I^{(0)}$ in the framework of AdS/CFT correspondence. This is explicit because when we rescale ζ into ζ/ϖ , equation (29) coincides with the equation of motion for a massless and chargeless scalar field in AdS_2 geometry [43].

Considering equation (27) in term of the order of ϖ , it's easy to obtain that the solution in the inner region near the matching region has the form

$$\psi_I(u) = a_I^{(0)} + \frac{a_I^{(0)}}{u-1} \mathcal{G}_R(\varpi) + \dots \quad (33)$$

where the dots denote the non-vanishing ϖ term, but they vanish in the limit of $\varpi \rightarrow 0$. Note that the solution of inner region is universal, meaning that it does not depend on the geometrical parameters. Then we will move on to explore the outer region solution in order to match the solutions.

3.1.2. Solution of outer region, matching and the shear viscosity

In this subsection, we shall work out the solution in the outer region, which can be expanded for both at asymptotical boundary and at the matching region so we can match the solution in outer region to that in inner region obtained in the above subsection.

Now, we expand the perturbation field of the outer region in the low ϖ limit as

$$\psi_O(u) = \psi_O^{(0)}(u) + \varpi \psi_O^{(1)}(u) + \varpi^2 \psi_O^{(2)}(u) + \dots \quad (34)$$

It's straightforward to get the leading order equation of (27)

$$u^2 f \psi_O^{(0)''}(u) + (3uf + uzf - u\theta f + u^2 f') \psi_O^{(0)'}(u) = 0, \quad (35)$$

from which we see that this equation only explicitly depends on the value of $\sharp = z - \theta$.

For arbitrary $z - \theta = \sharp$, the general solution of (35) is

$$\begin{aligned} \psi_O^{(0)}(u) \\ = a_O^{(0)} + \int_1^r e^{\int_1^{K[2]} -\frac{2-\sharp-\sharp^2-(2+2\sharp)K[1]^{\sharp}+(\sharp^2+3\sharp)K[1]^{2+2\sharp}}{K[1](2+\sharp-(2+2\sharp)K[1]^{\sharp}+\sharp K[1]^{2+2\sharp})} dK[1]} b_O^{(0)} dK[2], \end{aligned} \quad (36)$$

where $K[i]$ ($i = 1, 2$) is the complete elliptic integral of the first kind. And $a_O^{(0)}$ and $b_O^{(0)}$ are integral constants which will be discussed soon. Although the above solution is not explicit for general \sharp , once we give the value of \sharp , we can obtain the concrete expression of the solution. Furthermore, we can take its behavior

near the matching region and the boundary. Specially, the asymptotical behavior of the above equation has the form

$$\psi_0^{(0)}(u)|_{u \rightarrow \infty} = (a_0^{(0)} + C_0 b_0^{(0)}) - \frac{b_0^{(0)}}{(\sharp+2)u^{\sharp+2}} + \dots \quad (37)$$

with the dots denoting the higher subleading terms than $u^{-(\sharp+2)}$, while near the matching region, the behavior of (36) is

$$\psi_0^{(0)}(u)|_{u \rightarrow 1} = -\frac{b_0^{(0)}}{(\sharp+1)(\sharp+2)(u-1)} + a_0^{(0)} + \tilde{C}_0 b_0^{(0)} + \dots \quad (38)$$

where the dots represent the vanishing term in the limit of $w \rightarrow 0$. Note that C_0 and \tilde{C}_0 are some functions of \sharp . Matching (33) and (38) will give us the following relations

$$\begin{aligned} b_0^{(0)} &= -(\sharp+1)(\sharp+2)\mathcal{G}_R(w)a_I^{(0)}, \\ a_0^{(0)} &= [1 + (\sharp+1)(\sharp+2)\tilde{C}_0]\mathcal{G}_R(w)a_I^{(0)}. \end{aligned} \quad (39)$$

