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Cosmology of Brane Models with Radion Stabilization

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Abstract

We analyze the cosmology of the Randall-Sundrum model and that of compact brane models in general in the presence of a radius stabilization mechanism. We find that the expansion of our universe is generically in agreement with the expected effective four dimensional description. The constraint (which is responsible for the appearance of non-conventional cosmologies in these models) that must be imposed on the matter densities on the two branes in the theory without a stabilized radius is a consequence of requiring a static solution even in the absence of stabilization. Such constraints disappear in the presence of a stablizing potential, and the ordinary FRW (Friedmann-Robertson-Walker) equations are reproduced, with the expansion driven by the sum of the physical values of the energy densities on the two branes and in the bulk. For the case of the Randall-Sundrum model we examine the kinematics of the radion field, and find that corrections to the standard FRW equations are small for temperatures below the weak scale. We find that the radion field has renormalizable and unsuppressed couplings to Standard Model particles after electroweak symmetry breaking. These couplings may have important implications for collider searches. We comment on the possibility that matter off the TeV brane could serve as a dark matter candidate.

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1 Introduction

It has been widely understood over the past two years that a very promising route towards reconciling the apparent mismatch of the fundamental scales of particle physics and gravity is to change the short distance behavior of gravity at distances much larger than the Planck length [1, 2, 3]. One prominent suggestion has been to lower the fundamental scale of gravity all the way to the weak scale by introducing large extra dimensions [1]. This possibility attracted much attention; in particular, the presence of large extra dimensions opens up the possibility for new cosmological scenarios for the early Universe, which has been the subject of study of several interesting papers [4-11]. While the possibility of the existence of large extra dimensions is very exciting on its own, the existence of a big hierarchy between the weak and the Planck scales requires the radius of the extra dimension to be much larger than its natural value (see [12] for a scenario in which this might naturally occur).

Randall and Sundrum (RS) suggested a different setup [13, 14], in which the extra dimensions are small, but the background metric is not flat along the extra coordinate; rather it is a slice of anti-de Sitter (AdS_5) space due to a negative bulk cosmological constant balanced by the tensions on the two branes of this scenario. In this case, the curved nature of the space-time causes the physical scales on the two branes to be different, and exponentially suppressed on the negative tension brane. This exponential suppression can then naturally explain why the physical scales observed are so much smaller than the Planck scale. For early papers related to this subject see [15, 16]; generalizations of the RS models can be found in [17-26], embeddings into supergravity or string theory are discussed in [27-32]. Some of the aspects of the cosmology of these models have been examined in [33-39], and phenomenological consequences in [40, 41].

Clearly, the cosmology of this model can be very different from ordinary inflationary cosmology in four dimensions. A more detailed study of the early cosmology of the Randall-Sundrum model and that of brane models in general (including the large extra dimension scenario of [1]) has been however hindered by one obstacle: it seemed that the *late* cosmology of brane models will generically deviate from the usual FRW Universe in the 4D theory on our brane. This would bring into question most of the results of the early papers about the cosmology of brane models, where it has been commonly assumed that the late cosmology (from BBN to present) of these models is given by the standard FRW cosmology. This conclusion is reached by applying the results of [33], who examined the solutions to Einstein's equations in five dimensions on an S^1/Z_2 orbifold, with matter included on the two branes, and with no cosmological constants in the bulk or on the brane. The result of this study was that the expansion of the brane does not reproduce the conventional FRW equations. This result has been applied to the RS model in [35, 36], and it was found that the cosmology of the model with an infinitely large extra dimension (where one lives on the positive tension brane, the "Planck brane") in fact reproduces the ordinary FRW Universe. However, for the model which solves the hierarchy problem (in which our universe lives on the negative tension brane, the "TeV brane") there is a crucial sign difference in the Friedmann equation. However, this conclusion about the wrong-signed Friedmann equation (and the general conclusion that brane models have unconventional late cosmologies) has been reached in a theory without an explicit mechanism for stabilizing the radius of the extra dimension, the radion field. It was already pointed out in [19, 33, 35] that the effect of a radion stabilizing potential could significantly alter this picture.

In this paper we confirm this intuition. We examine the cosmology of brane models, in particular that of the RS model after the radius is stabilized (for example by the mechanism suggested by Goldberger and Wise [19] or Luty and Sundrum [20], but the details of the stabilization mechanism will be irrelevant). We explain that the origin of the unconventional cosmologies is *not* due to a breakdown of the effective 4D theory (neither is it due to the appearance of a negative tension brane in the case of the RS model), but rather to a constraint that the matter on the hidden and visible branes are forced to obey in order to ensure a static radion modulus. This constraint ensures a static solution for the radius even in the absence of a stabilization mechanism. Once such a constraint on the matter content appears in a theory, all the results about non-conventional cosmologies mentioned above follow. However, if the radius is stabilized, such constraints disappear (they turn into an equation determining how the radius is shifted from its minimum due to the presence of matter on the branes), and the ordinary FRW equations are recovered for the effective 4D theory for temperatures below the weak scale. We also examine the kinematics and dynamics of the radion field of the RS model in the presence of a stabilization force. We find that its natural mass scale is of the order of the weak scale, and that the shift in the radius is tiny for temperatures below the TeV scale. We also find that the radion TeV suppressed interactions with the Standard Model (SM), which results in renormalizable couplings to Standard Model fields after electroweak symmetry breaking. The existence of these couplings ensures that the radion can decay before nucleosynthesis, and may also have important consequences for collider searches.

We also comment on the possibility of matter in the bulk or on the Planck brane serving as dark matter candidates. Matter in the bulk naturally has TeV suppressed couplings to SM fields, so its annihilation cross section may result in interesting relic densities today. Matter on the Planck brane was presumably never in thermal equilibrium after inflation, and it is therefore not clear whether it could have interesting relic densities.

This paper is organized as follows: in Section 2, we present the effective 4D description of the RS model. We explain the physical reasons why the constraint (which we derive in Appendix A together with an approximate solution in the bulk for the case without a stabilized radion) should become irrelevant in the presence of a stable radion and find the effective 4D action for the radion-graviton system. In Section 3, we present a detailed solution to the 5 dimensional Einstein equations confirming the physical intuition of Section 2: once the radius is stabilized, the ordinary FRW Universe is reproduced. A similar solution to the case with vanishing background cosmological constants is derived in Appendix A, illustrating that our arguments are generic for brane models. In Section 4, we discuss the kinematics and dynamics of the radion field based on the effective 4 dimensional action obtained in Section 2. We also find the couplings of the radion field to the Standard Model particles. In Section 5, we discuss the issues related to (dark) matter on the hidden brane or in the bulk. We conclude in Section 6. Appendix A gives an approximate solution to the Einstein equations for the RS model with matter on the branes, but no stabilization mechanism for the radion. Appendix B discusses the cosmology of brane models with vanishing cosmological constants. In Appendix C, we derive the radion mass in the Goldberger-Wise model for radius stabilization. In Appendix D, we show that the Hubble law in the effective theory is consistent with the Newtonian force law between particles on the TeV and Planck branes, while in Appendix E we calculate the interaction strength of matter in the bulk to matter on the Planck and the TeV branes.

2 The Effective Four Dimensional Theory

In this section we construct the effective 4 dimensional equations of motion of the RS model. We demonstrate, that the constraint on the matter on the two branes (which is derived explicitly in Appendix A) in the theory without a stabilized radius can also be obtained from the effective theory. This constraint is a consequence of requiring a static solution to the equations of motion without a stabilization mechanism.

Throughout this paper (except in Appendix B) we consider the RS model perturbed by matter on the two branes. The metric is given by

$$\begin{aligned} ds^2 &= n(y,t)^2dt^2 - a(y,t)^2(dx_1^2+dx_2^2+dx_3^2) - b(y,t)^2dy^2, \\ &\equiv \tilde{g}_{AB}(x,y)dx^Adx^B. \end{aligned} \tag{2.1}$$

The two branes are located at $y=0$ and at $y=1/2$. The Einstein tensor for this metric is given by

$$\begin{aligned} G_{00} &= 3\left[\left(\frac{\dot a}{a}\right)^2 + \frac{\dot a\dot b}{ab} - \frac{n^2}{b^2}\left(\frac{a''}{a}+\left(\frac{a'}{a}\right)^2 - \frac{a'b'}{ab}\right)\right], \\ G_{ii} &= \frac{a^2}{b^2}\left[\left(\frac{a'}{a}\right)^2 + 2\frac{a'}{a}\frac{n'}{n} - \frac{b'n'}{bn} - 2\frac{b'a'}{ba} + 2\frac{a''}{a} + \frac{n''}{n}\right] + \frac{a^2}{n^2}\left[-\left(\frac{\dot a}{a}\right)^2 + 2\frac{\dot a}{a}\frac{\dot n}{n} - 2\frac{\ddot a}{a} + \right. \\ &\left. \frac{\dot b}{b}\left(-2\frac{\dot a}{a}+\frac{\dot n}{n}\right) - \frac{\ddot b}{b}\right], \\ G_{05} &= 3\left[\frac{n'}{n}\frac{\dot a}{a} + \frac{a'\dot b}{ab} - \frac{\dot a'}{a}\right], \\ G_{55} &= 3\left[\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot a}{a}\left(\frac{\dot a}{a}-\frac{\dot n}{n}\right)+\frac{\ddot a}{a}\right)\right]. \end{aligned} \tag{2.2}$$

