Spacetime Physics from Worldvolume Brane Physics

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Spacetime Physics from Worldvolume Brane Physics

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Abstract

In this lectures we review some aspects of the worldvolume D-brane low-energy effective field theories of type IIA and type IIB string theory as well as of M-theory. The algebraic approach to analyzing the physics of the interesection of M-branes is also presented. These notes represent an extended version of the lectures given at the spring school on QFT, Supersymmetry and Supergravity, Călimăneşti, 1998.

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Contents

1   Introduction

At present, there is a widely spread belief in the high energy theorist community that one has to go beyond QFT in order to unify all the fundamental interactions present in Nature. The most promising candidate for the unifying theory, until five years ago, was string theory. The perturbation theory revealed that actually there are five distinct string theories of which spectra contain a massless spin 2 particle identified with the graviton. Moreover, it was shown using the vertex operator technique, that the interaction theory of strings provide us with finite scattering amplitudes. This is in fact the major achievement of string theory, namely that it leads to an (ultraviolet) finite (unitary) perturbative theory of gravity.

There are, however, serious drawbacks of string theory. One of them is the spacetime dimensionality. Quantum Lorentz invariance imposed by high energy phenomena to which string theory is applied and supersymmetry fix the spacetime dimension at ten. It turned out that there is no way in which a unique four dimensional physics be extracted from string theory. Another drawback is the number of string theories since all of them are legitimate to claim the title of an unique theory.

In 1994, a resolution of the uniqueness of string theory was put forward. The idea behind it is that one can interpret the five string theories as being different facets of a unique unknown underlying theory, called *M-theory* which should lie in eleven dimensions. In this picture string theories represent only phases of M-theory describing it in different regimes. Most striking, they appear to be related each other by some special relations called *dualities* which are of a limited number of types: T-, S-, U- and Mirror dualities. Basically, these dualities relate the string theories and their compactifications to lower dimensions in a non-perturbative way. (T-duality is an exception since it relates weak coupling limits of different theories.) They may map a weak-coupled theory to a strong-coupled one, or a theory compactified on a small size manifold with another one compactified on a large size manifold, or combinations of these. Moreover, some manifolds may appear as being equivalent or mirrored to each other.

Since the dualities rely on the existence nonperturbative aspects of string theories and since there is no nonperturbative string theory at present, the dualities remain merely conjectural statements. However, one can make several checks of them based on computations which involve objects that can be found in perturbation theory and remain unaffected by the changing of the coupling constant. These objects are protected by supersymmetry: they belong to some short representation of the supersymmetry algebra which has a constant dimensionality under the continuous variation of any parameter of the theory. To obtain these magic objects one has to use the *low energy effective action* approach to string theory. By definition, the *effective action* is such that if the tree level scattering amplitudes are computed into its framework, these coincide with the S-matrix elements involving the massless states of strings. The latter ones, as mentioned previously, can be computed using the vertex operators. We should mention that there is an alterna-

tive approach to the effective action based on σ-models. [1] An effective action contains an infinite number of terms which can be organized according to the number of spacetime derivatives contained in each term. The terms which contain the lowest number of derivatives and which give the dominant contribution to the scattering amplitude when the external massless particles have small energy and momenta form the *low-energy effective action*. Thus, this action describes a field theory for massless states of string perturbative spectrum which are identified with the massless particles of the 'real world'. It is obvious by now the interest in the massless states and in the scattering processes in which they are involved. The main reason behind this is the fact that the string theory contains a natural length parameter $\sqrt{\alpha'}$ which should be of the order of $10^{-33}cm$ if the string theory is to describe the correct scale of interactions. Therefore, the massive modes in string theory should have masses of order $m \sim 10^{35}cm \sim 10^{19}$ GeV which is far from the reach of the present accelerators.

The low energy effective actions for strings are all known: they are the four types of supergravities in ten dimensions. The non-perturbative objects that are involved in dualities represent solutions of the equations of motion of supergravities. Most of them are higher dimensional solutions. Generically, they are known under the name of *branes*, but their classification includes particles, strings, waves, monopoles and hypersurfaces. Some of them are solitonic objects carrying charges that are preserved by spacetime topology. Others are considered fundamental objects. A very important class of branes consists of the so-called *D-branes* which represent hypersurfaces on which open strings can end.

In these lectures we will review the physics of these branes from the worldvolume point of view. The review is divided as follows. In Sec.II we present some basics of effective field theory approach and on the appearance of topological defects in such theories. The general features are illustrated in the case of a scalar field theory. The main motivation for studying topological defects is that the branes can play the same role in effective theories of strings. In Sec.III we shortly describe branes in type IIA and type IIB string theories since they admitt D-branes. In the Sec.IV we focus on the branes of M-theory. It is believed that the

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[1]It is fair to say that a rigorous effective action should be obtained just in a well defined field theory which, unfortunately, does not exists for strings.

low-energy limit of M-theory is the eleven dimensional supergravity. From this, one can obtain superstring theories by performing appropriate compactifications and dualities. The two distinct branes of M-theory are the membrane and the fivebrane. In the last section we introduce the basic ideas in investigating the intersection of M-branes from an algebraic point of view.

String theory, M-theory, branes, supersymmetry and supergravity are topics of high interest at present on which a vast literature has been written. Therefore, it is impossible to give an exhaustive bibliography on the issues addressed in this review. However, at various stages in writing these lectures we have found useful some classical books, reviews and articles. From these we mention here mostly the lectures and the introductory papers rather than original papers, with the clear idea in mind of leading the interested and perhaps unfamiliarised reader to the field. The choice of below references is incomplete and is rather a matter of taste.

For an introduction to perturbative string theory, one should see the classical references [1, 2, 3] and the recent reviews [4, 5, 9]. Nice presentations of branes can be found in [6, 7, 8]. For non-perturbative aspects of string theories and string dualities one should consult [11, 12, 20, 22, 28, 38]. Reviews of topics as effective worldvolume actions of D-branes, relation between D-branes and M(atrix) theory, quantum field theories from D-branes can be found in the following literature [10, 14, 15, 13]. The connection between branes and black holes is presented in [16]. Branes as solutions of supergravity is the subject of [17, 18, 19]. For introduction in M-theory, M-branes ane M-brane intersections the reader is relegated to [21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37]. The results about the algebraic approach to intersecting branes can be found in [39]. An introduction to string theory is also given by W. Troost in his lectures while a presentation of supersymmetry can be found in A. Van Proeyen's lectures at this school.

2 Topological defects in QFT

In this section we present the basic ideas that underlie the study of topological defects of quantum field theories. Since the dynamics of perturbative massless modes of strings is described by an effective field theory, we firstly review its philosophy. Next we will show how one can describe the dynamics of fluctuations around classical solutions of scalar field theory in four dimensions. We will end this section by reviewing the topological defects in supersymmetric field theories.

2.1 Philosophy of The Effective Field Theories

The low-energy effective field theory represents a very important tool to extract information from string theory. This theory describes the dynamics of perturbative massless states of strings. Among these, there are states that can be associated to the massless bosons that mediate the interactions in standard model *and* to the graviton. This method is effective if one limits the scale of interactions at small values.

To illustrate the basic ideas of effective field theories, let us consider the quantum electrodynamics which describe the physics based on electrons and photons. The Lagrangian L_{QED} describes scattering processes like $ee \to ee$, $\gamma\gamma \to \gamma\gamma$, $e\gamma \to e\gamma$, and so on. Now if we consider only those scattering processes which take place at energies under m_e, which is the electron mass, electrons are never produced. Therefore, we can integrate the electron degrees of freedom in the functional integral

$$\int \frac{[d\psi][d\bar{\psi}][dA_\mu]}{Vol.gauge} e^{iS_{QED}}, \tag{1}$$

where the integration volume is divided by the volume of gauge group. In this way we pass from the Lagrangian L_{QED} to the Lagrangian of the effective theory

$$L_{eff} \sim -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + B(F_{\mu\nu}D^\mu A^\nu) + C(\cdots) + \cdots, \tag{2}$$

where the natural scale $(m_e)^{-n}$ is given by n and the terms in brackets are nonrenormalizable in power counting sense. This example exemplifies the steps one should take in writing any effective field theory: i) identify the physical degrees of freedom; ii) integrate

out some of them according to the energy scale of the process. Note that in (2) we cannot compute $B, C, \ldots$ exactly because, in general, the involved integrals cannot be computed.

The same approach can be used in the case of quantum gravity. In this case one should consider that Einstein-Hilbert action represents just the leading term in the low-energy description of gravity that incorporates short distance degrees of freedom

$$\int d^x \sqrt{g} R + \cdots, \tag{3}$$

where the dots represent terms that are suppressed by $(m_P)^{-k}$. m_P is the Planck mass and k is the natural scale.

One can think that string theory is also a low energy expansion but this time about a point particle theory. The massless spin one particles belonging to the string spectrum reproduce a Yang-Mills theory in $\alpha' \to 0$ limit, while the massless spin two state has, in the same limit, the effective action identical with the Einstein-Hilbert action of gravitation.

To extract the low-energy field theory from string theory one can use two alternative approaches. The first one is based on string scattering amplitudes of massless states. These can be computed using vertex operators. Then one has to write down a Lagrangian that reproduces the scattering amplitudes in a perturbative fashion considering that at each order the Lagrangian is invariant with respect to all the symmetries of the string theory. It is important to notice that only using the full set of symmetries which include general coordinate invariance, gauge symmetries and supersymmetries one can derive a unique Lagrangian which furnishes the low-energy field-theory. The second method is based on σ-model approach and consists in doing the computations to all orders in σ-model perturbation theory and to lowest order in string theory.

2.2 Topological Defects in d=4 Scalar Field Theory

Another important tool in investigating string theory is represented by the classical solutions of the corresponding low-energy effective field theories. The dynamics of these solutions can be derived considering small fluctuations. The solutions themselves represent topological defects in spacetime.

