ICTP Lectures on Large Extra Dimensions

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Abstract

I give a brief and elementary introduction to braneworld models with large extra dimensions. Three conceptually distinct scenarios are outlined: (i) Large compact extra dimensions; (ii) Warped extra dimensions; (iii) Infinite-volume extra dimensions. As an example I discuss in detail an application of (iii) to late-time cosmology and the acceleration problem of the Universe.

Based on lectures given at: Summer School on Astroparticle Physics and Cosmology Triese, Italy, June 17 – July 5, 2002¹

¹A part of these lecture were also delivered at Fifth J.J. Giambiagi Winter School of Physics "Precision Cosmology", July 28 – August 1, 2003, Buenos Aires, Argentina.

Disclaimer

Models with large extra dimensions have been studied very actively during the last few years. There are thousands of works dedicated to the subject and any attempt of detailed account of those developments would require enormous efforts. The aim of the present work is to give a brief and elementary introduction to basic ideas and methods of the models with large extra dimensions and braneworlds.

The work is based on lectures delivered at ICTP Summer School on Astroparticle Physics and Cosmology for students with an introductory-level knowledge in classical and quantum fields, particle physics and cosmology. The scope and extent of the lectures were restricted by the goals of the School. I apologize to those researchers who's advanced and original contributions to the subject could not be reflected in these lectures.

1 Introduction

The magnitude of gravitational force F between two macroscopic objects separated at a distance r obeys the inverse-square law, $F \sim r^{-2}$. This would not be so if the world had an $N \geq 1$ extra spatial dimensions that are similar to our three – in that case we would instead measure $F \sim r^{-(2+N)}$. Similar arguments hold for microworld of elementary particles. For instance, we know from accelerator experiments that electromagnetic interactions of charged particles obey the inverse-square law.

However, experimental capabilities are limited and so is our knowledge of the validity of these laws of nature. For instance, it has not been established how gravity behaves at distances shorter than 10^{-4} cm, or at distances larger than 10^{28} cm. All we know is that for 10^{-4} cm $\leq r \leq 10^{28}$ cm the inverse square law provides a good description of nonrelativistic gravitational interactions, but laws of nature might be different outside of that interval. Likewise, we are certain that electromagnetic interactions obey the inverse-square law all the way down to distances of order 10^{-16} cm, but they might change somewhere below that scale.

At present it is not clear how exactly these laws of nature might change. There is a possibility that they will change according to the laws of higher-dimensional space if extra dimensions exist. However, it is fair to wonder why should one think in the first place that the world might have extra dimensions? I will give below major theoretical arguments that motivated an enormous amount of research in the field of extra dimensions.

The first scientific exploration of the idea of extra dimensions was by Kaluza [1] and Klein [2]. They noticed that gravitational and electromagnetic interactions, since so alike, could be descendants of a common origin. However, amazingly enough, the unified theory of gravity and electromagnetism was possible to formulate only in space with extra dimensions. Subsequently, non-Abelian gauge fields, similar to those describing weak and strong interactions, were also unified with Einstein's gravity in models with extra dimensions. Therefore, the first reason why extra dimensions were studied was:

• Unification of gravity and gauge interactions of elementary particles.

So far we have been discussing classical gravitation. However, quantization of gravity is a very nontrivial task. A candidate theory of quantum gravity, string theory (M-theory), can be formulated consistently in space with extra six or seven dimensions; Hence, the second reason to study extra dimensions:

• Quantization of gravitational interactions.

All the extra dimensions considered above were very small, of the planckian size and therefore undetectable. A new wave of activity in the field of extra dimensions came with the framework of Arkani-Hamed, Dimopoulos and Dvali (ADD) [3] who observed that the Higgs mass hierarchy problem can be addressed in models with *large extra dimensions*. Because the extra dimensions are large in the ADD framework, their effects can be measurable in future accelerator, astrophysical and table-top experiments. Moreover, these models can be embedded in string theory framework [4]. Subsequently Randall and Sundrum proposed a model with warped extra dimension [5] that also provides an attractive setup for addressing the Higgs mass hierarchy problem and for studying physical consequences of extra dimensions. Thus, the third reason is:

• Higgs mass hierarchy problem.

Another type of hierarchy problem is the problem of the cosmological constant. The latter is very hard to address unless one of the conventional notions such as locality, unitarity, causality or four-dimensionality of space-time is given up. In that regard, theories with *infinite volume* extra dimensions [6] – the only theories that are not four-dimensional at very low energies – were proposed as a candidate for solving the cosmological constant problem [7, 8]. Hence the fourth reason is:

• Cosmological constant problem.

In what follows I will discuss some of the developments in extra dimensional theories listed above.

2 Introduction to Kaluza-Klein Theories

Extra spatial dimensions are not similar to our three dimensions in the Kaluza-Klein (KK) approach. Instead, the extra dimensions form a *compact* space with certain compactification scale L. For instance, one extra dimension can be a circle of radius L, or simply an interval of size L. For more than one extra dimensions this space could be a higher dimensional sphere, torus or some other manifold. In general, D-dimensional space-time in the KK approach has a geometry of a direct product $M^4 \times X^{D-4}$ where M^4 denotes four-dimensional Minkowski space-time, and X^{D-4} denotes a compact manifold of extra dimensions – called an *internal manifold*².

What is implied in the KK approach is that there is a certain dynamics in Ddimensional space-time that gives rise to preferential compactification of the extra (D-4)-dimensions leaving four minkowskian dimensions intact. The geometry $M^4 \times X^{D-4}$ should be a solution of D-dimensional Einstein equations.

Let us now discuss what are the physical implications of the compact extra dimensions. Based on common sense it is clear that at distance scales much larger than L the extra dimensions should not be noticeable. They only become "visible" when one probes very short distances of order L.

²The X^{D-4} does not have to be a manifold in a strict mathematical definition of this notion (see examples below) however, we will use this name most of the time for simplicity.

To discuss these properties in detail we start with a simplest example of a real scalar field in (4 + 1)-dimensional space-time. In the the paper we use the mostly positive metric [- + + + + ...]. The Lagrangian density takes the form

$$\mathcal{L} = -\frac{1}{2} \partial_A \Phi \partial^A \Phi, \qquad A = 0, 1, 2, 3, 5.$$
(1)

Here the field $\Phi(t, \vec{x}, y) \equiv \Phi(x_{\mu}, y)$, $\mu = 0, 1, 2, 3$, depends on four-dimensional coordinates x_{μ} as well as on an extra coordinate y. The extra dimension is assumed to be compactified on a circle S^1 of radius L. Therefore, the five-dimensional space-time has a geometry of $M^4 \times S^1$. In this space the scalar field should be periodic with respect to $y \to y + 2\pi L$:

$$\Phi(x,y) = \Phi(x,y+2\pi L).$$
(2)

Let us now expand this field in the harmonics on a circle

$$\Phi(x,y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{iny/L}.$$
(3)

(Note that $\phi_n^*(x) = \phi_{-n}(x)$). Substituting this expansion into (1) the Lagrangian density (1) can be rewritten as follows

$$\mathcal{L} = -\frac{1}{2} \sum_{n,m=-\infty}^{+\infty} \left(\partial_{\mu} \phi_n \partial^{\mu} \phi_m - \frac{nm}{L^2} \phi_n \phi_m \right) e^{i(n+m)y/L} , \qquad (4)$$

while the action takes the form

$$S = \int d^4x \int_0^{2\pi L} dy \mathcal{L} = -\frac{2\pi L}{2} \int d^4x \sum_{n=-\infty}^{+\infty} \left(\partial_\mu \phi_n \partial^\mu \phi_n^* + \frac{n^2}{L^2} \phi_n \phi_n^* \right).$$
(5)

On the right hand side of the above equation we performed integration w.r.t. y. The resulting expression is an action for an infinite number of four-dimensional fields $\phi_n(x)$. To study properties of these fields it is convenient to introduce the notation

$$\varphi_n \equiv \sqrt{2\pi L} \phi_n \,. \tag{6}$$

The latter allows to rewrite the action in the following form

$$S = \int d^4x \left[-\frac{1}{2} \partial_\mu \varphi_0 \partial^\mu \varphi_0 \right] - \int d^4x \sum_{k=1}^{+\infty} \left(\partial_\mu \phi_k \partial^\mu \phi_k^* + \frac{k^2}{L^2} \phi_k \phi_k^* \right)$$
(7)

Therefore, the spectrum of a compactified theory consists of:

• A single real massless scalar field, called a *zero-mode*, φ_0 ;

• An infinite number of massive complex scalar fields with masses inversely proportional to the compactification radius, $m_k^2 = k^2/L^2$.

All the states mentioned above are called the Kaluza-Klein modes. At low energies, i.e., when $E \ll 1/L$ only the zero mode is important; while at higher energies $E \gtrsim 1/L$ all the KK modes become essential.

As a next step we consider a (4+1)-dimensional example of Abelian gauge fields. An additional ingredient, compared to the scalar case, is the local gauge invariance the consequences of which we will emphasize below.