Then putting the above relations in to (37), we can write asymptotic form of $h_y^\chi(u)$ as

$$\begin{aligned} \psi_0(u)|_{u \rightarrow \infty} &= a_I^{(0)} \left[1 + (\sharp+1)(\sharp+2)(\tilde{C}_0 - C_0)\mathcal{G}_R(w) + \dots \right] \\ &\quad + (\sharp+1)a_I^{(0)}\mathcal{G}_R(w)[1 + \dots]u^{-(\sharp+2)} + \dots \\ &= a_I^{(0)} [1 + A(\sharp)\mathcal{G}_R(w) + \dots] \\ &\quad + (\sharp+1)a_I^{(0)}\mathcal{G}_R(w)[1 + \dots]u^{-(\sharp+2)} + \dots \end{aligned} \quad (40)$$

where in the second line we have defined the constant function $A(\sharp) = (\sharp+1)(\sharp+2)(\tilde{C}_0 - C_0)$. Then taking account of (22) and (23) and setting $a_I^{(0)} = 1$, we obtain the ratio of shear viscosity to the entropy density is

$$\begin{aligned} \eta/s &= -\lim_{\omega \rightarrow 0} \frac{\text{Im} G_{xy,xy}^R(\omega)}{\omega s} \\ &= -\lim_{\omega \rightarrow 0} \frac{1}{\omega s} \text{Im} \left[\frac{1}{16\pi G} \sqrt{r_h^{4+2z-4\theta} u^{2+2z-4\theta} r_h^\theta u^{2+\theta}} \right. \\ &\quad \times f[-(\sharp+1)(\sharp+2)\mathcal{G}_R(w)(1 + \mathcal{O}(w))u^{-(\sharp+3)}] \Big|_{u \rightarrow \infty} \\ &= \frac{r_h^{2-\theta}}{16\pi G s} = \frac{1}{4\pi}, \end{aligned} \quad (41)$$

where we have substituted $\mathcal{G}_R(w) = i\alpha\omega = \frac{i\omega}{(\sharp+1)(\sharp+2)r_h^z}$ in the second line. Note that the result in AdS case with $z = 1$ and $\theta = 0$ discussed in [44] can be recovered by our case with $\sharp = 1$. So we conclude that in HV background, the KSS bound $\eta/s = \frac{1}{4\pi}$ always hold at zero temperature, independent on the geometrical exponents.

3.2. Shear viscosity at finite temperatures

At finite temperature, we do not have AdS_2 geometry near the horizon because the first order of the expansion of the redshift now dominants. But we will take some approximation to work out the solutions. In details, we will solve the equation in the near region with $\omega < u\omega \ll 1$ and outer region with $u \gg 1$, and then match the solutions in the near horizon region $u - 1 \ll 1$.¹ We rewrite equation (24) as

¹ Note that in the coordinate r , the near region is $r_h\omega < r\omega \ll 1$ and the outer region is $r > r_h$ while the matching near horizon region is $r - r_h \ll r_h$ [47].

$$\psi''(u) + \left(\frac{3+z-\theta}{u} + \frac{f'}{f} \right) \psi'(u) + \frac{\omega^2}{r_h^{2z} u^{2z+2} f^2} \psi(u) = 0. \quad (42)$$

3.2.1. The matching near horizon region

In the matching near region, we have $u - 1 \ll 1$ and $f(u) = f'(r_h)(u - 1) + \dots$ with $f'(r_h) = 4\pi T$, which gives us the leading order of equation as (42) as

$$\psi''(u) + \left(\frac{1}{u-1} \right) \psi'(u) + \frac{\omega^2}{(4\pi T r_h^z)^2 (u-1)^2} \psi(u) = 0. \quad (43)$$

The solution to the above equation is $\psi(u) = C_1(u-1)^{-i\omega/4\pi T r_h^z} + C_2(u-1)^{i\omega/4\pi T r_h^z}$. To regularize, we choose the infalling solution by setting $C_2 = 0$. Then in the low frequency limit, the solution behaves as

$$\psi(u) = C_1 \left(1 - \frac{i\omega}{4\pi T r_h^z} \log(u-1) \right). \quad (44)$$