To grasp the basic ideas, let us consider the simplest example of a scalar field theory

in d=4 described by the following Lagrangian

$$L = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}g^2[\phi^2 - \frac{m^2}{g^2}]^2, \tag{4}$$

where we consider the spacetime signature $(-,+,+,+)$ and $\mu, \nu = 0,1,2,3$. The theory given by (4) has two different vacua with $m_{vac} = \pm\frac{m}{g}$. The equations of motion derived from the Lagrangian above admits a static solution (known in the equivalent $1+1$ dimensions under the name of 'kink') given by

$$\phi_{cl}(z) = \frac{m}{g}tanh\frac{mz}{\sqrt{2}} \tag{5}$$

which interpolates between the vacuum at $z \rightarrow -\infty$ corresponding to $-m/g$ and the vacuum at $z \rightarrow +\infty$ corresponding to $+m/g$. If we represent the energy density as a function of z it has a maximum of m^3/g^2 at $z = 0$. This classical static solution is thus a soliton and it is heavy in the perturbation theory. From the geometrical point of view it represents a membrane which extends along the x,y directions in the coordinates we have chosen.

It is interesting to describe the dynamics of the membrane. To this end let us consider small fluctuations around the static solution (5)

$$\phi(t,x,y,z) = \phi_{cl}(z) + \delta\phi(t,x,y,z). \tag{6}$$

Inserting (6) in (4) we obtain the following expansion of the Lagrangian around the static solution

$$L = L(\phi_{cl}) - \frac{1}{2}\partial_i(\delta\phi)\partial^i(\delta\phi) + [\frac{\partial^2}{\partial z^2} + m^2 - 3m^2tanh^2(\frac{mz}{\sqrt{2}})]\delta\phi + O[(\delta\phi^3)], \tag{7}$$

where $i = t,x,y$. In the Lagrangian above the third term represents the action of a definite differential operator on the variation of the scalar field. Let us derive its spectrum. Since the operator is homogeneous in x and y we can separate the variables

$$\delta\phi(t,x,y,z) = B(t,x,y)Z(z) \tag{8}$$

and we arrive at the following equation

$$[\frac{d^2}{dz^2} + m^2 - 3m^2tanh^2(\frac{mz}{\sqrt{2}})]Z(z) = \omega Z(z), \tag{9}$$

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where ω denotes the eigenvalues of the differential operator. The spectrum $\{\omega\}$ is positive definite and consists in a discrete set at small values of ω and a quasi-continuum set at large values. The smallest acceptable eigenvalue is $\omega_0 = 0$. The corresponding eigenfunction is $Z_0(z) = \phi'_{cl}(z)$ and it gives a free theory for $\delta\phi$. It is also responsible for breaking the translational invariance of the original theory along z axis. Thus, the unbroken symmetries are the translations parallel with the membrane, i. e. along x and y axis. The fact that the symmetry is broken signals us the presence of a Nambu-Goldstone boson. Indeed, this corresponds to $Z_0(z)$ and is transverse to the membrane.

The low-energy dynamics of the topological defect can be obtained by taking into account only the zero mode oscillations which can be studied at long or short distances. Since we would like to keep only finite terms in expansion, let us deal with the long distance case. The Lagrangian (7) with (8) becomes

$$L = L(\phi_{cl}) - \frac{1}{2} Z_0^2(z) \partial_i B \partial^i B + \cdots = -Z_0^2(1 + \frac{1}{2}\partial_i B \partial^i B). \qquad (10)$$

Integrating this Lagrangian we arrive at the following first order action

$$S = -T \int dt dx dy (1 + \frac{1}{2}\partial_i B \partial^i B + \cdots), \qquad (11)$$

where the membrane tension is given by the integration of $Z_0(z)$

$$T = - \int dz L(\phi_{cl}) = \int dz Z_0^2(z). \qquad (12)$$

The action (11) describes the dynamics of the infinitesimal fluctuations of zero mode. To obtain the dynamics of finite fluctuations we have to take into account all terms in expansion. The form of the formal series in B tells us that the following action can do the job

$$S = -T \int dt dx dy \sqrt{1 + h^{ij}\partial_i B \partial_j B}, \qquad (13)$$

where $h^{ij}(t, x, y)$ is the three dimensional induced metric on the membrane worldvolume. We can recast (13) in a manifest covariant form. Indeed, if we consider that we have the parametric description of the topological defect

$$x^0 = t$$

$$x^1 = x$$
$$x^2 = y$$
$$x^3 = B(t, x, y), \tag{14}$$

employing a general parametrization $x^\mu(\sigma_0, \sigma_1, \sigma_2)$ where $\{\sigma_0, \sigma_1, \sigma_2\}$ are coordinates on the worldvolume, we can write the action of finite fluctuations as

$$S = -T \int d^{2+1}\sigma \sqrt{-det(\partial_i x^\mu \partial_j x^\nu h_{\mu\nu})}. \tag{15}$$

In deriving (15) we have used the following property of 3×3 determinant

$$det(g_{ij} + t_i t_j) = det g_{ij}(1 + t^j t_j), \tag{16}$$

where $t^i = g^{ik}t_k$. The above action describes the long distance dynamics of membrane, i.e. it does not see the 'thick' of the membrane. It shows that the geometry completely determines the dynamics of the topological defect. It is important to notice that (15) is invariant under 3d diffeomorphisms, while the original theory (4) is invariant under 4d diffeomorphisms. This illustrate a generic feature, namely that *the topological defects break down the symmetry of the background.* As a consequence, there are Nambu-Goldstone (NG) bosons associated to the broken bosonic symmetries. In the case at hand, the NG field is $X_3(\sigma)$ associated to the transverse momentum to the membrane. A p-dimensional topological defect embedded in a D-dimensional spacetime also breaks the symmetries of the original theory. These symmetries are in general diffeomorphisms, gauge symmetries and supersymmetries, but they can also be discrete symmetries of different types. The topological defect would break the diffeomorphisms from D to $(p+1)$ which correspond to the translational symmetries of worldvolume. The rest of D-$(p+1)$ broken diffeomorphisms will appear as an NG bosons. The other symmetries can be totally broken or can be partially preserved. Also, due to the presence of the NG bosons and other worldvolume tensor fields, new gauge symmetries may arise.

If we are interested in short-distance membrane dynamics, we need to consider all higher oscillation modes beside ω_0 as well as the interactions of them with zero mode. The action will be described by an infinite sum of terms containing the curvature of the membrane. In this case the geometry determines the dynamics, too.

2.3 Topological Defects in SUSY Field Theories

Let us generalize the main result of the previous section, the action (15), to a supersymmetric field theory. To this end, assume that the SUSY field theory admits a topological defect (classical solution of the equations of motion) with the same properties as the ones considered in the previous section, and that this geometrical object is embedded in the target superspace of the theory. Take for superspace the coordinates $Z^M = (X^\mu, \Theta)$, where $X^\mu = \{X^0, X^1, \ldots, X^D\}$ are the bosonic coordinates and Θ's are their fermionic superpartners. Also consider that the topological defect is $p + 1$ dimensional and that its worldvolume is parametrized by the bosonic real variables $\{\sigma^0, \sigma^1, \ldots, \sigma^p\}$. The embedding of the topological defect in the target superspace is thus given by $(X^\mu(\sigma), \Theta(\sigma))$.

A generalization of the dynamics of the oscillations around the classical p-brane to the super-p-brane can be given in terms of super-momenta which generalize $\partial_i x^\mu$ terms in (15). Their expression enter the 1-superforms that replace dx^μ and is given in terms of worldvolume 1-forms by

$$dx^\mu \to dX^\mu - i\bar{\Theta}\Gamma d\Theta = \Pi_i^\mu d\sigma^i. \tag{17}$$

Then the worldvolume action of the finite zero modes of the superbrane is

$$S = -T \int d^{p+1}\sigma \sqrt{-det\Pi_i^\mu \Pi_j^\nu h_{\mu\nu}}, \tag{18}$$

where $h_{\mu\nu}$ is the induced worldvolume metric.

The original theory is invariant under bosonic diffeomorphisms which regard X^m coordinates and under target space supersymmetry

$$\delta X^\mu = i\bar{\epsilon}\Gamma^\mu \Theta$$
$$\delta\Theta = \epsilon, \tag{19}$$

where Γ's form a D-dimensional representation of Clifford algebra and ϵ is some infinitesimal fermionic parameter. We can argue that the superbrane breaks the diffeomorphisms of the bosonic subspace down to $Gl(p+1)$ in a similar manner to the bosonic case. What about the supersymmetry? Is it completely broken or not? To have a linearly realized supersymmetry we need an equal number of bosonic and fermionic degrees of freedom. In D-dimensions, a p-brane leaves D-$(p + 1)$bosonic degrees of freedom corresponding to

the transverse directions to the worldvolume of the brane. The rest of $(p+1)$ degrees of freedom correspond to the coordinates associated to the translational symmetry of the worldvolume action. Thus, in order to have unbroken supersymmetry, we should have the number of fermionic degrees of freedom equals to D-$(p+1)$. This clearly depends on D and p. Therefore, let us take an example.

Consider $N = 1$ $d = 11$ supersymmetry and assume that we have a p-brane. The number of bosonic degrees of freedom is $11 - (p+1) = 10 - p$. The fermionic degrees of freedom can be counted by going to the on-shell formulation. Since in $d = 11$ we have 32 fermionic components one has on-shell 16 components. By equating $10-p = 16$ we see that $p = -6$ which is a nonsense! A first conclusion is that there is no topological defect that preserves some of the supersymmetries unless there is some other symmetry that reduces further the number of fermionic degrees of freedom. Such of symmetry fortunately exists. It is a fermionic gauge symmetry and it is called *kappa-symmetry* or *k-symmetry*. Its general form is

$$\delta_k X^\mu = i\bar{\Theta}\delta_k\Theta$$
$$\delta_k\Theta = \cdots, \tag{20}$$

where $k(\sigma)$ is the infinitesimal gauge parameter and the dots are in place of some specific terms depending on the theory. The most remarkable property of k-symmetry is that it kills half of the fermionic degrees of freedom, as in the string theory.