Let us start with the Lagrangian density

$$\mathcal{L} = -\frac{1}{4g_5^2} F_{AB} F^{AB} , \qquad (8)$$

where the dimensionality's are set as follows: $[A_B] = [mass], [g_5^{-1}] = [mass]$. As in the previous example we assume compactification on a circle S^1 of radius L and periodic boundary conditions on the fields. We decompose $F_{AB}^2 = F_{\mu\nu}^2 + 2(\partial_{\mu}A_5 - \partial_5 A_{\mu})^2$, and expand the fields A_{μ} and A_5 in the harmonics on a circle

$$A_{\mu}(x,y) = \sum_{n=-\infty}^{+\infty} A_{\mu}^{(n)}(x) e^{iny/L}, \quad A_{5}(x,y) = \sum_{n=-\infty}^{+\infty} A_{5}^{(n)}(x) e^{iny/L}.$$
(9)

As in the scalar example we integrate w.r.t. y to calculate the effective 4d action

$$S = \int d^4x \int_0^{2\pi L} dy \mathcal{L} \equiv \int d^4x \, \mathcal{L}_4 \,. \tag{10}$$

Using gauge transformation the expression for \mathcal{L}_4 can be cast in the following form

$$\mathcal{L}_{4} = -\frac{1}{4g_{4}^{2}} \left\{ F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + 2\sum_{k=1}^{+\infty} \left[F_{\mu\nu}^{(k)} F^{*(k)\mu\nu} + \frac{2k^{2}}{L^{2}} A_{\mu}^{(k)} A^{*(k)\mu} \right] + 2(\partial_{\mu} A_{5}^{(0)})^{2} \right\} . (11)$$

Therefore, we conclude that the spectrum of the compactified model consists of the following states:

- A zero-mode a massless gauge field $A^{(0)}_{\mu}$ with the gauge coupling $g_4^2 = g_5^2/(2\pi L)$;
- Massive KK gauge bosons with the mass $m_k^2 = k^2/L^2$;
- Massless scalar field $A_5^{(0)}$.

A few words on local gauge invariance are in order here. The five-dimensional model is invariant under five-dimensional local gauge transformations $A_B(x, y) \rightarrow A_B(x, y) + \partial_B \alpha(x, y)$. After compactification the five-dimensional gauge transformations reduce to an infinite number of *four-dimensional* gauge transformations – one for each KK level $A_{\mu}^{(n)}(x) \to A_{\mu}^{(n)}(x) + \partial_{\mu}\alpha^{(n)}(x)$. However, only the zero-mode is massless gauge field, all the higher KK modes are massive. This can be interpreted as a consequence of the Higgs mechanism taking place on each massive KK level where a massless gauge field "eats" one massless scalar $A_5^{(n)}$ and becomes a massive gauge field with 3 physical degrees of freedom. On the massless level there is a 4d massless gauge field with 2 physical degrees of freedom plus one real massless scalar $A_5^{(0)}$.

Finally we come to the main subject of this section and consider a (4 + 1)dimensional example of gravity. It demonstrates how 4d Einstein gravity can be unified with electromagnetism in a 5d theory — the original proposal of Kaluza and Klein.

The 5d action takes the form

$$S = \frac{M_*^3}{2} \int d^4x dy \sqrt{G} R_5 \,. \tag{12}$$

As in the previous examples the space is $M^{(4)} \times S^1$ and we expand fields in the harmonics on a circle of radius L

$$G_{AB}(x,y) = \sum_{n=-\infty}^{+\infty} G_{AB}^{(n)}(x) e^{iny/L}.$$
 (13)

In what follows we will concentrate on the zero mode $G_{AB}^{(0)}$ neglecting all the massive modes.

Let us introduce the notations

$$G_{\mu\nu}^{(0)} = e^{\phi/\sqrt{3}} (g_{\mu\nu}(x) + e^{-\sqrt{3}\phi} A_{\mu} A_{\nu}), \qquad (14)$$
$$G_{\mu5}^{(0)} = G_{5\mu}^{(0)} = e^{-2\phi/\sqrt{3}} A_{\mu}, \\G_{55}^{(0)} = e^{-2\phi/\sqrt{3}}.$$

Using these expressions we find the 4d action for the zero mode fields

$$S_{\rm zm} = M_*^3 \pi L \int d^4 x \sqrt{g} \left(R_4(g) - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} e^{-\sqrt{3}\phi} F_{\mu\nu}^2 \right) \,. \tag{15}$$

Recalling that the conventional 4D action for gravity has a form

$$\frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{g} R_4(g) \,, \tag{16}$$

we find that $M_{\rm Pl}^2 = M_*^3 2\pi L$. As a result, the Newton constant $G_N = (8\pi M_{\rm Pl}^2)^{-1}$ can be related to the higher dimensional scale and the compactification radius

$$G_N = \frac{1}{16\pi^2 M_*^3 L} \,. \tag{17}$$

The main result of the above discussion is that four-dimensional gauge and gravitational fields have a common origin in five-dimensional gravitational field.

Let us count physical degrees of freedom. A four-dimensional massless graviton has 2 physical degrees of freedom (pdf's); A four-dimensional massless gauge boson has also 2 pdf's, and a real scalar has 1 pdf. Total is 5 pdf's, in agreement with 5 pdf's of a massless five-dimensional graviton ³.

Let us now turn to the massive KK levels. The analysis is similar to that of gauge fields but more cumbersome. Nevertheless, the main results can be summarised as follows. There is a massive graviton with the mass $m_k^2 = k^2/L^2$ at each k'th level. These gravitons acquire masses via the Higgs mechanism – one massless graviton (2 pdf's) "eats" 1 massless gauge boson (2 pdf's) and one real scalar (1pdf) – this makes one massive 4D graviton that has 5 pdf's. The massive gravitational KK modes are charged under the massless gauge field. The charges are determined as $q_k \sim k/LM_{\rm Pl} \sim m_n/M_{\rm Pl}$. At the linearized level gauge transformations do not mix with each other different KK levels, however, this mixing shows up once the nonlinear interactions of gravitational theory are taken into account [9].

3 Introduction to Braneworlds

The idea that our (3 + 1)-dimensional world could be realized as a 3d surface in higher dimensional space was actively discussed in the context of general relativity the 1960th and 1970th.

A first particle physics application of this idea was put forward by Rubakov and Shaposhnikov [10] and independently by Akama [11].

In this section, following [10], we consider a toy example of the braneworld where the main mechanism of localisation can explicitly be worked out.

We start with a scalar field in 5-dimensions with the following Lagrangian density

$$\mathcal{L} = -\frac{1}{2} \partial_A \Phi \partial^A \Phi - \frac{\lambda}{2} \left(\Phi^2 - \eta^3 \right)^2 .$$
 (18)

The Lagrangian is invariant under the \mathbf{Z}_2 transformations $\Phi \to -\Phi$, however, the vacua of the theory are not — under the \mathbf{Z}_2 the two vacua $\Phi = \pm \eta^{3/2}$ interchange. Therefore, the \mathbf{Z}_2 is spontaneously broken. As a result, there should exist domain walls. We find the following domain wall (kink) solution to the classical equation of motion

$$\Phi_{\rm cl}(y) = \eta^{3/2} \tanh\left(\sqrt{\lambda}\eta^{3/2} y\right) \equiv \eta^{3/2} \tanh\left(m_0 y\right) \,. \tag{19}$$

³In general, the total number of independent components of a rank 2 symmetric tensor in *D*-dimensions is D(D+1)/2, however, only D(D-3)/2 of those correspond to physical degrees of freedom of a *D*-dimensional massless graviton; the remaining extra components are the redundancy of manifestly gauge and Lorentz invariant description of the theory.

Transverse to the domain wall space is one dimensional, hence, domain wall is a codimension one object. Its worldvolume has 3 spatial coordinates, therefore, it is also called a 3-brane.

Let us discuss certain properties of the solution. The tension of the wall is its surface energy density $T = \int dy H(\Phi_{\rm cl}) = \int dy T_{00}(\Phi_{\rm cl})$, where H denotes the Hamiltonian and T_{00} denotes the 00 component of the stress tensor. The tension is determined as follows

$$T \sim \frac{m_0^3}{\lambda} \sim \sqrt{\lambda} \eta^{3/2} \eta^3$$
 (20)

Below we would like to understand what are the excitations that live on the brane worldvolume. According to the braneworld idea [10],[11], in a realistic construction, those excitations should be identified with the Standard Model particles. For this purpose we perform the following decomposition

$$\Phi(x,y) = \Phi_{\rm cl}(y) + \delta\Phi(x,y).$$
(21)

Then we find that the 5d equations have a solution

$$\delta\Phi(x,y) = \left(\frac{d\Phi_{\rm cl}}{dy}\right)\rho(x), \qquad (22)$$

where the four-dimensional field ρ satisfies the equation

$$\partial_{\mu}^{2}\rho = 0.$$
 (23)

Therefore, ρ is nothing but a massless four-dimensional mode. The wavefunction of this mode is proportional to $d\Phi_{\rm cl}/dy$ and vanishes outside of the brane. Therefore, this mode is localized on a brane. This excitation is just a Nambu-Goldstone boson of spontaneously broken translation invariance along the y direction.