Here primes (dots) denote derivatives with respect to y (t). The Einstein equation is given by $G_{ab}=\kappa^2 T_{ab}$, where T_{ab} is the energy-momentum tensor, and $\kappa^2=\frac{1}{2M^3}$, where M is the five dimensional Planck scale. There is a contribution to T from the bulk cosmological constant of the form

$$T_{ab}^{bulk} = \tilde{g}_{ab}\Lambda, \tag{2.3}$$

and from the branes

$$T_a^{b,brane} = \frac{\delta(y)}{b}\text{diag }(V_* + \rho_*, V_* - p_*, V_* - p_*, V_* - p_*, 0) + \frac{\delta(y-\frac{1}{2})}{b}\text{diag }(-V+\rho, -V-p, -V-p, -V-p, 0), \tag{2.4}$$

where V_* is the (positive) tension of the "Planck" brane at $y = 0$, ρ_* and p_* are the density and pressure of the matter included on the positive tension brane (where we assume an equation of state $p_* = w_*\rho_*$) as measured with respect to the metric $\tilde{g}$, and ρ and p are the density and pressure of the matter on the negative tension brane (the "TeV" brane), again measured with respect to $\tilde{g}$. Hereafter quantities measured with respect to the metric $\tilde{g}$ are refered to as "bare " quantities. We know that in the limit $\rho, p, \rho_*, p_* \longrightarrow 0$ we want to recover the static Randall-Sundrum solution* of the form

$$n(y) = a(y) = e^{-|y|m_0 b_0}, \tag{2.5}$$

where the relations between Λ, V_*, V and m_0 are given by

$$V_* = \frac{6m_0}{\kappa^2} = -V,$$
$$\Lambda = -\frac{6m_0^2}{\kappa^2}. \tag{2.6}$$

The effective 4D Planck scale is then given by

$$(8\pi G_N)^{-1} = M_{Pl}^2 \equiv \frac{1-\Omega_0^2}{\kappa^2 m_0}, \tag{2.7}$$

where the notation for the present-day value of the warp factor

$$\Omega_0 \equiv e^{-m_0 b_0/2} \tag{2.8}$$

has been introduced.

Investigation of the cosmology of brane models has shown that in order to find solutions to Einstein's equations with matter on the branes there appears to be a constraint between the matter on the two branes, if one requires that the extra dimension remains static [33, 34]. The appearance of such constraints, as explained in Appendix A for the case of the RS model, generically leads to non-conventional cosmologies. The constraint between the "bare" matter density ρ_* on the Planck brane, and "bare" matter density ρ on the TeV brane for the case of the RS model is found in Appendix A to be†

$$\rho_* = -\rho\Omega_0^2. \tag{2.9}$$

*Note that in this paper we use a different notation than Refs. [13, 14]. The parameter k of [13, 14] is denoted here by m_0, while the radius of the extra dimension r_c is denoted by b_0. For the coordinate along the extra dimension we use y instead of Φ, and for the 5D Planck scale we use M^3 instead of $2M^3$. In our notation $\kappa^2 = 1/2M^3$. We also place the branes at $y = 0$ and at $y = \frac{1}{2}$ instead of 0 and π.

†For the case of one extra dimension with two branes at $y = 0$ and $y = 1/2$ and with vanishing cosmological constants this constraint is of the form $\rho_* a(0) = -\rho a(\frac{1}{2})$ [33].

In addition, the computation in Appendix A of the Hubble parameter, H, for the induced metric at the Planck brane implies that $\rho_* > 0$. So the above constraint implies that the energy density on the TeV brane must be negative. Since this is at odds with phenomenology, it is important to understand the physical origin of these constraints before concluding that this model (and brane models in general) is phenomenologically unacceptable.

In Appendix A it is shown that although there is a constraint between the Planck brane and TeV–brane energy densities, an effective 4D theory does exist. For example, for small perturbations in the energy densities the two branes expand at practically the same rate, and by an amount that agrees with the effective theory expectation. This is consistent with what one expects in a 4D effective theory where there is a (approximate) uniform expansion of the two branes. This suggests that the origin of the unconventional cosmologies is the above–mentioned constraint, rather than the fact that the TeV brane has negative tension, or that there is a sick 4D effective theory.

In this section the 4D effective theory is constructed from two equivalent approaches. First, we directly average the 5D Einstein equations over the bulk to obtain some 4D equations. Alternatively, we average the 5D Einstein action over the bulk to obtain a 4D effective action. From both approaches we obtain the following picture: without a radion potential and for generic energy densities on the two branes, the radion runs off to infinity. This can be avoided, again without a radion potential, by tuning the two energy densities in precisely the manner prescribed by Eq. (2.9). Thus the above constraint equation is just a consequence of requiring that the radion modulus remains static in the absence of a stabilizing potential. Once the radion is stabilized the above constraint becomes irrelevant, and one obtains the conventional late cosmology.

2.1 The Averaged Einstein Equations

In this subsection we average the 5D Einstein equations over the bulk in order to demonstrate that without a stabilizing potential the system is over–constrained once we require that the radion modulus is static. To perform the averaging, we linearize the metric about the RS solution:

$$\begin{aligned} a(y,t) &= a(t)\Omega(y,b(t))\,(1+\delta\bar{a}(y,t)) \\ n(y,t) &= \Omega(y,b(t))\,(1+\delta\bar{n}(t,y)) \\ b(t,y) &= b(t)\,(1+\delta b(y,t))\,. \end{aligned} \tag{2.10}$$

Here we keep the time dependence $b(t)$ in the warp factor Ω, and we will use the notation

$$\Omega \equiv \Omega(y,b(t)) = e^{-m_0 b(t)|y|/2}\ ,\ \Omega_b \equiv \Omega(1/2,b(t))\ . \tag{2.11}$$

The value of Ω_b, when $b = b_0 = \textit{constant}$, is then given by Ω_0. In this expansion we allow $\delta f_i = \delta\bar{a}$, $\delta\bar{n}$, δb to be of the form $\delta f = \delta f(\rho(t), \rho_*(t), y)$ and we only assume that $\delta f \sim O(\rho, \rho_*)$. This last assumption is reasonable, for in the limit that ρ, $\rho_* \to 0$ we should recover the RS solution. Thus an expansion in δf_i is equivalent to an expansion in ρ, ρ_*. It is then sufficient to work to linear order in δf_i. Terms such as $d\delta a/dt \sim \dot{\rho} \sim \rho^{3/2}$ are higher order in ρ, ρ_* and are neglected, while we work to all orders in $b(t)$ and its time derivatives.

We also include a radion potential

$$a^3 V_r(b) \equiv -b(t) \int dy \Omega^4 \mathcal{L}_R = b(t) \int dy \Omega^4 T^0_0 \tag{2.12}$$

in the computations presented here, and set $V_r = 0$ when desired. Here $\mathcal{L}_R$ is some non–specified bulk dynamics responsible for generating a potential. Although our focus in this section is the constraint obtained without a radion potential, we include it here for later application.

Recall that in classical electromagnetism on a manifold without boundary one integrates $\nabla \cdot E = \rho$ over the manifold to conclude that the sum of the charges must vanish. So here, to see if there is a *topological* constraint on the component energy densities, we might try integrating the analog of Gauss' Law in Einstein's theory. Namely, consider

$$\int dy\ \Omega^4 G^0_0 = \kappa^2 \int dy\ \Omega^4 T^0_0 \ . \tag{2.13}$$

We shall see that this equation does not lead to a topological constraint. Rather, this equation, combined with the average of the other Einstein equations demonstrates that the constraint on the energy densities follows from requiring a static extra dimension even when there is no radion potential.

Substituting the above expansion, Eq.(2.10), into Eq. (2.13) and integrating gives

$$\frac{\dot{a}^2}{a^2} + (m_0 b)\frac{\Omega_b^2}{1-\Omega_b^2}\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{(m_0 b)^2}{4}\frac{\Omega_b^2}{1-\Omega_b^2}\frac{\dot{b}^2}{b^2} = \frac{\kappa^2 m_0}{3}\frac{1}{1-\Omega_b^2}\left(\rho_* + \rho\Omega_b^4 + V_r(b)\right) + \epsilon^2 \tag{2.14}$$

where $\epsilon^2 = O((\delta\bar{a})^2, (\delta\bar{n})^2, (\delta b)^2)$. Note that there are no corrections to H^2 linear in the perturbations. Thus the corrections to the Hubble formula from the details of the bulk geometry are important only at high temperatures. Also note that this equation reduces to the conventional FRW Hubble law when the energy density in the radion is small (i.e. $\dot{b}$ and V_r are negligible) compared to the other sources. (Recall that $8\pi G_N = \kappa^2 m_0/(1-\Omega_0^2)$). Further, in this limit the expansion rate is set by $\rho_* + \rho\Omega_b^4$, which is the sum of the physically measured energy densities on the Planck and TeV branes, respectively. For small oscillations of the radion, the expansion of the universe is ordinary FRW. Thus the bulk average of the G_{00} equation resulted in the 4D Hubble Law.