We would like to implement k-symmetry to the superbrane worldvolume theory in order to have some supersymmetry preserved. The action (18) is not invariant under (20) and to compensate its variation one has to add an extra term called *Wess-Zumino term*, such that the total action

$$S_T = S + S_{WZ}, \tag{21}$$

is invariant under $p + 1$ diffeomorphisms, k-symmetry and some supersymmetry. Note, however, that while the variation of S_T is zero under these symmetries, the Lagrangian varies to a total derivative. In general, *when the Lagrangian is not invariant but quasi-invariant, i. e. invariant up to a total derivative, a central charge is present in the algebra of its symmetries.*

Let us return to our example. Due to k-symmetry, the number of on-shell bosonic degrees of freedom is halfed. Therefore, there is a topological defect that preserves some supersymmetries for $p = 2$. The corresponding field theory is the theory of worldvolume fields and it is $2 + 1$ dimensional. Therefore, we have $N = 8$ susy with scalar fields and fermions and there are sixteen realized supercharges, which is half of the number of supersymmetries of the background. This illustrates another general feature of extended solutions (or topological defects), namely that they *preserve 1/2 of the supersymmetries of the background theory as a consequence of k-symmetry.*

We can picture how supersymmetry determines the possible values of spacetime and of extended objects and the field realization of the corresponding theories. This picture is called the *old brane scan* and can be put into the form of a table (1). On the vertical axis is represented the spacetime dimension while on the horizontal one is represented the worldvolume dimension of the brane. In this table we represented only the field theories based on scalar multiplets (some of them come also from vector multiplets). For a complete version of this scan, see [18].

The question mark stands for the 9-dimensional brane of d=11 supergravity, which is in fact a boundary. Such an object should have a very interesting dynamics since it is very heavy and fills almost the entire space. Thus it represents a strong source for the gravitational field. It appears that it is related to the massive supergravity in ten dimensions. The relation that governs the objects present in (1) is

$$d - (p + 1) = \frac{n_\Theta}{4}, \tag{22}$$

where in the l.h.s. are the bosonic degrees of freedom while in the r.h.s. are the fermionic ones. The denominator 4 comes from considering that on-shell the number of background fermionic degrees of freedom n_Θ should be taken in half and that k-symmetry halves one more the on-shell fermionic degrees of freedom.

Table 1: The old brane scan.

$d\uparrow$	0	1	2	3	4	5	6	7	8	9	10	11
11	.			•						?		
10	.		•			•						
9	.	•			•							
8	.			•								
7	.			•								
6	.		•		•							
5	.	•		•								
4	.		•	•								
3	.	•	•									
2	.	•										
1	.											
0	.	.	.	.	.	.	.	.	.	.	.	.
	0	1	2	3	4	5	6	7	8	9	10	11 $p+1\rightarrow$

3 Branes in Type II String Theories

We present in this section the properties of the branes (topological defects) in type II string theories. We will firstly review the low-energy effective field theory of strings. Then we list some of the properties of branes that couple to NS-NS massless states of string perturbation spectra. There are also branes that carry RR charges, called D-branes, which will be discussed in the end.

3.1 Type IIA and Type IIB Effective String Theories

Anomaly cancellation in string theory fixes the spacetime dimensionality to ten. In $d = 10$ one can have $N = 2$ and $N = 1$ supersymmetry algebra. $N = 2$ superstring theories belong to the so called type II class, and there are two of them: type IIA which is non-chiral and type IIB which is chiral.

In order to give a low-energy effective field theory description of these two theories

one has to identify firstly the massless string modes. These are given by the perturbation string theory. The bosons belong to NS-NS and R-R sectors. The NS-NS sector is common for both theories and it contains the following fields

$$\left.\begin{array}{cc} g & 35 \\ B_{(2)}/\tilde{B}_{(6)} & 28 \\ \phi & 1 \end{array}\right\} \text{IIA/IIB NS-NS},$$

where g is the metric, ϕ is the dilaton and B is the 2-form associated to the antisymmetric tensor field. The field content of R-R sector for type IIA theory is

$$\left.\begin{array}{cc} A_{(1)}/\tilde{A}_{(7)} & 8 \\ A_{(3)}/\tilde{A}_{(5)} & 56 \end{array}\right\} \text{IIA R-R},$$

where A's are antisymmetric tensor potentials of which duals are denoted by tilde. The number of degrees of freedom bring in by $A_{(1)}$ is 8 and by $A_{(3)}$ is 56. The RR-sector of type IIB theory contains the fields

$$\left.\begin{array}{cc} A_{(0)}/\tilde{A}_{(8)} & 1 \\ A_{(2)}/\tilde{A}_{(6)} & 26 \\ A(4)(chiral) & 35 \end{array}\right\} \text{IIB R-R},$$

where the number of degrees of freedom is 1 for the form $A_{(0)}$, 26 for $A_{(2)}$ and 35 for $A_{(4)}$ which has a self-dual strength form. The spinors belong to the NS-R and R-NS sectors. For IIA theory they come in the following form

$$\left.\begin{array}{cc} \psi_\mu^\alpha/\psi_{\mu\alpha} & 56+56 \\ \lambda_\alpha/\lambda^\alpha & 8+8 \end{array}\right\} \text{IIA NS-R,R-NS},$$

where $\psi_\mu^\alpha, \psi_{\mu\alpha}$ are spin-3/2 gravitino fields of opposite chirality and λ's are the associated spinors. The ψ's have each 56 degrees of freedom while λ's have 8 degrees of freedom. The NS-R and R-NS sectors of type IIB string theory, due to the same choice of the left/right vacuum states, lead to a chiral theory with

$$\left.\begin{array}{cc} \psi_\alpha^{i\mu} & 2\times56 \\ \lambda_{i\alpha} & 2\times8 \end{array}\right\} \text{IIB NS-R,R-NS},$$

where the number of degrees of freedom is 2×56 for gravitini and 2×8 for spinors. All the four sector of the theories are tensor products of left and right sectors because type

II strings are closed strings. (For details on string theory see the references indicated in the introduction.)

The dynamics of the bosonic massless modes is described by an effective action which is the action of the corresponding supergravity and is given by

$$S = -\frac{1}{16\pi G} \int d^{10}x \sqrt{g} \{ e^{-2\phi}[R + 4(\nabla\phi)^2 - \frac{1}{2}H^2] - \sum_p \frac{1}{2(p+1)!}F_{p+1}^2 \} + S_{CS}, \quad (23)$$

where $F_{p+2} = dA_{p+1}$, $H = dB$ and $p = 0, 2$ in the type IIA case and $p = -1, 1, 3$ in the type IIB case. We note that, due to the presence of $A_{(4)}$ which has a self-dual 5-form field strength, type IIB theory was originally formulated in terms of the equations of motion. The Eq.(23) describes type IIB theory only if the self-duality condition is imposed by hand. At present, no covariant and local action that describes type IIB supergravity is known such that self-duality results as an equation of motion, but some progress in this direction was recently reported (see for example [32].)

Some remarks are in order here. We firstly note that the fields coming from NS-NS sector have a factor of $e^{-2\phi}$ in front. This factor do not appear for R-R fields and that has important consequences on the dualities among string theories. Secondly, the only arbitrary dimensionful parameter of the type II string theories is α', and therefore the Newton constant is expressed in terms of it, $G \sim \alpha'^4$. Thirdly, we remark that while there are perturbative string states carrying a charge with respect to the NS-NS fields, i. e. winding states and Kaluza-Klein momentum states when there is at least one compact direction, there are no such of states carrying R-R charges. In the end, we emphase that the last terms in (23) is of the form

$$S_{CSIIA} = \frac{1}{2} \int A_{(2)} \wedge H_{(4)} \wedge H_{(4)} \quad , \quad S_{CSIIB} = \frac{1}{2} \int A_{(4)} \wedge H_{(3)} \wedge H_{(3)}, \quad (24)$$

but for simplicity we will discuss only (23) without these Chern-Simons terms.

There are classical solutions of the equations of motion of the low-energy effective string theories which represent extended supersymmetric objects, called *branes*. When they have p translational spacelike Killing vectors they are called p-branes. Some of them can be interpreted as solitons and they play an important role in the non-perturbative theory. Some other represent elementary excitations with respect to some perturbation

formulation. In general they carry the charges of an antisymmetric tensor field. This can be understood if one thinks to an $A_{(p+1)}$-form as a generalization of the ordinary $A_{(1)}$ electromagnetic postential. The latter minimally couples in $d = 4$ to a particle (0-brane) via the action

$$q_e \int dx^\mu A_\mu, \tag{25}$$

where q_e is the electric charge. There is also the magnetic coupling to the 'dual' of A_μ (obtained in fact by dualizing the field strength $F = dA \to \tilde{F} \to \tilde{F} = d\tilde{A} \to \tilde{A}$) of the form

$$q_m \int dx^\mu \tilde{A}_\mu. \tag{26}$$

These couplings can easily be generalized to an $A_{(p+1)}$ potential form which has the dual $\tilde{A}_{(p+1)} = A_{(d-p-3)}$. The couplings are given by

$$q_p \int A_{(p+1)}, \tag{27}$$

where the electric charge is given by the integration over an $S^{d-(p+2)}$ sphere of the dual field strength

$$q_p = \int_{S^{d-(p+2)}} *F_{(p+2)}, \tag{28}$$

and the magnetic coupling is given by

$$q_{\tilde{p}} \int A_{(d-p-3)} \tag{29}$$

with the magnetic charge

$$q_{\tilde{p}} = \int_{S^{p+2}} F_{(p+2)}. \tag{30}$$

As an important remark, note that if $\tilde{p} = p$ the brane could carry electric and magnetic charges. Since $\tilde{p} = d - p - 4$ this implies that $p = \frac{d}{2} - 2$. Example of such solutions are the string $(p = 1)$ in d=6 and the 3-brane in d=10.

The electric and magnetic charges obey the generalized Dirac condition

$$q_{d-p-4} q_p = 2\pi n, \tag{31}$$

where n is an integer. Eq.(31) shows that a duality that exchanges the electric and magnetic potentials also exchanges the weak and strong coupling constants. If the coupling

is done through the field strength, the corresponding object carry no charge. Indeed, this generalizes to higher dimensions the well known coupling of the neutron with the electromagnetic field in four dimensional QED, i.e $\bar{\psi}\gamma^\mu\gamma^\nu\psi F_{\mu\nu}$.