3.2.2. The near region

The region with $\omega < u\omega \ll 1$ is our near region, in which the equation becomes

$$\psi''(u) + \left(\frac{3+z-\theta}{u} + \frac{f'}{f} \right) \psi'(u) = 0. \quad (45)$$

Its solution can express as

$$\psi(u) = C_3 + C_4 \int \frac{du}{fu^{3+z-\theta}}. \quad (46)$$

Near horizon with $u \rightarrow 1$ and $f(u) = 4\pi T(u-1)$, the solution (46) behaves as

$$\psi(u) = C_3 + C_4 \int \frac{du}{4\pi T(u-1)} = C_3 + \frac{C_4}{4\pi T} \log(u-1), \quad (47)$$

while, at large radius with $u \gg 1$ and $f(u) \rightarrow 1$, it becomes

$$\psi(u) = C_3 + C_4 \int \frac{du}{u^{3+z-\theta}} = C_3 - \frac{C_4}{u^{2+z-\theta}}. \quad (48)$$

3.2.3. Outer region, matching solution and the shear viscosity

In the asymptotic of the outer region $u \gg 1$, it is easy to get $f'(u) \rightarrow 0$ and $f(u) \rightarrow 1$, so the perturbation equation can be simplified as

$$\psi''(u) + \left(\frac{3+z-\theta}{u} \right) \psi'(u) + \frac{\omega^2}{r_h^{2z} u^{2z+2}} \psi(u) = 0. \quad (49)$$

We rewrite the above equation in the coordinate $u = 1/u$ and obtain the following solution to the equation of motion

$$\psi = \left(\frac{\omega u^z}{zr_h^z} \right)^p \left[C_5 J_p \left(\frac{\omega u^z}{zr_h^z} \right) + C_6 Y_p \left(\frac{\omega u^z}{zr_h^z} \right) \right], \quad (50)$$

where J and Y are first and second kind of Bessel function with $p = \frac{z-\theta+2}{2z}$, respectively. According to the feature of Bessel function, in the low frequency limit, the leading order the above solution is

$$\psi = \tilde{C}_6 + \tilde{C}_5 \left(\frac{\omega}{r_h^z} \right)^{2p} u^{2zp} = \tilde{C}_6 + \tilde{C}_5 \left(\frac{\omega}{r_h^z} \right)^{\frac{z-\theta+2}{z}} \frac{1}{u^{z-\theta+2}}. \quad (51)$$

Now we are ready to match the inner and outer solutions in matching region. This can be achieved by identifying (47) with (44), and (48) with (51), which gives us the following relations between the coefficients

$$\begin{aligned} \tilde{C}_6 &= C_3 = C_1, \quad C_4 = -\frac{i\omega}{r_h^z} C_1, \\ \tilde{C}_5 \left(\frac{\omega}{r_h^z} \right)^{\frac{z-\theta+2}{z}} &= \frac{i\omega}{r_h^z} C_1. \end{aligned} \quad (52)$$

Thus, the leading order of the asymptotic solution (51) at low frequency limit is

$$\psi = C_1 + C_1 \left(\frac{i\omega}{r_h^z} \right) \frac{1}{u^{z-\theta+2}}, \quad (53)$$

where we will consider the normalizability of the solution near the horizon, i.e. we set $C_1 = 1$. Having the above behavior, with the similar algebraic computation using (22) and (23), it is straightforward to obtain the shear viscosity

$$\eta = \frac{r_h^{2-\theta}}{16\pi G}. \quad (54)$$

The shear viscosity is the same as the result (41) at zero temperature, so that the KSS bound is also fulfilled at finite temperature as we expect.