Let us now introduce fermions. For this we add to the Lagrangian the following two terms

$$\Delta \mathcal{L} = i \bar{\Psi} \Gamma^M \partial_M \Psi - h \Phi \bar{\Psi} \Psi, \qquad (24)$$

where Ψ denotes a 5-dimensional Dirac fermion. The equation of motion for the fermion in the background of the domain wall reads as follows:

$$i\Gamma^M \partial_M \Psi - h \Phi_{\rm cl} \Psi = 0.$$
⁽²⁵⁾

This equation has a normalizable solution of the following form

$$\Psi_{\rm zm}(x,y) = e^{-\int_0^y h \Phi_{\rm cl}(z) dz} \chi_L(x) , \qquad (26)$$

where χ_L denotes a four-dimensional massless chiral mode

$$i\Gamma^{\mu}\partial_{\mu}\chi_L = 0, \qquad \chi_L = (1 - \gamma_5)\chi/2.$$
 (27)

From this expression we see that the wavefunction of this mode vanishes outside of the brane. Therefore, one obtains a four-dimensional chiral mode that is localized on the worldvolume⁴.

Summarizing, in a simple construction described above scalars and fermions can be localized on a brane. However, for realistic model building one should in addition perform two major steps:

(i) Localize gauge fields on a brane;

(ii) Obtain four-dimensional gravity on the brane.

A mechanism for gauge field localisation within the field theory context was proposed by Dvali and Shifman [12]. It is based on the observation that gauge field can be in the confining phase the bulk while being in the broken phase on a brane; then confining potential prevents the low energy brane gauge fields to propagate into the bulk. This mechanism is discussed in details in Refs. [12].

Localisation of gauge fields is a rather natural property of D-branes in closed string theories [13] – the gauge fields emerge on a brane as fluctuations of open strings that are attached to the brane and do not exist in the bulk.

As to the issue (ii), below we discuss three distinct mechanisms by mens of which the laws of 4d gravity can be obtained on a brane.

4 Braneworlds with Compact Extra Dimensions

One way to obtain 4d gravity on a brane is to combine the braneworld idea with the idea of KK compactification. This, as was proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) [3], opens up new possibilities to solve the Higgs mass hierarchy problem and gives rise to new predictions that can be tested in accelerator, astrophysical and table-top experiments. Moreover, the framework can be embedded in string theory [4].

The main ingredients of a simplest ADD scenario are:

- Standard Model particles are localized on a 3-brane, while gravity spreads to all 4 + N dimensions.
- The fundamental scale of gravity M_* , and the ultraviolet (UV) scale of the Standard Model, are around a few TeV or so. This can eliminate the Higgs mass hierarchy problem.
- N extra dimensions are compactified.

⁴There also exists a solution with an opposite chirality that is not localized on a brane.

The action for a simplest ADD model takes the form:

$$S_{\rm ADD} = \frac{M_*^{2+N}}{2} \int d^4x \int_0^{2\pi L} d^N y \sqrt{G} R_{(4+N)} + \int d^4x \sqrt{g} (T + \mathcal{L}_{\rm SM}(\Psi, M_{\rm SM}))$$
(28)

where $M_* \sim (1 - 10)$ TeV, g(x) = G(x, y = 0), $T + \langle \mathcal{L}_{SM} \rangle = 0$, the latter condition is a usual fine-tuning of the cosmological constant.

Technical simplifications which are adopted above but that can be easily lifted are as follow:

(1) The Brane width is taken to be zero (generically, the natural scale for the brane width could be M_*^{-1} .)

(2) Brane fluctuations are neglected (these are Nambu-Goldstone bosons which couple to matter derivatively).

(3) All extra dimensions have equal size L (in general, different extra dimensions could have different sizes).

(4) Only gravity can propagate in the bulk (in general, other fields could also live in the bulk, in fact there are attractive scenarios with right-handed Neutrino living in the bulk [14].)

Let us first study the properties of 4d gravity in the ADD scenario. The low effective 4d action for a zero mode takes the form

$$\frac{M_*^{2+N}}{2} \int d^4x \int_0^{2\pi L} d^N y \sqrt{G} R_{(4+N)} \to \frac{M_*^{2+N} (2\pi L)^N}{2} \int d^4x \sqrt{g_{\rm zm}} R_{\rm zm} \,, \qquad (29)$$

hence, we should define the 4d Planck mass

$$M_{\rm Pl}^2 = M_*^{2+N} (2\pi L)^N \,. \tag{30}$$

Postulating that the quantum gravity scale is at $M_* \sim \text{TeV}$ we find what should be the size of extra dimensions

$$L \sim 10^{-17+30/N} \,\mathrm{cm}\,.$$
 (31)

For one extra dimension, N = 1, one gets $L \sim 10^{13}$ cm, this is excluded within the ADD framework since gravity below 10^{13} would have been higher dimensional. For N = 2 we get $L \sim 10^{-2}$ cm; this particular case is very interesting since it predicts modification of the 4d laws of gravity at submillimiter distances – the subject of active experimental studies. For larger N the value of L should decrease; but even for N = 6 L is very large compared to $1/M_{\rm Pl}$.

Two static sources on the brane interact with the following nonrelativistic gravitational potential

$$V(r) = -G_N m_1 m_2 \sum_{n=-\infty}^{+\infty} |\Psi_n(y=0)|^2 \frac{e^{-m_n r}}{r}, \qquad (32)$$

where $\Psi_n(y=0)$ denotes the wavefunction of n'th KK mode at a position of a brane and $m_n = |n|/L$. If $r \gg L$ from the above expression we find

$$V(r) = -\frac{G_N m_1 m_2}{r} \,. \tag{33}$$

This recovers the conventional 4d law of Newtonian dynamics. In the opposite limit, i.e., when $r \ll L$ one gets

$$V(r) = -\frac{m_1 m_2}{M_*^{2+N} r^{1+N}}.$$
(34)

That is the law of (4 + N)-dimensional gravitational interactions. Therefore, the laws of gravity are modified at distances of order L.

Selected topics of the ADD phenomenology:

- Gauge coupling unification. In a conventional 4d theory the renormalization group running of the gauge coupling constants is logarithmic. This changes in higher dimensions where the power-law running takes place [17]. As was shown by Dienes, Dudas and Gherghetta [18], the power-law running is what gives rise to an accelerated unification of the strong, weak and electromagnetic couplings at a scale around M_* in braneworlds with compact extra dimensions.
- Missing energy signals in accelerator experiments. The SM particles are localized on a brane only up to some energy scale that is comparable to M_* . At about that scale the SM particles could in principle escape into the bulk. This would provide missing energy signals in accelerator experiments. Another missing energy signal can be due to emission of KK gravitons into the bulk, see detailed discussions in Refs. [3],[15].
- Energy loss by stars via emission of light KK gravitons. In the 6d ADD model the KK gravitons are light, $m_{KK} \sim L^{-1} \sim 10^{-4}$ eV. Therefore, these gravitons can be emitted in the interior of astrophysical objects the temperature of which exceeds 10^{-4} eV. As a result, these objects, such as stars, can coll down due to the process of emission of the KK gravitons into the bulk. Each KK graviton emission if $M_{\rm Pl}$ suppressed. However, because of the the high-multiplicity of KK graviton the net result for the emission rate is suppressed by $1/M_*^2$. Unless this rate is small enough, a star would cool down faster than it should by emitting these KK gravitons. This puts a lower bout on M_* in a 6d theory to be 50 TeV or so [3, 16].
- Cosmological implications. There exist new scenarios of inflation and Baryogenesis within the braneworld context. These scenarios manifestly use properties of branes. For instance, inflation on "our brane" can be obtained if another brane falls on top of "our brane" in the early period of development of the brane-universe [19]. The potential that is created by another brane in "our

world" can be viewed as the conventional inflationary potential. Baryon asymmetry of a desired magnitude can also be produced during the collision of these two branes [20]. For more recent developments see Refs. [21],[22], [23], [24].

5 Braneworlds with Warped Extra Dimensions

In this section we describe another way of obtaining 4d gravity on a brane. It is based on a phenomenon of *localisation of gravity* discovered by Randall and Sundrum (RS) [5].

We start with a so-called RS II model that has a single brane embedded in 5dimensions bulk with negative cosmological constant. The action of the model is written as follows:

$$S_{\rm RS} = \frac{M_*^3}{2} \int d^4x \int_{-\infty}^{+\infty} dy \sqrt{G} (R_5 - 2\Lambda) + \int d^4x \sqrt{g} (T + \mathcal{L}_{\rm SM}(\Psi, M_{\rm SM})) , \quad (35)$$

where Λ denotes the negative cosmological constant and T is the brane tension.