In general, a brane that represents an elementary excitation contains a singularity. Therefore, it appears as an 'electric' object and its singularity plays the role of a source of the field. The 'magnetic' branes are usually not singular. They are solitons and their charge is protected by the topology of spacetime.

An important class of supersymmetric p-branes consists in the so-called *extremal p-branes*. They play a crucial role in providing evidence for string dualities. Extremal branes are those branes that saturate a relation of Bogomoln'yi-Prasad-Sommerfel (BPS) type between mass and charge. This relation is a consequence of supersymmetry of the theory. The fields of BPS objects belong to some short representation of the background superalgebra. The dimension of this representation does not change when any of the parameters of the theory, like the coupling constant, varies continuously. It follows that the properties of the BPS objects remain the same at strong coupling as at weak coupling. This reveals their non-perturbative character.

In reviewing some of the properties of the branes we made no distinction between their different types. We note now that while from the tensorial point of view the branes share similar properties, string perturbation theory distinguishes between NS-NS branes, i. e. those that couple to fields belonging to the NS-NS sector, and R-R branes, also called Dp-branes. The latter form the subject of the next subsection.

3.2 R-R Charges and D-Branes

Dp-branes can be defined as hyperplanes on which open strings can end. In string theory, they can be easily obtained by imposing Dirichlet boundary conditions on p-spatial directions and Neumann boundary conditions on the rest. They can also be deduced in the low-energy effective field theory as extended non-perturbative solutions. From counting the number of fermionic and bosonic degrees of freedom of the branes, we see that the following relation holds in in type II theories: $8 - (d - (p+1)) = p + 1$. This relation shows that there is an excedent of bosonic degrees of freedom that equals the number of

transverse components of an worldvolume $U(1)$ vector potential. This vector represents a massless excitation that couples to the endpoints of the open string and force it to end on the brane. Thus we conclude that the dynamics of a D-brane can be described in terms of the excitations of the open string.

Let us see how the dynamics of the low-energy massless modes of the open string can be described. If we impose Dirichlet boundary conditions on $9 - p$ directions X^i

$$\partial_\tau X^i|_{\sigma=0,\pi} = 0, \tag{32}$$

where (τ, σ) are the timelike, respectively spacelike, parameters of the string worldsheet, $0 \leq \sigma \leq \pi$ and $i = p + 1, \ldots, 9$. Eq.(32) is equivalent to

$$X^i(\tau, \sigma = 0) = x_0^i \quad , \quad X^i(\tau, \sigma = \pi) = x_\pi^i, \tag{33}$$

where x_0^i, x_π^i are some fixed constants. This means that zero modes of the open strings do not depend on the X^i's coordinates. The fact that zero modes of the strings are not dynamical in these directions implies that the corresponding low-energy fields do not depend on these coordinates, and therefore they belong to a representation of $SO(1, p)$ rather than $SO(1, 9)$. The effective field theory for open string with Neumann boundary conditions is $N = 1$ supergravity coupled to super Yang-Mills with gauge group $SO(32)$. The fields of this theory belong to $\mathbf{8_v} + \mathbf{8_-}$ representation of $SO(1, 9)$. The effect of introducing Dirichlet boundary conditions instead of Neumann boundary conditions along $p + 1$ directions is that of decomposing $\mathbf{8_v} + \mathbf{8_-}$ states under $SO(p - 1)$. This means that the low-energy effective theory is the dimensional reduction of super Yang-Mills theory in d=10 to $p + 1$ dimensions.

D-branes break half of the supersymmetries of the background theory since this is the maximum number allowed by the open string theory with Dirichlet boundary conditions. Far away from the D-brane it seems to be two generators of spacetime supersymmetry, however, they are related to each other on the worldsheet of the opens string localized near the D-brane. Indeed, one of the effects of Dirichlet boundary conditions is to implement spacetime parity reversals on right or left moving modes of type I string. If the D-brane lies on $X^1, \ldots X^p$ directions, then the effect of this operation on supersymetry is given by

$$Q_L = \pm P_{p+1} \cdots P_9 Q_R, \tag{34}$$

where $P_i = \Gamma_i \Gamma_{11}$ is the operator that anticommutes with any Γ_i and commutes with all others and $\Gamma_{11} = \Gamma_0 \cdots \Gamma_9$ is the chirality matrix. Since Q_R is antichiral it follows that

$$Q_L = \pm \Gamma_0 \cdots \Gamma_p Q_R. \tag{35}$$

However, we should also take into account that Neumann boundary conditions along the rest of directions impose that the two operators be equal

$$Q_L = Q_R. \tag{36}$$

Applying Γ_{11} to (35) and using (36) we conclude that, if the theory is nonchiral (type IIA) $p = -1, 1, 3, 5, 7$ while if it is chiral (type IIB) $p = 0, 2, 4, 6, 8$ These are the possible D-branes in type II string theory, which should be supplemented with a D9-brane in IIA case.

Let us analyze now the low-energy effective field theory of D-branes. Take a fundamental open string attached to a Dp-brane. It naturally couples to the antisymmetric tensor of NS-NS sector, B. Accordingly, there is a gauge transformation of B of the form $\delta B = d\Lambda$ which produces the following variation of the action

$$\delta \int_{\Sigma_2} B = \int_{\partial \Sigma_2} \Lambda \tag{37}$$

where $\partial \Sigma_2$ is the intersection of the string worldvolume with the brane worldvolume Σ_{p+1} according to the figure below
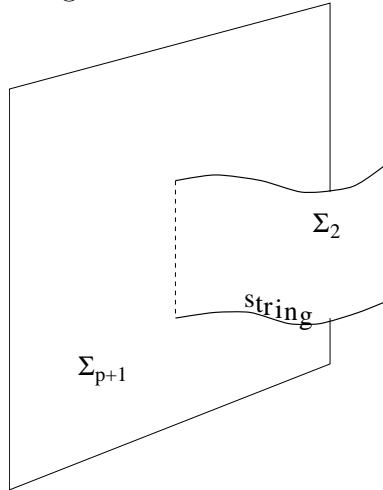
String ending on D-brane. The dotted line is $\partial \Sigma_2$.

The variation (37) is cancelled by a term

$$\int_{\partial \Sigma_2} A,$$

where A is a worldvolume vector potential transforming as $\delta A = \Lambda$ under the gauge transformation in the background. Thus, the gauge invariant field strength should take into account both background as well as worldvolume fields, and it is of the unique form $F = dA - B$. This suggests that the string endpoint generating A is a source needed for gauge invariance reasons. Now if we take into account the R-R tensor fields, the Wess-Zumino terms needed to have k-symmetry invariance as discussed in the previous sections, should be modified in order to be invariant under the above gauge symmetry. β-function calculations fix the action to the so called Dirac-Born-Infeld form

$$S = T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-det(G_{\alpha\beta} + F_{\alpha\beta})} + \mu_p \int d^{p+1}\sigma \sum_q A_q \wedge e^F, \qquad (38)$$

where A_q are R-R forms, μ_p is the R-R charge, $G_{\alpha\beta}$ is the induced worldvolume metric and

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + B_{\alpha\beta}. \qquad (39)$$

Here, A_α are the components of the $U(1)$ vector potential and $B_{\alpha\beta}$ is the pull-back of NS-NS field. Eq. (38) describes the dynamics of the bosonic degrees of freedom of a Dp-brane. It can be generalize to the supersymmetric case by embedding the worldvolume in superspace. Let us list some of the properties of D-branes.

1. They are BPS states and therefore preserve 1/2 of the supersymmetry of the background.

2. Their tension equals their R-R charge and it is given by

$$T_p = \mu_p \sim \frac{1}{g_s}.$$

   Therefore, they are heavy objects at small couplings but lighter than ordinary solitons which have tensions proportional to $1/g_s^2$.

3. The static force between parallel branes cancel, and thus we can put them on the top of each other.

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Table 2: The D-brane scan.

$d \uparrow$													
11	.												
10	.	●	●	●	●	●	●	●	●	●	●	●	8×8
9	.												
8	.												
7	.												
6	.	●	●	●	●	●	●						4×4
5	.												
4	.	●	●	●	●								2×2
3	.	●	●	●									1×1
2	.												
1	.												
0	.	.	.	.	.	.	.	.	.	.	.	.	.
	0	1	2	3	4	5	6	7	8	9	10	11	$p+1 \rightarrow$

If several branes are on the top of each other, new massless gauge bosons arise from the strings that were previously stretched among branes. The gauge symmetry is enhanced from $U(1) \times U(1) \times \cdots$ to $U(n)$ for a stack of n-branes.

There is an equation governing the equality between the the number of bosonic and the number of fermionic worldvolume fields in terms of spacetime and super D-brane worldvolume dimensions similar to the one derived for generic super p-branes. It reads as

$$d - (p+1) + (p-1) = \frac{n_\Theta}{4}, \tag{40}$$

where the first term in the l.h.s. comes from the scalar fields, the second one comes from the extra vector fields and the terms in the r.h.s represent the fermionic degrees of freedom. From it we can table the D-branes in the table bellow named the *D-brane scan*. The products denote the number of supercharges preserved in each targetspace dimension.

4   Branes in M-Theory

In this section we are going to review the extended solutions of supergravity in d=11 dimensions. At present, it is believed that this theory represents the low-energy effective field theory of a more general eleven dimensional quantum theory, named *M-Theory* which should unify all string theories in a consistent manner. Beside the above assumption, nothing else is known about M-theory except, maybe, a non-covariant formulation of it in terms of D0-branes (particles) known under the name of *M(atrix) theory* (see the references). From M-theory point of view, string theories represent d=10 dimensional different phases, related among themselves by dualities. There is a continuously increasing number of evidences that support the dualities which are merely conjenctures. A rigorous proof of them would involve mastering of string theories at both perturbative *and* non-perturbative level. Although no non-perturbative string theory has been put forward yet, there are a number of tools that allow us to check dualities. They are useful in investigating M-theory, too, and they are based on effective field theories, branes and supersymmetry.

