Intersection Rules for S-Branes

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Abstract

We give a model-independent derivation of general intersecting rules for spacelike branes (S-branes) in arbitrary dimensions d. This is achieved by directly solving bosonic field equations for supergravity coupled to a dilaton and antisymmetric tensor fields with minimal ansätze. We compare the results with those in eleven-dimensional supergravity and other solutions.

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There has been much interest in time-dependent and spacelike brane solutions (S-branes) of supergravities in eleven and ten dimensions because of its possible connection with tachyon condensations and dS/CFT correspondence [1, 2]. These theories are the low-energy limits of the string theories and supposedly unifying M-theory of strings. Time-dependent solutions are investigated rather recently and not much is known on these solutions. It is thus important to better understand these classical p-brane solutions.

The single p-brane solutions (Sp-branes) have been discussed in refs. [1, 3, 4, 5] for low-energy effective supergravities (see also refs. [6, 7] for related solutions). Following the usual convention, Sp-branes are used for those with $(p + 1)$-dimensional Euclidean world-volume. It has then been noted that the more general solutions can be understood as intersecting ones of these fundamental p-branes [8, 9] and the rules how the branes intersect with each other are given in analogy to the usual branes [10]–[14]. Although the "rules" are consistent with most of the known solutions, it is not clear if there are any other solutions than those given by these rules. The questions we would like to ask here are how general these rules are and how severely they restrict the solutions for supergravities in $d = 11$ and lower dimensions.

A systematic approach to formulating the rules for the way how they can intersect has been derived for the usual branes in [13, 14]. The purpose of this note is to extend this work to the S-branes and clarify what ansätze are really necessary. In particular, we derive the intersection rules from the general approach. We show that the rules are simple consequences of the field equations, which can be easily integrated and the consistency of the solutions reduces the problem of solving the field equations to an algebraic one.

The results of our analysis turn out to be consistent with the superposition rules in ref. [8] for $d = 11$ supergravity, but our results apply to more general supergravity coupled to a dilaton and antisymmetric tensors. We show that the requirement that the fields for the each brane be independent is sufficient to give the solutions and intersection rules.

Let us start with the general action for gravity coupled to a dilaton ϕ and m different n_A-form field strengths:

$$I = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^{m} \frac{1}{2n_A!} e^{a_A \phi} F_{n_A}^2 \right]. \tag{1}$$

This action describes the bosonic part of $d = 11$ or $d = 10$ supergravities; we simply drop

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ϕ and put $a_A = 0$ and $n_A = 4$ for $d = 11$, whereas we set $a_A = -1$ for the NS-NS 3-form and $a_A = \frac{1}{2}(5 - n_A)$ for forms coming from the R-R sector.[1] To describe more general supergravities in lower dimensions, we should include several scalars, but for simplicity we disregard this complication in this paper.

From the action (1), one derives the field equations

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \sum_A \frac{1}{2n_A!}e^{a_A\phi}\left[n_A\left(F_{n_A}^2\right)_{\mu\nu} - \frac{n_A - 1}{d - 2}F_{n_A}^2 g_{\mu\nu}\right],$$

$$\Box\phi = \sum_A \frac{a_A}{2n_A!}e^{a_A\phi}F_{n_A}^2,$$

$$\partial_{\mu_1}\left(\sqrt{-g}e^{a_A\phi}F^{\mu_1\cdots\mu_{n_A}}\right) = 0,$$

$$\partial_{[\mu}F_{\mu_1\cdots\mu_{n_A}]} = 0. \tag{2}$$

The last equations are the Bianchi identities.

We take the following metric for our system:

$$ds_d^2 = -e^{2u_0}dt^2 + \sum_{\alpha=1}^p e^{2u_\alpha}dy_\alpha^2 + e^{2B}d\Sigma_{k,\sigma}^2, \tag{3}$$

where $d = p + k + 1$, the coordinates $y_\alpha, (\alpha = 1, \ldots, p)$ parametrize the p-dimensional world-volume directions and the remaining coordinates of the d-dimensional spacetime are the time t and coordinates on k-dimensional spherical ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces, whose line elements are $d\Sigma_{k,\sigma}^2$. Since we are interested in time-dependent solutions, all the functions appearing in the metrics as well as dilaton ϕ are assumed to depend only on the time t. The Ricci tensors for the metric (3) are

$$R_{00} = -\sum_{\alpha=1}^p[\ddot{u}_\alpha + (\dot{u}_\alpha)^2 - \dot{u}_\alpha\dot{u}_0] - k(\ddot{B} + \dot{B}^2 - \dot{B}\dot{u}_0),$$

