# Brane collisions and braneworld cosmology

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Recently, we proposed a new braneworld model<sup>1)</sup> in which the collision of a brane universe and a vacuum bubble coming from the extra-dimension is utilized as a trigger of brane big-bang. In this article, mainly reviewing this model, we briefly summarize cosmological braneworld scenarios in which collision of branes plays an important role.

#### §1. Introduction

Braneworld scenario<sup>2)</sup> especially of the Randall-Sundrum type<sup>3)</sup> have recently attracted much attention. In particular, cosmological models in this scenario have been studied actively<sup>4), 5)</sup>. It has been shown that braneworld cosmology seems to be consistent with the 4-dimensional conventional cosmology at least on scales much lower than that corresponding to the brane tension, provided bulk Weyl curvature is sufficiently small. However, it is not clear whether braneworld cosmology can predict any evidences for the existence of extra-dimension(s) which are testable in near future observations. Further, it is still not evident whether braneworld models actually have a great advantage over the conventional ones. Under such a current situation, it is an important direction of research to seek for an alternative scenario in which the existence of extra-dimension(s) plays an essential role.

As one of such attempts, the present authors recently proposed a new cosmological braneworld model<sup>1)</sup> in which an inflation occurs on a boundary brane driven by small mismatch between the bulk vacuum energy and the brane tension, and the nucleation of a true vacuum bubble becomes a trigger of the big-bang in the braneworld. One of the distinctive features in our model is that the bubble nucleation occurs in extra-dimension(s). Not only does such a vacuum bubble coming from extra-dimension heat up the brane universe through the colliding process, but also provide simultaneously an anti-de Sitter bulk of the Randall-Sundrum setup, reducing the effective cosmological constant on the brane to zero.

In this article, we shall briefly summarize a colliding brane cosmology. We first give a brief review of colliding brane models so far proposed in the next section. Then, in section 3, we illustrate our brane big-bang model proposed in Ref.<sup>1)</sup>. In section 4), we discuss problems in our model, some of which may be common in any types of colliding brane models.

#### §2. Colliding brane cosmology

Recently there have appeared several interesting works in which collision of branes is actively used. Among them, the idea called brane inflation, in which the interbrane separation plays the role of an inflaton field, is rather promissing to

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work<sup>6)</sup>. On the other hand, one of the most ambitious proposals is the ekpyrotic model<sup>7)</sup>, which aims at solving major cosmological problems without the use of inflation. A severe criticism<sup>8)</sup> to it and a number of calculations of density fluctuations with the controversial claim have also been made<sup>9)</sup>.

Also in the context of Horava-Witten theory, an intriguing possibility that the collision of a visible brane and a bulk moving brane generates the baryon asymmetry on a visible brane universe has been proposed very recently by Bastero-Gil et.al<sup>10</sup>.

Within the context of the Randall-Sundrum scenario, some models to consider bubble nucleation in the bulk have been discussed in several different contexts<sup>11</sup>. For example, an idea to realize the Randall-Sundrum setup by a collision of bubbles was discussed by Gorsky and Selivanov<sup>12</sup>, where the bubbles nucleate through the Schwinger process in some external field.

Bucher  $^{13)}$  proposed an interesting model in which anti-de Sitter bubbles appear as a result of a false vacuum decay  $^{14)}$  and a collision of the two nucleated bubbles create a hot big bang universe, giving a possible origin of a homogeneous and isotropic bulk and brane geometry. Density perturbations in this model have also been calculated  $^{15), 16)}$  to show that the scale-invariant spectrum does not easily arise. But the result does not immediately exclude the possibility of this model because the amplitude due to the effect of the bubble wall fluctuations tends to be very tiny. Our model we review in the next section has several similarities with this colliding bubble model.

It should be commented that in general relativity, brane (shell) collisions have been discussed in a number of literatures<sup>17)</sup>. The formalism for treating collision of gravitating shells developed in conventional general relativistic context has been extended to more general cases with an eye for applications in braneworld cosmology<sup>18)</sup>. However, concerning perturbations of colliding brane models, as far as the present authors know, such a formalism taking self-gravity of colliding branes into account has not been developed yet. Most of perturbation calculations in colliding brane models have been made by ignoring self-gravity of colliding branes, or after reducing the system in question to effective 4-dimensional theories with a scalar field which mimics fluctuation of a moving brane.

