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## Intersection Rules for $p$ -Branes

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### Abstract

We present a general rule determining how extremal branes can intersect in a configuration with zero binding energy. The rule is derived in a model independent way and in arbitrary spacetime dimensions  $D$  by solving the equations of motion of gravity coupled to a dilaton and several different  $n$ -form field strengths. The intersection rules are all compatible with supersymmetry, although derived without using it. We then specialize to the branes occurring in type II string theories and in M-theory. We show that the intersection rules are consistent with the picture that open branes can have boundaries on some other branes. In particular, all the D-branes of dimension  $q$ , with  $1 \leq q \leq 6$ , can have boundaries on the solitonic 5-brane.

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## 1 Introduction

There has been recently considerable progress in the study of classical solutions of supergravities in 10 and 11 dimensions which are the low-energy effective field theories of string theories and (the would-be) M-theory. These solutions play a key rôle for probing the duality conjectures [1, 2, 3] which appear to relate between them all the string theories and M-theory. It is therefore important to gain better understanding of these classical  $p$ -brane solutions.

In Type II string theories, there are two kinds of  $p$ -branes, those charged under the NSNS fields and those carrying RR charge. The first ones correspond to elementary states of string theory (for the 1-brane [4]) and to purely solitonic objects (for the 5-brane [5]), while the second ones have been shown to be described by D-branes [6, 7]. In the case of M-theory, since it is still at a conjectural level, the 2- and the 5-brane do not have any description involving elementary quantum objects (however, the 5-brane has been conjectured to behave as a D-brane for open elementary membranes [8, 9, 10]).

In their low-energy effective field theory description, the single  $p$ -brane solutions have been described in [11, 12]. Although they look quite similar, especially in the Einstein frame, they already show a very different behaviour depending on their coupling to the dilaton. M-branes (the  $p$ -branes of M-theory), due to the absence of a dilaton in the theory, have a regular horizon<sup>1</sup>, while  $D = 10$   $p$ -branes all have a naked curvature singularity at the location of the “would-be” horizon (one exception is the self-dual RR 3-brane which does not couple to the dilaton of the type IIB theory).

Intersections of  $p$ -branes of any kind are objects of growing interest, mainly because when enough branes intersect a stabilization of the dilaton and of the moduli is achieved. This leads to a finite semiclassical entropy of the resulting black hole after compactification has been carried over. In some cases, it has then been possible to identify and count the microscopic states responsible for that entropy, using D-brane technology, in complete agreement with the semiclassical result [14, 15, 16, 17, 18].

It is interesting in its own respect to derive a general rule stating how  $p$ -branes can intersect. Heuristic arguments involving string theory representation of the branes and duality have been given in [8, 9, 7]. These arguments rely heavily on the D-brane picture and on the requirement that the configuration preserves some supersymmetries. In [19], the solutions of [20] are interpreted as intersecting M-branes of the same kind, following the  $p-2$  intersection rule for  $p$ -branes. Based on this work and on other known

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<sup>1</sup> Actually the maximally extended manifold of the 2-brane is much similar to the one of an extreme Reissner-Nordström black hole, with a curvature singularity hidden by a horizon, while the 5-brane manifold is completely regular [13].

solutions, in [21] Tseytlin formulates the harmonic superposition rule for intersecting  $p$ -branes (see also [22, 23]), starting from 11 dimensions and deriving the intersection rules from compactification and dualities. From the arguments of [8, 9, 10], the 5- and the 2-brane in  $D = 11$  are taken to intersect on a string<sup>2</sup>. From their supersymmetry conditions and using T-duality the resulting intersecting  $p$ -brane bound states have been recently classified in [27], where M-branes and D-branes are considered. A derivation of the intersection rules not based on supersymmetry arguments has been given in [28], asking that  $p$ -brane probes in  $p'$ -brane backgrounds feel no force and can thus create bound states with vanishing binding energy. Still, the model for the  $p$ -brane probe is different if this one is an M-brane, a D-brane or an NSNS-brane.