$$R_{\alpha\beta} = e^{2(u_\alpha - u_0)}[\ddot{u}_\alpha - \dot{u}_\alpha\dot{u}_0 + \sum_{\gamma=1}^p \dot{u}_\gamma\dot{u}_\alpha + k\dot{B}\dot{u}_\alpha]\delta_{\alpha\beta},$$

$$R_{ab} = \left\{e^{2(B-u_0)}[\ddot{B} + k\dot{B}^2 - \dot{B}\dot{u}_0 + \sum_{\alpha=1}^p \dot{u}_\alpha\dot{B}] + \sigma(k-1)\right\}\bar{g}_{ab}, \tag{4}$$

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[1]There may be Chern-Simons terms in the action, but they are irrelevant in our following solutions.

where $\bar{g}_{ab}$ is the metric for the hypersurface $\Sigma_{k,\sigma}$. Here and in what follows, a dot denotes a derivative with respect to t.

For the field strengths, we take the most general ones consistent with the field equations and Bianchi identities. The value for an electrically charged Sq-brane (whose world-volume is $(q+1)$-dimensional) is given by

$$F_{t\alpha_1\cdots\alpha_{q+1}} = \epsilon_{\alpha_1\cdots\alpha_{q+1}}\dot{E}, \quad (n_A = q+2), \tag{5}$$

where $\alpha_1, \cdots, \alpha_{q+1}$ stand for the tangential direction to the Sq-brane. The magnetic case is given by

$$F^{\alpha_{q+2}\cdots\alpha_p a_1\cdots a_k} = \frac{1}{\sqrt{-g}}e^{-a\phi}\epsilon^{t\alpha_{q+2}\cdots\alpha_p a_1\cdots a_k}\dot{\tilde{E}}, \quad (n_A = d-q-2) \tag{6}$$

where $a_1, \cdots, a_k$ denote the coordinates of the k-dimensional hypersurface $\Sigma_{k,\sigma}$. The functions E and $\tilde{E}$ are again assumed to depend only on t.

The electric field (5) trivially satisfies the Bianchi identities but the field equations are nontrivial. On the other hand, the field equations are trivial but the Bianchi identities are nontrivial for the magnetic field (6).

We will solve the field eqs. (2) with the simplifying ansatz

$$-u_0 + \sum_{\alpha=1}^{p} u_\alpha + kB = 0, \tag{7}$$

which simplifies the field equations (2) considerably. For both cases of electric (5) and magnetic (6) fields, we find that the field eqs. (2) are cast into

$$-\ddot{u}_0 + (\dot{u}_0)^2 - \sum_{\alpha=1}^{p}(\dot{u}_\alpha)^2 - k\dot{B}^2 = \frac{1}{2}\dot{\phi}^2 + \sum_A \frac{d-q_A-3}{2(d-2)}S_A(\dot{E}_A)^2, \tag{8}$$

$$\ddot{u}_\alpha = -\sum_A \frac{\delta_A^{(\alpha)}}{2(d-2)}S_A(\dot{E}_A)^2, \quad (\alpha = 1, \cdots, p), \tag{9}$$

$$\ddot{B} + \sigma(k-1)e^{2u_0-2B} = \sum_A \frac{q_A+1}{2(d-2)}S_A(\dot{E}_A)^2, \tag{10}$$

$$\ddot{\phi} = \sum_A \frac{\epsilon_A a_A}{2}S_A(\dot{E}_A)^2, \tag{11}$$

$$\left(S_A\dot{E}_A\right)^{\cdot} = 0, \tag{12}$$

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where A denotes the kinds of q_A-branes and we have defined

$$S_A \equiv \exp\left(\epsilon_A a_A \phi - 2\sum_{\alpha \in q_A} u_\alpha\right),\tag{13}$$

and

$$\delta_A^{(\alpha)} = \begin{cases} d - q_A - 3 \\ -(q_A + 1) \end{cases} \text{ for } \begin{cases} y_\alpha \text{ belonging to } q_A-\text{brane} \\ \text{otherwise} \end{cases},\tag{14}$$

and $\epsilon_A = +1(-1)$ corresponds to electric (magnetic) fields. For magnetic case we have dropped the tilde from E_A. Equations (8), (9) and (10) are the $00, \alpha\alpha$ and ab components of the Einstein equation in (2), respectively. The last one is the field equation for the field strengths of the electric fields and/or Bianchi identity for the magnetic ones. It is remarkable that both the electric and magnetic cases can be treated simultaneously just by using the sign ϵ_A. This is because the original system (1) has the S-duality symmetry under

$$g_{\mu\nu} \to g_{\mu\nu}, \quad F_{n_A} \to e^{-a_A \phi} * F_{n_A}, \quad \phi \to -\phi.\tag{15}$$