In what follows we will review the basics of d=11 supergravity. Then we will review the M2 and M5 branes and finally we will shortly present the algebraic approach to the intersection of M-branes.

4.1   Scales in String Theory

M-theory, which one expects to unify all fundamental interactions, is unknown at present. Anyway, there are reasons to consider that its low-energy effective action should be the d=11 supergravity. One argument that supports this hypothesis is the fact that type IIA and heterotic $E_8 \times E_8$ superstrings can be obtained from d=11 supergravity by appropriate compactifications on one spacelike dimension.

Let us denote the fundamental string length in ten dimensions by l_s and the Planck length in ten and eleven dimensions by l_{10} and l_{11}, respectively. Now, if we want to compactify d=11 supergravity to obtain type IIA supergravity, we should consider one of the spacetime directions, say X^{11} as being a circle of radius R. In the limit when the fundamental string length $\alpha' = l_s^2$ goes to zero, it follows from Newton's constant in d=10

$G^2 = g_s^2 \alpha'^4$ that $l_s > l_{10}$, when g_s is the string coupling. On the other hand, the string coupling is given by the vacuum expectation value $g_s = e^{<\phi>}$ which is related to the component of the metric in d=11 dimensions $G_{11,11} = e^{2\sigma}$ by $e^{<\phi>} = e^{\frac{3}{2}\sigma}$. Thus, in the perturbative regime, i. e. $g_s < 1$, we conclude that

$$R \sim g_s^{\frac{2}{3}} l_{11}. \tag{41}$$

If we want to relate d=11 units to the string units we can use the fact that the Newton constant in d=10 string theory and the one from compactified d=11 theory should coincide

$$G_{10} \sim g_s^2 l_{10}^8 \sim \frac{G_{11}}{R} = \frac{l_{11}^9}{R} \tag{42}$$

from which we see that

$$l_{11} \sim g_s^{\frac{1}{3}} l_s \quad , \quad R \sim g_s l_s. \tag{43}$$

The conclusion is that at $g \ll 1$, both the eleven dimensional Planck length and the radius of the eleventh dimension are small compared to the string scale and therefore the following relation holds

$$l_s > l_{10} > l_{11} > R. \tag{44}$$

In the interval $l_s > l_{10}$ the gravity from the low-energy effective field theory can be decoupled from the super-Yang-Mills component of the theory as was recently shown in [34, 35, 36] using D-brane technology. In this limit another duality was conjectured, between the supergravity on AdS spacetime and CFT on its boundary.

Let us take a closer look at the compactification of d=11 supergravity. If we take the large R limit in the compactified theory, the Kaluza-Klein modes propagating in this direction become light. But the compactified theory is type IIA supergravity. Therefore, in this theory there should be states of which masses behave like

$$m = \frac{1}{R} \sim \frac{1}{g_s l_s}. \tag{45}$$

These are pointlike solitonic objects with vanishing masses in the strong coupling and carrying R-R charge.

A simple dimensional analysis gives us the relation between the tension of M2-brane and the tension of the fundamental string. Since the tension of M2-brane is $T_m = E/A =$

l_{11}^{-3} where E is the energy and A is the area, and the tension of the fundamental string is given by $T_s = E/L = l_s^{-2}$, it follows that the two tensions are related by $T_m = g_s^{-1}T_s^{3/2}$.

4.2 Effective Field Theory of M-theory: d=11 supergravity

As we have already noticed, one can interpret the eleven dimensional supergravity as the low energy effective field theory of M-theory. There is a huge literature on this topic going back to 1978 when d=11 sugra was discovered. The eleven dimensions represent a limitation on the dimensionality of spacetime in which supergravity can exist. This limitation is twofold: on physical grounds it reflects the fact that no massless fields with spin greater than 2 and no more than one graviton is permitted. From mathematical point of view, it is related to the Clifford algebras in $(1, d-1)$ dimensions and to the fact that gravitational interactions are incompatible with higher spin gauge fields. d=11 supergravity is also the unique quantum field theory in the sense that only N=1 supersymmetry is allowed and the field content is fixed.

The action of d=11 supergravity is given by

$$S = \int d^1 1\sqrt{-g}[k^{-2}R - \frac{1}{2}F_{MNPQ}F^{MNPQ} + kA \wedge F \wedge F + \bar{\psi}_M \Gamma^{MNP}D_N\psi_P] + (4fermi \ \ terms), \tag{46}$$

where k is the Einstein constant, A is the 3-form gauge potential and ψ is the 3/2 gravitino. Indeed, if we count the degrees of freedom we see that we should add to the $\frac{d(d-3)}{2} = 44$ transverse traceless spatial metric components $\frac{(d-2)(d-30(d-4)}{3!} = 84$ transverse spatial components A_{MNP} of the 3-form to obtain $2^{[\frac{d}{2}]-1} \times (d-3) = 128$ degrees of freedom of the fermionic spinor-vector gravitino.

If we want to obtain type IIA supergravity, we have to perform a Kaluza-Klein compactification. The Kaluza-Klein ansatze reads

$$ds^2 = e^{-\frac{2}{3}\phi}dx^\mu dx^\nu g_{\mu\nu} + e^{\frac{4}{3}}(dy - dx^\mu A_\mu) \tag{47}$$

for the d=11 line element and

$$A = \frac{1}{6}dx^\mu dx^\nu dx^\rho A_{\mu\nu\rho} + \frac{1}{2}dx^\mu \wedge dx^\nu \wedge dy A_{\mu\nu} \tag{48}$$

for the three form. Here, $\mu, \nu, \rho = 0, 1, \ldots, 9$ and y is the compactified dimension $y \sim y + 2\pi R$. Before going on and discuss the extended solutions of this theory, let us make a detour to the supersymmetry and see what its role is in the d=11 theory that we study.

Let us consider the supersymmetry algebra of d=11 supergravity and focus on the spinor charge anticommutators because they represent the relevant part for our discussion

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^M)_{\alpha\beta} P_M, \tag{49}$$

where C is the (real, antisymmetric) charge conjugation matrix, Γ's form a representation of Clifford algebra and P_M are the components of momenta. Let us suppose that there is some quantum theory that realizes the algebra (49) and label the vacuum of this theory by $|0>$. Since it is supersymmetric invariant we should have

$$Q_\alpha |0> = 0. \tag{50}$$

Assume that in this theory there is some (massless) state that preserves some fraction ν of the supersymmetry. Then the following relation should hold

$$<\nu|\{Q_\alpha, Q_\beta\}|\nu> \geq 0. \tag{51}$$

To determine the number ν we firstly remark that (49) implies that

$$det(\Gamma^M P_M) = (P^2)^{16} = 0, \tag{52}$$

which means that the momentum of these states is small. If we go to a frame where

$$P_M = \frac{1}{2}(-1, 1, 0, \ldots, 0) \tag{53}$$

and if we chose the Majorana representation (in which $C = \Gamma^0$), we can recast (49) in the following form

$$\{Q_\alpha, Q_\beta\} = \frac{1}{2}(1 - \Gamma_{01})_{\alpha\beta}, \tag{54}$$

where $\Gamma_{01} = \Gamma_0 \Gamma_1$. (It is easy to verify that $\Gamma_{01}^2 = 1$ which implies that its eigenvalues are ± 1. Also, since $Tr\Gamma_{01} = 0$, half of the eigenvalues are +1 and half are -1.) Now, for the states with zero eigenvalue, (54) gives us

$$\Gamma_{01}\epsilon = \pm\epsilon. \tag{55}$$

Thus, from all that was said above, we see that $\nu = 1/2$ and that massless states $|\nu>$ preserve half of the supersymmetry algebra (49).

These states can be associated to the massless superparticles which have the following action in Minkowski vacuum

$$S[X, \Theta, e] = \int d\tau \frac{1}{e}(\dot{X}^M - i\bar{\Theta}\Gamma^M \Theta)^2, \tag{56}$$

where $e(\tau)$ is an independent worldline scalar density, $X^M(\tau)$ are the bosonic fields and $\Theta_\alpha(\tau)$ are their fermionic superparteners. The action (56) is invariant under (rigid) superPoincaré transformations

$$\delta X^M = i\bar{\epsilon}\Gamma^M \Theta$$
$$\delta \Theta = \epsilon \tag{57}$$

with the spinor parameter $\epsilon(tau)$. The superparticle does not break all supersymmetries since there is also a kappa-symmetry of the action

$$\delta_k X^M = i\bar{\Theta}\Gamma^M \delta_k \Theta$$
$$\delta_k \Theta = \not{P}k(\tau). \tag{58}$$

If we use the light-cone gauge in which $\Gamma = \Theta = 0$, where $\Gamma^\pm = \frac{1}{2}(\Gamma^0 \pm \Gamma^{10})$, we see that the gauge fixed theory has sixteen linearly realized symmetries. Thus, the superparticle breaks half of the supersymmetries of the background field theory which has (49) as a symmetry.