Repeating the above averaging procedure for the G_{ij} equation gives, for $\dot{b}_0 = 0$:

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} = -8\pi G_N \left(p_* + p\Omega_0^4 - V_r(b_0)\right) + \epsilon^2 \tag{2.15}$$

This is just the FRW "pressure " equation. As with the G_{00} equation, there are no corrections of $O(\epsilon)$.

The *unaveraged* linearized G_{55} equation is given by (again for $\dot{b}_0 = 0$) :

$$3\frac{\Omega'}{\Omega}(3\delta a' + \delta n') - 3\frac{b_0^2}{\Omega^2}\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = -\kappa^2 b^2 \left(2\Lambda\delta b - T^5_5\right) + \epsilon^2 \ . \tag{2.16}$$

Here we *do not* average over the fifth coordinate. If this is performed, then on the LHS quantities such as δa and δn appear, whose value depends on the detailed form of the solution in the bulk. The results of Appendix A could then be used to obtain the same constraint obtained below in Eq. (2.21). As seen below, more generality is obtained by not averaging the G_{55} equation. Next the "jump equations" for $\delta a'$ and $\delta n'$, derived in [33], given by

$$\delta a'|_0 = -\frac{1}{6}\kappa^2 b\left(\rho_* + \delta b V_*\right) \ , \ \delta a'|_{1/2} = \frac{1}{6}\kappa^2 b\left(\rho + \delta b V\right) \ , \tag{2.17}$$

$$\delta n'|_0 = \frac{1}{6}\kappa^2 b\left(3p_* + 2\rho_* - \delta b V_*\right) \ , \ \delta n'|_{1/2} = -\frac{1}{6}\kappa^2 b\left(3p + 2\rho - \delta b V\right) \ , \tag{2.18}$$

are inserted into Eq. (2.16) to obtain (for example at $y = 0$)

$$-\frac{1}{6}\kappa^2 m_0(-\rho_* + 3p_*) - \left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = -\frac{1}{3}\kappa^2 T^5_5|_0 \ . \tag{2.19}$$

In the following we show that without a radion potential the three equations (2.14), (2.15) and (2.19) imply a constraint between the matter densities if a static solution ($b = constant$) is imposed. The important point about Eq. (2.19) is that in the absence of a radion potential the system is over–constrained. To see this, we can eliminate the $a(t)$ dependence by using the other two equations, Eqs. (2.14) and (2.15). For a static solution for b, i.e. $\delta b = 0$ and $\dot{b}_0 = 0$, and with $V_r = T_{55} = 0$, this gives

$$-\frac{1}{6}\kappa^2 m_0(-\rho_* + 3p_*) = \frac{1}{6}\frac{\kappa^2 m_0}{1 - \Omega_0^2}\left(\rho_* - 3p_* + (\rho - 3p)\Omega_0^4\right) \tag{2.20}$$

which simplifies to

$$(-3p + \rho)\Omega_0^4 = (3p_* - \rho_*)\Omega_0^2. \tag{2.21}$$

In order that this equation remains consistent with the two conservation of energy equations, Eqs. (A.7) and (A.8), a further fine tuning (for $w \neq 1/3$) $w = w_*$ is needed. So the above constraint is then

$$\rho_* = -\Omega_0^2\rho \ , \tag{2.22}$$

which is the same constraint obtained from the explicit solutions Appendix A, Eq. (A.13). Alternatively, the same constraint could have been obtained by comparing the *unaveraged* G_{55} equations at $y = 0$ and $y = 1/2$ when $T_{55} = 0$.

This discussion then demonstrates the origin of the constraint: it is a consequence of requiring $b = constant$ without a radion potential. With these assumptions the system is over-determined, and a fine tuning of the energy densities is required to maintain a static solution in the bulk.

2.2 The Effective Four Dimensional Action

Below we derive the effective action for the four dimensional theory. We will demonstrate that the above explanation for the origin of the constraint equation can also be obtained from this 4D action. This should be expected, since without a stabilizing potential the radion modulus is massless and appears in the 4D effective theory. Consequently, the physics of maintaining a static modulus should also be obtained in the effective theory.

To obtain the effective action, in the following we perturb the metric about the RS solutions as in Eqs. (2.10). We compute to $O(\delta f)$ and to all order in $b(t)$. Terms such as, e.g. $d\delta a/dt \sim \dot{\rho} \sim \rho^{3/2}$, are higher order in ρ, ρ_* and are neglected. The effective Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{eff} &= -M^3 \int_{-1/2}^{1/2} dy \sqrt{g}(R + \Lambda/M^3) \\ &\quad - a^3 V_r(b) + a^3 \mathcal{L}_{Pl}(a, \Psi_0^{(Pl)}, \ldots) + a^3 \Omega_b^4 \mathcal{L}_{TeV}(\Omega_b a, \Psi_0^{(TeV)}, \ldots) \ , \end{aligned} \tag{2.23}$$

where $V_r(b)$ is the radion potential, and $\mathcal{L}$ denote the Lagrangians of the matter fields on the Planck and TeV branes, expressed in terms of the bare (not rescaled) fields, bare masses and induced metric. With our approximations the 4D effective action is then computed to be

$$\begin{aligned} S_{eff} &= -\frac{3}{\kappa^2 m_0} \int dt a^3 \left((1-\Omega_b^2)\frac{\dot{a}^2}{a^2} + m_0 b \Omega_b^2 \frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{(m_0 b)^2}{4}\Omega_b^2 \frac{\dot{b}^2}{b^2} \right) - \int dt a^3 V_r(b) \\ &\quad + \int dt a^3 \mathcal{L}_{Pl} + \int dt a^3 \Omega_b^4 \mathcal{L}_{TeV}. \end{aligned} \tag{2.24}$$

We have expressed the induced metric on the two branes as $\tilde{g}_{\mu\nu} = \Omega_b^2 \,\mathrm{diag}(1, -a^2, -a^2, -a^2)$. Since the curvature scalar

$$\mathcal{R}_{(4)}(a) = -6\frac{\ddot{a}}{a} - 6\left(\frac{\dot{a}}{a}\right)^2 \tag{2.25}$$

integrating the second term by parts results in a more conventional looking action

$$\begin{aligned} S_{eff} &= -\frac{1}{2\kappa^2 m_0} \int dt a^3 (1-\Omega_b^2) \mathcal{R}_{(4)}(a) + \int dt a^3 \left(\frac{3}{4}\frac{1}{\kappa^2 m_0}(m_0 b)^2 \Omega_b^2 \frac{\dot{b}^2}{b^2} - V_r(b) \right) \\ &\quad + S_{Pl}^M + S_{TeV}^M \end{aligned} \tag{2.26}$$

where S^M are the matter actions on the two branes.

It is now straightforward to compute the b equation of motion. The point is that due to the dependence of Ω_b on b, the presence of the matter on the branes generates a potential for b. To see this, we compute the variation of the above action with respect to b, noting that (we assume) S^M depends on b only through the warp factor Ω_b. Thus the contribution of the matter fields to the b equation is

$$\frac{\delta S^M}{\delta b} = \frac{\delta S^M}{\delta \tilde{g}^{\mu\nu}} \frac{\delta \tilde{g}^{\mu\nu}}{\delta b} = -\sqrt{\tilde{g}} \tilde{T}_{\mu\nu} \tilde{g}^{\mu\nu} \frac{\Omega_b'}{\Omega_b} = -\sqrt{g} \frac{\partial}{\partial b} \frac{1}{4} \tilde{T} \Omega_b^4 \ , \tag{2.27}$$

where $\tilde{g}_{\mu\nu} = \Omega_b^2 \,\mathrm{diag}(1, -a^2, -a^2, -a^2)$, and $\tilde{T}$ is the trace of the stress tensor[‡] in terms of the bare fields and bare masses, and is equal to $\rho - 3p$ for a perfect fluid. Thus the matter fields generate an effective potential for b that is

$$V_{eff}(b) = \frac{1}{4}\left(\rho_* - 3p_* + (\rho - 3p)\Omega_b^4\right) \ , \tag{2.28}$$

where we have added for later convenience a b independent contribution from the Planck brane (which does not contribute to the b equation).

