Brane World Astronomy

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Abstract. Unified theories suggest that space is intrinsically 10 dimensional, even though everyday phenomena seem to take place in only 3 large dimensions. In "Brane World" models, matter and radiation are localized to a "brane" which has a thickness less than $\approx (\mathrm{TeV})^{-1}$ in all but the usual three dimensions, while gravity propagates in additional dimensions, some of which may extend as far as submillimeter scales. A brief review is presented of some of these models and their astrophysical phenomenology. One distinctive possibility is a gravitational wave background originating in the mesoscopic early universe, at temperatures above about 1 TeV and on scales smaller than a millimeter, during the formation of our 3-dimensional brane within a 10-dimensional space.

I UNIFIED THEORY

The Standard Model of strong, weak and electromagnetic interactions includes all the forms of mass-energy so far observed in nature, other than gravity. It is based on a relativistic quantum field theory of interacting fermion and boson fields, with forces arising from Yang-Mills vector gauge fields, propagating in a 3+1-dimensional spacetime. Gravity is formulated in a completely different way, using General Relativity, as a classical theory of dynamical spacetime itself: "Spacetime tells mass-energy how to move, and mass-energy tells spacetime how to curve."

Even though there is no direct inconsistency or disagreement of these theories with experiment or with each other, there is widespread dissatisfaction with the inelegance of this dualistic situation. It is suspected by those who believe in the unity of the natural world that there might be a single unified theory, derivable from simple principles of symmetry, which will appear in an appropriate mathematical limit as the Standard Model fields propagating in a General Relativistic spacetime.

Major steps have been taken recently in the construction of a unified theory; one can cite several triumphs of a theoretical nature, such as the ability to count precisely the quantum mechanical degrees of freedom of spacetime itself in certain special black hole spacetimes [1]. We may now also have observed [2–4] the first real-world phenomenon which specifically calls upon new quantum-gravity unification physics outside of "Standard Model Plus GR": the Cosmological Constant (or "Dark Energy"). Thus there is real hope that the Theory of Everything may become a real, testable physical theory. However it is not clear how to complete the most important step, connecting the fundamental theory to the real world which is so well described by "Standard Model Plus GR." The current best candidate for a Theory of Everything, "M theory", is formulated in ten spatial dimensions instead of three, and has no direct, distinctive connections with any real-world experiment.

Recent developments in M theory suggest that there may be an intermediate level of structure associated with dimensional reduction, which has spawned a wide variety of proposed designs for new "Brane World" models [6–13]: In these models, the fields of the Standard Model are confined to an approximately three-dimensional wall or "brane" imbedded in an extended ten-dimensional space or "bulk", which is described by adding extra dimensions to General Relativity. The brane has a thickness smaller than the TeV scale of current particle experiments, while the extra dimensions of the bulk can be as large as the 10^{-2}eV scale of current gravity experiments. This paper is a brief overview of brane worlds and some new effects they might produce in astrophysics.

Brane world models aim to short-cut the connection between fundamental theory and phenomena. They introduce a kind of "effective theory" as a conceptual bridge— a parametrized model broadly motivated by structures in the fundamental theory, which can be used to calculate new phenomena at low energy. Although it is not clear that this strategy will work in the long run, it has certainly broken a logjam in thinking and has spawned many intriguing new theoretical predictions and experimental tests.

II EXTRA DIMENSIONS

Direct detailed data from accelerators confirm the 3+1-dimensional behavior of Standard Model quantum fields directly to the current experimental energies of about 100 GeV, and with some modest extrapolation to about 1 TeV. That is, any effects of a fourth spatial dimension had better not appear in particle interactions unless it is on a length scale much smaller than $(1\ \mathrm{TeV})^{-1}$.

The ideas [14,15] of the logarithmic running of couplings in standard supersymmetric grand unification (SUSY GUTs) suggest an extension to much higher energies. Renormalization group calculations allow extrapolation of the observed strong, weak and electromagnetic couplings as a function of energy; the three curves intersect at the supersymmetric grand unification scale, around 10^{16} GeV. This nontrivial intersection, which has been confirmed by increasingly precise accelerator experiments, is often cited as evidence for the unification scheme. In SUSY GUTs, the Standard Model structure including 3+1-D field theory is preserved up to this much higher energy scale.