In this paper we find  $p$ -brane intersection rules purely from the (bosonic) equations of motion of the low-energy theory<sup>3</sup>. Moreover, these rules are even model-independent. What we actually do is to solve the equations of motion for a particular ansatz which has as a consequence extremality and zero binding energy. Not only the harmonic superposition rule is recovered, but also a constraint on the way the different  $p$ -branes mutually intersect. This constraint depends on the coupling to the dilaton of the field strength under which they are charged. The dependence of the intersection rules on the branes we are considering is thus reduced to this single characteristic. The ansatz we take, essentially reducing all the different functions of transverse space to  $\mathcal{N}$  independent ones, where  $\mathcal{N}$  is the number of intersecting branes, also implies preservation of some of the supersymmetries.

It is worth pointing out the intersection rule between the solitonic 5-brane and the D-branes. If  $q$  is the dimension of the D-brane, then its intersection with the NSNS 5-brane has dimensionality  $q - 1$  (provided  $1 \leq q \leq 6$ ; however D-branes with  $q \geq 7$  are for some aspects pathological, at least from the classical point of view). It is thus tempting to speculate that the solitonic 5-brane acts as a D-brane for the D-branes. This representation of the solitonic 5-brane is consistent with the dimensional reduction of the  $D = 11$  5-brane, which is seen as a D-brane for open membranes. Indeed, a 2-brane ending on a 5-brane can be related, after compactification on a transverse direction, to a D2-brane ending on a solitonic 5-brane. Also, the intersection rules for D-branes between themselves are compatible with the picture of an open  $q$ -brane having boundaries on a  $(q + 2)$ -brane [8].

<sup>2</sup> There is a different solution to  $D = 11$  supergravity involving a 2-brane inside a 5-brane [24, 25, 26]. Since it preserves 1/2 of the supersymmetries, it is clearly an intersection of a different kind of the ones considered here.

<sup>3</sup> While this was being completed, a paper [29] appeared where a similar approach to this problem is considered.

The paper is organized as follows: in Section 2 a general model inspired from the bosonic sector of  $D = 10$  or  $11$  supergravity is presented, along with the successive ansätze on the metric which allow us to find a solution to the equations of motion. The condition for this solution to be consistent yields the intersection rules. In Section 3, we specialize to M-theory and string theories and rederive the intersection rules for all the branes occurring in these theories. In Section 4, we speculate about the implications of a unified picture of all the known branes, most notably for what concerns state counting in black hole entropy problems.

## 2 General intersecting $p$ -brane solution

As a starting point, we take a very general action including gravity, a dilaton and  $\mathcal{M}$  field strengths of arbitrary form degree and coupling to the dilaton. The action reads:

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left( R - \frac{1}{2} (\partial\phi)^2 - \sum_A \frac{1}{2n_A!} e^{a_A \phi} F_{n_A}^2 \right), \quad A = 1 \dots \mathcal{M} \quad (1)$$

The metric is expressed in the Einstein frame.

Although we take the space-time to have a generic dimension  $D$ , this action is most suitable for describing the bosonic part of  $D = 10$  or  $D = 11$  supergravities. In fact, in (1) we did not write the various Chern-Simons type terms which occur in these theories. Nevertheless, the solutions we will present below (in section 3) are also consistent solutions of the full equations of motion including Chern-Simons terms.

To describe lower dimensional supergravities, we should include several scalar fields as in [30], but since we are indeed mainly interested in  $D = 10$  or  $11$  we prefer to keep only one scalar field for simplicity. Also, we will nevertheless be able to consider these lower dimensional cases by compactification.

In order to specialize to  $D = 10$ , we will have to take  $a_A = -1$  for the NSNS 3-form field strength and  $a_A = \frac{1}{2}(5 - n_A)$  for the field strengths coming from the RR sector. For  $D = 11$  we will simply have to put  $a_A = 0$  for the 4-form.