For general ρ and ρ_*, the minimum of this potential is at $b \to \infty$. In fact with $\dot{b} = 0$ and $V_r = 0$, the b equation is

$$-\frac{3\Omega_b^2}{\kappa^2}\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2\right) = -\frac{m_0}{2}(3p - \rho)\Omega_b^4 - \frac{3\Omega_b^2}{\kappa^2}\left(\frac{\dot{a}}{a}\right)^2 \ , \tag{2.29}$$

which simplifies to

$$(3p - \rho)\Omega_b^4 = \frac{6\Omega_b^2}{\kappa^2 m_0}\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) \ . \tag{2.30}$$

We note that this is just the average of the G_{55} equation at the location of the TeV brane. Using Eqs. (2.14) and (2.15) the above equation simplifies to

$$(3p - \rho)\Omega_b^4 = -\frac{\Omega_b^2}{1 - \Omega_b^2}\left(3p_* - \rho_* + (3p - \rho)\Omega_b^4\right) \ . \tag{2.31}$$

For generic ρ and ρ_* the solution to this equation is $\Omega_b \to 0$. Since $\Omega_b = e^{-m_0 b/2}$, $b \to \infty$. That is, the branes want to blow apart. There is another solution, however, and that is to allow a fine tune between ρ_* and ρ. In fact, inspecting the above equation implies that $3p_* - \rho_* = -\Omega_b^2(3p - \rho)$ is also a static solution. Combining this with the conservation of energy equations for ρ, ρ_*, then implies the earlier constraint $\rho_* = -\rho\Omega_b^2$.

So this demonstrates that the constraint $\rho_* = -\rho\Omega_b^2$ is quite general, as it does not depend on the details of the solutions in the bulk. More importantly, this effective action computation explicitly shows that the constraint directly follows from requiring that the radion modulus is static even when there is no stabilizing potential. From this perspective it is clear that with a radion potential ρ_* and ρ will not be required to be correlated. For uncorrelated ρ and ρ_*, the branes want to go off to infinity; this however, will be balanced by the restoring force from the radion potential. What was once a constraint equation for ρ and ρ_*, in the presence of the radion potential becomes an equation determining the new equilibrium point: $\Delta b \sim O(\rho, \rho_*)$.

[‡]Thus we see that b couples to the trace of the stress tensor, and has "dilaton"-like couplings.

3 The Solution to the Einstein Equations in the presence of a Stabilizing Potential

We have seen in the previous section that the constraint on the matter on the two branes is a consequence of the fact that without radius stabilization one has an extra light field in the spectrum, whose equations of motion are static only for correlated values of the matter densities. In this section, we will show that the conclusions reached above are in agreement with the detailed solutions of the 5 dimensional Einstein equations, once the radius is stabilized. To study this problem, we assume that a five dimensional potential $U(b)$ has been generated in the 5D theory by some mechanism (for example see [19]). Then the equations of motion in the bulk are given by

$$\begin{aligned} &G_{00} = \kappa^2 n^2 \left(\Lambda + U(b)\right), \quad G_{ii} = -\kappa^2 a^2 \left(\Lambda + U(b)\right), \\ &G_{05} = 0, \quad G_{55} = -\kappa^2 b^2 \left(\Lambda + U(b) + bU'(b)\right), \end{aligned} \tag{3.1}$$

with G_{AB} given in 2.2. To simplify the solution of the equations, we assume that the mass of the radion is very heavy, and that near it's minimum U is given approximately by $U(b) = M_b^5 (\frac{b-b_0}{b_0})^2$, where b_0 is the stabilized value of the radius, and the parameter M_b^5 is proportional to the radion mass m_{radion}. In order to understand what is happening to these equations we assume for a moment that M_b is the largest mass scale in the theory. Then the solution to the last equation in Eq. (3.1) is simply given by $b = b_0$, with no other constraint on a and n. With this solution we also find that $U(b_0)$=0. Thus in the presence of a heavy radion field the relevant Einstein equations are the 00 and ii components, with the radius fixed to be at the stable value b_0. This already shows how the constraint is eliminated. One of the equations of motion, which played a vital role in establishing the correlation between the matter on the two branes is simply not appearing, it is automatically satisfied in the presence of the stable radius.

The physics of this is that if the radion is very heavy, it can always adjust in an infinitesimal way to satisfy this equation. As an analogy one can think of two charged spheres which gravitationally attract but electrically repel each other. There will be a static solution only, if the force from the electric charges exactly cancels the force from the gravitational attraction – thus there is a relation between the masses and charges of the two spheres. If we connect a spring with an extremely large spring constant between the two spheres, however, the only condition we will have is that the spring be at its own equilibrium point, and the other forces are canceled by infinitesimal changes in the length of the spring.

Thus what we are left to show is that the first two equations, with the radion set to b_0 have solutions for arbitrary values of matter perturbations. To simplify the calculation, we perturb around the RS solution with cosmological constants δV and δV_* instead of matter densities. The appropriate Hubble law for matter densities can be obtained by the substitution $\delta V \to \rho$, $\delta V_* \to \rho_*$ and $H \to \dot{a}_0(t)/a_0(t)$ (to lowest order in the perturbations). We solve the equation using the ansatz

$$a(t,y) = e^{Ht} e^{-|y| m_0 b_0}(1 + \delta a(y)), \quad n(t,y) = e^{-|y| m_0 b_0}(1 + \delta a(y)), \quad b = b_0. \tag{3.2}$$

This way the 00 and the ii Einstein equations reduce to a single equation of the form

$$H^2 b_0^2 = a(y)a''(y) + a'(y)^2 - 2m_0^2 b_0^2 a(y)^2, \tag{3.3}$$

where $a(y) = e^{-|y|m_0 b_0}(1 + \delta a(y))$. Plugging back this ansatz and expanding to first order in $\delta a(y)$ we obtain the equation

$$\delta a''(y) - 4b_0 m_0 \delta a'(y) = b_0^2 H^2 e^{2|y|m_0 b_0}. \tag{3.4}$$

This equation is a linear inhomogeneous ordinary differential equation, which can be solved by the standard rules. The general solution is given by

$$\delta a(y) = \frac{\alpha}{4m_0 b_0}(e^{4|y|b_0 m_0} - 1) - \frac{H^2}{4m_0^2}(e^{2|y|b_0 m_0} - 1), \tag{3.5}$$

where the overall constant has been fixed such that $\delta a(0) = 0$. The remaining two constants, α and H^2 have to be fixed such that the jumps of this function at the two branes reproduce the matter perturbation that we are including. The result is given by

$$H^2 = \frac{\kappa^2 m_0}{3(1 - \Omega_0^2)}(\delta V_* + \delta V \Omega_0^4), \tag{3.6}$$

and the value of the other constant α is given by

$$\alpha = \frac{\kappa^2 b_0}{6(1 - \Omega_0^2)}(\delta V_* \Omega_0^2 + \delta V \Omega_0^4). \tag{3.7}$$

We note that Eq. (3.6) is the standard Hubble law formula with correct normalization for the physically observed energy density $\delta V_* + \Omega_0^4 \delta V$. The consistency of this solution requires $\delta a \ll 1$, which by inspection implies that δV_* is much less than the intermediate scale $\Omega_0^2 M_{Pl}^4$, and that $\delta V \Omega_0^4 \ll (\mathrm{TeV})^4$. Thus the 4D effective theory should be valid when these conditions are satisfied, in particular if the temperature on the TeV brane is below the TeV scale.

The lesson is as expected: radion stabilization removes the constraint between the matter fields on the two branes, there is a solution to Einstein's equations for any value of the perturbations, and the Hubble constant is given by the expression expected from the effective 4D description of the theory. To show that our conclusions are generic for brane models, we derive a solution similar to the one presented in this section in Appendix A for the case of vanishing cosmological constants (the case considered in [33]) in the presence of a stabilized radius.

4 Radion Kinematics and Dynamics

In this Section we concentrate on the radion field and point out some implications of the 4D effective action, Eq. (2.24). First we discuss the cosmological consequences of (2.24), and then we examine the couplings of the radion to the SM fields and the consequences of these couplings.

4.1 Radion Cosmology

Recall that for the metric (dropping the δa, δn and δb perturbations of Section 2, since they only contribute at $O((\delta f)^2)$)

$$
\begin{aligned}
a(t,y) &= a(t)e^{-m_0 b(t)|y|} \\
n(t,y) &= e^{-m_0 b(t)|y|} \\
b(t,y) &= b(t)
\end{aligned} \tag{4.1}
$$

the 4D effective action is

$$
\begin{aligned}
S_{eff} &= -\frac{3}{\kappa^2 m_0}\int dta^3\left((1-\Omega_b^2)\frac{\dot{a}^2}{a^2}+m_0 b\Omega_b^2\frac{\dot{a}}{a}\frac{\dot{b}}{b}-\frac{(m_0 b)^2}{4}\Omega_b^2\frac{\dot{b}^2}{b^2}\right)-\int dta^3 V_r(b) \\
&\quad +S_{Pl}^M+S_{TeV}^M\ .
\end{aligned} \tag{4.2}
$$

From this equation we learned in Section 2.2 that without a stabilizing potential a constraint must be imposed on the different energy densities to maintain a constant radion modulus. In this Section we include a stabilizing potential V_r.