The limits on the dimensionality of gravity are much weaker. Gauss' law tells us that the gravitational force falls off as r^{1-N_s} where N_s is the total number of spatial dimensions. Experiments [16] (motivated in part by brane world models) now confirm Newton's inverse square law (and hence $N_s = 3$) from astronomical scales down to hair's-width distances of about 250μm. This submillimeter scale is is however still vastly larger than those probed by the Standard Model fields; 0.3 mm corresponds to an energy of about 0.003 eV.

The "Cosmological Constant Problem" may be related to effects of extra dimensions. One way to state this problem (there are many) begins with the observation that the zero point fluctuations of Standard Model quantum fields on a given scale E, if they couple to gravity, correspond to a gravitating vacuum energy density of magnitude E^4. The observed vacuum energy (or Dark Energy) density is about equal to the critical density of the universe, which is about $(10^{-2}\text{eV})^4$. But the success of the Standard Model requires the presence of the field fluctuations at least up to TeV scales. (Above that energy, it is possible for fermion and boson contributions to cancel exactly due to supersymmetry. Below that energy, we know that the system is not supersymmetric in today's vacuum). As presently formulated, the theory requires an offset of the zero energy level of the vacuum magically tuned to a precision of 17×4 orders of magnitude.

If for some reason the coupling of gravity to zero point modes were strongly suppressed above 10^{-2}eV, the gravitating energy of the vacuum would come out about right. The corresponding length scale, 0.1mm, lies just below the current experimental tests for gravitational coupling, but will become accessible with the next generation experiments. (Note that the predictions of the brane worlds with extra dimensions on these scales, which are discussed below, naitvely have the wrong sign to solve the cosmological constant problem, since with $N_s > 3$ the gravitational force increases faster than r^{-2} on small scales).

A coincidence worth mentioning is that the gravitational timescale associated with matter at an energy density of $\rho \approx (\text{TeV})^4$ is $(G\rho)^{-1/2} \approx 1mm/c$. This coincidence is important in cosmology because it means that the uncertainty associated with the possible geometrical effects of gravity propagating in extra dimensions start at about the same place as the uncertainties associated with the possible new physics beyond the TeV scale. If there is indeed new physics at the 0.1mm scale responsible for the cosmological constant, then this also would explain the coincidence between the age of the universe when the cosmological constant starts to dominate the mass-energy (that is, about now), and the typical lifetimes of stars; that is, there may be a derivable reason why $\rho_{vacuum} \approx M_{Planck}^4 (M_{proton}/M_{Planck})^6$.

Aside from the cosmological constant, there is the corresponding Hierarchy Problem in particle physics itself: if the fundamental scale is 10^{19} GeV, how is the "light" TeV scale preserved in all orders of all interactions? (Related to this: what is the origin of Large Numbers of astrophysics, which derive from the large ratio m_{Planck}/m_{proton}?) The traditional approach is to invoke supersymmetry above the TeV scale to preserve the hierarchy, and to explain the large numbers as due to the logarithmically running couplings. In some of the new schemes however, the Planck scale is a kind of illusion; physics is fundamentally different above the ≈ 10 TeV scale, which is the only fundamental scale in the theory. The large numbers in these schemes arise from taking modest numbers to large powers; the gravitational force in ten dimensional space falls off as r^{-9}!

III EXTRA DIMENSIONS IN UNIFIED THEORY

The candidate Theory of Everything, sometimes called M theory (or supersymmetric superstring theory, or matrix theory, etc., depending on the limit and the context), consistently includes both quantum mechanics and general relativity, and possibly includes the Standard Model. It provides a framework for computing statistically the entropy of certain black holes from first principles. A hallmark of the theory is a formal melding and blurring of the distinction between string (and particle) degrees of freedom, and geometrical degrees of freedom. Powerful dualities are exploited to show the equivalence of different formulations and between large and small scales and strong and weak coupling limits. In spite of the fluid character of the ideas, one central property seems to become more firmly established with time: the theory exists only in 10 fundamental spatial dimensions.

The central idea for dealing with the 7 "extra" dimensions is "compactification": we are unaware of the extra dimensions because they are much smaller than the three normal large space dimensions. For example, a three dimensional tube can appear two dimensional if its walls are thin enough, and even one dimensional if it is long and thin enough. As the above remarks indicate, the size of the extra dimensions for Standard Model field propagation must be smaller than $(\mathrm{TeV})^{-1}$, and smaller than $(10^{16}\mathrm{GeV})^{-1}$ if SUSY GUT ideas are right.