### §3. Brane big-bang brought by bulk bubble

Now we shall illustrate our idea<sup>1)</sup>. In our model, 5-dimensional bulk spacetime is supposed to nucleates in a false vacuum phase with a single positive tension brane at the fixed point of  $\mathbb{Z}_2$ -symmetry. The false vacuum bulk can be locally Minkowski or de Sitter space. The pre-existing brane is in an inflationary phase because of the mismatch between the bulk vacuum energy and the brane tension. This inflationary phase would last forever if there were no mechanism to terminate it. However, since the bulk is initially in a false vacuum state, a true vacuum anti de Sitter-bubble (AdSbubble) spontaneously nucleates in the bulk as a result of the false vacuum decay via quantum tunneling<sup>14)</sup>. This AdS-bubble expands in the false vacuum bulk. If the transition occurs with the highest symmetry, the nucleated bubble has the common center which respects the symmetry of the bulk-brane system. However, even if the transition with the highest symmetry is the most probable process as discussed in Ref.<sup>19)</sup>, quantum fluctuations lead to displacement of the position of the nucleation from the center of the symmetry. Then, because the surface of the Ads-bubble expands just like a de Sitter space, the bubble eventually hits the inflationary brane universe. The point is that the intersection of the brane and the bubble is spacelike. Thus, when the bubble hits the brane, the energy of the bubble wall can be converted into radiation on the brane unless it dissipates into the bulk. Furthermore, the effective cosmological constant on the brane is reduced with the true vacuum energy chosen to be the negative value which balances the tension of the brane. As a result, the inflation comes to an end, and the brane can be thermalized through this colliding process. It is worth noting that the brane is instantaneously heated up at the colliding surface beyond the horizon scale of the brane. Although such a type of thermalization appears a causality violation from the viewpoint of the observers on the brane, it is a natural consequence of the bubble nucleation in the bulk (outside the brane). We call this collision hypersurface a "big-bang surface." If the brane inflation lasts long enough before the collision, the big-bang surface can become homogeneous and isotropic. Then, in the future of the big-bang surface, the brane evolves as a radiation dominated Friedmann-Lemaitre-Robertson-Walker (FLRW) universe.

After the collision, the bulk around the brane becomes anti-de Sitter spacetime and the gravity is effectively localized on the brane by the Randall-Sundrum mechanism. Since the true vacuum energy is lower than that in the false vacuum, this model allows a creation of anti-de Sitter bulk from de Sitter or Minkowski-bulk<sup>14</sup>. The whole story is summarized in Fig. 1.

In this model the type of the resultant FLRW brane universe depends on the location of the bubble nucleation in the bulk; It can be open, closed, or flat, if the separation  $\Delta$  between two centers of the bubble and the brane is in spacelike, timelike, or null separation, respectively. Furthermore it can be shown that, for the open and closed FLRW brane universe, the spatial curvature radius  $a_i$  at the moment of the brane big-bang is related to the magnitude of  $\Delta$  and the brane's curvature radius  $\alpha_B$  as  $|\Delta|a_i \approx \alpha_B \ell$  for case (a):  $\ell^{-2} \gg \alpha_B^{-2}$ , and  $|\Delta|a_i \approx \alpha_B^2$  for case (b):  $|\ell^{-2}| \leq \alpha_B^{-2}$ , with  $\ell$  being the curvature radius of the false vacuum bulk. It turns out that in order to solve the flatness problem,  $\Delta$  must satisfy

$$\frac{|\Delta|}{\alpha_B} \lesssim 10^{-32} \left(\frac{\sigma_W \ell^5}{\alpha_W}\right)^{1/4}, \quad \frac{|\Delta|}{\alpha_B} \lesssim 10^{-32} \left(\sigma_W \alpha_B^4 \frac{\alpha_B}{\alpha_W}\right)^{1/4}, \quad (3.1)$$

for case (a) and case (b), respectively. Here  $\alpha_W$  and  $\sigma_W$  denote the bubble wall's radius and tension. Thus, for the flatness, the bubble nucleation must be confined very close to the symmetry center. For this, we need to introduce some interaction between the boundary brane and the tunneling field  $\phi$ . As a simple way to introduce the degree of freedom that controls the strength of such an interaction, we consider model having a  $U(\phi)$  potential localized on the brane as well as a potential  $V(\phi)$  in