The presence of the interaction $\dot{a}\dot{b}$ implies that in this coordinate basis the physical radion is mixed with the massless graviton. To separate the fields, it is convenient to perform a conformal transformation on the metric:

$$
\begin{aligned}
a(t) &= f(b(\bar{t}))\ \bar{a}(\bar{t}) \\
dt &= f(b(\bar{t}))\ d\bar{t}\ .
\end{aligned} \tag{4.3}
$$

We want the action in the new basis to contain no cross terms $\dot{a}\dot{b}$. This fixes f to be

$$
f(b)=\left(\frac{1-\Omega_0^2}{1-\Omega_b^2}\right)^{1/2}\quad , \tag{4.4}
$$

where recall that Ω_0 is the present–day value of Ω_b. Note that for small perturbations of Ω_b away from its present–day value, $f(b)=1-\Omega_0^2 m_0\delta b/2+\cdots$. So for small perturbations, i.e. $m_0\delta b\ll 1$, the difference between the Einstein frame and the original frame is incredibly small, $O(10^{-30})$. The difference between the two frames is then only important for large departures in Ω_b from its current value.

Then the 4D effective action in this frame is :

$$
\begin{aligned}
S &= \int d\bar{t}\bar{a}^3(\bar{t})\left(-\frac{3}{\kappa^2 m_0}(1-\Omega_0^2)\left(\frac{\dot{\bar{a}}^2}{\bar{a}^2}-\frac{1}{4}(m_0 b)^2\frac{\Omega_b^2}{(1-\Omega_b^2)^2}\frac{\dot{b}^2}{b^2}\right)-f(b)^4 V_r(b)\right) \\
&\quad +\int d\bar{t}\bar{a}^3 f(b)^4\mathcal{L}_{Pl}(\bar{a},\Psi_0^{(Pl)},...)+\int d\bar{t}\bar{a}^3 f(b)^4\Omega_b^4\mathcal{L}_{TeV}(\Omega_b\bar{a},\Psi_0^{(TeV)},...).
\end{aligned} \tag{4.5}
$$

In this action the radion appears as a scalar field with a non–trivial (b–dependent) kinetic term. In the absence of sources the radion potential is

$$
\overline{V}_r(b)=f(b)^4 V_r(b). \tag{4.6}
$$

In the presence of sources one can show similarly to the arguments in Section 2.2 that this is modified to

$$\overline{V}_{r,eff}(b) = \overline{V}_r + \frac{f(b)^4}{4}\left(\rho_* - 3p_* + (\rho - 3p)\Omega_b^4\right) \ . \tag{4.7}$$

It is also convenient to define a new scale

$$\Lambda_W = \Omega_0 M_{Pl} \ . \tag{4.8}$$

Then $\Lambda_W \sim \mathcal{O}(\text{TeV})$. Inspecting the above Einstein frame Lagrangian, we see that for small perturbations away from $\Omega_b = \Omega_0$, the canonically normalized radion ϕ is

$$m_0 b(t) = \sqrt{\frac{2}{3}}\frac{\phi(t)}{\Lambda_W}(1-\Omega_0^2) \sim \sqrt{\frac{2}{3}}\frac{\phi(t)}{\Lambda_W} \ . \tag{4.9}$$

In the following the Ω_0^2 corrections are neglected, so the second expression on the RHS is used.

In this frame the Hubble law has a simple expression and interpretation: (This can also be obtained from performing the conformal rescaling in Eq. (2.14))

$$\begin{aligned}\frac{\dot{\bar{a}}^2}{\bar{a}^2} &= \frac{8\pi G_N}{3}\left(f(b)^4(\rho_* + \rho_{vis}) + \frac{1}{4}\frac{3}{8\pi G_N}(m_0 b)^2\left(\frac{\Omega_b}{1-\Omega_b^2}\right)^2\frac{\dot{b}^2}{b^2} + \overline{V}_r(b)\right) \\ &= \frac{8\pi G_N}{3}\left(\frac{1}{2}\dot{\phi}^2 + \overline{V}_r(\phi) + f(b)^4(\rho_* + \rho_{vis})\right) \ .\end{aligned} \tag{4.10}$$

Here and throughout G_N is the present–day value of Newton's constant, and the second expression is only valid for b close to b_0. We see from this equation that as long as the energy density in the radion does not dominate the energy density in $\rho_{vis} = \rho\Omega_b^4$, in the Einstein frame the universe expands as in the usual cosmology. Since by assumption the oscillations are small, in the original frame we also have FRW expansion since then $a(t) = \bar{a}(\bar{t}) \times (1 + \text{small osc.})$.

In the absence of sources ($\rho = \rho_* = 0$) we require that the potential truly stabilizes the radion. That is, we require $\dot{b} = 0$ is a solution. Inspecting the b equation we see that a static solution in this case is possible only if $\overline{V}' = 0$. If we also require that a is static in this limit then the a equation of motion implies that $\overline{V} = 0$. Since $f \neq 0$ and $f' \neq 0$, at the local minimum we must have $V = 0$ and $V' = 0$. With this knowledge, the radion mass obtained from the above action is then given by

$$m_r^2 = \frac{2}{3m_0^2}\frac{\overline{V}_r''(b_0)}{M_{Pl}^2\Omega_0^2}(1-\Omega_0^2)^2 \ . \tag{4.11}$$

Near the local minimum it is then convenient to expand V_r as

$$\begin{aligned}V_r(b) &= \frac{3}{4}m_r^2\left(\frac{m_0 b_0}{1-\Omega_0^2}\right)^2\Omega_0^2 M_{Pl}^2\left(\frac{b-b_0}{b_0}\right)^2 \\ &= \frac{1}{2}m_r^2(\phi-\phi_0)^2 \ .\end{aligned} \tag{4.12}$$

For the Goldberger–Wise stabilizing mechanism, $V'' \sim M_{Pl}^2 \times O(\mathrm{TeV})^4$ (see Appendix B), so from the above formula $m_r \sim O(\mathrm{TeV})$. This is not surprising, since a bulk scalar field with $O(M_{Pl})$ bare mass appears in the 4D effective theory as a Kaluza–Klein tower of scalars, where the lightest field has an $O(\mathrm{TeV})$ mass.

In the following the action for the canonically normalized radion ϕ is then truncated at

$$S_{radion} = \frac{1}{2}\int d^4x\sqrt{g}\left((\partial\phi)^2 - m_r^2(\phi-\phi_0)^2\right) \ . \tag{4.13}$$

This will be the appropriate action for describing small displacements of the radion from its minimum.

In Section 2 it was argued that the special constraint between the energy densities on the two branes disappears once the radion is stabilized. Using the Einstein frame action, Eq.(4.5), the b equation is

$$\frac{3}{16\pi G_N} m_0^2 b \frac{\Omega_b^2}{(1-\Omega_b^2)^2}\left(\frac{\ddot{b}}{b} + 3\frac{\dot{\bar{a}}}{\bar{a}}\frac{\dot{b}}{b} - \frac{m_0 b}{2}\frac{\dot{b}^2}{b^2}\frac{1+\Omega_b^2}{1-\Omega_b^2}\right) = -\overline{V}'_{r,eff} \tag{4.14}$$

This equation can be used to recover the earlier constraint $\rho_* = -\Omega_b^2\rho$ when $\overline{V} = 0$ and $\dot{b} = 0$ is imposed. Once this constraint is not satisfied, it is clear from inspecting the above equation that the restoring force term $\overline{V}'$ will balance the expansion force term. More concretely, plugging V_r, Eq. (4.12), into the above equation and neglecting the $\dot{b}$ and $\ddot{b}$ terms gives

$$\frac{\Delta b}{b_0} = \frac{1}{3}\frac{1-\Omega_0^2}{m_0 b_0}\frac{(\rho_{vis} - 3p_{vis})}{m_r^2\Omega_0^2 M_{Pl}^2} \tag{4.15}$$

where Δb is the distance between the minimum of the effective potential with and without matter, and we assume that the radion was at its minimum at the start of matter–domination (MD). Also $\rho_{vis} = \Omega_b^4\rho$, etc. is the physically measured energy density on the TeV brane. To obtain this formula it was also assumed that the Planck brane is empty ($\rho_* = 0$). To include $\rho_* \neq 0$, in the above formula replace $\rho_{vis} \to \rho_{vis} + \Omega^2\rho_*$, etc. The consistency of neglecting the $\dot{b}$ and $\ddot{b}$ terms requires $\Delta b/b \ll 1$, for then $\dot{b} \sim \dot{\rho} \sim \rho^{3/2}$ is suppressed compared to the Δb term.