The equation of motion derived from this action takes the form (the Gibbons-Hawking surface term in the action is implied and hereafter we put $\mathcal{L}_{SM} = 0$ for simplicity)

$$M_*\sqrt{G}\left(R_{AB} - \frac{1}{2}G_{AB}R\right) = -M_*^3\Lambda\sqrt{G}G_{AB} + T\sqrt{g}\,g_{\mu\nu}\,\delta^{\mu}_A\,\delta^{\nu}_B\,\delta(y)\,. \tag{36}$$

In our conventions the brane is located in extra space at the y = 0 point. The above equations have a solution with a flat 4d worldvolume

$$ds^{2} = e^{-|y|/L} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^{2}, \qquad (37)$$

where $\eta_{\mu\nu} = \text{diag}(-+++)$ is the four-dimensional flat space metric, and we introduced the following notations

$$L \equiv \sqrt{-\frac{3}{2\Lambda}}, \qquad T = \frac{3M_*^3}{L}. \tag{38}$$

The values of Λ and T have to be carefully adjusted to each other for this solution to exist. Although the coordinate y runs in the interval $(-\infty, +\infty)$ nevertheless, the physical size of extra dimension is finite:

$$\int_{-\infty}^{+\infty} dy \sqrt{G} \sim L \,. \tag{39}$$

The primary question that we would like to address is how does gravity look like on the brane? For this let us consider graviton fluctuations:

$$ds^{2} = \left(e^{-|y|/L}\eta_{\mu\nu} + h_{\mu\nu}(x,y)\right)dx^{\mu}dx^{\nu} + dy^{2}.$$
(40)

We decompose $h_{\mu\nu}(x,y) \equiv u(y)\tilde{h}_{\mu\nu}(x) = u(y)\epsilon_{\mu\nu}\exp(ipx)$ with $p^2 = -m^2$. As a result the equation for the function u takes the following form:

$$\left(-m^2 e^{|y|/L} - \partial_y^2 - \frac{2}{L}\delta(y) + \frac{1}{L^2}\right)u(y) = 0.$$
(41)

For a zero-mode $m^2 = 0$ this equation simplifies and the solution can be found easily:

$$u(y) = \operatorname{const.} e^{-|y|/L} \,. \tag{42}$$

Hence the interval for the zero-mode factorizes as follows

$$ds^{2} = e^{-|y|/L} \tilde{g}_{\mu\nu}(x) dx^{\mu} dx^{\nu} + dy^{2}, \qquad (43)$$

where we used the notations $\tilde{g}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \tilde{h}_{\mu\nu}(x)$.

It is important to emphasize that the five dimensional action is integrable w.r.t. y for the zero-mode:

$$\frac{M_*^3}{2} \int d^4x \int_{-\infty}^{+\infty} dy \sqrt{G} R_{(5)} \to \frac{M_*^3(2L)}{2} \int d^4x \sqrt{\tilde{g}} \tilde{R} \,. \tag{44}$$

The result of this integration is a conventional 4d action. Hence we find a relation between the 4d Planck mass and M_*

$$M_{\rm Pl}^2 = M_*^3(2L) \tag{45}$$

This looks similar to the relation between the fundamental scale M_* , the size of extra dimension L and the Planck mass $M_{\rm Pl}$ in the ADD model with one extra dimension. The similarity is due to the fact that the effective size of the extra dimension that is felt my the zero-mode graviton is finite $\sim L$ as in the ADD as well as in the RS models.

Besides the zero-mode there are an infinite number of KK modes [5]. Since the extra dimension is not compactified the KK modes have no mass gap. In the zero-mode approximation used in (44) these states were neglected. However, at short distances << L the effects of those modes become important. This can be seen by calculating a static potential between sources on a brane. The result reads:

$$V(r) = -\frac{G_N m_1 m_2}{r} \left(1 + \frac{(2L)^2}{r^2}\right).$$
(46)

The second term in the parenthesis is due to the exchange of KK modes. We see that this term becomes dominant when $r \leq L$.

The above construction with the localized graviton can be used for a new solution of the hierarchy problem. This is achieved in a so-called RS I model [25].

The model contains two branes that are placed at the endpoints of an interval of a certain size. One brane, called the "hidden brane", has positive tension and the other one, called "visible brane", has negative tension. The equation of motion for this model looks as follows:

$$M_*\sqrt{G}\left(R_{AB} - \frac{1}{2}G_{AB}R\right) - M_*^3\Lambda\sqrt{G}G_{AB} = T_{\text{hid}}\sqrt{g_{\text{hid}}} g_{\mu\nu}^{\text{hid}} \delta_A^\mu \delta_B^\nu \delta(y) + T_{\text{vis}}\sqrt{g_{\text{vis}}} g_{\mu\nu}^{\text{vis}} \delta_A^\mu \delta_B^\nu \delta(y - y_0), \qquad (47)$$

were we used the notations

$$g_{\mu\nu}^{\text{hid}}(x) = G_{\mu\nu}(x, y=0), \quad g_{\mu\nu}^{\text{vis}}(x) = G_{\mu\nu}(x, y=y_0).$$
 (48)

As we mentioned above, the y direction is compactified on an orbifold S_1/\mathbb{Z}_2 and y runs in the interval $[-y_0, y_0]$. One can check that there exists the following static solution to the equations of motion

$$ds^{2} = e^{-|y|/L} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^{2}.$$
(49)

The next step is find out fluctuations about this classical background. For this we proceed as in the RS II case. The derivation is straightforward and the result is that the tensor $\eta_{\mu\nu}$ should be replaced as $\eta_{\mu\nu} \to \bar{g}_{\mu\nu}(x)$, where

$$g_{\mu\nu}^{\rm hid}(x) = \bar{g}_{\mu\nu}(x), \quad g_{\mu\nu}^{\rm vis}(x) = e^{-|y_0|/L} \bar{g}_{\mu\nu}(x).$$
 (50)

Let us now look at what this leads to. For this we turn to the matter part of the Lagrangian. In the RS I it is assumed that the Standard Model fields are localized on a negative tension brane, i.e., at $y = y_0$. As a representative SM field we consider the Higgs field ϕ . We obtain:

$$\int d^4x \sqrt{g_{\text{vis}}} \left\{ g_{\text{vis}}^{\mu\nu} (D_{\mu}\phi)^+ (D_{\nu}\phi) - \lambda (|\phi|^2 - v_0^2)^2 \right\} \rightarrow \int d^4x \sqrt{\bar{g}} \left\{ \bar{g}^{\mu\nu} (D_{\mu}\phi)^+ (D_{\nu}\phi) - \lambda (|\phi|^2 - e^{-y_0/L} v_0^2)^2 \right\}.$$
(51)

Hence the Higgs VEV on a visible brane is rescaled by an exponential factor $v = e^{-y_0/2L}v_0$. Thus, all masses on the visible brane are suppressed by this exponential factor as compared to their natural values

$$m^2 = e^{-y_0/L} m_0^2 \,. \tag{52}$$

If $m_0 \sim M_{\rm Pl}$, then in order to get $m \sim \text{TeV}$ one needs $y_0/L \sim 100$. Therefore, small hierarchy in y_0/L gives rise to large hierarchy between m and m_0 .

The hierarchy problem is solved at the expense of fine tunning of the tension of the hidden brane to the tension of the visible brane and both these tensions to the bulk cosmological constant. A possible way to avoid the fine tuning is to use the stabilization mechanism proposed by Goldberger and Wise [26]. Another interesting scenario, studied by Karch and Randall [27] emerges when the tension and bulk cosmological constant are slightly detuned so that the worldvolume has AdS_4 geometry. Regretfully, detailed discussion of these developments goes beyond the scope of the present lectures.

Selected topics of the RS phenomenology:

- Missing energy signals in accelerator experiments. The SM particles are localized on a brane only up to some energy scale that is comparable with M_* . At about that scale the SM particles could be emitted into the bulk. As in the ADD case, this would provide missing energy signals in accelerator experiments. See Ref. [28] for details.
- Gauge coupling unification. In a conventional 4d theory the renormalization group running of the gauge coupling constants is logarithmic. As we discussed before, this changes in flat higher dimensions, the power-law running takes place [17]. However, in the RS case the extra 5th dimension is not flat. This affects dramatically the gauge coupling running which can still be logarithmic as was discussed in Refs. [29], [30].

6 Braneworlds with Infinite Volume Extra Dimensions

In this section we consider the third known mechanism of obtaining 4d gravity on a brane [6]. This mechanism is different from the previously discussed once since it allows the volume of the extra space to be infinite.

$$V_N \equiv \int d^N y \sqrt{G} \to \infty \,. \tag{53}$$

The motivation for constructing the models with infinite-volume extra dimensions are as follow:

- The size of extra dimensions does not need to be stabilized since the extra dimensions are nether compactified nor warped.
- Because of the presence of infinite-volume extra dimensions gravity is modified at large distances. This gives rise to new solutions for late-time cosmology and acceleration of the universe.
- Although the supersymmetry should be broken on a brane where the SM fields live, nevertheless unbroken supersymmetry can be maintained in the bulk since it has an infinite volume

The 5d model with these properties was proposed in [6]

$$S_{\rm DGP} = \frac{M_*^3}{2} \int d^4x \int_{-\infty}^{+\infty} dy \sqrt{G} R_5 + \int d^4x \sqrt{g} \left(\frac{M_{\rm Pl}^2}{2} R_4 + T + \mathcal{L}_{\rm SM}(\Psi, M_{\rm SM})\right).$$
(54)

(The Gibbons-Hawking surface term is implied above). The main postulates in this approach are:

- The SM fields are localized on a brane while gravity is propagating everywhere in extra dimensions
- The UV cutoff of the SM $M_{\rm SM} \sim M_{\rm GUT} \gg M_*$.
- The brane width is assumed to be $1/M_{\rm SM}$.
- $T + \langle \mathcal{L}_{SM} \rangle = 0$, this is a usual fine tunning of the cosmological constant but the latter can be relaxed in higher co-dimensions [7, 8], in which case the model can be embedded into string theory [31].