Return now to the supersymmetry in d=11. Let us express the Lagrangian of the action (46) in terms of vielbeins and write down the indices (for pedagogical reasons). The result is

$$
\begin{aligned}
L &= -\frac{1}{2}eR(\omega) - \frac{1}{2}e\bar{\psi}_M \Gamma^{MNR} D_N(\omega)\psi_R - \frac{1}{48}eF_{MNRS}F^{MNRS} \\
&\quad - \frac{i}{\sqrt{2}}(12)^3\epsilon^{M_1\cdots M_{11}}F_{M_1\cdots M_4}F_{M_5\cdots M_8}A_{M_9\cdots M_{11}} - \frac{\sqrt{2}}{384}e(\bar{\psi}_M \Gamma^{MNRSTLN}\psi_N + 12\bar{\psi}^R \Gamma^{ST}\psi^L)F_{RSTL} \\
&\quad + (\psi^4(terms),
\end{aligned} \tag{59}
$$

where $e = det(e_M{}^A)$, $M, N, \ldots = 0, 1, \ldots, 10$ are spacetime indices and $A, B, \ldots = 0, 1, \ldots, 10$ are tangent-space indices. Also

$$\omega_{MAB} = \omega^0_{MAB} + \frac{1}{8}\bar{\psi}^P \Gamma_{PMABQ}\psi^Q \tag{60}$$

where ω^0_{MAB} is the spin connection in the absence of supersymmetry and D_M are the components of covariant derivative. We also used the notation

$$\Gamma^{M_1 \cdots M_n} = \Gamma^{[M_1} \cdots \Gamma^{M_n]}, \tag{61}$$

where the square brackets denote antisymmetrization. The transformations of e, ψ and A under supersymmetry are given by

$$\begin{aligned}
\delta e^A_M &= \frac{1}{2} \bar{\epsilon} \Gamma^A \psi_M \\
\delta A_{MNR} &= -\frac{\sqrt{2}}{8} \bar{\epsilon} \Gamma_{[MN} \psi_{R]} \\
\delta \psi_M &= [D_M(\omega) + \frac{\sqrt{2}}{288} (\Gamma_M^{RSLT} - 8\delta_M^R \Gamma^{SLT}) F^0_{RSLT}] \epsilon,
\end{aligned} \tag{62}$$

where F^0_{MNRS} is the supercovariant field strength, given by

$$F^0_{RSLT} = 24 \partial_{[M} A_{NRS]} + \frac{3}{\sqrt{2}} \bar{\psi}_{[M} \Gamma_{NR} \psi_{S]}. \tag{63}$$

The equations of motion derived from (59) are non-linear since they express the interactions. We are interested in finding solutions of these equations that preserve some fraction of the supersymmetry. The bosonic solutions have ψ_M vanishing. From (62) this implies that the supersymmetric variation of E and A vanish, too. Therefore, the only condition for having a supersymmetric solution is

$$\delta \psi_M = [D_M(\omega) + \frac{\sqrt{2}}{288} (\Gamma_M^{RSLT} - 8\delta_M^R \Gamma^{SLT}) F^0_{RSLT}] \epsilon = 0. \tag{64}$$

Spinor parameters ϵ_0 satisfying this equation are called *Killing spinors*. Now if we choose a gauge in which $F = 0$ we have

$$\delta \psi_M = D_M(\omega) \epsilon_0 = 0. \tag{65}$$

As we can see from (64) or (65) the Killing spinor condition imposes severe restrictions on the gravitational field. Indeed, applying $D_M(\omega)$ twice one obtains from (65)

$$D_M D_N \epsilon_0 = 0. \tag{66}$$

It is important to notice that under the constraint (65), the equations of motion of supergravity have the following supersymmetric solution

$$ds^2 = dudv + k(\xi, u) du^2 + d\xi \cdot d\xi, \tag{67}$$

where ξ are Cartezian coordinates for E^9 and k is an arbitrary function of u and harmonic on E^9. The solution (67) is called a 'M-wave' and it describes an asymptotically flat region at $k \to 0$ as $|\xi| \to \infty$. If this metric is used to compute D_M, (65) reduces to $\Gamma_v \epsilon = 0$, where $v = t + x_1$ which is equivalent to $\Gamma_{01} \epsilon = \epsilon$. The field ϵ is assumed to be constant st infinity and its value is taken to be the zero-eigenvalue of $\{Q, Q\}$.

We can conclude from the above analysis that we have fluctuations of the classical solutions described by Goldstone fields whose effective action is the superparticle action in the light-cone gauge. We can use the fluctuations of M-waves to find the M-branes.

4.3 Branes of M-Theory

From the d=11 supergravity action (46) we derive the equations of motion using the usual variational principle. For the antisymmetric tensor field they are given by

$$d * F + \frac{1}{2} F \wedge F = 0, \tag{68}$$

from which we can compute the following charge

$$U = \int_{\partial M_s} (*F + \frac{1}{2} A \wedge F), \tag{69}$$

where ∂M_s is the boundary at infinity of an arbitrary infinite spacelike 8-dimensional subspace of d=11 spacetime. Similarly, from the Bianchi identity

$$dF = 0, \tag{70}$$

we can deduce another charge that is conserved under it, namely

$$V = \int_{\partial \tilde{M}_5} F, \tag{71}$$

where the surface integral is taken over the boundary at infinity of a spacelike 5-dimensional subspace. These charges can be related to an *central charge* extension of the supersymmetry algebra

$$\{Q, Q\} = C(\Gamma^M P_M + \Gamma^{MN} U_{MN} + \Gamma^{MNPQR} V_{MNPQR}, \tag{72}$$

where C is the charge conjugation matrix, M_{MN} and V_{MNPQR} are 2-form and 5-form charges related to U and V, respectively. The l. h. s. of (72) has 528-components since

the spinors of d=11 supergravity have 32-components. On the r. h. s. we also have 528 components as follows: 11 for P_M, 55 for U_{MN} and 462 for V_{MNPQR}. This sets the match between the number of components in the two sides of (72).

The supersymmetry algebra (72) represents a modified version of (49). The physical reason for this modification is the presence of some extended physical objects in supergravity theory. These are a 2-brane (called M2-brane) and a 5-brane (called M5-brane) respectively.

Consider firstly an $M2$-brane. It arises as a solution of the bosonic part of the action (46) under the 'electric' ansatz. By choosing appropriate coordinates, we can write this solution in the following form

$$
\begin{aligned}
ds^2 &= [R^2(-dt^2 + d\sigma^2 + d\rho^2) + 4k^{\frac{1}{3}}R^{-2}dR^2] + k^{\frac{1}{3}}d\Omega_7^2 \\
&+ k^{\frac{1}{3}}[(1-R^3)^{-\frac{1}{3}} - 1][4R^{-2}dr^2 + d\Omega_7^2] \\
A_{\mu\nu\rho} &= R^3\epsilon_{\mu\nu\rho},
\end{aligned}
\tag{73}
$$

where the worldvolume coordinates are $x^\mu = (t, \sigma, \rho)$ and $d\Omega_7^2$ is the line element on the unit 7-sphere, corresponding to the boundary ∂M_8 and k is an specific integration constant. This form of the solution has several advantages. Firstly, it displays explicitly the inside horizon. At $R = 0$ with $R < 0$ being the interior, the light-cones does not "flip over". Secondly, we can see from (73) that at $R \to 1$ (the asymptotic exterior) the solution is flat, while at $R = 0$ (near thee horizon) the last product in the metric vanishes and what is left is the standard form of the metric on $(AdS)_4 \times S^7$. From this we can see that the memebrane interpolates between flat space vacuum at $R \to 1$ and $(AdS)_4 \times SS^7$ at the horizon.. Inside the horizon one eventually encounters a *timelike* singularity as $R \to -\infty$. The nature of this singularity suggests that the solution should be coupled to a δ-function and gives the interpretation of the membrane as an *electrically* charged object. Indeed, the membrane represents the unique matter that consistently couple to d=11 supergravity. Thus, the electric source that couples to supergravity is described by the fundamental membrane action. Its bosonic part is given by

$$
S_2 = -T_2 \int_{\Sigma_3} d^{2+1}\sigma \sqrt{-det(\partial_\mu X^M \partial_\nu X^N g_{MN}(X))} + Q_2 \int_{\sigma_3} \frac{1}{3!}\partial_\mu X^M \partial_\nu X^N \partial_\rho X^R \epsilon^{\mu\nu\rho} A_{MNR}(X).
\tag{74}
$$

If the tension and the electric charge are equal, $Q_2 = T_2$, it turns out that the action is k-symmetric. Varying the action above with respect to A_3, we obtain the following current

$$J^{MNR} = Q_2 \int_{\Sigma_3} \delta^3(Y - X(\sigma))dX^M \wedge dX^N \wedge dX^R, \tag{75}$$

that enters the r. h. s. of the equation of motion (68). Thus we can compute the charge as a volume integral of the source

$$U = \int_{M_8} *J(3) = Q_2. \tag{76}$$

The action (74) is invariant under the truncation of the first two terms in the r. h. s. of the algebra (72) and the central charge is obtained by an integration over the 2-cycle occupied by the membrane in spacetime

$$Z^{MN} = Q_2 \int dX^M \wedge dX^N. \tag{77}$$

Now if we are going to see if a membrane preserves the supersymmetry or not, we take a Majorana representation of the Dirac matrices ($C = \Gamma^0$) and fix the membrane in, say, 12 plane. We thus take only Z^{12} different from zero. The truncation of (72) in this setting is given by

$$\{Q, Q\} = P^0 + \Gamma^{012}Z_{12}, \tag{78}$$

and the l.h.ss. of the equation above is clearly positive. P^0 should be nonzero since the sign of Z^{12} can be flipped if the membrane is replaced by an antimembrane. For $P^0 = 0$ we have the vacuum,, while for $P^0 > 0$ we have that

$$P^0 \geq |Z_{12}| \sim T_2 \geq |Q_2|. \tag{79}$$

If the membrane is stable it saturates the bound, i. e. $T_2 = |Q_2|$. In this case (78) becomes

$$\{Q, Q\} = P^0[1 \pm \Gamma^{012}]. \tag{80}$$

For spinors that satisfy $\Gamma^{012}\epsilon = \pm\epsilon$, since $(\Gamma^{012})^2 = 1$ and $Tr\Gamma^{012} = 0$, results that the dimension of the zero-eigenvalue eigenspaces of $\{Q, Q\}$ is sixteen. The conclusion is that the membrane saturating the bound (79) preserves half of the supersymmetry of the vacuum.