From Eq. (4.15) it is seen that there is no shift in b during a radiation dominated era ($w = 1/3$) from the dominant component.* Hence the physical implications of this shift are much smaller than what one would have naively thought. During this era, however, the energy density contained a small component of non–relativistic matter, which will cause b to shift. Since both Newton's constant and SM particle masses depend on the vev of the radion, a substantial shift in these quantities during any era after BBN would lead to a BBN somewhat different from the standard BBN cosmology. The success of standard BBN then implies that these changes must be small, and this is found to be the case here. To estimate

*This is because the stress tensor of radiation is traceless. In what follows we neglect the quantum corrections to Tr T.

the shift since the start of BBN, we note that the energy density in non–relativistic matter just before the start of BBN was $\rho_{NR} \sim (T_{BBN}/T_0)^3 \rho_{c,0} \sim 10^{20}$ eV^4, where we have used $T_{BBN} \sim 10$ MeV, $\rho_{c,0} \sim 10^{-10}$ eV^4. Substituting $\rho_{vis} - 3p_{vis} \sim \rho_{NR}$ into Eq. (4.15) gives

$$\frac{\delta G_N}{G_N} \sim \Omega_0^2 m_0 \delta b \sim 10^{-30} \times 10^{-28} \left(\frac{\mathrm{TeV}}{m_r}\right)^2 . \tag{4.16}$$

We also require that the weak scale has not shifted by more than $\mathcal{O}(10\%)$ since BBN, which implies $\Delta b/b < \mathcal{O}(1/m_0 b_0)$. From Eq. (4.15) this implies

$$m_r > 10^{10} \frac{(\mathrm{eV})^2}{\mathrm{TeV}} \sim \mathcal{O}(10^{-2}) \text{ eV}. \tag{4.17}$$

Of course, it is obvious from the discussion in Section 4.2 that accelerator experiments provide a much stronger constraint on the radion mass than the above result.

One general drawback of the RS model seems to be that there are two fine-tunings required to obtain a static solution (see Eq. (2.6)). One of these tunings is clearly equivalent to the usual cosmological constant problem, however, it seems that there is a second equally bad tuning appearing, making the model less attractive. We will argue that (as already emphasized by [19, 43]) the second tuning was only a consequence of looking for a static solution without a stabilized radion, and disappears in the presence of a stabilization mechanism. To see this, assume that the radius is stabilized, and we perturb the brane tensions as $V \to V + \delta V$ and $V_* \to V_* + \delta V_*$ so that the fine-tune relations Eq. (2.6) no longer hold. From the previous discussions about stabilizing the radion, we find that the first perturbation generates an effective potential for b: $\delta V_{eff} \sim \delta V \Omega_b^4$. This perturbation has two effects: to shift b, and to increase the 4D cosmological constant. The 4D cosmological constant is canceled by appropriately choosing δV_*, so one fine–tuning equivalent to the 4D cosmological constant problem remains. But if δV is small compared to both V and the typical mass scale in the stabilizing potential, then from Eq. (4.15) we find that the shift in b is tiny. Thus once the radion is stabilized, only one fine–tuning of $\mathcal{O}(M_{Pl}^{-4})$ is required (equivalent to canceling the 4D cosmological constant). The other tuning required in order to get the correct hierarchy of scales is clearly only of $\mathcal{O}(10^{-1} - 10^{-2})$. The conclusion is that there is only one severe fine–tuning in the RS model, equivalent to the cosmological constant problem.

We next argue that there is no moduli problem associated with the radion. A potential moduli problem can occur from two sources. The first is that coherent oscillations of the radion can overclose the universe well–before BBN, ruining the success of BBN. In the next section we will see that after EWSB, the radion has renormalizable interactions with the SM fields. So after EWSB it promptly decays, long before the start of BBN. So by the start of BBN there is no energy in the coherent oscillations of the radion. The second concern is that the above shift in b may introduce a correction to the Hubble Law which would modify the time evolution of our universe. This too could spoil the success of standard BBN cosmology. We shall find that the correction is sufficiently small that this is not a concern.

The shift in b will add energy to the radion potential. As the universe cools this energy will not red shift like matter, since it is not a coherent oscillation. Rather, as the universe cools b will follow its instantaneous minimum since the cooling is adiabatic (for $m_r \gg H$). The energy stored in the radion potential then appears as a ρ^2 correction to the Hubble law. This could affect the time evolution of the universe. For a general equation of state, we find from the above expressions for Δb and V_r that

$$\frac{V_r(\Delta b)}{\rho_{vis}} \sim \frac{(3p_{vis} - \rho_{vis})^2}{m_r^2 \Lambda_W^2 \rho_{vis}} \sim \frac{\rho_{NR}^2}{m_r^2 \Lambda_W^2 \rho_{vis}}. \tag{4.18}$$

Using the value of $\rho_{NR} \sim 10^{20}$ eV^4 when $T \sim 10$ MeV, then the LHS is $\ll 1$ if $m_r > 10^{-6}$eV which is weaker than the collider constraints. So there is no moduli problem from the shift in the radion.

We conclude this section by arguing that a stabilizing potential is in fact required in order that the SM particle masses have not changed significantly since the start of MD. (Of course direct accelerator searches require that the radion is heavier than $\mathcal{O}$(10–100) GeV.) The point is that without a stabilizing potential, the source terms on the TeV wall generate a potential for b, which wants the walls to separate. We can estimate the rate of expansion for b by setting the stabilizing potential to zero in (4.14) (which does not mean that the full $\overline{V}_{eff}$ vanishes). Inspecting that equation one can find that the $\dot{a}\dot{b}$ term never dominates the expansion, thus the radion is not slowly rolling. One finds that the $\ddot{b}$ and $\dot{b}^2$ terms in (4.14) are comparable, and this results in an expansion rate

$$\frac{\dot{b}}{b} \sim H \frac{M_{Pl}}{\Lambda_W} \tag{4.19}$$

for b close to b_0. Thus the fractional change in the radius will be significant ($\mathcal{O}(1)$) over time scales much shorter than the Hubble time, which results in significant changes in the SM particle masses during MD, completely changing the predictions for BBN. Thus the success of BBN cosmology requires that the radion is stabilized before the onset of BBN.

4.2 Couplings of the Radion to SM Particles

The radion field interacts with SM particles, and the size of these interactions determines whether the radion can decay quickly enough to avoid overclosure by the start of BBN. We shall see that the radion has renormalizable and and 1/TeV suppressed interactions with the SM fields.

In the RS scenario, the radion has renormalizable interactions with SM fields after electroweak symmetry breaking (EWSB). This is because the mass scales on the TeV brane depend on the radion modulus by

$$< h >= v = v_0 e^{-\frac{1}{\sqrt{6}} \frac{\phi}{\Lambda_W}} \ . \tag{4.20}$$

But since this warping contains the radion modulus, this dependence introduces renormalizable couplings of the radion to matter once EWSB occurs. For example, in the Yukawa interaction we expand the Higgs vev as in Eq.(4.20) to obtain

$$\lambda_{ij} h \overline{\psi}_i \psi_j \rightarrow \lambda_{ij} v \overline{\psi}_i \psi_j - \lambda_{ij} \frac{1}{\sqrt{6}} \frac{v}{\Lambda_W} \phi \overline{\psi}_i \psi_j + \cdots \ . \tag{4.21}$$

So if present, this results in renormalizable interactions between the radion and the SM particles, scaled only by v/Λ_W in addition to the usual factors of Yukawa couplings or gauge couplings. Since the radion mixes with both the neutral Higgs and with the massless graviton (though the latter is negligible compared to the radion-Higgs mixing), we must work in a basis where the radion does not mix with these fields.

Next the action for the radion and Higgs interactions is derived, and the physical eigenstates and their couplings to SM fields are identified. We parameterize the metric on the TeV brane as

$$\tilde{g}_{\mu\nu}(x, y = 1/2) = \Omega_b^2(x) g_{\mu\nu}(x) \ , \tag{4.22}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) \ , \tag{4.23}$$

$$\Omega_b(x) = e^{-m_0 b(x)/2} \tag{4.24}$$

and in the bulk as

$$\begin{aligned} \tilde{g}_{\mu,\nu}(x, y) &= e^{-2m_0 b(x)|y|} g_{\mu\nu}(x) \ , \\ g_{55}(x, y) &= -b^2(x) \ . \end{aligned} \tag{4.25}$$

This expression then includes the fluctuations in the radion and the zero–mode graviton. Inserting this into the 5D action and integrating over the extra dimension results in the following 4D effective action

$$S_{eff} = \int d^4x \sqrt{g} \left(-\frac{1}{2} \frac{(1 - \Omega_b^2)}{\kappa^2 m_0} \mathcal{R}_{(4)}(g) + \frac{3m_0}{4\kappa^2} \Omega_b^2 (\partial b)^2 - V_r(b) \right) + S_{TeV} + S_{Pl} \ . \tag{4.26}$$