What kind of gravity is described by this model? Let us look at the gravitational part of (54). We introduce the quantity

$$r_c \sim M_{\rm Pl}^2 / M_*^3$$
, (55)

When $r_c \to \infty$ the 4d term dominates, in the opposite limit $r_c \to 0$ the 5d term dominates. Therefore we expect that for $r \ll r_c$ to recover the 4d laws on the brane, while for $r \gg r_c$ 5d laws.

The 4d Ricci scalar $R_4 = R_4(g(x))$ is constructed out of the induced metric on a brane

$$g_{\mu\nu}(x) \equiv G_{\mu\nu}(x, y=0)$$
. (56)

The Standard Model (SM) fields are confined to the brane. Note that the SM cutoff should not coincide in general with M_* and, in fact, is assumed to be much higher in our case. For simplicity we suppress the Lagrangian of SM fields. The braneworld origin of the action (54) and parameters M_* , $M_{\rm Pl}$ were discussed in details in Refs. [6, 7, 32].

Let us first study the non-relativistic potential between two sources confined to the brane. For a time being we drop the tensorial structure in the gravitational equations and discuss the distance dependence of the potential. We comment on the tensorial structure below.

The static gravitational potential between the sources in the 4-dimensional worldvolume of the brane is determined as:

$$V(r) = \int G_R(t, \vec{x}, y = 0; 0, 0, 0) dt , \qquad (57)$$

where $r \equiv \sqrt{x_1^2 + x_2^2 + x_3^2}$ and $G_R(t, \vec{x}, y = 0; 0, 0, 0)$ is the retarded Green's function (see below). Let us turn to Fourier-transformed quantities with respect to the world-volume four-coordinates x_{μ} :

$$G_R(x,y;0,0) \equiv \int \frac{d^4p}{(2\pi)^4} e^{ipx} \tilde{G}_R(p,y) .$$
 (58)

In Euclidean momentum space the equation for the Green's function takes the form:

$$\left(M_*^3(p^2 - \partial_y^2) + M_{\rm Pl}^2 p^2 \,\delta(y) \right) \tilde{G}_R(p, y) = \delta(y) \,. \tag{59}$$

Here p^2 denotes the square of an Euclidean four-momentum $p^2 \equiv p_4^2 + p_1^2 + p_2^2 + p_3^2$. The solution with appropriate boundary conditions takes the form:

$$\tilde{G}_R(p,y) = \frac{1}{M_{\rm Pl}^2 p^2 + 2M_*^3 p} \exp(-p|y|) , \qquad (60)$$

where $p \equiv \sqrt{p^2} = \sqrt{p_4^2 + p_1^2 + p_2^2 + p_3^2}$. Using this expression and Eq. (57) one finds the following (properly normalized) formula for the potential

$$V(r) = -\frac{1}{8\pi^2 M_{\rm Pl}^2} \frac{1}{r} \left\{ \sin\left(\frac{r}{r_c}\right) \operatorname{Ci}\left(\frac{r}{r_c}\right) + \frac{1}{2} \cos\left(\frac{r}{r_c}\right) \left[\pi - 2\operatorname{Si}\left(\frac{r}{r_c}\right)\right] \right\} , (61)$$

where $\operatorname{Ci}(z) \equiv \gamma + \ln(z) + \int_0^z (\cos(t) - 1) dt/t$, $\operatorname{Si}(z) \equiv \int_0^z \sin(t) dt/t$, $\gamma \simeq 0.577$ is the Euler-Mascheroni constant, and the distance scale r_c is defined as follows:

$$r_c \equiv \frac{M_{\rm Pl}^2}{2M_*^3} \,.$$
 (62)

In our model we choose r_c to be of the order of the present Hubble size, which is equivalent to the choice $M_* \sim 10 - 100$ MeV. We will discuss phenomenological compatibility of such a low quantum gravity scale below. It is useful to study the short distance and long distance behavior of this expression.

At short distances when $r \ll r_c$ we find:

$$V(r) \simeq -\frac{1}{8\pi^2 M_{\rm Pl}^2} \frac{1}{r} \left\{ \frac{\pi}{2} + \left[-1 + \gamma + \ln\left(\frac{r}{r_c}\right) \right] \left(\frac{r}{r_c}\right) + \mathcal{O}(r^2) \right\} .$$
(63)

Therefore, at short distances the potential has the correct 4d Newtonian 1/r scaling. This is subsequently modified by the logarithmic *repulsion* term in (63).

Let us turn now to the large distance behavior. Using (61) we obtain for $r >> r_c$:

$$V(r) \simeq -\frac{1}{8\pi^2 M_{\rm Pl}^2} \frac{1}{r} \left\{ \frac{r_c}{r} + \mathcal{O}\left(\frac{1}{r^2}\right) \right\}$$
 (64)

Thus, the long distance potential scales as $1/r^2$ in accordance with laws of 5d theory.

We would like to emphasize that the behavior (60) is intrinsically higher-dimensional and is very hard to reproduced in conventional four-dimensional field theory. Indeed, the would be four-dimensional inverse propagator should contain the term $\sqrt{p^2}$. In the position space this would correspond in the Lagrangian to the following pseudo-differential operator

$$\hat{\mathcal{O}} = -\partial_{\mu}^2 + \frac{\sqrt{-\partial_{\mu}^2}}{r_c} \,. \tag{65}$$

We are not aware of a consistent four-dimensional quantum field theory with a finite number of physical bosons which would lead to such an effective action.

4D gravitational interactions in the present model are mediated by a resonance graviton with the lifetime $\tau \sim r_c$. The resonance-mediated gravity was first discussed in Refs. [33, 34, 35] in a different context. Yet another scenario in which the large distance gravity is modified due to the mass of a graviton was proposed and studied by the Oxford group [36].

Finally we would like to comment on the tensorial structure of the graviton propagator in the present model. In flat space this structure is similar to that of a massive 4d graviton [6]. This points to the van Dam-Veltman-Zakharov (vDVZ) discontinuity [37, 38]. However, this problem can in general be resolved by at least two methods. In the present context one has to use the results of [39] where it was argued that the vDVZ discontinuity which emerges in the lowest perturbative approximation is in fact absent in the full nonperturbative theory. The application of the similar arguments to our model leads to the result which is continuous in $1/r_c$. This is discussed in details in Ref. [40]. Thus, the vDVZ problem is an artifact of using the lowest perturbative approximation ⁵.

In general, the simplest possibility to deal with the vDVZ problem, as was suggested in Ref. [32], is to compactify the extra space at scales bigger than the Hubble size with r_c being even bigger, but we do not consider this possibility here.

1. Cosmological solutions: Below we will mainly be interested in the geometry of the 4d brane-world and follow Ref. [41]. For the completeness of the presentation let us first recall the full 5d metric of the cosmological solution. The 5d line element is taken in the following form:

$$ds^{2} = -N^{2}(t,y) dt^{2} + A^{2}(t,y) \gamma_{ij} dx^{i} dx^{j} + B^{2}(t,y) dy^{2}, \qquad (66)$$

where γ_{ij} is the metric of a 3 dimensional maximally symmetric Euclidean space, and the metric coefficients read [44]

$$N(t,y) = 1 + \epsilon |y| \ddot{a} (\dot{a}^{2} + k)^{-1/2},$$

$$A(t,y) = a + \epsilon |y| (\dot{a}^{2} + k)^{1/2},$$

$$B(t,y) = 1,$$
(67)

where a(t) is 4d scale factor and $\epsilon = \pm 1$. Knowing the braneworld intrinsic geometry is all what matters as far as 4d observers are concerned. This geometry is given in the above solution. Taking the y = 0 value of the metric we obtain the usual 4d

⁵Note that the continuity in the graviton mass in (A)dS backgrounds was demonstrated recently in Refs. [45, 46]. We should emphasize that we are discussing the continuity in the classical 4d gravitational interactions on the brane. There is certainly the discontinuity in the full theory in a sense that there are extra degrees of freedom in the model. These latter can manifest themselves at quantum level in loop diagrams [47].

Friedmann-Lemaître-Robertson-Walker (FLRW) form (enabling to interpret t as the cosmic time on the braneworld)

$$ds^{2} = -dt^{2} + a^{2}(t) dx^{i} dx^{j} \gamma_{ij}, \qquad (68)$$

$$= -dt^{2} + a^{2}(t) \left(dr^{2} + S_{k}^{2}(r) d\psi^{2} \right), \qquad (69)$$

where $d\psi^2$ is an angular line element, k = -1, 0, 1 parametrizes the brane world spatial curvature, and S_k is given by

$$S_k(r) = \left\{ \begin{array}{l} \sin r & (k=1) \\ \sinh r & (k=-1) \\ r & (k=0) \end{array} \right\}$$
(70)

In the present case, the dynamics is generically different from the usual FLRW cosmology, as shown in [44]. The standard first Friedmann equation is replaced in our model by

$$H^{2} + \frac{k}{a^{2}} = \left(\sqrt{\frac{\rho}{3M_{\rm Pl}^{2}} + \frac{1}{4r_{c}^{2}}} + \epsilon \frac{1}{2r_{c}}\right)^{2},\tag{71}$$

where ρ is the total cosmic fluid energy density. We have in addition the usual equation of conservation for the energy-momentum tensor of the cosmic fluid given by

$$\dot{\rho} + 3H(p+\rho) = 0$$
. (72)

Equations (71) and (72) are sufficient to derive the cosmology of our model. In particular using these relations one can obtain a second Friedmann equation as in standard cosmology.