Fig. 1. The conformal diagram shows the brane big-bang scenario. The dotted line hemisphere represents the instanton of the system. A true vacuum AdS-bubble is nucleated and expands in the false vacuum bulk bounded by an inflating de Sitter brane. The expanding AdS-bubble eventually collides with a portion of the de Sitter brane. The case that the separation  $\Delta$  of the two centers is spacelike is illustrated. In this case, the intersection, i.e., the big-bang surface, has a hyperbolic geometry  $\mathbb{H}^3$  and an open FLRW brane universe is realized after the brane big-bang. This geometry is glued along the boundary surfaces, except  $\mathcal{I}_{AdS}$  and  $\mathcal{I}_{Min}^{\pm}$ , onto a copy of itself with  $\mathbb{Z}_2$ -symmetry being satisfied.

the bulk. Define a parameter  $\nu$  which controls the strength of the interaction by

$$\nu := -4\partial_y \log \alpha \big|_{brane} - 2 \frac{V'[\phi]}{U'[\bar{\phi}]} \big|_{brane} + \frac{1}{2} U''[\bar{\phi}] \big|_{brane} \,, \tag{3.2}$$

with y being the transverse coordinate to the brane. Then, on the assumption that the effect of the gravitational back reaction is small, perturbing the most symmetric (i.e.,  $\Delta = 0$  case) instanton solution  $\bar{\phi}$ , we can estimate the probability distribution of the off-centred bubble nucleation as a function of  $\Delta$ :

$$P(\Delta) \propto e^{-S_E[\bar{\phi}]} \exp\left(-\frac{1}{2}\alpha_B^4 \nu M^5 \Delta^2\right)$$
 (3.3)

Here  $S_E[\bar{\phi}]$  is the Euclidean action for the symmetric solution  $\bar{\phi}$ , and M denotes an energy scale of the bulk scalar field, being related to the difference of the vacuum energy between true and false vacua:  $\delta V \approx M^5$ . From this, we can find that the required concentration (3.1) of the nucleation point can be realized if the tunneling field has an appropriate potential localized on the brane.

However, we have to care the following point. The interaction between the brane and the tunneling field inevitably increases the effective mass-squared of the perturbation modes corresponding to bubble wall fluctuations. When there is no interaction, this effective mass squared is negative and the wall fluctuation grows until it hits the brane. If this mass squared becomes positive, the fluctuation modes are stabilized, and the bubble wall never hit the brane. Hence, the interaction should be strong enough to force the bubble nucleate near the center of the symmetry but weak enough to let the bubble fluctuation grow. Our model thus requires one fine tuning to adjust the strength of this interaction.

These constraints on the model parameters are summarized in Fig. 2, where besides the conditions mentioned above, we also have taken into account the following two requirements: sufficiently high reheating temperature for the standard nucleosynthesis to proceed successfully, and the recovery of Newton's law up to 1mm.



Fig. 2. The constraints on the parameters  $(m_5, M)$  are shown in the units of  $m_{pl}$ , where  $m_5$  is 5-dimensional Planck mass. The parameters in the shaded region are allowed for  $O(\nu \ell) \approx 1$ . FG denotes the constraint that the bubble collide with the brane and result in the sufficiently flat universe, and **NW** denotes the constraint that the Newton's' law be valid on scale larger than 1mm. **RH** is the constraint on the reheating temperature.

## §4. Discussion

Concerning the constraints for the model parameters, we can find in Fig. 2 that still a wide region in the parameter space is not excluded. However, the result must interpreted carefully. The interaction strength needs to be tuned additionary. Unfortunately we could not find a natural explanation for this parameter tuning within our simple model. We expect future new invention on this point.

In Ref.<sup>1)</sup>, the amplitude of density fluctuations of this model was not estimated. The analysis of density perturbations will bring another meaningful constraint on the model parameters. To apply a similar analysis done for Bucher's model<sup>15), 16)</sup> to this model, however further extension of the formalism is necessary.