From this action we find that for small fluctuations about $b = b_0$, the canonically normalized radion ϕ is

$$m_0 b(x) = \sqrt{\frac{2}{3}} \frac{\phi(x)}{\Lambda_W} \tag{4.27}$$

which is the same as Eq. (4.9) for small Ω_0. (The small difference between this and Eq. (4.9) is due to the latter normalization being defined in the Einstein frame.) The SM action on the TeV brane is

$$\begin{aligned} S_{TeV} &= \int d^4x \sqrt{\tilde{g}} \left(\tilde{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - V(H) + i\bar{\psi}_i \tilde{e}^{a\mu} \gamma_a D_\mu \psi_i - \lambda_{ij} H \psi_i \psi_j + h.c. \right) \\ &= \int d^4x \sqrt{g} \Omega_b^4 \left(\Omega_b^{-2} g^{\mu\nu} D_\mu H^\dagger D_\nu H - V(H) + i\Omega_b^{-1} \bar{\psi}_i e^{a\mu} \gamma_a D_\mu \psi_i - \lambda_{ij} H \psi_i \psi_j + h.c. \right) \ , \end{aligned} \tag{4.28}$$

where for simplicity only the fermion and Higgs interactions are included. It is straightforward to include other interactions, for example, the Yang-Mills kinetic terms. We assume a Higgs potential of the form

$$V(H) = \lambda (H^\dagger H - \frac{1}{2} v_0^2)^2 \ . \tag{4.29}$$

To obtain canonical normalization of the kinetic terms in Eq.(4.28) we rescale $H \to \Omega_b^{-1} H$ and $\psi \to \Omega_b^{-3/2}\psi$. The effect of this is to renormalize $v_0 \to \Omega_b v$, and to introduce higher dimension operators between the radion and the SM fields. That is, the new SM action is

$$\begin{aligned} S_{eff} &= \int d^4x\sqrt{g}\left(g^{\mu\nu}D_\mu H^\dagger D_\nu H - V(H,\phi) + i\bar{\psi}_i e^{a\mu}\gamma_a D_\mu \psi_i - \lambda_{ij} H\psi_i\psi_j + h.c.\right) \\ &\quad + S_{radion} + \int d^4x\sqrt{g}\left(\frac{1}{\sqrt{6}}\frac{\partial\phi}{\Lambda_W}\left(H^\dagger DH + h.c.\right) + \left(\frac{\partial\phi}{\sqrt{6}\Lambda_W}\right)^2 H^\dagger H\right) \\ &\quad + \int d^4x\sqrt{g}\left(\frac{3}{2\sqrt{6}\Lambda_W} i\bar{\psi}_i\gamma^\mu\psi\partial_\mu\phi\right) + S_{Pl} \ . \end{aligned} \tag{4.30}$$

Note that in the kinetic terms the radion has dimension 5 and dimension 6 interactions with fermions and bosons. The Higgs potential after the rescaling is

$$V(H,\phi) = \lambda\left(H^\dagger H - \frac{1}{2}v_0^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{\Lambda_W}}\right)^2 \ . \tag{4.31}$$

Note that this confirms the intuition that the source of the radion coupling to SM particles is the conformal breaking sector. This potential generically results in mass mixing between the Higgs and the radion. Inspecting the final action, Eq.(4.30), we also see that after EWSB there is kinetic mixing between the radion and Higgs. So in order to verify the claim that after EWSB the radion has renormalizable interactions with the SM fields, we must identify the coupling of the SM fields to the physical eigenstates made out of H^0 and ϕ, and in particular verify that the gauge eigenstate H^0 is not a mass eigenstate.

Expanding $H^0 = (v + h + iH_I)/\sqrt{2}$, the ϕ and H^0 equations are

$$0 = m_r^2(\phi - \phi_0) + \lambda\gamma v_0 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{\Lambda_W}}(v^2 - v_0^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{\Lambda_W}}) \tag{4.32}$$

and

$$0 = \lambda v(v^2 - v_0^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{\Lambda_W}}) \ , \tag{4.33}$$

where the scaling factor γ is defined by

$$\gamma \equiv \frac{v}{\sqrt{6}\Lambda_W} = \frac{1}{\sqrt{6}}\left(\frac{v}{\Omega_0 M_{Pl}}\right) \ . \tag{4.34}$$

Note that γ is fixed by the precise value of Ω_0. So the vacuum that breaks EWS is

$$\begin{aligned} <\phi> &= \phi_0 \\ v^2 &= v_0^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi_0}{\Lambda_W}} \ . \end{aligned} \tag{4.35}$$

This is physically reasonable, since the radion and Higgs potential is the sum of two positive definite quantities, which has a local minimum when each term separately vanishes. This also demonstrates that at tree–level the fermion and gauge boson mass spectrum is the same here as in the SM, since the vev of ϕ only determines v, and otherwise does not contribute to any of those masses.

In the original gauge basis the mass matrix for (ϕ, h) is

$$\begin{pmatrix} m_r^2 + 2\gamma^2\lambda v^2 & 2\gamma\lambda v^2 \\ 2\gamma\lambda v^2 & 2\lambda v^2 \end{pmatrix} . \tag{4.36}$$

In this basis, however, the kinetic terms are not diagonal, but are given by

$$\mathcal{L}_{kin} = \frac{1}{2}(1+\gamma^2)(\partial\phi)^2 + \frac{1}{2}(\partial h)^2 + \gamma\partial\phi\partial h. \tag{4.37}$$

The kinetic mixing is undone by the field redefinition[†] $\phi - \phi_0 = \bar{\phi}$ and $h = \bar{h} - \gamma\bar{\phi}$. In the $(\bar{\phi}, \bar{h})$ basis the mass matrix is also diagonalized. So $(\bar{\phi}, \bar{h})$ are also mass eigenstates, with mass squared m_r^2 and $2\lambda v^2$, respectively [‡]. Note that these are unchanged from their naive expectation. The relation between the gauge and physical basis is then

$$\phi = \phi_0 + \bar{\phi} \tag{4.38}$$

and

$$h = \bar{h} - \gamma\bar{\phi} . \tag{4.39}$$

Thus the couplings of the radion to SM fields are similar to the neutral Higgs, and may be important for collider phenomenology since γ may not be small. We point out that the linear couplings of the radion to SM fields may also be obtained from :

$$\bar{\phi}\frac{\delta S_{SM}}{\delta\phi}|_{\phi_0} = \bar{\phi}\frac{\delta S_{SM}}{\delta\tilde{g}^{\mu\nu}}\frac{\delta\tilde{g}^{\mu\nu}}{\delta\phi} = -\bar{\phi}\sqrt{\tilde{g}}\tilde{T}_{\mu\nu}\tilde{g}^{\mu\nu}\frac{\Omega'_\phi}{\Omega_\phi} = \gamma\frac{\bar{\phi}}{v}T . \tag{4.40}$$

So to $O(\bar{\phi})$ the radion couples to the trace of the stress–tensor. To this order, this is equivalent to the coupling of the radion to the SM in the large extra dimension scenario, except that here the suppression is TeV rather than M_{Pl}.

This discussion then demonstrates that at the EW scale the theory contains two neutral, CP even, spin–0 particles coupled to the SM fermions and gauge bosons. The tree–level mass parameters are the same, however, as in the SM, since the vev of the radion does not break electroweak symmetry. After EWSB the radion has both mass and kinetic mixing with the neutral Higgs, which after diagonalization results in renormalizable interactions between the physical radion and the SM fields. These interactions are scaled by $v/(\sqrt{6}\Lambda_W)$, so the radion is more weakly (or more strongly!) coupled to SM fields than the neutral Higgs. We also point out that the radion also couples to the kinetic energy terms, and these

[†] Note, that the radion also has kinetic mixing with the graviton. Due to the weak dependence of M_{Pl} on b, however, this mixing is proportional to Ω_0^2, and is thus negligible compared to the Higgs mixing considered here.

[‡]This is also seen if in Eq. (4.28) we rescale the fields by the vev of Ω_b instead of the full quantum Ω_b.

operators become important at EW energies. It is then interesting to determine the current experimental limits on the radion mass, and to what extent future collider experiments can distinguish between the radion and neutral Higgs boson. We note that these important experimental issues are completely determined by the precise value of Ω_0.

The dominant decay of the radion is to $\bar{t}t$, ZZ or WW if kinematically allowed, otherwise to $\bar{b}b$. At e^+e^- colliders it can be produced in association with Z, so its collider signatures are similar to the neutral Higgs, but with suppression or enhancement of the rate. The Higgs may also decay to two radions (if kinematically allowed), and may be competitive with its SM decay modes.

The cosmological implications of this is that the radion can decay to SM particles before the start of BBN. Of course, a requirement on early cosmology for this to occur is that by the start of BBN the radion was close to its present–day value. Then the lifetime of the radion was small enough, and the energy density in coherent oscillations of the radion was be transfered to radiation by the start of BBN.

We also note that if matter is present on the Planck brane, the coupling of the radion to that brane is $O(1/M_{Pl})$ or smaller (see Appendix E). Consequently, the radion has a very small branching fraction to decay on the Planck brane. This means that in the early universe the coherent oscillations of the radion would predominantly dump their energy into the TeV brane, and only transfer a small fraction $O((\mathrm{TeV}/M_{Pl}))^2$ to the Planck brane. It also implies that during the time that b was close to its present–day value, matter on the Planck brane was not in thermal equilibrium with the matter on the TeV brane.

We conclude this section by noting, that (just as for the case of the KK gravitons) the radion of the RS model has very different properties from the radion of the large extra dimensions of [1]. In the case of large extra dimensions, the lower bound on the mass of the radion is 10^{-3} eV from the gravitational force measurements. In these models there is also an upper bound on the radion mass, which is obtained by requiring that the curvature radius is smaller than the physical size of the extra dimensions. This upper bound varies between 10^{-2} eV for two extra dimensions to 20 MeV for six extra dimensions. Thus in these models the radion is typically very light, with mass between 10^{-3} eV and 20 MeV, while its couplings to the SM fields are suppressed by M_{Pl}. In the RS model however, the radion has unsuppressed couplings to the SM fields, which result in a lower bound of order $10-100$ GeV for the radion mass, while there is no upper bound from the curvature constraint. Thus one can see, that the radion in these two models has very different properties, and they should be easy to distinguish experimentally.