The equations of motion (EOM) can be put in the following form:

$$R^\mu{}_\nu = \frac{1}{2} \partial^\mu \phi \partial_\nu \phi + \sum_A \frac{1}{2n_A!} e^{a_A \phi} \left( n_A F^{\mu\rho_2 \dots \rho_{n_A}} F_{\nu\rho_2 \dots \rho_{n_A}} - \frac{n_A - 1}{D - 2} \delta^\mu_\nu F_{n_A}^2 \right), \quad (2)$$

$$\square \phi = \sum_A \frac{a_A}{2n_A!} e^{a_A \phi} F_{n_A}^2, \quad (3)$$

$$\partial_{\mu_1} (\sqrt{-g} e^{\alpha_A \phi} F^{\mu_1 \dots \mu_{n_A}}) = 0. \quad (4)$$

The last set of equations has to be supplemented by the statement that the  $n_A$ -form is the field strength of an  $(n_A - 1)$ -form potential. The same condition is obtained imposing the Bianchi identities (BI) to the field strengths:

$$\partial_{\mu_1} F_{\mu_2 \dots \mu_{n_A+1}} = 0. \quad (5)$$

We now specialize to a particular form of the metric, which is a slight generalization of the  $p$ -brane ansatz:

$$ds^2 = -B^2 dt^2 + \sum_{i,j} C_{(i)}^2 \delta_{ij} dy_i dy_j + G^2 \delta_{ab} dx^a dx^b, \quad (6)$$

where  $y_i$  are compact coordinates with  $i, j = 1 \dots p$ ,  $a, b = 1 \dots D - p - 1$  and  $B, C_{(i)}$  and  $G$  depend only on the overall transverse coordinates  $x^a$ . Since we will allow for multi-center solutions, we cannot postulate spherical symmetry in the overall transverse space. However, we will eventually recover spherical symmetry when all the branes are located at the same point in transverse space. Also, we take a diagonal metric, thus excluding after compactification the presence of KK momentum or KK monopoles.

The overall transverse space has dimension equal to  $D - p - 1$ . We have to recall at this stage that for the branes of dimension  $q$ , with  $q$  less than  $p$ , present in the solution, we have to consider a ‘‘lattice’’ of such  $q$ -branes in the transverse compact directions and then to average over them assuming that the compact directions are very small.

Considering now the  $n$ -form field strengths, we can generally make two kinds of ansätze. The electric ansatz is done asking that the BI are trivially satisfied, while for the magnetic one we ask instead that the EOM for the field strength are trivially satisfied. A correct electric ansatz for a  $q + 2$  form corresponding to an electrically charged  $q$ -brane is the following:

$$F_{i_1 \dots i_{q+2}} = \epsilon_{i_1 \dots i_{q+2}} \partial_a E, \quad E = E(x^a). \quad (7)$$

One can easily check that it satisfies (5).

For a magnetically charged  $q$ -brane, one needs an ansatz for a  $D - q - 2$  form. A good one which automatically satisfies the EOM (4) is:

$$F^{i_{q+1} \dots i_p a_1 \dots a_{D-p-2}} = \frac{1}{\sqrt{-g}} e^{-a\phi} \epsilon^{i_{q+1} \dots i_p a_1 \dots a_{D-p-1}} \partial_{a_{D-p-1}} \tilde{E}, \quad \tilde{E} = \tilde{E}(x^a). \quad (8)$$

Also the dilaton depends only on overall transverse space,  $\phi = \phi(x^a)$ .

Let us now discuss in some detail the next ansätze that we will make in order to solve the EOM (2)–(4). First of all, consider how many independent functions we have at hand. As far as the metric is concerned, one must have at least  $\mathcal{N}$  different functions in the set  $(B, C_{(1)}, \dots, C_{(p)})$  in order to be able to make the distinction between the  $\mathcal{N}$  intersecting branes (from now on, we slightly change the notations; the index  $A$  runs over all the branes, magnetic and electric). These functions are supplemented by the metric component relative to the overall transverse space  $G$ , the dilaton  $\phi$  and the  $\mathcal{N}$  functions appearing in the field strengths,  $E_A$  or  $\tilde{E}_A$ . Our ultimate goal will be to reduce the number of independent functions to  $\mathcal{N}$ , which, leading to the harmonic superposition rule, is interpreted as the requirement of vanishing binding energy for the  $p$ -brane bound state. A first step towards this goal is to impose the following constraint:

$$BC_{(1)} \dots C_{(p)} G^{D-p-3} = 1. \quad (9)$$

This constraint effectively expresses the metric component  $G$  as a function of the others, and can be interpreted physically as enforcing extremality (it can be checked on a single  $p$ -brane solution [12]). A consequence of this relation is that the  $R^a{}_b$  components of the Einstein equations (2) are reduced to algebraic ones. We will see that they indeed play a central rôle.