From eq. (12) one finds

$$S_A \dot{E}_A = c_A,\tag{16}$$

where c_A is a constant. With the help of eq. (16), we find that eqs. (9) and (11) give

$$\begin{aligned} \dot{u}_\alpha &= -\sum_A \frac{\delta_A^{(\alpha)}}{2(d-2)} c_A E_A + c_\alpha, \\ \dot{\phi} &= \sum_A \frac{\epsilon_A a_A}{2} c_A E_A + c_\phi, \end{aligned}\tag{17}$$

where c_α and c_ϕ are integration constants. Let us next define

$$g(t) = (u_0 - B)/(k-1).\tag{18}$$

We find from (7)

$$B = g - \frac{1}{k-1}\sum_{\alpha=1}^p u_\alpha, \qquad u_0 = kg - \frac{1}{k-1}\sum_{\alpha=1}^p u_\alpha,\tag{19}$$

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Using (17), we get

$$\dot{B} = \dot{g} + \sum_A \frac{q_A + 1}{2(d-2)} c_A E_A - \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha, \tag{20}$$

$$\dot{u}_0 = k\dot{g} + \sum_A \frac{q_A + 1}{2(d-2)} c_A E_A - \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha, \tag{21}$$

Substituting (16) and (20) into (10), we obtain

$$\ddot{g} + \sigma(k-1)e^{2(k-1)g} = 0, \tag{22}$$

which yields

$$\dot{g}^2 + \sigma e^{2(k-1)g} = \beta^2, \tag{23}$$

where β is an integration constant. The solution to eq. (23) is given by

$$g(t) = \begin{cases} \frac{1}{k-1} \ln \frac{\beta}{\cosh[(k-1)\beta(t-t_1)]} & : \sigma = +1, \\ \pm\beta(t-t_1) & : \sigma = 0, \\ \frac{1}{k-1} \ln \frac{\beta}{\sinh[(k-1)\beta(t-t_1)]} & : \sigma = -1, \end{cases} \tag{24}$$

where t_1 is another integration constant.

Substituting eqs. (17), (22) and (23) into (8) yields

$$(k-1)\left(\sum_A \frac{q_A+1}{2(d-2)} c_A E_A - \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha \right)^2 + \sum_{\alpha=1}^{p} \left(\sum_A \frac{\delta_A^{(\alpha)}}{2(d-2)} c_A E_A - c_\alpha \right)^2$$

$$+ \frac{1}{2}\left(\sum_A \frac{\epsilon_A a_A}{2} c_A E_A + c_\phi \right)^2 + \sum_A \frac{c_A}{2} \dot{E}_A - k(k-1)\beta^2 = 0. \tag{25}$$

This equation must be valid for arbitrary functions E_A of t. From the E_A-independent part of eq. (25), one finds

$$\frac{1}{k-1}\left(\sum_{\alpha=1}^{p} c_\alpha \right)^2 + \sum_{\alpha=1}^{p} c_\alpha^2 + \frac{1}{2} c_\phi^2 = k(k-1)\beta^2. \tag{26}$$

We can then rewrite eq. (25) as

$$\sum_{A,B} \left[M_{AB} \frac{c_A}{2} - \delta_{AB} \left\{ \left(\frac{1}{E_A} \right)^{\cdot} + \frac{2\tilde{c}_A}{E_A} \right\} \right] \frac{c_B}{2} E_A E_B = 0, \tag{27}$$

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where

$$M_{AB} = \sum_{\alpha=1}^{p} \frac{\delta_A^{(\alpha)} \delta_B^{(\alpha)}}{(d-2)^2} + (k-1)\frac{(q_A+1)(q_B+1)}{(d-2)^2} + \frac{1}{2}\epsilon_A a_A \epsilon_B a_B, \tag{28}$$

$$\tilde{c}_A = \sum_{\alpha \in q_A} c_\alpha - \frac{1}{2}c_\phi \epsilon_A a_A. \tag{29}$$

Since M_{AB} is constant, eq. (27) cannot be satisfied for arbitrary functions E_A of t unless the second term inside the square bracket is a constant. Requiring this to be a constant tells us that the function E_A must satisfy

$$\left(\frac{1}{E_A}\right)^{\cdot} + \frac{2\tilde{c}_A}{E_A} = \tilde{c}_A N_A, \tag{30}$$

or

$$E_A = \frac{e^{\tilde{c}_A(t-t_A)}}{N_A \cosh \tilde{c}_A(t-t_A)}, \tag{31}$$

where N_A is a normalization factor and t_A is an integration constant. In this way, the problem reduces to the algebraic equation (27) supplemented by (30) without making any assumption other than (7).