In the sense that our universe is realized inside a single nucleated bubble, our new

scenario has a common feature with the one-bubble open inflation  $^{20), 21}$ . However, in the one-bubble open inflation, since the bubble wall, namely, the boundary surface of the old inflationary phase, is timelike, the universe inside the nucleated bubble is curvature dominant from the beginning. Hence the flatness problem is not solved without the second inflationary epoch. On the other hand, in the present scenario, the bubble nucleation occurs not on the brane but inside the bulk. The boundary surface of the inflation on the brane is spacelike, hence the brane universe can be sufficiently flat without introducing a second inflation.

If we interpret our scenario on the viewpoint of the 4-dimensional effective theory, it does not look quite natural. The phase transition occurs beyond the horizon scale in a completely synchronized manner. It will be necessary to consider a slightly complicated situation in order to explain such a process without assuming the existence of extra-dimension(s). This means that our scenario gives a new paradigm opened for the first time in the context of the braneworld.

As a simple case, we considered a single bubble nucleation in the bulk. There is, however, a possibility that many bubbles nucleate. If nucleation of multi bubbles can also be confined near the bulk symmetric center, then the situation will be similar to the single bubble case. As vacuum bubbles expand, they immediately collide with each other after the nucleation and continue to expand as a single bubble. The bubble collision may produce inhomogeneities on the bubble wall. However the rapid expansion of the bubble wall will erase such inhomogeneities by the time of the brane big-bang. If the bulk field potential  $V(\phi)$  has a number of different vacua and bubbles are nucleated in the different vacuum phases, the collision of the nucleated bubbles may produce topological defects of lower dimension, which could generate large inhomogeneity on the brane universe through the collision. But provided again that the nucleation of bubbles is confined very near the center, one can expect that the abundance of such lower dimensional topological defects in a horizon scale of the FLRW brane is reduced sufficiently by the de Sitter-like expansion of the bubble wall, as the standard inflation solves the problem of unwanted relics.

Our scenario has a lot in common with Bucher's model<sup>13)</sup> in the sense that the bubble collision in higher dimensional spacetime brings a big-bang to our 4dimensional world realized on the brane. Although the basic idea is quite similar, these two models have many different aspects. In Bucher's model, the flatness problem is solved by the large separation between the nucleation centers of the two bubbles when the initial bulk is Minkowskian. When the initial bulk is de Sitter space, the two bubble centers must be located at almost anti-podal points of the bulk each other. The place of collision is not special at all before the collision occurs, and hence the tension of the brane formed after the bubble collision is brought by the colliding bubbles. On the other hand, in our model the universe starts with a small bulk initially, and the brane with positive tension exists as a target for the bubble wall to collide. As a consequence of this, the localization of the nucleation center of the colliding bubble became necessary instead of the large separation of the bubbles, or requirement for the two bubbles to locate at almost anti-podal points each other.

Following point is thought to be a common problem arising in any cosmological

colliding brane models. The mechanism of energy transfer at brane collision process has not been made clear yet. In our model, we simply assumed that the energy of the bubble is completely converted into the energy of radiation on the brane. However, the process of collision will be heavily model dependent, and it is easy to imagine other possibilities. For example, the collision might be elastic, and then the bubble bounces into the bulk. The energy of the bubble wall may completely dissipate into the bulk as radiation. The energy dissipating into the bulk may produce the Weyl components of the bulk gravity. Then, they affect the evolution of the FLRW brane as dark radiation, whose energy density must be suppressed compared to that of the fields localized on the brane. Otherwise, the standard big-bang nucleosynthesis would not work. To avoid this problem, our scenario may require a certain mechanism which realizes the efficient energy conversion from the bubble wall to the fields localized on the brane. This might be possible, for example, if a sufficiently large number of light fields which couple to the bulk scalar field reside on the brane. If the inverse of the bulk curvature radius is larger than the reheating temperature, most of the KK modes of the bulk fields will not be excited by the collision. Then, the number of relevant degrees of freedom localized on the brane is larger than that in the bulk. In such a situation, once the equi-partition among these relevant degrees of freedom is established, the relative contribution from dark radiation is suppressed. Alternatively, we might be able to construct a model in which such a relic dark radiation is diluted by a fairly short period inflation like thermal inflation<sup>22</sup>) implemented by the potential on the brane. To further investigate this issue, one needs to specify the details of the model, which will be supplied once we can embed brane collision model in more fundamental theories such as string theory or M-theory.

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