Because of (9), the EOM considerably simplify and become:

$$\partial_a \partial_a \ln B = \frac{1}{2} \sum_A \frac{D - q_A - 3}{D - 2} S_A (\partial_a E_A)^2, \quad (10)$$

$$\partial_a \partial_a \ln C_{(i)} = \frac{1}{2} \sum_A \frac{\delta_A^{(i)}}{D - 2} S_A (\partial_a E_A)^2, \quad (11)$$

$$\begin{aligned} & \frac{\partial_a B \partial_b B}{B^2} + \sum_i \frac{\partial_a C_{(i)} \partial_b C_{(i)}}{C_{(i)}^2} + (D - p - 3) \frac{\partial_a G \partial_b G}{G^2} + \delta_{ab} \partial_c \partial_c \ln G = \\ & = \frac{1}{2} \sum_A S_A \partial_a E_A \partial_b E_A - \frac{1}{2} \partial_a \phi \partial_b \phi - \delta_{ab} \frac{1}{2} \sum_A \frac{q_A + 1}{D - 2} S_A (\partial_c E_A)^2, \end{aligned} \quad (12)$$

$$\partial_a \partial_a \phi = -\frac{1}{2} \sum_A \varepsilon_A \alpha_A S_A (\partial_a E_A)^2, \quad (13)$$

$$\partial_a (S_A \partial_a E_A) = 0, \quad S_A = (BC_{(i_1)} \dots C_{(i_{q_A})})^{-2} e^{\varepsilon_A \alpha_A \phi}. \quad (14)$$

We have  $\delta_A^{(i)} = D - q_A - 3$  if the direction  $y_i$  is longitudinal to the brane labelled by  $A$  and  $\delta_A^{(i)} = -(q_A + 1)$  if it is transverse.  $\varepsilon_A = (+)$  if the corresponding brane is electrically charged, and  $\varepsilon_A = (-)$  if it is magnetic, and in this case we dropped the tilde from  $E_A$ .

The following step is to reduce the number of independent function to  $\mathcal{N}$ . We make it in such a way that the only relevant equations will be the (14), the other determining only algebraic coefficients. The ansatz we take is the following:

$$E_A = l_A H_A^{-1}, \quad S_A = H_A^2, \quad (15)$$

where  $l_A$  is a constant to be determined later. It follows that (14) directly reduce to:

$$\partial_a \partial_a H_A = 0, \quad (16)$$

thus characterizing the solution by  $\mathcal{N}$  harmonic functions, each corresponding to the charge of a particular  $p$ -brane. Actually, the most general solution of (16) is:

$$H_A = 1 + \sum_k \frac{c_A Q_{Ak}}{|x^a - x_k^a|^{D-p-3}}, \quad (17)$$

i.e. a multicenter solution, which existence is a consequence of the no-force condition [28] between parallel branes satisfying the ansätze we took here.

The functions  $B$ ,  $C_{(i)}$  and  $e^\phi$  are taken to be products of the  $H_A$ , and the consistency of the equations (10), (11), (12) and (13) requires the second relation of the ansatz (15).

If we take:

$$\ln B = - \sum_A \frac{D - q_A - 3}{D - 2} \alpha_A \ln H_A, \quad (18)$$

$$\ln C_{(i)} = - \sum_A \frac{\delta_A^{(i)}}{D - 2} \alpha_A \ln H_A, \quad (19)$$

$$\phi = \sum_A \varepsilon_A \alpha_A \ln H_A, \quad (20)$$

leading also to:

$$\ln G = \sum_A \frac{q_A + 1}{D - 2} \alpha_A \ln H_A, \quad (21)$$

the  $p + 2 \geq \mathcal{N} + 1$  equations (10), (11) and (13) imply the  $\mathcal{N}$  conditions:

$$\alpha_A \partial_a \partial_a \ln H_A + \frac{1}{2} l_A^2 (\partial_a \ln H_A)^2 = 0. \quad (22)$$