Let us turn to the 5-brane solution of the d=11 supergravity. This solution represents a magnetic 5-brane and in the same coordinates as the ones used in the case of the membrane, it reads as

$$
\begin{aligned}
ds^2 &= R^2 dx^\mu dx^\nu \eta_{\mu\nu} + k^{\frac{2}{3}} \big[\frac{4R^{-2}}{(1-R^6)^{\frac{8}{3}}} dR^2 + \frac{d\Omega^2_4}{(1-R^6)^{\frac{2}{3}}} \\
F_{m_1 \ldots m_4} &= 3k\epsilon_{m_1 \ldots m_4 p} \frac{y^p}{r^5}
\end{aligned}
\tag{81}
$$

where $\mu, \nu = 0, \ldots, 5$ are the worldvolume indices and $m_1,, \ldots, m_4, p = 6, \ldots, 11$ are transverse space indices. The surface at $R = 0$ represents a nondegenerate horizon. The light-cones maintain their timelike orientation when crossing the horizon as in the membrane case. These is an important symmetry of the 5-brane, namely a discrete izometry $R \to -R$. This allows us to identify the spacetime regions $R \le 0$ and $R \ge 0$ which means that the sets $-1 < R \le 0$ and $0 \le R < 1$ should also be identified. The asymptotic limits $R \to -1$ and $R \to +1$ are indistinguishable and they describe regions of flat geometry. Thus there is no singularity in the 5-brane. However, there is a throat at the horizon with the $(AdS)_7 \times S^4$ geometry down the throat instead of $(AdS)_4 \times S^7$ in the membrane solution. The 5-brane spacetime (81) is geodesically complete, so the solution is completely non-singular which shows that the charge carried by the 5-brane is a magnetic charge. Indeed, the field strength is purely transverse as one can see from (81), so no electric charge is present. Since in the integral there is just one orientation that gives a contribution (the surface transversal to the d=6 worldvolume), we can write down the charge of the 5-brane

$$
V = \int_{\partial M_{5T}} d\Sigma^m_{(4)} \epsilon_{mnpqr} F^{npqr},
\tag{82}
$$

where $d\Sigma^m_{(4)}$ is the surface integral over the boundary of M_{5T} which is the transverse space. This charge is preserved by topological properties of spacetime, and thus $M5$ is a *solitonic* solution of supergravity.

The M5-brane field content consists in five bosons associated to the transversal coordinates (which are also Nambu-Goldstone bosons of the broken translational invariance and worldvolume fields), and eight fermions. In order to fix the mismatch of the bosonic and fermionic degrees of freedom, one should also add a self-dual two-form B ($*dB = dB$). One can then write down the low energy effective action which has the following bosonic

sector

$$S = \int d^6\sigma [\sqrt{-det(g + i\tilde{H})} + \frac{1}{4\sqrt{(\partial a)^2}}\tilde{\mathcal{H}}^{IJ}H_{IJK}\partial^K a]$$

$$+ \int_{WV}(\mathcal{C}_6 + \frac{1}{2}H \wedge \mathcal{C}_{(3)}), \tag{83}$$

where $\mathcal{C}_6$ is the pull-back of C_6, $\mathcal{C}_3$ is the pull-back of $C_{(3)}$, a is a non-dynamical field, and

$$\tilde{\mathcal{H}}^{IJ} = \frac{1}{6\sqrt{(\partial a)^2}}\epsilon^{IJKLMN}\partial_K a H_{LMN}$$

$$\tilde{H}^{IJ} = \frac{1}{\sqrt{-detg}}g_K^I g_K^J \tilde{\mathcal{H}}^{KL}, \tag{84}$$

where $I, J = 0, \ldots, 5$. We see that the covariant action (83) is clearly non-local due to the presence of a.

Let us count the number of supersymmetries preserved by the 5-brane. To this end we are going to use the same procedure as for the membrane. We first go to a Majorana representation and fix the brane in some directions, say 12345,, with $Y_{12345} \neq 0$. Let us define $Y = Y_{12345}/P^0$. The algebra corresponding to (72) and truncated to the first and last term in r. h. s. is given by

$$\{Q, Q\} = P^0[1 - \Gamma_{012345}]. \tag{85}$$

In Majorana representation Q's are real, so we can talk about the positiveness of the above commutator. Since this should be positive, using the properties of Γ_{012345} we see that Y should satisfy

$$|Y| \leq 1. \tag{86}$$

Now, if the bound of the above equation is saturated, we have sixteen zero eigenvalues of $\{Q, Q\}$ if the parameters of the corresponding susy transformation satisfy

$$\Gamma_{012345}\epsilon = \epsilon. \tag{87}$$

From that we conclude that the M5-brane preserves half of the supersymmetries of the background.

Before ending this paragraph, let us make some remarks. Let us introduce for M2- and M5-branes the following ADM energy density

$$E = \int_{\partial M_T} d^{d-p-1}\Sigma^M (\partial^n h_{mn} - \partial_m h_b^b), \tag{88}$$

where ∂M_T is the boundary transverse space and h_{mn} are the components of the metric in the asymptotically flat expansion $g_{MN} = \eta_{MN} + h_{MN}$. Here, $m = p, \ldots, 10$ runs over the transverse diresction. The relation above is valid for any p-brane solution. In this setting the Bogomol'ny bound read for electric and magnetic branes as follows

$$E \geq \frac{2}{\sqrt{\Delta}} U \quad , \quad \textbf{electric bound}$$
$$E \geq \frac{2}{\sqrt{\Delta}} V \quad , \quad \textbf{magnetic bound}, \tag{89}$$

where Δ is some specific number and U and V are the electric and magnetic charges, respectively. Both M2- and M5-branes saturate this bound, i. e.

$$E_2 = U \quad , \quad E_5 = V, \tag{90}$$

which make these objects BPS-states. This is another way to see that the branes of M-theory preserve half of the supersymmetries of the background.

5   Intersecting Branes

As we saw in the precedent sections, all low-energy effective actions of string theories as well as d=11 supergravity, admit extended solutions. These are characterized by some supersymmetry and by a charge. The solutions that preserve half of the supersymmetry of the background and satisfy a BPS-bound are particularly important in investigating the non-perturbative aspects of superstrings. In particular, the Dp-branes play a central role.

We have described diverse quantum field theories corresponding to branes and we have seen that these are supersymmetric field theories that realize half of the supersymmetries of the background. Let us turn our attention to some other field theories that emerge from branes, more exactly from the intersection of branes. We will limit in this section only to the branes of M-theory.

For the present purposes, it is useful to write the brane solutions of d=11 supergravity, namely the ones given previously in (73) and (81) in *isotropic coordinates*. The M2-brane takes the following form

$$
\begin{aligned}
ds^2 &= H^{\frac{1}{3}}[H^{-1}(-dt^2 + dx_1^2 + dx_2^2) + (dx_3^2 + \cdots dx_{10}^2)] \\
F_{t12m} &= \frac{c}{2}\frac{\partial_\alpha H}{H^2} \quad, \quad H = H(x_3, \ldots x_{10}) \ , \ \nabla^2 H = 0 \ , \ c = \pm 1,
\end{aligned}
\tag{91}
$$

where H is an harmonic function depending on the transverse coordinates. If we take

$$
H = 1 + \frac{a}{r^6} \quad, \quad r = |\vec{x}|
\tag{92}
$$

we have just one membrane with the worldvolume oriented along the hyperplane $\{0, 1, 2\}$ and located at $r = \sqrt{x_3^2 + \cdots x_{10}^2} = 0$. If we generalize (92) to several centres

$$
H = 1 + \sum_{I=1}^{k} \frac{a_I}{r_I^6} \quad, \quad r_I = |\vec{x} - \vec{x}_I|
\tag{93}
$$

we describe k parallel M2-branes located at positions $\vec{x}_I$.

The M5-brane solution is given by

$$
\begin{aligned}
ds^2 &= H^{\frac{2}{3}}[H^{-1}(-dt^2 + dx_1^2 + \cdots + dx_5^2) + (dx_6^2 + \cdots + dx_{10}^2)] \\
F_{\alpha_1 \cdots \alpha_n} &= \frac{c}{2}\epsilon_{\alpha_1 \cdots \alpha_5}\partial_{\alpha_5} H \quad, \quad H = H(x_6, \ldots, x_{10}) \quad, \quad c = \pm 1,
\end{aligned}
\tag{94}
$$

where $\epsilon_{\alpha 1 \ldots \alpha_5}$ is the flat d=5 alternating symbol. For a single M5-brane one chooses

$$
H = 1 + \frac{a}{r^4} \quad, \quad r = |\vec{x}|,
\tag{95}
$$

while for k parallel M5-branes we have

$$
H = 1 + \sum_{I=1}^{k} \frac{a_I}{r_I^4} \quad, \quad r_I = |\vec{x} - \vec{x}_I|.
\tag{96}
$$

Let us denots an M2-brane localized in $\{0, 1, 2\}$ directions and an M5-brane localized in $\{0, 1, 2, 3, 4, 5\}$ directions by

$$
\begin{array}{ccccccccccc}
M2 & 1 & 2 & - & - & - & - & - & - & - & - \\
M5 & 1 & 2 & 3 & 4 & 5 & - & - & - & - & -
\end{array}
$$

For the intersection of two M2-branes we can have different configurations according to their displacement in spacetime like, for example

$$
\begin{array}{ccccccccccc}
M2 & 1 & 2 & - & - & - & - & - & - & - & - \\
M5 & - & - & 3 & 4 & - & - & - & - & - & -
\end{array}
$$

or

$$
\begin{array}{ccccccccccc}
M2 & 1 & 2 & - & - & - & - & - & - & - & - \\
M5 & - & 2 & 3 & - & - & - & - & - & - & -
\end{array}
$$

The difference between the above configurations is that in the first case the intersection contains just one point, the origin of the hyperplanes, while in the second case it contains the real axis. Therefore, we will label the first configuration by $(0|M2M2)$ and the second one by $(1|M2M2)$, respectively. Examples of other intersections of branes of d=11 supergravity are

$$
\begin{array}{ccccccccccc}
M5 & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\
M5 & 1 & 2 & 3 & - & - & 6 & 7 & - & - & -
\end{array}
$$

and

$$
\begin{array}{ccccccccccc}
M2 & 1 & 2 & - & - & - & - & - & - & - & - \\
M5 & 1 & - & 3 & 4 & 5 & 6 & - & - & - & -
\end{array}
$$

84

labeled, according to our notations, by $(3|M5M5)$ and $(1|M2M5)$, respectively. The corresponding analytic solutions can be found by "overlapping" (91) and (94).