Equation (71) with $\epsilon = 1$ and $\rho = 0$ has an interesting self-inflationary solution with a Hubble parameter given by the inverse of the crossover scale r_c . This can be easily understood looking back at the action (54) where it is apparent that the intrinsic curvature term on the brane appears as a source for the bulk gravity, so that with appropriate initial conditions, this term can cause an expansion of the brane world without the need of matter or cosmological constant on the brane. This self-inflationary solution is the key ingredient for our model to produce late time accelerated expansion⁶. Before discussing in detail this issue let us first compare our cosmology with the the standard one.

We first note that the standard cosmological evolution is recovered from (71) whenever $\rho/M_{\rm Pl}^2$ is large compared to $1/r_c^2$, so that the early time cosmology of our model is analogous to standard cosmology. In this early phase equation (71) reduces, at leading order, to the standard 4d Friedmann equation given by

$$\frac{H^2 + \frac{k}{a^2}}{3M_{\rm Pl}^2} = \frac{\rho}{3M_{\rm Pl}^2} \,. \tag{73}$$

⁶Note that the nonzero 4d Ricci scalar on the brane makes a seemingly negative contribution to the brane tension [48, 44]. In this case, we consider a non fluctuating brane which is placed at the \mathbf{R}/\mathbf{Z}_2 orbifold fixed point.

The late time behavior is however generically different, as was shown in [44]: when the energy density decreases and crosses the threshold $M_{\rm Pl}^2/r_c^2$, one either has a transition to a pure 5d regime (see e.g. [49, 50]) where the Hubble parameter is linear in the energy density ρ (this happens for the $\epsilon = -1$ branch of the solutions), or to the self inflationary solution mentioned above (when $\epsilon = +1$). This latter is the case we would like to investigate in more detail in the rest of this work and we set $\epsilon = +1$ from now on. In terms of the Hubble radius (and for the flat Universe) the crossover between the two regimes happens when the Hubble radius H^{-1} is of the order of the crossover length-scale between 4d and 5d gravity, that is r_c . If we do not want to spoil the successes of the ordinary cosmology, we have thus to assume the r_c is of the order of the present Hubble scale H_0^{-1} .

The conservation equation (72) is the same as the standard one, so that a given component of the cosmic fluid (non relativistic matter, radiation, cosmological constant...) will have the same dependence on the scale factor as in standard cosmology. For instance, for a given component, labeled by α , which has the equation of state $p_{\alpha} = w_{\alpha}\rho_{\alpha}$ (with w_{α} being a constant) one gets from (72) $\rho_{\alpha} = \rho_{\alpha}^{0}a^{-3(1+w_{\alpha})}$ (with ρ_{α}^{0} being a constant). The Friedmann equation (71) can be rewritten in term of the red-shift $1 + z \equiv a_{0}/a$ as follows:

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{k} (1+z)^{2} + \left(\sqrt{\Omega_{r_{c}}} + \sqrt{\Omega_{r_{c}} + \sum_{\alpha} \Omega_{\alpha} (1+z)^{3(1+w_{\alpha})}} \right)^{2} \right\},$$
(74)

where the sum is over all the components of the cosmic fluid. In the above equation Ω_{α} is defined as follows:

$$\Omega_{\alpha} \equiv \frac{\rho_{\alpha}^{0}}{3M_{\rm Pl}^{2}H_{0}^{2}a_{0}^{3(1+w_{\alpha})}} , \qquad (75)$$

while Ω_k is given by

$$\Omega_k \equiv \frac{-k}{H_0^2 a_0^2} , \qquad (76)$$

and Ω_{r_c} denotes

$$\Omega_{r_c} \equiv \frac{1}{4r_c^2 H_0^2} \,. \tag{77}$$

In the rest of this paper, as far as the cosmology of our model is concerned we will consider a non-relativistic matter with density Ω_M in which case equation (74) reads⁷.

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{k} (1+z)^{2} + \left(\sqrt{\Omega_{r_{c}}} + \sqrt{\Omega_{r_{c}} + \Omega_{M} (1+z)^{3}} \right)^{2} \right\}.$$
 (78)

⁷Notice that we have set the cosmological constant on the brane to zero, and will do so until the end of this work since we are interested here in producing an accelerated Universes without cosmological constant.

We can compare this equation with the conventional Friedmann equation:

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{k} (1+z)^{2} + \Omega_{M} (1+z)^{3} + \Omega_{X} (1+z)^{3(1+w_{X})} \right\} .$$
(79)

Here, in addition to the matter and curvature contributions we have included the density of a dark energy component Ω_X with equation of state parameter w_X . When $w_X = -1$, the dark energy acts in the same way as a cosmological constant, and the corresponding Ω_X will be denoted as Ω_{Λ} in the following. Comparing (78) and (79) we see that Ω_{r_c} acts similarly (but not identically, as we will see below) to a cosmological constant.

The z = 0 value of equation of equation(78) leads to the normalization condition:

$$\Omega_k + \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_M}\right)^2 = 1, \tag{80}$$

which differs from the conventional relation

$$\Omega_k + \Omega_M + \Omega_X = 1 . \tag{81}$$

For a flat Universe $(\Omega_k = 0)$ we get from equation (80)

$$\Omega_{r_c} = \left(\frac{1 - \Omega_M}{2}\right)^2 \text{ and } \Omega_{r_c} < 1.$$
(82)

This shows in particular that for a flat Universe, Ω_{r_c} is always smaller than Ω_X , nevertheless, as will be seen below, the effects of Ω_{r_c} and Ω_X can be quite similar. Figure 1 shows the different possibilities for the expansion as a function of Ω_M and Ω_{r_c} .

2. Cosmological Tests: We would like to discuss now, in a qualitative way, a few cosmological tests and measurements. We do not expect that the current experimental precision would enable us to discriminate between the prediction of our model and the ones of standard cosmology. However, the future measurements might enable to do so.

In order to compare the outcome of our model with various cosmological tests we need first to summarize some results. In the FLRW metric (68), we define, as usual (see e.g.[51]), the transverse, H_0 independent (dimensionless), comoving distance d_M :

$$d_M = \frac{S_k\left(\sqrt{|\Omega_k|}d_C\right)}{\sqrt{|\Omega_k|}}, \quad \text{if } \Omega_k \neq 0 ,$$

$$d_M = d_C, \qquad \text{if } \Omega_k = 0 ,$$
(83)

where d_C is defined as follows:

$$d_C = \int_0^z H_0 \frac{dx}{H(x)} \,. \tag{84}$$



Figure 1: Different possibilities for the expansion as a function of Ω_M and Ω_{r_c} . The solid line denotes a flat universe (k = 0), with Ω_{r_c} obtained through equation (82). The Universes above the solid line are closed (k = 1), the universes below are open (k = -1). The Universes above the dashed line avoid the big bang singularity by bouncing in the past.

From the expression for d_M one gets the $(H_0 \text{ independent and dimensionless})$ luminosity distance d_L and the $(H_0 \text{ independent})$ angular diameter distance d_A given by

$$d_L = (1+z)d_M, \tag{85}$$

$$d_A = \frac{d_M}{1+z}.$$
(86)

These definitions can be used on the same footing both in standard and in our cosmological scenarios (as they stand above, they only rely on the geometry of the four-dimensional Universe seen by the radiation which is the same in both cases). The only difference is due to the expression for H(z) which enters the definition of d_C ; one should choose either equation (79) or (78) depending on the case considered. Whenever we want to distinguish between the two models, we will put a *tilde* sign to the quantities corresponding to our model (e.g. \tilde{d}_L).

2.1. Supernovae Observations: The evidence for an accelerated universe coming from supernovae observation relies primarily on the measurement of the apparent magnitude of type Ia supernovae as a function of red-shift. The apparent magnitude m of a given supernova is a function of its absolute magnitude \mathcal{M} , the Hubble constant H_0 and $d_L(z)$ (see e.g. [52]). Considering the supernovae as standard candles, \mathcal{M} is the same for all supernovae, so is H_0 ; thus, we need only to compare $d_L(z)$ in our model with that in standard cosmology. Figure 2 shows the luminosity distance d_L as a function of red-shift in standard cosmology (for zero and non-zero cosmological constant) and in our model. This shows the expected behavior: our model mimics the cosmological constant in producing the late-time accelerated expansion. However, as is also apparent from this plot, for the same flat spatial geometry and the same amount of non-relativistic matter, our model does not produce exactly the same acceleration as a standard cosmological constant, but it rather mimics the one obtained from a dark energy component with $w_X > -1$.