By virtue of (16), these conditions in turn imply:

$$\alpha_A = \frac{1}{2}l_A^2. \quad (23)$$

The last set of equations (12) becomes:

$$\begin{aligned} \sum_{A,B} \alpha_A \alpha_B \partial_a \ln H_A \partial_b \ln H_B & \left[ \frac{(D-q_A-3)(D-q_B-3)}{(D-2)^2} + \sum_i \frac{\delta_A^{(i)} \delta_B^{(i)}}{(D-2)^2} + \right. \\ & \left. + (D-p-3) \frac{(q_A+1)(q_B+1)}{(D-2)^2} + \frac{1}{2} \varepsilon_A a_A \varepsilon_B a_B \right] = \sum_A \alpha_A \partial_a \ln H_A \partial_b \ln H_B. \end{aligned} \quad (24)$$

For this set of equations to be satisfied for independent  $H_A$ , one has two sets of algebraic conditions to satisfy: the first set contains a condition for each brane and fixes the factor  $\alpha_A$ , and the second set contains a condition for each pair of distinct  $p$ -branes, and fixes their intersection rules.

If (24) is rewritten as:

$$\sum_{A,B} (M_{AB} \alpha_A - \delta_{AB}) \alpha_B \partial_a \ln H_A \partial_b \ln H_B = 0, \quad (25)$$

then the first set of conditions is given by  $M_{AA} \alpha_A = 1$ , which yields:

$$\alpha_A = \frac{D-2}{\Delta_A}, \quad (26)$$

where:

$$\Delta_A = (q_A+1)(D-q_A-3) + \frac{1}{2} a_A^2 (D-2). \quad (27)$$

This completes the description of the solution, which is indeed a superposition of single branes according to the ‘‘harmonic superposition rule’’ formulated in [21]:

$$\begin{aligned} B &= \prod_A H_A^{-\frac{D-q_A-3}{\Delta_A}}, & C^{(i)} &= \prod_A H_A^{-\frac{\delta_A^{(i)}}{\Delta_A}}, \\ G &= \prod_A H_A^{\frac{q_A+1}{\Delta_A}}, & e^\phi &= \prod_A H_A^{\varepsilon_A a_A \frac{D-2}{\Delta_A}}, \\ E_A &= \sqrt{\frac{2(D-2)}{\Delta_A}} H_A^{-1}. \end{aligned} \quad (28)$$



Let us now look at the second set of equations implied by (25),  $M_{AB} = 0$  for  $A \neq B$ . Suppose that the two branes involved, characterized by  $q_1$  and  $q_2$ , intersect over  $\bar{q} \leq q_1, q_2$  dimensions. Define also  $\bar{p} = q_1 + q_2 - \bar{q} \leq p$ . Then we have:

$$\begin{aligned} M_{12} &= \frac{1}{(D-2)^2} \{ (D-q_1-3)(D-q_2-3) + \bar{q}(D-q_1-3)(D-q_1-3) + \\ &\quad - (q_1-\bar{q})(D-q_1-3)(q_2+1) - (q_2-\bar{q})(q_1+1)(D-q_2-3) + \\ &\quad + (p-\bar{p})(q_1+1)(q_2+1) + (D-p-3)(q_1+1)(q_2+1) \} + \frac{1}{2} \varepsilon_1 a_1 \varepsilon_2 a_2 \\ &= (\bar{q}+1) - \frac{(q_1+1)(q_2+1)}{D-2} + \frac{1}{2} \varepsilon_1 a_1 \varepsilon_2 a_2. \end{aligned}$$

We thus have an equation giving the number of dimensions on which two branes intersect, depending on their own dimension and on their respective coupling to the dilaton:

$$\bar{q} + 1 = \frac{(q_A+1)(q_B+1)}{D-2} - \frac{1}{2} \varepsilon_A a_A \varepsilon_B a_B. \quad (29)$$

### 3 Intersections in $D = 11$ and $D = 10$ supergravities

We now specialize (29) to cases of interest in M-theory and string theory.

#### 3.1 Intersection of M-branes

For  $D = 11$  supergravity, the 4-form field strength gives rise to electric 2-branes and magnetic 5-branes. Since there is no dilaton in this theory, we simply take  $a = 0$  for all the branes we will consider.