The $(0|M2M2)$ solution is given by

$$
\begin{aligned}
ds^2 &= (H_1 H_2)^{\frac{1}{3}} [-(H_1 H_2)^{-1} dt^2 + H_1^{-1}(dx_1^2 + dx_2^2) \\
&+ H_2^{-1}(dx_3^2 + dx_4^2) + (dx_5^2 + \cdots + dx_{10}^2)] \\
F_{t12\alpha} &= \frac{c_1}{2} \frac{\partial_\alpha H_1}{H_1^2}, F_{t34\alpha} = \frac{c_2}{2} \frac{\partial_\alpha H_2}{H_2^2}
\end{aligned}
\tag{97}
$$

where $H_i = H_i(x_5, \ldots, x_{10})$, $\nabla^2 H_i = 0$, $c_i = \pm 1$, $\alpha = 5, \ldots, 10$ and $i = 1, 2$. Here, H_i are harmonic in the coordinates $\{x_5, \ldots, \xi_{10}\}$. If we take these functions of the following form

$$
H_i = 1 + \frac{a_i}{r_i^4} \ , \ r_i = |\vec{x} - \vec{x}_i|,
\tag{98}
$$

we have a membrane oriented in $\{1, 2\}$ plane, at $\vec{x}_1$ and another one in the $\{3, 4\}$ plane at $\vec{x}_2$ which overlap in a point and are orthogonal to each other. A generalization is given by

$$
H_i = 1 + \sum_{I=1}^{k_i} \frac{a_{i,I}}{r_{i,I}^4} \ , \ r_{i,I} = |\vec{x} - \vec{x}_{i,I}|
\tag{99}
$$

which describes k_1 parallel membranes with $\{1, 2\}$ orientation and $\vec{x}_{1,I}$ positions and k_2 parallel membranes with $\{3, 4\}$ orientation and located at $\vec{x}_{2,I}$.

The $(1|M2M5)$ solution is given by

$$
\begin{aligned}
ds^2 &= H_1^{\frac{2}{3}} H_2^{\frac{1}{3}} [H_1^{-1} H_2^{-1}(-dt^2 + dx_1^2) + H_1^{-1}(dx_2^2 + \cdots dx_5^2) \\
&+ H_2^{-1}(dx_6^2) + (dx_7^2 + \cdots + dx_{10}^2)] \\
F_{6\alpha\beta\gamma} &= \frac{c_1}{2} \epsilon_{\alpha\beta\gamma\delta} \partial_\delta H_1 \ , \ F_{t16\alpha} = \frac{c_2}{2} \frac{\partial_\alpha H_2}{H_2^2} \ , \ H_i = H_i(x_7, \ldots, x_{10}),
\end{aligned}
\tag{100}
$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the d=4 flat space alternating symbol.

The $(3|M5M5)$ solution is given by

$$
\begin{aligned}
ds^2 &= (H-1H-2)^{\frac{2}{3}} [(H_1 H_2)^{-1}(-dt^2 + dx_1^2 + \cdots + sx_3^2) + H_1^{-1}(dx_4^2 + dx_5^2) \\
&+ H_2^{-1}(dx_6^2 + dx_7^2) + (dx_8^2 + dx_9^2 + dx_{10}^2)] \\
F_{67\alpha\beta} &= \frac{c_1}{2} \epsilon_{\alpha\beta\gamma} \partial_\gamma H_1 \ , \ F_{45\alpha\beta} = \frac{c_2}{2} \epsilon_{\alpha\beta\gamma} \partial_\gamma H_2 \ , \ H_i = H_i(x_8, x_9, x_{10}).
\end{aligned}
\tag{101}
$$

Let us analyze the supersymmetries of these intersections. They are made out of 1/2 susy preserved by each object which an associated constraint of the type

$$
\Gamma \epsilon = e,
\tag{102}
$$

where Γ is some antisymmetric product of gamma matrices and satisfies $\Gamma^2 = 1$ and $Tr\Gamma = 0$. Now let us take two such matrices, Γ and Γ' associated to two branes that intersect and let us denote the carge/tension rations by ζ and ζ', respectively. The supersymmetry algebra contains an anticommutator of the form

$$\{Q, Q\} = P^0[1 + \zeta\Gamma + \zeta'\Gamma']. \tag{103}$$

In a real representation, the positivity bound imposes some constraints on ζ and ζ' depending of the commutation relation $[\Gamma, \Gamma']$.

If $\{\Gamma, \Gamma'\} = 0$ the following relation holds

$$(\zeta\Gamma + \zeta'\Gamma')^2 = \zeta^2 + \zeta'^2. \tag{104}$$

The bound $\zeta^2 + \zeta'^2 \le 1$ is equivalent to

$$T \ge |Z| + |Z'|, \tag{105}$$

where Z and Z' are the charges of the two branes. If this bound is saturated, (103) can be written as

$$\{Q, Q\} = 2P^0[\zeta\Pi + \zeta'\Pi'], \tag{106}$$

where $\Pi = (1/2)(1 - \Gamma)$ and $\Pi' = (1/2)(1 - \Gamma')$ commute. A zero eigenvalue eigenspinor of $\{Q, Q\}$ must be annihilated by both of Π and Π'. That implies that the following relations should be simultaneously satisfied

$$\Gamma\epsilon = \epsilon \quad , \quad \Gamma'\epsilon = \epsilon. \tag{107}$$

Now since $Tr(\Gamma\Gamma') = 0$, the two matrices Γ and Γ' can be simultaneously brought to the form

$$\begin{aligned}\Gamma &= diag(\overbrace{1, \ldots, 1}^{16}, \overbrace{-1, \ldots, -1}^{16}) \\ \Gamma' &= diag(\overbrace{1, \ldots, 1}^{8}, \overbrace{-1, \ldots, -1}^{8}, \overbrace{1, \ldots, 1}^{8}, \overbrace{-1, \ldots, -1}^{8}).\end{aligned} \tag{108}$$

Thus, this structure preserves 1/4 supersymmetry.

At present there is no theory that describes the worldvolume field theory of an intersection of branes in a consistent manner. However, we note that a configuration of two

intersecting branes should appear in the worldvolume field theory as a soliton preserving half of the worldvolume supersymmetry. In this algebraic approach the spacetime interpretation of a worldvolume p-brane preserving 1/2 supersymmetry is completely encoded in the worldvolume supersymmetry algebra. Remarkably, all 1/4 supersymmetric intersections can be obtained as 1/2 supersymmetric solutions of the worldvolume field equations of various M-branes.

Let us take the maximal central charge extension of d=6 worldvolume supertranslations algebra of M5-branes

$$\{Q_\alpha^I, Q_\beta^J\} = \Omega^{IJ} P_{\alpha\beta} + Y_{[\alpha\beta]}^{[IJ]} + Z^{(IJ)} \ , \ \Omega_{IJ} Y^{[IJ]} = 0, \tag{109}$$

where $\alpha, \beta = 1, \ldots, 4$ is an index of $SU^*(4) \simeq Spin(5)$ and $I, J = 1, \ldots 4$ are $Sp(2)$ indices. Here, Ω^{IJ} is an $Sp(2)$ invariant antisymmetric tensor, Y is a worlvolume 1-form and Z is a worldvolume self-dual 3-form. These charges are associated to the 1/2 supersymmetric string and 3-brane, respectively. However, there are more charges than corresponding worldvolume objects, which suggest that in the worldvolume physics, the spacetime identification: 1-p-form $\leftrightarrow$ 1-object no longer holds. In this case, the number of charges are given by the respresentations of $Sp(2)$ and the interpretation is that $Sp(2)$ representations provide the information needed to reconstruct the branes within branes, the intersecting branes, etc. To clarify the content of the above statement, let us consider $Sp(2)$ as the double cover of $SO(5)$ transverse to the M5-brane worldvolume. Then the space components of the 1-form Y define a vector in the transverse five dimensional space. The associated worldvolume 1-brane can be considered as the boundary of an M2-brane in M5-brane. But the **5** representation of $Sp(2)$ can be interpreted as a 4-form in the transverse space, and thus the associated 1-brane should be interpreted as the intersection of the M5-brane with another M5-brane. The time components of Y can be viewed as the space components of of a dual 5-form which can be interpreted as an M5-brane inside another M5-brane. In the case when **5** representation of $Sp(2)$ defines a 1-form in the transverse space, the interpretation of the time components of Y is that of a 6-brane and the configuration described is that of a 5-brane inside a 6-brane. The corresponding '6-brane' of M-theory is in fact the Kaluza-Klein (KK) monopole, so we have an M5-brane inside KK-monopole. Similarly, taking the **5** representation to define a 4-form, the time components of Y give

us an M5-brane inside an '9-brane' which is in fact the M-boundary.

In a similar manner one can interpret the components of the 3-form Z. The same reasoning can be applied to M2-brane if one starts with the N=8 d=3 worldvolume superalgebra

$$\{Q^i_\alpha, Q^j_\beta\} = \delta^{ij} P_{(\alpha\beta)} + Z^{(ij)}_{(\alpha\beta)} + \epsilon_{\alpha\beta} z^{[ij]} \ , \ \delta_{ij} Z^{(ij)} = 0, \tag{110}$$

where Q^i are eight d=3 Majorana spinors and $i = 1, \ldots, 8$. Due to the $SO(8)$ automorphism group of this algebra, the 0-form and 1-form centralcharges transform as chiral $SO(8)$ spinors.

We are not going to elaborate further on this topic, but rather refer the reader to ([39]). We note, however, that the worlvolume p-brane solitons which describe intersecting branes carry p-form charges which can be expressed as integrals of local charge densities. For example, in the case of the following intersections

$$
\begin{array}{ccccccccccc}
M2 & 1 & 2 & - & - & - & - & - & - & - & - \\
M2 & - & - & 3 & 4 & - & - & - & - & - & - \\
\end{array}
$$

and

$$
\begin{array}{ccccccccccc}
M2 & 1 & - & - & - & - & 6 & - & - & - & - \\
M5 & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\
\end{array}
$$

the charges are given by

$$Z^{34} = \int_{M_2} dx^3 \wedge dx^4 \ , \quad Y^6 = \int_{W_4} dx^6 \wedge H, \tag{111}$$

where W_4 is spanned by x^2, x^3, x^4, x^5 and H is the worldvolume 3-form field-strength of M_5.

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