2.2. Comparison with dark energy: We want here to compare the predictions of our model to the ones of standard cosmology with a dark energy component. For this purpose we choose a reference standard model given by standard cosmology with the parameters $\Omega_{\Lambda} = 0.7$, $\Omega_M = 0.3$ and k = 0 (and denote the associated quantities with the superscript ^{ref}, e.g. d_L^{ref}). Figures 3 and 4 show respectively the luminosity distance $d_L(z)$ and $d_C(z)H(z)$ (Alcock-Paczynski test, see e.g. [53]) for various cases, showing that with precision tests, one should be able to discriminate between our model and a pure cosmological constant.

2.3. Cosmic Microwave Background (CMB): It is well known that in standard cosmology, the location of points of constant luminosity distance at small z is degenerated in the plane (Ω_M , Ω_Λ). This degeneracy can be lifted through CMB observations. Figure 5 shows that this is the case as well in our model (Which



Figure 2: Luminosity distance as a function of red-shift for ordinary cosmology with $\Omega_{\Lambda} = 0.7, \Omega_M = 0.3, k = 0$ (dashed line), $\Omega_{\Lambda} = 0, \Omega_M = 1, k = 0$ (solid line), and dark energy with $\Omega_X = 0.7, w_X = -0.6, \Omega_M = 0.3, k = 0$ (dotted-dashed line) and in our model (dotted line) with $\Omega_M = 0.3$ and a flat universe (for which one gets from equation (82) $\Omega_{r_c} = 0.12$ and $r_c = 1.4H_0^{-1}$).



Figure 3: Plot of $d_L(z)/d_L^{ref}(z)$ for various models of dark energy with constant equation of state parameters w_X in standard cosmology (solid lines) as compared with the outcome of the model consider in this paper (dashed and dotted lines). All plots correspond to flat universes with $\Omega_M = 0.3$ (solid lines, and dotted line), and $\Omega_M = 0.27$ (dashed line).



Figure 4: Plot of $H(z)d_C(z)/H^{ref}(z)d_C^{ref}(z)$ (Alcock-Paczynski test) for various models of dark energy with constant equation of state parameters w_X in standard cosmology (solid lines) as compared with the outcome of the model considered in this paper (dashed and dotted lines). All plots correspond to flat universes, with $\Omega_M = 0.3$ (solid lines, and dotted line), and $\Omega_M = 0.27$ (dashed line).

should not be too much of a surprise, considering the similarities between early cosmology in the two models, as well as between the luminosity distances vs red-shift relations). The solid lines of figure 5 are lines of constant \tilde{d}_L at red-shift z = 1; the dotted lines are lines of constant $\sqrt{\Omega_M} d_A$ at red-shift z = 1100. This latter quantity roughly sets the position of the first acoustic peak in the CMB power spectrum, since its inverse measures the angular size on the sky of a physical length scale at last scattering proportional to $1/\sqrt{\Omega_M}$ (as is at first approximation the sound horizon at last scattering). Eventually figure 6 shows the angular diameter distance d_A , at z = 1100, of standard cosmology divided by \tilde{d}_A in our model, as a function of w_X , for a flat universe and $\Omega_M = 0.3$. This shows that, for the same content of matter (and a flat universe), the first Doppler peak in our model will be slightly on the left of the one obtained in standard cosmology with a pure cosmological constant.

One might wonder whether it is possible to obtain the similar cosmological scenario in purely four-dimensional theory by introducing additional generally covariant terms in the Einstein-Hilbert action. The conventional local terms which can be added to the 4d theory contain higher derivatives.

$$M_{\rm Pl}^2 \sqrt{g} \left(R + \alpha \frac{R^2}{M_{\rm Pl}^2} + \dots \right)$$
 (87)

Whatever the origin of these terms might be their contributions should be suppressed at distances bigger than millimeter. That is required by existing precision gravitational measurements. This implies that at distances of the present Hubble size their



Figure 5: The Solid lines are lines of equal luminosity distance (in our model), $\tilde{d}_L(z=1)/d_L^{ref}(z=1)$, at red-shift z=1, the contours are drawn at every 5% level. The dashed line corresponds to a flat universe. The dotted line are line of equal $\sqrt{\Omega_M}\tilde{d}_A(z)$ for z=1100, the contours are drawn at every 5% level.



Figure 6: Angular diameter distance d_A at z = 1100 of standard cosmology divided by $\tilde{d}_A(z = 1100)$ in our model, as a function of w_X for a flat universe and $\Omega_M = 0.3$

contributions are even more suppressed. For instance, from the requirement that the contribution of the R^2 term to the Newtonian interaction be sub-dominant at distances around a centimeter implies that the relative contribution of the R^2 term at the Hubble scale is suppressed by the factor $(\text{cm}^2 H_0^2) \sim 10^{-56}$. The contributions of other higher terms are suppressed even stronger.

It seems that the only way to accommodate this unusual behavior in a would-be pure 4d theory of gravity is to introduce terms with fractional powers of the Ricci scalar, for instance, such as the term $\sqrt{g R}$. However, it is hard to make sense of such a theory.

Therefore, we conclude that the scenario discussed in the previous sections is intrinsically high-dimensional one.

3. Constraints In the preset framework such a low five-dimensional Planck scale is compatible with all the observations [32]. In fact, at distances smaller that the present horizon size the brane observer effectively sees a single 4d graviton which is coupled with the strength $1/M_{\rm Pl}$ (instead of a 5d graviton coupled by the $1/M_*^{3/2}$ strength).

As it was shown in [32] the high energy processes place essentially no constraint on the scale M_* . This can be understood in two equivalent ways, either directly in five-dimensional pictures, or in terms of the expansion in 4d modes.

As was shown above, in five-dimensional language the brane observer at high energies sees graviton which is indistinguishable from the four-dimensional one; for short distances the propagator of this graviton is that of a 4d theory

$$\tilde{G}_R(p, y = 0) \propto \frac{1}{p^2}$$
 (88)

Moreover, this state couples to matter with the $1/M_{\rm Pl}^2$ strength. Therefore in all the processes with typical momentum $p \ll 1/r_c$ the graviton production must go just like in 4d theory. For instance, the rate of the graviton production in a process with energy E scales as

$$\Gamma \sim \frac{E^3}{M_{\rm Pl}^2} \,. \tag{89}$$

The alternative language is that of the mode expansion. From the point of view of the four-dimensional brane observer a single 5-dimensional massless graviton is in fact a continuum of four-dimensional states, with masses labeled by a parameter m

$$G_{\mu\nu}(x,y) = \int dm \ \phi_m(y) \ h^{(m)}_{\mu\nu}(x) \ . \tag{90}$$

The crucial point is that the wave-functions of the massive modes are suppressed on the brane as follows

$$|\phi_m(y=0)|^2 \propto \frac{1}{4 + m^2 r_c^2}$$
 (91)

This is due to the intrinsic curvature term on the brane which "repels" heavy modes off the brane [32, 54]. As a result their production in high-energy processes on the brane is very difficult. Let us once again consider bulk graviton production in a process with energy E (e.g. star cooling via graviton emission at temperature T of order E). This rate is given by [32]

$$\Gamma \sim \frac{E^3}{M_*^3} \int_0^{m_{\text{max}}} dm \ |\phi_m(0)|^2 .$$
 (92)

Here the integration goes over the continuum of bulk states up to a maximum possible mass which can be produced in a given process $m_{\text{max}} \sim E$. However, since heavier wave-functions are suppressed on the brane by a factor $\frac{1}{m^2 r_c^2}$, the integral is effectively cut-off at $m \sim 1/r_c$, which gives for the rate

$$\Gamma \sim \frac{E^3}{M_*^3 r_c} \sim \frac{E^3}{M_{\rm Pl}^2}$$
 (93)

This is in agreement with Eq. (89) and in fact coincides with the rate of production of a single four-dimensional graviton, which is totally negligible. Thus high-energy processes place no constraint on scale M_* [32].

Due to the same reason cosmology places no bound on the scale M_* . Indeed, the potential danger would come from the fact that the early Universe may cool via graviton emission in the bulk, which could affect the expansion rate and cause deviation from an ordinary FLRW cosmology. However, due to extraordinarily suppressed graviton emission at high temperature, the cooling rate due to this process is totally negligible. Indeed in radiation-dominated era, the cooling rate due to graviton emission is

$$\Gamma \sim \frac{T^3}{M_{\rm Pl}^2} \,. \tag{94}$$

At any temperature below $M_{\rm Pl}$ this is much smaller that the expansion rate of the Universe $H \sim T^2/M_{\rm Pl}$. Thus essentially until $H \sim M_*^3/M_{\rm Pl}^2$ (which only takes place in the present epoch) Universe evolves as "normal".

The only constraint in such a case comes from the measurement of Newtonian force, which implies $M_* > 10^{-3}$ eV (this will be discussed in more detail elsewhere).

4. Dissipation: In the previous sections we established that classically the asymptotic form of the 4d metric on the brane is that of de Sitter space. Here we would like to ask the question whether this asymptotic form can be modified due to quantum effects. This could happen if there is dissipation of the energy stored in the expectation value of the 4d Ricci scalar into other forms which either can radiate into the bulk or be red-shifted away on the brane. Below we shall identify such a mechanism of potential dissipation.