The relation (29) becomes:

$$\bar{q} + 1 = \frac{(q_A+1)(q_B+1)}{9}. \quad (30)$$

This rule gives the expected results, confirming the ones in [19, 21]: we have  $\bar{q} = 0$  for  $2 \cap 2$  (i.e. two membranes intersect on a point),  $\bar{q} = 3$  for  $5 \cap 5$  and  $\bar{q} = 1$  for  $2 \cap 5$ . This last result is a support to the conjecture that open membranes can end on the magnetic 5-brane. All these rules are valid for each pair of branes in the configuration, so one can build along these lines any bound state of more than two branes.

Let us notify that the derivation in section 2 implicitly assumes that the overall transverse space is asymptotically flat. Configurations such that  $D - p - 3 \leq 0$  are

thus excluded. In the  $D = 11$  framework, we see that the configuration of two 5-branes intersecting on a string [22], though allowed by straightforward supersymmetry arguments, is automatically excluded.

### 3.2 Intersection of D-branes

In  $D = 10$  the field strengths arising from the RR sector of the superstring couple to the dilaton in such a way that  $\varepsilon a = \frac{1}{2}(3 - q)$  both for electrically and magnetically charged  $q$ -branes. For two such D-branes, (29) can be recast in the following form:

$$q_A + q_B - 2\bar{q} = 4, \quad (31)$$

which was already used in [23]. In its original derivation [7], which requires the intersection of D-branes to preserve some supersymmetries, the r.h.s of (31) is  $0 \pmod 4$ , thus allowing for a larger number of intersections; however,  $q_A + q_B - 2\bar{q} = 0$  is just the superposition of two similar branes, while if  $q_A + q_B - 2\bar{q} = 8$ , again the overall transverse space is not asymptotically flat.

To summarize, if we adopt the notation  $q_A \cap q_B = \bar{q}$ , we have the following intersections:

- in type IIA theory:  $6 \cap 2 = 2$ ,  $4 \cap 4 = 2$ ,  $4 \cap 2 = 1$ ,  $4 \cap 0 = 0$  and  $2 \cap 2 = 0$ . Note for instance that the configuration  $6 \cap 0$  is not allowed and that  $6 \cap 4$  gives 3 in spite of the fact that the transverse space is only 2 dimensional.
- in type IIB theory:  $5 \cap 3 = 2$ ,  $5 \cap 1 = 1$ ,  $3 \cap 3 = 1$ ,  $3 \cap 1 = 0$ . Note here that  $1 \cap 1$  gives  $-1$ , which could be reasonable in a Euclidean setting in which D-instantons are indeed present.

Note that from the results above we have the rule  $(q+2) \cap q = q-1$  for D-branes (with  $1 \leq q \leq 3$ ) in agreement with [8].

### 3.3 Intersection of NSNS branes with other branes

Let us first look at the intersections of the NSNS branes between themselves. The NSNS 3-form field strength couples to the dilaton with  $a = -1$ , thus leading to  $1 \cap 1 = -1$ ,  $1 \cap 5 = 1$  and  $5 \cap 5 = 3$  (even if in this case  $D - p = 3$ ) for the NSNS 1- and 5-brane, in agreement with the  $SL(2, Z)$  duality of type IIB theory.

It is more instructive to check the intersection rules between NSNS branes and D-branes. For the “elementary” string, we get:

$$1_{NS} \cap q_{RR} = 0, \tag{32}$$

which states nothing else than the fact that open strings end on the D-branes. For the solitonic 5-brane, we have [28]:

$$5_{NS} \cap q_{RR} = q - 1, \tag{33}$$

leading us to speculate that the NSNS 5-brane can effectively act as the locus on which the boundaries of open D-branes are constrained, i.e. it is a D-brane for D-branes. This picture has to be confirmed by a calculation which goes beyond the purely classical approach we are taking here.

Note that we did not take into account D-branes with  $q > 6$  since from the classical solution point of view they are rather pathological, yielding automatically a transverse space of spatial dimension lower than 3. See however [27] for a classification of intersecting branes in  $D - p = 2$  and 3.