The idea of compactification was investigated by Kaluza and Klein in the 1920's as a way to unify gravity and electromagnetism. They showed the geometrodynamics of an additional very small dimension could appear at low energies as an electromagnetic field tensor. Extra dimensions lead to new predicted degrees of freedom in fields— for example an intrinsically massless field creates a "Kaluza-Klein tower" of new, effectively massive excitations corresponding to harmonics of states propagating around the short new directions. Altough traditionally the extra dimensions and new effects associated with M-theory have generally been assumed to happen close to the Planck scale, this is not neccesarily the case; the most notable result of brane world models is that the extra dimensions may be very large, possibly even infinite in extent.

M-theory is known to contain structures that offer suggestive clues to compactification. Features called branes appear which have lower dimensionality than the whole space. They form sites where the fundamental objects, one dimensional strings, can terminate, suggesting that in a low energy theory the gauge interactions may be confined to a lower dimensional surface embedded in the ten dimensional space. Branes are often thought of as classical, defect- or soliton-like structures resembling cosmic domain walls, with a surface tension and an internal vacuum energy larger than that in the surrounding higher dimensional background space. (Since the latter can be negative, the background space can be a higher-dimensional Anti-de Sitter solution even though our universe has a positive energy density). New types of excitations, corresponding to degrees of freedom such as displacement of the brane in the higher dimensions, correspond to propagating modes and new types of particles that might be observed.

One of the most spectacular discoveries in unification theory is Maldacena's AdS5/CFT correspondence: $N = 4$ supergravity in extended 5D Anti-deSitter space is exactly equivalent to a conformal field theory on its 4D boundary, which can be regarded as just ordinary Minkowski space. Here is a concrete example of a quantum gravity theory with all the richness associated with fields in five dimensions, all the details of which map onto the behavior of a conformal field theory in the standard 4D spacetime [5].

A related earlier idea called "holography" was inspired by the thermodynamics and information content of black holes. The conjecture is that all 3D fields are actually encoded by some theory acting on a 2D surface. We know that black hole entropy is given by a constant times the surface area of its (two-dimensional) event horizon. This means that a finite (indeed countable) amount of data on a two-dimensional surface (roughly, a few bytes per Planck area) must suffice to specify everything going on in the three-dimensional volume of space within it. The holography conjecture is that this applies to the whole universe— that three-dimensional space in some sense is an illusion, that the actual behavior is in some more fundamental sense two dimensional.

IV BRANE WORLDS

Brane worlds start with the idea that the familiar Standard Model fields are confined to a wall or "brane". This structure has three large dimensions but a thickness TeV^{-1} or smaller in the other dimensions. Gravity on the other hand can propagate much farther into one or more larger dimensions (called the "bulk"). The brane can be thought of as a stable classical defect embedded in a highly symmetric space of more dimensions, usually Anti-deSitter. Within this framework there are many options.

In some models, one or two extra dimensions can be of surprisingly large size [6–8], as they are only constrained by the direct experimental gravitational probes of the order of a few hundred microns. Most of the "large" extra dimensions however must be much smaller than this. Elaborations of brane worlds have been explored; for example, some have multiple branes which interact gravitationally. In others, there are different branes for different Standard Model fermion fields, with bosons allowed to travel in the bulk between them.

In one interesting class of brane-world models ("nonfactorizable geometries", [9–12]), the extra dimensions can be even larger, but the larger embedding space is highly curved, which traps gravitons in a bound state close to a brane. (Such geometries are said to have a "warp factor" (!)). The curvature radius of higher-dimensional (e.g. Anti-deSitter) space is again on a mesoscopic scale, which may be as large as ≈ 0.2 mm. Macroscopic black holes can be pictured as thin pancakes stuck to the brane, with only three large dimensions. The AdS space is Poincaré invariant and is itself a stable solution, so the setup is dynamically self-consistent, the kind of structure which might develop naturally from a defect in fields in higher