An observer in de Sitter space is submerged in a thermal bath with nonzero temperature due to Hawking radiation from the de Sitter horizon. The temperature of this radiation is $T \sim H$. The crucial point is that the energy stored in this radiation can dissipate into the bulk in the form of vary long-wavelength graviton emission from the brane. To estimate the rate of this dissipation we can use Eq. (94) with $T \sim H$. The corresponding change of the brane energy density in the absence of other forms of matter and radiation is given by:

$$\frac{d\rho_{\rm eff}}{dt} = -\frac{H^3}{M_{\rm Pl}^2} \rho_{\rm eff} , \qquad (95)$$

where $\rho_{\rm eff} \equiv M_{\rm Pl}^2 \langle R \rangle$ and the Hubble parameter can be written as $H^2 \propto \langle R \rangle$. The corresponding decay time is huge $\tau \sim 10^{137}$ sec. Therefore, the 4d metric eventually asymptotes to flat Minkowski space. Note the crucial difference from the conventional 4d de Sitter space where the vacuum energy cannot dissipate anywhere due to the Hawking radiation. In our case the existence of infinite volume bulk is vital.

5. Infinite Volume and String Theory If the recent observations on the cosmological constant are confirmed it may be extremely nontrivial to describe the accelerated Universe within String Theory [55, 56]. To briefly summarize the concerns let us consider a generic theory with extra dimensions. Usually one is looking for a ground state of the theory with compactified or warped extra dimensions. In both of these cases there is a length scale which defines the volume of the extra space. This scale cannot be bigger than a millimeter [3]. Therefore, at larger distances a conventional 4-dimensional space is recovered. Astrophysical observations indicate that this latter asymptotes to the state of 4-dimensional accelerated expansion similar to 4d de Sitter. In which case the following two problems may emerge [55, 56]:

• An observer in dS space sees a finite portion of the space bounded by event horizon. In fact, the four-dimensional dS interval can be transformed into the form:

$$ds_{\rm dS}^2 = -\left(1 - H^2 u^2\right) d\tau^2 + \frac{du^2}{(1 - H^2 u^2)} + u^2 d\Omega_2 .$$
 (96)

An observer is always inside of a finite size horizon. As was argued in [55] physics for any such an observer is described by a finite number of degrees of freedom⁸. On the other hand, there are an infinite number of degrees of freedom in String Theory and it is not obvious how String Theory can be reconciled with this observation.

⁸Indeed, the number of degrees of freedom inside the region bounded by the horizon is finite. Moreover, physics of the exterior of the horizon can in principle be encoded into the information on the horizon. This latter, according to the Beckenstein-Hawking formula, has finite entropy and, therefore, supports a finite number of degrees of freedom.

• Another related difficulty is encountered when on tries to define the String Theory S-matrix on dS space. As we mentioned above, we could think of dS space as a cavity with a shell surrounding it. This shell has nonzero temperature. Thus, particles in the cavity are immersed in a thermal bath and, moreover, there are no asymptotic states of free particles required for the definition of the S-matrix. It was shown recently that these problems generically persist [57, 58] in quintessence models of the accelerating Universe.

Both of these difficulties are related to the fact that in dS space the comoving volume of the region which can be probed in the future by an observer is finite (the same discussion applies to any accelerating Universe with -1 < w < -2/3, where the equation of state is $p = w\rho$).

The theories with infinite-volume extra dimensions might evade these difficulties. The reason is that the accelerating Universe in this case can be accommodated in a space which is not simply 4-dimensional dS. In fact, as we argued in previous sections, although the space on the brane looks like de Sitter space for long time, it will asymptote to space with no dS horizon in the infinite future.

Let us discuss briefly these issues.

We start by counting the number of degrees of freedom which are in contact with a braneworld observer. It is certainly true that an observer on the brane is bounded in the world-volume dimensions by a dS horizon. However, there is no horizon in the transverse to the brane direction. Thus, any observer on a brane is in gravitational contact with infinite space in the bulk. In this case, the infinite number of bulk modes of higher dimensional graviton participate in 4d interactions on the brane [6, 32]. Therefore, the number of degrees of freedom needed to describe physics on the brane is infinite.

The problem of definition of the S-matrix might be more subtle. Below we present a simplest possibility. The key observation is that the metric (67) in the bulk is nothing but the metric of flat Minkowski space. Indeed, performing the following coordinate transformation [59]:

$$Y^{0} = A \left(\frac{r^{2}}{4} + 1 - \frac{1}{4\dot{a}^{2}}\right) - \frac{1}{2}\int dt \,\frac{a^{2}}{\dot{a}^{3}}\partial_{t}\left(\frac{\dot{a}}{a}\right) ,$$

$$Y^{i} = A x^{i} ,$$

$$Y^{5} = A \left(\frac{r^{2}}{4} - 1 - \frac{1}{4\dot{a}^{2}}\right) - \frac{1}{2}\int dt \,\frac{a^{2}}{\dot{a}^{3}}\partial_{t}\left(\frac{\dot{a}}{a}\right) ,$$
(97)

where $r^2 = \eta_{ij} x^i x^j$ and $\eta_{ij} = \text{diag}(1, 1, 1)$, the metric takes the form:

$$ds^{2} = -(dY^{0})^{2} + (dY^{1})^{2} + (dY^{2})^{2} + (dY^{3})^{2} + (dY^{5})^{2}.$$
(98)

The brane itself in this coordinate system transforms into the following boundary conditions:

$$-(Y^{0})^{2} + (Y^{1})^{2} + (Y^{2})^{2} + (Y^{3})^{2} + (Y^{5})^{2} = \frac{1}{H_{0}^{2}},$$

$$Y^{0}(t, y = 0) > Y^{5}(t, y = 0)$$
. (99)

Therefore, the space to the right of the brane is transformed to Minkowski space with the boundary conditions (99).

On this space the S-matrix could be defined as there are asymptotic *in* and *out* states of free particles. The same procedure can be applied to the metric on the left of the brane. However, the brane space-time being de Sitter, one encounters the same problems to define in and out states for scattering products localized on the brane. This is true as long as one neglects dissipation discussed in section 7, due to which the whole space-time will asymptote to Minkowski space-time for which the mentioned problems do not persist.

Summarizing, the models with infinite-volume extra dimensions might be a useful ground for describing an accelerating Universe within String Theory. In addition we point out that these models allow to preserve bulk supersymmetry even if SUSY is broken on the brane [35, 60]. Further cosmological studies of these models can be found in Refs. [61], [62], [63], [64], [65].

6. Massive gravity and perturbation theory

It has been known for some time [39] that perturbation theory in massive gravity breaks down at a scale that is parametrically lower than an ultraviolet cutoff of the theory. This breakdown can be traced to nonlinear graviton self-interaction diagrams [40], and can be interpreted as strong interaction of longitudinal polarizations of a massive graviton [66]. A simple way to see the breakdown of perturbation theory is to look at the tree-level trilinear graviton vertex diagram. In the nonlinear level one has two extra propagators which could provide a singularity in the graviton mass m_g up to $1/m_g^8$. Two leading terms $1/m_g^8$ and $1/m_g^6$ do not contribute so the result contains only the $1/m_g^4$ singularity [40]. This leads to breakdown of perturbation theory, and for a Schwarzschield source of mass M the breakdown happens at a scale $\Lambda_m \sim m_g/(Mm_g/M_{\rm Pl}^2)^{1/5}$ [39], [40]. As we mentioned above, this can also be understood in terms of interactions of longitudinal polarizations of a massive graviton becoming strong [66]. For pure gravitational sector itself, that can be thought of as a source with $M = M_{\rm Pl}$, the corresponding scale Λ_m reduces to $m_g/(m_g/M_{\rm Pl})^{1/5}$ [66]. Using the freedom in addition higher nonlinear terms this scale can be made only as big as $m_g/(m_g/M_{\rm Pl})^{1/3}$ [66].

In Ref. [40] it has been shown that similar non-linear diagrams lead to the precocious breakdown of perturbation theory in the model of Ref. [6] already at the tree-level. However, this breakdown was argued to be an artifact of using perturbative expansion in G_N which is ill-defined in that case. Moreover, it was argued in Ref. [40] that the *re-summation* of tree-level diagrams should lead to consistent results. This was confirmed by comparing a number of exact solutions of the model of Ref. [6] with their perturbative counterparts, showing that the perturbative results do not reproduce correctly the results of exact calculations. Therefore, as long as the classical theory is concerned, the model of Ref. [6] has no strong coupling problem when it is treated with consistent methods. However, recently it was argued in Refs. [67] and [68] that the strong coupling problem could come back in loops – the theory becoming strongly interacting at the quantum level. The question of quantum loops, however, is very subtle in the present context for the following reason: there is a connection between the ultraviolet (UV) and infrared (IR) physics in this model and the ultraviolet completion of the model is not know. Hence, discussing quantum loops at low-energies in a theory with UV/IR connection without knowing the UV physics might lead to ambiguous results. In particular, choice of a given low-energy prescription in the loops could lead to implicit assumptions about the UV theory because of the UV/IR connection. In this regard, the question whether the precocious breakdown of perturbation theory in the loops is an artifact of a low-energy method used or it is a fundamental drawback of the theory, remains open, to the authors knowledge. This issue will be discussed in detail elsewhere.

Acknowledgments

I am grateful to the organizers for warm hospitality and for creating very productive and enjoyable atmosphere.

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