## 4 Conclusion and discussion

The aim of this paper was to uncover general intersection rules for the  $p$ -branes which would go beyond their distinction between NSNS branes, D-branes and even M-branes. Indeed, the charges carried by NSNS branes or by D-branes appear to be related by U-duality<sup>4</sup> [1], while they are realized in a completely different way in string theory. As a consequence, the microscopic state counting of the black hole entropy is only possible for very particular configurations. The conjecture that some branes could act as D-branes for other branes, already formulated in [8], might allow some steps ahead.

A picture in which the M-branes are treated in a way much similar to the D-branes in string theory has also been used to perform some counting of states in [31, 32]. This is also consistent with the relation between M-theory and the string theories. Here we have shown that, from the point of view of the classical solutions, in which U-duality is a true symmetry of the equations of motion, if we interpret the result (32) as a hint for the existence of D-branes for strings, then we have also to suppose the existence of

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<sup>4</sup> To be complete, one should also add all the charges generated by the KK reduction. We did not take them into account here for simplicity.

D-branes for higher branes. The consistency of the open membrane picture in M-theory with compactification also requires this.

We can calculate the mass of the solutions given in (28) by using the formula for the ADM mass:

$$M = -\frac{L^p \Omega_{D-p-2} r^{D-p-2}}{8\pi G_D} [\sum_i \partial_r C_{(i)} + (D-p-2)\partial_r G]_{|r \rightarrow \infty}. \quad (34)$$

Using (28) and the fact that  $H_A \rightarrow 1$  when  $r \rightarrow \infty$ , one obtains:

$$M = \sum_A M_A, \quad (35)$$

where  $M_A = Q_A$  in suitable units for each constituent  $p$ -brane. The result (35) comes essentially from the second ansatz (15), assuring zero binding energy, while the extremality condition for each brane is due to the first ansatz (9).

One can check that the configurations (28) are supersymmetric. The exact amount of preserved supersymmetry depends on each particular configuration and is at least equal to  $1/2^N$ . The interest of the approach followed here is its independence with respect to the model and the space-time dimension. On the contrary, the approach based on supersymmetry depends heavily on the particular supergravity model considered.

Since the harmonic functions have the form  $H_A = 1 + \frac{\varepsilon_A Q_A}{r^{D-p-3}}$ , we can calculate the behaviour of the area and of the dilaton at the 'horizon'. For the area:

$$\begin{aligned} A_{D-p-2} &= \Omega_{D-p-2} r^{D-p-2} C_{(1)} \dots C_{(p)} G^{D-p-2} \\ &= \Omega_{D-p-2} r^{D-p-2} B^{-1} G \\ &= \Omega_{D-p-2} r^{D-p-2} \prod_A H_A^{\frac{D-2}{\Delta_A}} \Big|_{r=0}. \end{aligned}$$

For  $D = 10$  or  $D = 11$  supergravities, we always have  $\frac{D-2}{\Delta_A} = \frac{1}{2}$  and thus we need 4 charges to have a 4 dimensional black hole with non-zero entropy and 3 charges for a 5 dimensional one. It is worth pointing out that the formula above states that these two cases are really the only ones which allow for a non-zero extremal entropy, at least in the framework of configurations in  $D = 11$  or 10 supergravity without internal momenta.

For the dilaton to be constant at the horizon, we see in (28) that one needs to check for each configuration that  $\sum_A \varepsilon_A \alpha_A \frac{D-2}{\Delta_A} = 0$ . One indeed always finds that when the area of the horizon is finite, the dilaton has a fixed value at the horizon.

It is puzzling that the entropy has a microscopic explanation only for particular configurations involving D-branes and momentum. Here, we considered extremal black

hole configurations built up exclusively by branes, i.e. without momenta in the internal directions. One such configuration which yields a 5 dimensional black hole is the highly symmetric  $2 \cap 2 \cap 2$  intersection in M-theory. After compactification and a chain of T-dualities, this solution is related to the “ $1_R \cap 5_R + \text{momentum}$ ” configuration of type IIB theory, which was used to perform a counting of microstates in [15, 16]. The counting critically uses the presence of a momentum in the bound state. It would be nice to be able to perform a counting of microstates for the configuration when it is expressed in its most simple form.

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