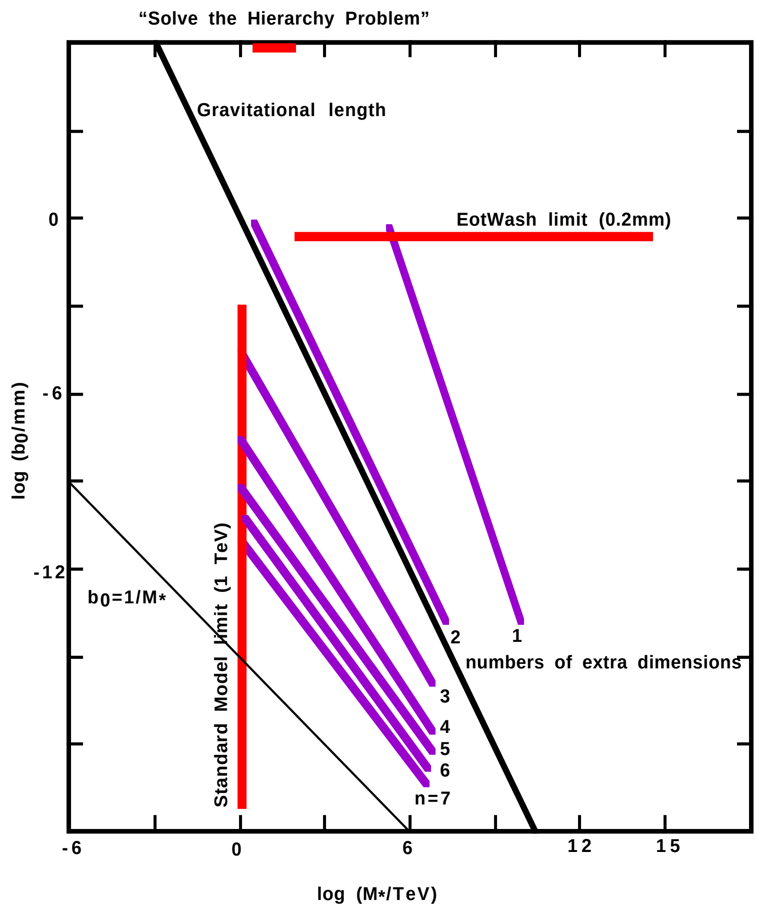

FIGURE 1. Summary of parameters for simple brane world models. It is assumed that there is a single unification scale M_* and that there are n extra dimensions of equal size b. The series of lines labeled $n = 1$ to 7 correspond to viable models with the right gravitational coupling at low energies. The "Gravitational length" line, degenerate with $n = 2$, denotes the Schwarzschild radius of a black hole with mean density M_*^4. The thresholds of direct current particle and gravity experiments are shown. Models with only one fundamental scale, which may "solve the hierarchy problem", lie not far beyond the reach of current accelerator constraints.

dimensions.

The apparent (usual) Planck mass in 3+1D, M_{Planck}, is related to the true fundamental scale M_* by $M^2_{Planck} \approx M^2_*(M^n_* V_n)$ where V_n is the volume of n extra dimensions (larger than M^{-1}_*) in which gravity propagates. Thus if there is one extra dimension much larger than the others, the mm limits from gravity experiments require a unification scale $M_* = 10^6$TeV or larger. If there are two extra large dimensions the mm limits give M_* close to the TeV scale. If we try to solve the hierarchy problem with a single M_* not too far above the TeV scale, this can be accomplished for $n \geq 2$ by choosing suitable extra dimension sizes; for 7 equally large extra dimensions, we might have gravity propagating in ten dimensions, seven of which have size $b \approx 10^{-10}$ mm. The range of options for a simple model with n extra bulk dimensions of the same size b_0 is illustrated in figure 1.

An interesting result is that the unification implied by the running-together of Standard Model coupling constants can still work in brane-world scenarios, but the three gauge couplings come together at a much reduced energy [17,18]. With the addition of extra dimensions *for the gauge fields* (as well as gravity), the renormalization of the fields produces a power-law dependence of coupling on energy (like gravity always had), so that they run together in a rather modest range of energy. For example, if the brane has a width in a single extra dimension of TeV^{-1} then at higher energy the Standard Model couplings rapidly converge and meet in a point at about 20 TeV. This is regarded as less elegant than the parameter-free running-together of SUSY at the 10^{16} GeV GUT scale but it may be the way nature works. These schemes thus hold out the attractive possibility of a unification scheme, even including gravity, with just one scale; it is even possible that we might find full quantum gravity effects accessible at the level of the next-generation accelerators. The famous "Desert" and the Planck scale, which have shaped so much discussion in the past, may be mirages.

V BRANE ASTRONOMY

The new fields and particles of these models might appear at accelerators in various manifestations. Some of these appear as "normal" new particle effects, such as excitation of Nambu-Goldstone modes of brane oscillations which would show up with the same signatures of missing energy and momentum as a weakly interacting scalar particle, or radion modes which might appear with signatures resembling (if not identical with) a Higgs scalar. Other possibilities include quirky signatures, such as multiple, evenly-spaced events produced as a particle traveling in the bulk punches periodically through the 3-brane. Null results in laboratory searches for measurable departures from Newton's inverse square law at short distances are an important constraint for $n = 1$ or 2.