Equation (27) has two implications if we take independent functions for the fields E_A. In this case, first putting $A = B$ in eq. (27), we learn that

$$c_A = \frac{2(d-2)\tilde{c}_A N_A}{\Delta_A}, \tag{32}$$

where

$$\Delta_A = (q_A+1)(d-q_A-3) + \frac{1}{2}a_A^2(d-2). \tag{33}$$

By use of eqs. (31) and (32), eqs. (17), (20) and (21) can be integrated with the results

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\begin{aligned}
u_0 &= kg(t) + \sum_A \frac{q_A+1}{\Delta_A} \ln \cosh \tilde{c}_A(t-t_A) + c_0 t + c_0', \\
u_\alpha &= -\sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln \cosh \tilde{c}_A(t-t_A) + \tilde{c}_\alpha t + c_\alpha', \\
B &= g(t) + \sum_A \frac{q_A+1}{\Delta_A} \ln \cosh \tilde{c}_A(t-t_A) + c_0 t + c_0', \\
\phi &= \sum_A \frac{(d-2)\epsilon_A a_A}{\Delta_A} \ln \cosh \tilde{c}_A(t-t_A) + \tilde{c}_\phi t + c_\phi',
\end{aligned} \tag{34}
$$

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where c'_α's are new integration constants and

$$c_0 = \sum_A \frac{q_A + 1}{\Delta_A}\tilde{c}_A - \frac{\sum_{\alpha=1}^p c_\alpha}{k-1}, \quad c'_0 = -\frac{\sum_{\alpha=1}^p c'_\alpha}{k-1}, \quad \tilde{c}_\alpha = c_\alpha - \sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A}\tilde{c}_A,$$

$$\tilde{c}_\phi = c_\phi + \sum_A \frac{(d-2)\epsilon_A a_A}{\Delta_A}\tilde{c}_A. \tag{35}$$

To fix the normalization N_A, we go back to eq. (13). Using (34), we find

$$S_A = [\cosh \tilde{c}_A(t-t_A)]^2 e^{\epsilon_A a_A c'_\phi - 2\sum_{\alpha \in q_A} c'_\alpha}, \tag{36}$$

which, together with (16) and (32), leads to

$$N_A = \sqrt{\frac{\Delta_A}{2(d-2)}} e^{\epsilon_A a_A c'_\phi/2 - \sum_{\alpha \in q_A} c'_\alpha}. \tag{37}$$

Our metric and other fields are thus finally given by

$$\begin{aligned}
ds_d^2 &= \prod_A [\cosh \tilde{c}_A(t-t_A)]^{2\frac{q_A+1}{\Delta_A}}\left[e^{2kg(t)+2c_0t+2c'_0}\left\{-dt^2 + e^{-2(k-1)g(t)}d\Sigma_{k,\sigma}^2\right\}\right. \\
&\quad \left. + \sum_{\alpha=1}^p \prod_A [\cosh \tilde{c}_A(t-t_A)]^{-2\frac{\gamma_A^{(\alpha)}}{\Delta_A}} e^{2\tilde{c}_\alpha t + 2c'_\alpha}dy_\alpha^2\right], \\
E_A &= \frac{e^{\tilde{c}_A(t-t_A)}}{N_A \cosh \tilde{c}_A(t-t_A)}, \quad \tilde{c}_A = \sum_{\alpha \in q_A} c_\alpha - \frac{1}{2}c_\phi \epsilon_A a_A. \tag{38}
\end{aligned}$$

where we have defined

$$\gamma_A^{(\alpha)} = \begin{cases} d-2 \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} y_\alpha \text{ belonging to } q_A-\text{brane} \\ \text{otherwise} \end{cases}. \tag{39}$$

These solutions contain $2p+2$ integration constants $c_\alpha, c'_\alpha(\alpha = 1, \cdots, p), c_\phi, c'_\phi$, and t_1 and t_A with β determined by eq. (26). Among these, c'_α can be removed by rescaling the coordinates, and t_1 by a shift of the time. Without any preference of the choice of other paremeters, we leave these as free parameters. Thus the general solutions can be constructed by the following rules: (1) All the directions are multiplied by $[\cosh \tilde{c}_A(t-t_A)]^{2\frac{q_A+1}{\Delta_A}}$, and in addition, (2) the overall transverse direction (time and k-dimensional space) has the form $e^{2c_0t}[-e^{2kg(t)}dt^2 + e^{2g(t)}d\Sigma_{k,\sigma}^2]$ up to the rescaling of the coordinates, (3) the coordinates belonging to the brane are multiplied by $[\cosh \tilde{c}_A(t-t_A)]^{-2\frac{d-2}{\Delta_A}}$. When these are specified

to $d = 11$ supergravity and the integration constants are chosen appropriately, these give the superposition rules discussed in ref. [8].