4. Conclusion and discussion

In this letter, via matching method, we have calculated the shear viscosity of a holographic charged non-relativistic effective field theory at both zero and finite temperature, which is dual to a charged HV gravity. We find that the ratio of shear viscosity and the entropy density is always $1/4\pi$ at any temperature as that found in RN-AdS black hole, which satisfies the KSS bound. Our result shows that the domain of universality of the ratio can be enlarged to include holographic HV effective field theory. Here we only consider the perturbations formula $e^{-i\omega t+i\vec{k}\cdot\vec{x}}$ in the limit with momentum $\vec{k} = 0$. It is worthwhile to calculate transport coefficients at finite \vec{k} . Also, we can calculate the shear viscosity of the other non-relativistic backgrounds, for example, the background with the Schrödinger symmetry [55].

Besides the shear viscosity, another important transport coefficient is the conductivity of the dual field theory. It is addressed in [56] that the electric conductivity at large frequency in three dimensional field theory dual to an AdS geometry approaches to be a constant. So it is of great interest to further investigate the conductivity in Lifshitz and HV gravitational theory. For the shear channel with the radial gauge, the vector perturbations, $g_{ti} = \bar{g}_{ti} + h_{ti}$, $A_i = \bar{A}_i + a_i$, $A_t = \bar{A}_t + b_t$ with $i = x, y$, control the conductivity, including electric conductivity, heat conductivity and thermoelectric conductivity of the dual boundary field theory [54]. Due to the $SO(2)$ symmetry in the x - y plane, one can only consider the perturbation in x direction, which satisfy the following equations of motion in linear order

$$\begin{aligned} 0 &= u^4 h_t''(u) + (5u^3 - u^3 z - u^3 \theta) h_t'(u) \\ &\quad + u^{z+\theta-1} [r_h^{2z-4}(z-\theta)\mu a_x'(u) + r_h^{2\theta-6}(z-\theta+2)\mu b_x'(u)], \end{aligned} \quad (55)$$

$$\begin{aligned} 0 &= u^2 f a_x''(u) + (u(3z-\theta-1)f + u^2 f') a_x'(u) \\ &\quad + u^{3-3z+\theta}(z-\theta)\mu r_h^{2-2z} h_t'(u) + \frac{\omega^2}{u^{2z} r_h^{2z} f} a_x(u), \end{aligned} \quad (56)$$

$$\begin{aligned} 0 &= u^2 f b_x''(u) + (u(z+\theta-3)f + u^2 f') b_x'(u) \\ &\quad + u^{5-z-\theta}(z-\theta+2)\mu r_h^{2-2z} h_t'(u) + \frac{\omega^2}{u^{2z} r_h^{2z} f} b_x(u), \end{aligned} \quad (57)$$

$$\begin{aligned} 0 &= u^{z+3} h_t'(u) + u^{2z+\theta-2} [r_h^{2z-4}(z-\theta)\mu a_x(u) \\ &\quad + r_h^{2\theta-6}(z-\theta+2)\mu b_x(u)]. \end{aligned} \quad (58)$$

Note that the last equation is a constraint equation from the xu -component of linearized Einstein equations.

The AC electric conductivity has been studied in [47,57] in Lifshitz black brane with $\theta = 0$, however, the authors turned off the perturbation b_x and only work with the simplified perturbation equation of a_x . They argued that when the Lifshitz exponent $z > 1$, the AC electric conductivity behaves with a (non-)power scaling in large frequency limit which is analogous to the phenomena found in some disorder realistic materials [58]. Recently, the DC conductivity dual to both a_x and b_x in the hyperscaling violation theory with additional massless axions has been disclosed in [59,60]. Considering both perturbations of the two Maxwell field, a new matrix computational method was proposed. By calculating the mixed DC thermoelectric conductivities, both linear dependent and quadratic dependent of the resistivity on the temperature can be recovered. So it would be very interesting to solve the coupled system (55)–(58) to disclose the properties of various AC conductivity of the system and extended the study to HV theory with momentum relaxation. We will address the results elsewhere in the near future.

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