As usual, astrophysical environments reach farther into parameter space. New weakly coupled species in these models are constrained in the same way axions are, using arguments based on energy losses from supernovae and red giants. The

“Kaluza-Klein tower” states can be particularly interesting. Massive KK modes of the graviton are a generic effect, and their cosmological production is an important constraint. They are produced thermally in the early universe, and only avoid causing an overclosure catastrophe in some cases because they can be very weakly coupled to the thermal particles on the brane. By the same token, for the right parameters they are a cold dark matter candidate. KK ladders of massive sterile neutrinos are a possible candidate for warm dark matter, and may display unusual nonthermal energy distributions induced by species oscillations. Brane worlds may bring important new insights into the cosmological constant [19] and inflation [20]. It is even possible for gravitational waves to travel faster than light since they can take a “short cut” across the bulk.

A more speculative phenomenon, which potentially reaches even farther into parameter space, is the classical production of gravitational waves in the early universe, which survive to the present as a nonthermal stochastic background [21]. Brane-world models suggest new sources of stochastic backgrounds: their new geometrical degrees of freedom can be coherently excited by symmetry breaking in the early universe, leading to gravitational radiation today at redshifted frequencies appropriate for new observatories such as LIGO and LISA [22,23]. New extra-dimensional effects remain important until the Hubble length $H^{-1} \approx M_{Planck}/T^2$ is comparable to the size or curvature radius b of the extra dimensions [8,24,25], or until the temperature falls below the new unification scale, whichever happens last.

Of particular interest are two new geometrical degrees of freedom common to many of these models: “radion” modes controlling the size or curvature of the extra dimensions [24,26], and new Nambu-Goldstone modes corresponding to inhomogeneous displacements of the brane in the extra dimensions [8,13]. Cosmological symmetry breaking can create large-amplitude, coherent classical excitations on scales of order H^{-1} as the configuration of the extra dimensions and the position of the brane settle into their present state.

The scalar modes of this distortion have long ago disappeared since they are on a very small scale (i.e., less than 1 mm times the redshift, or about the size of the solar system today), but the tensor modes might be observable. Extra dimensions with scale between 10 Å and 1 mm, which enter the 3+1-D era at cosmic temperatures between 1 and 1000 TeV, produce backgrounds with energy peaked at observed frequencies in the LISA band, between 10^{-1} and 10^{-4} Hz. The background is detectable above instrument and astrophysical foregrounds if initial metric perturbations are excited to a fractional amplitude of $\approx 10^{-3}$ or more. As shown in Figure 2, brane world models which “solve the hierarchy problem” naturally produce backgrounds in the range of frequencies encompassed by LISA for all the viable cases, $n = 2$ to 7. Ground based detectors (LIGO, VIRGO, TAMA, GEO), probing higher frequencies, reach extra dimensions down to 10^{-15} mm and unification energies up to 10^{13} GeV.

Thus it is possible that gravitational wave astrophysics might “see” outside of the four dimensions of ordinary spacetime, and trace the details of how the three