The solution given in ref. [3] is reproduced if we restrict these to a single S-brane and choose the integration constants as

$$c_\gamma = \alpha - \frac{a}{\chi}c_1, (\gamma = 1, \cdots, p); \quad c_\gamma = -\frac{p}{q-1}\left(\alpha - \frac{a}{\chi}c_1\right), (\gamma = p+1, \cdots, p+q-k);$$

$$c_\phi = \frac{d-2}{q-1}a\alpha + \frac{2p}{\chi}c_1; \quad c_\gamma' = \frac{1}{\chi}\left(\ln\frac{(d-2)\chi\alpha^2}{(q-1)b^2} - ac_2\right), (\gamma = 1, \cdots, p);$$

$$c_0' = c_b' = c_\gamma' = -\frac{p}{\chi(q-1)}\left(\ln\frac{(d-2)\chi\alpha^2}{(q-1)b^2} - ac_2\right), (\gamma = p+1, \cdots, p+q-k), \quad (40)$$

with $q \to p-1, p \to p+q-k, \epsilon = -1$ and $\chi = 2p + (d-2)a^2/(q-1)$.[2] Equation (26) reduces to

$$\frac{pc_1^2}{\chi} + \frac{(d-2)\chi\alpha^2}{2(q-1)} - k(k-1)\beta^2 = 0, \tag{41}$$

and the normalization is determined to be

$$N_A = \frac{(q-1)b}{2(d-2)\alpha}, \tag{42}$$

in complete agreement with ref. [3].

The second condition following from eqs. (27) is $M_{AB} = 0$ for $A \neq B$. This leads to the intersection rules for two branes: If q_A-brane and q_B-brane intersect over $\bar{q}(\leq q_A, q_B)$ dimensions, this gives

$$\bar{q} = \frac{(q_A+1)(q_B+1)}{d-2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B. \tag{43}$$

Remember that the world-volume of q-branes lies in $(q+1)$-dimensional space and not in time. For eleven-dimensional supergravity, we have electric S2-branes, magnetic S5-branes and no dilaton $a_A = 0$. The rule (43) tells us that S2-brane can intersect with S2-brane over a '0-brane' ($\bar{q} = 0$) (which actually lives in 1-dimensional space) and with S5-brane over a 'string' ($\bar{q} = 1$) (2-dimensional space), and S5-brane can intersect with S5-brane over '3-brane' ($\bar{q} = 3$) (4-dimensional space), again in agreement with refs. [8]. In particular, our results show that there is no other intersecting solution as long as we

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[2]Here c_1 and c_2 on the rhs are those used in ref. [3] and should not be confused with our $c_\gamma, (\gamma = 1, 2)$.

treat the functions E_A with different index A as independent. If this condition is relaxed, there may be other solutions.

For all the spacelike Dq-brane solutions in type II superstrings, we find

$$\epsilon a = \frac{3-q}{2},\tag{44}$$

which tells us that the intersection rule is

$$\bar{q} = \frac{q_A + q_B}{2} - 2.\tag{45}$$

These solutions do not preserve any supersymmetry. In fact, they are supposed to correspond to branes with Dirichlet boundary conditions in the time direction, and hence describe configurations which exist only for a fixed time. These also contain singularity somewhere in time. Closer examination of their properties would be quite interesting.

We note that our derivation is a simple generalization of the general method developed in ref. [14]. It is quite satisfying to see that this is so useful method. There are also an important class of time-dependent solutions called null-branes [15, 16], which preserve supersymmetry. It would be also interesting to apply our method to these solutions.

To summarize, we have given quite a general model-independent derivation of the superposition rules in arbitrary dimensions. The intersection rules simply follow from the field equations if we require that the functions E_A with different index A be independent. In all cases, the algebraic eq. (27) (together with (30)) must be satisfied, and this equation should be most useful to examine possible solutions. We hope to discuss various properties of these solutions using the hints from dualities implied by underlying string dynamics elsewhere.

Acknowledgement

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