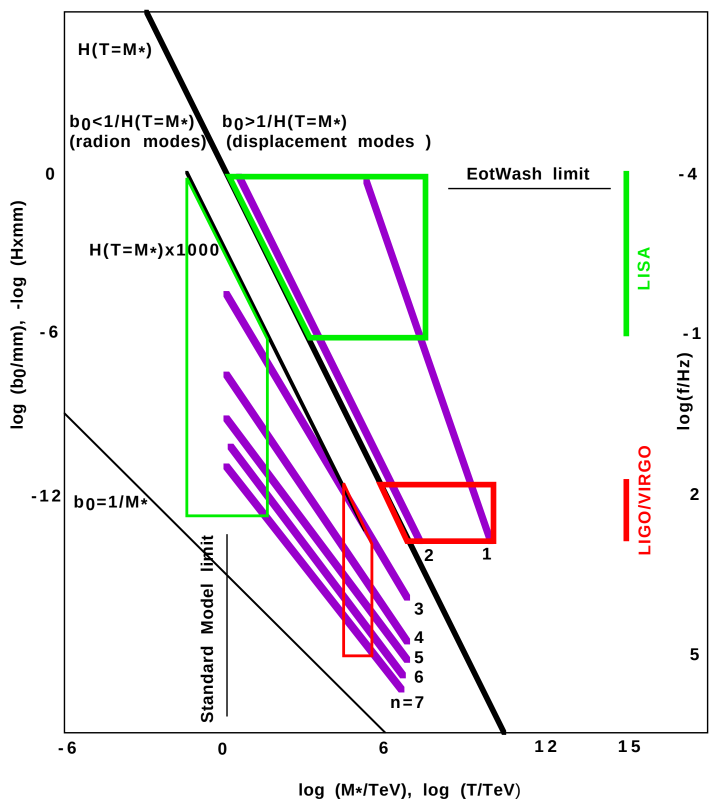

FIGURE 2. Summary of the new parameter space of extra dimensions that will be probed by gravitational-wave interferometers. Boxes indicate the corresponding regions of these parameters which may give rise to detectable mesoscopic gravitational radiation backgrounds in the LISA and LIGO bands. Heavy-line boxes show the displacement mode parameters, lighter-line boxes show the radion mode parameters. These regions extend well beyond those already constrained by gravitational experiments, direct particle production, or other astrophysical constraints. Theories which "solve the hierarchy problem" have M_* close to the Standard Model limit, and all of the viable ones ($2 \leq n \leq 7$) could possibly produce an observable background of one type or the other in the LISA band.

spatial dimensions settled into their present shape, in brane worlds that cannot be tested by any other known technique (see Figure 2). The background spectrum also contains information about a regime of cosmic history not preserved by any other relic (Figure 3).

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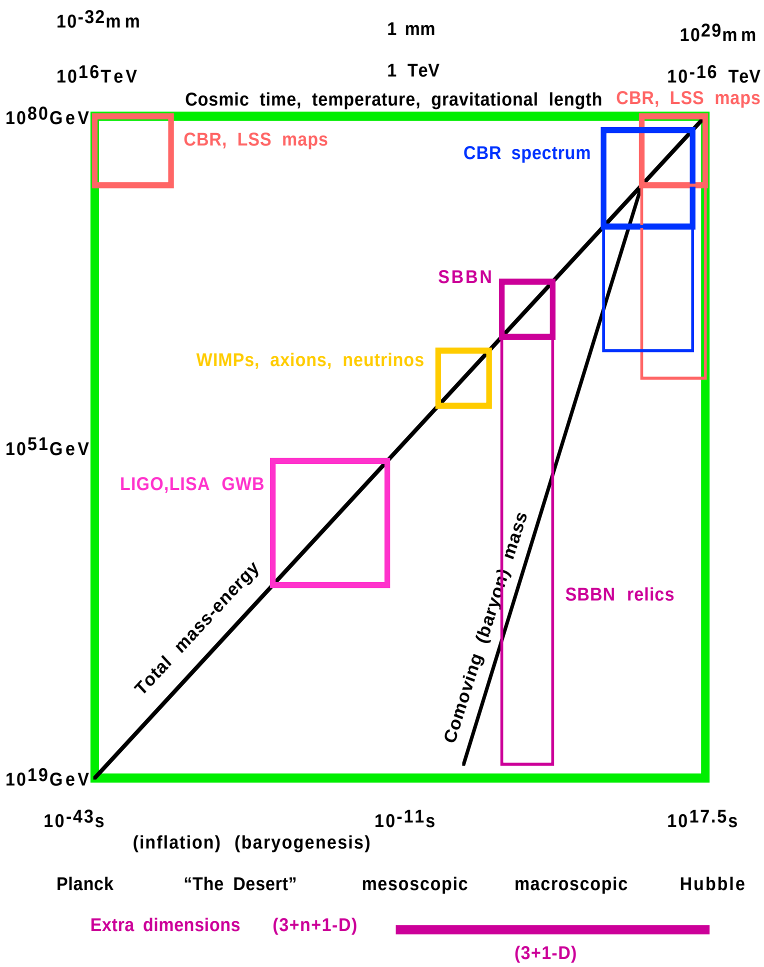

FIGURE 3. Summary of evidence about cosmic history, showing scale (in terms of total mass-energy) versus cosmic time/temperature. Boxes are labled by the technique used to constrain events in each domain of time and scale, including the microwave background anisotropy and spectrum, cosmological nucleosynthesis, and dark matter production. The box labeled LIGO,LISA GWB shows the hitherto unexplored region of mesoscopic phenomena which will either be opened up or constrained by gravitational wave astronomy— the universe earlier than 1 TeV, when it may have had more than three spatial dimensions.