5 Dark Matter on the Planck Brane or in the Bulk?

The expansion rate of the universe is driven by three sources: matter living on the TeV brane, matter living on the Planck brane, and matter living in the bulk:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\left(\rho_* + \rho\Omega_0^4 + \rho_{bulk}\right) \ , \tag{5.1}$$

where $\rho_{bulk} = b_0 \int_{-\frac{1}{2}}^{\frac{1}{2}} \Omega(y)^4 \rho_{bulk}(y) dy$, with $\rho_{bulk}(y)$ being the five-dimensional bulk energy density. (Recall that for late–cosmology it is a good approximation to set $V_r = 0$ and $\dot{b} = 0$ in the Hubble law. Also the time dependence in b due to the shift in b contributes $O(\rho^2)$ to H^2 which is negligible for late–cosmology).

We see from Eq. (5.1) that the appearance of ρ_* and ρ_{bulk} in the expansion rate is potentially dangerous. This is because the natural scale on the Planck brane is $O(M_{Pl}^4)$, and so this energy density could easily overclose the universe. But we will argue below that natural mechanisms exist to suppress the energy density on the Planck wall well–below its natural value. But of course a careful computation within a specific model for early cosmology is required to establish that these suppressions are indeed suffucient.

Since the natural scale on the Planck brane is M_{Pl}^4, we see from the Hubble Law that there are two different energy scales appearing in the effective 4D theory, which is at first surprising. This suggests that observers living on the TeV brane will measure Planck–mass particles living on the Planck brane to really have Planckian masses, even though the cutoff on the TeV brane is O(TeV). In fact, this is confirmed in Appendix D by a computation of the Newtonian force law between a particle on the Planck brane and a particle on the TeV brane. We find that the Newtonian force law between particles of bare mass m_1 and m_2 located at y_1 and y_2 along the extra dimension is given by

$$F_N = G_N m_1 \Omega(y_1) m_2 \Omega(y_2)/r \ . \tag{5.2}$$

So the Newtonian force law and Hubble Law for matter living on different branes are consistent.

Thus one can see from (5.1), that there are two new candidates for dark matter: matter in the bulk and matter on the Planck brane. The radion is not a candidate for dark matter, since as we have seen in the previous section, it has a short lifetime (of the order M_{weak}^{-1}), and is therefore not abundant today.

Matter in the bulk can naturally be dark matter, since its natural mass scale is $\mathcal{O}(TeV)^*$, and it has interactions with the SM fields suppressed by the TeV scale. Thus it could have been in thermal equilibrium with the TeV brane when the temperature of the Universe was $\mathcal{O}(TeV)$. If these bulk fields are unstable, then they will predominantly decay to SM fields, rather than matter living on the Planck brane, since there the couplings are suppressed by M_{Pl}. However, if those bulk fields are stable, and since it is natural for their couplings to SM particles to be TeV suppressed (see Appendix E), their annihilation cross section may turn out to lead to interesting relic densities today, which may serve as dark matter.

Another interesting possibility is to have matter on the Planck brane serve as dark matter. The natural mass scale for particles on the Planck brane is either M_{Pl} or 0 (massless). In either case one might worry that matter on the Planck brane would overclose the Universe. However, matter there is very weakly coupled to bulk fields, since its couplings are suppressed due to the small wavefunction of the bulk fields at the Planck brane, in addition

*A 5D scalar with $O(M_{Pl})$ bare mass appears in the 4D effective theory as a Kaluza-Klein tower of scalars. The lightest mode has a mass of O(TeV)[18].

to the suppression of M_{Pl} for non–renormalizable operators. We illustrate this in Appendix E, where we calculate the couplings of bulk scalars to the Planck (and the TeV) branes. Therefore matter on the Planck brane has presumably never been in thermal equilibrium with our brane after inflation. Even if massive matter there was in a thermal equilibrium with the TeV brane (or if the temperatures on the two branes were once comparable), its density is suppressed by a Boltzmann factor, and is therefore greatly suppressed. Such a suppression would however not happen for massless fields on the Planck brane (unless the temperature itself on the Planck brane is for some reason much smaller than on the TeV brane), thus the presence of such massless fields is strongly disfavored, which is a constraint for model building. It would be worthwhile to investigate whether non-thermal production of Planck brane matter could result in interesting relic densities [42] and thus serve as dark matter.

6 Conclusions

In this paper we have considered the effect of a stabilization mechanism for the radion on the cosmology of the RS model. We have found that the previously discovered unconventional cosmologies result from the fact that the radion was not stabilized in the basic RS scenario. The constraint between the matter on the two branes was a consequence of trying to find a static solution to the radion equations of motion without actually stabilizing it. Once a stabilization mechanism is added, the constraint disappears, and the ordinary four dimensional FRW Universe is recovered at low temperatures if the radion is stabilized with a mass of the order of its natural scale M_{weak}.

In this paper we have also pointed out that after electroweak symmetry breaking the radion has renormalizable couplings to matter on the TeV brane. These interactions are uniquely determined by the precise value of the warp factor. The production and detection of the radion are then similar to the detection of the SM Higgs, but with enhanced or suppressed rates. An important consequence of these interactions for cosmology is that there is no radion moduli problem, as the radion decays into SM particles long before the start of BBN. We have also speculated that matter in the bulk or on the Planck brane are new candidates for dark matter.

Note Added

While this paper was being concluded, several papers related to this subject appeared [44, 45, 46], however none of these papers consider the central issue of our paper, the effect of radion stabilization on the cosmology of the negative tension brane. Ref. [44] derives the four dimensional gravity equations in a covariant formalism, and concludes similarly to [35, 36]: the expansion of the positive tension brane is conventional, however, there is a sign difference for the case of the negative tension brane. Refs. [45, 46] consider the cosmology of the positive tension brane. Ref. [45] concludes that after inflation one can always end up with the “correct” expanding solution found in [35]. Ref. [46] finds the exact solutions in the bulk to the Einstein equations with matter on the branes and background cosmological constants. The solution presented in Appendix A of this paper is a perturbative expansion of the exact solution of [46], obtained independently from [46].

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Appendix

A The Solution without a Stabilized Radius

In this Appendix we find an approximate solution to Einstein’s equations in the RS model with matter on the branes for the case when the radius is not stabilized. The main purpose of this is to illustrate the generic feature of all such solutions of brane models without a stable radius: there appears to be a constraint between matter on the two branes. This is similar to the relations found in [33, 34, 38]. It has been suggested [33] that such relations are of topological origin and are very generic to brane world scenarios. In the Sections 2 and 3 we argue that the real origin of this constraint is the requirement of a stable radius even in the absence of a stabilizing potential. Otherwise the extra dimension itself will expand or contract. The presence of the stabilizing potential then prevents the expansion of the extra dimension, and a phenomenologically acceptable cosmology follows.

In order to find a solution we assume the form of the metric to be

$$ds^2 \;=\; n(y,t)^2 dt^2 - a(y,t)^2 (dx_1^2 + dx_2^2 + dx_3^2) - b^2 dy^2, \tag{A.1}$$

that is we assume we have a time-independent constant radion b even though there is no stabilization mechanism. We also assume that the matter density on the branes is small compared to the brane tensions $\rho, \rho_* \ll V$. Then one can find a solution as a perturbative series in ρ around the RS solution:

$$a(y,t) = a_0(t)e^{-|y|b_0 m_0}(1 + \sum_{l=1}^{\infty} \rho_*(t)^l f_l(y)),$$
$$n(y,t) = e^{-|y|b_0 m_0}(1 + \sum_{l=1}^{\infty} \rho_*(t)^l g_l(y)). \tag{A.2}$$

One could start with a more general ansatz, where instead of explicitly expanding in $\rho_*(t)$ we expand in arbitrary functions of time. However, the "jump equations" discussed below will immediately tell us that this function of time is proportional to $\rho_*(t)$.

Since $\rho_* \ll V$, we will keep only terms that are linear in ρ_*, thus our ansatz for the solution will be

$$a(y,t) = a_0(t)e^{-|y|b_0 m_0}\left(1 + \rho_*(t) f(y)\right),$$
$$n(y,t) = e^{-|y|b_0 m_0}\left(1 + \rho_*(t) g(y)\right). \tag{A.3}$$

We know from [33], that the jumps of a and n are related to the density and pressure in the following way:

$$\frac{[a'(0,t)]}{a(0,t)b_0} = -\frac{\kappa^2}{3}\left(V + \rho_*(t)\right) \ ,$$
$$\frac{[n'(0,t)]}{n(0,t)b_0} = \frac{\kappa^2}{3}\left(-V + \rho_*(t)(2+3w_*)\right), \tag{A.4}$$

where $[h]$ denotes the jump in the function $[h(0,t)] = (h(0+\epsilon) - h(0-\epsilon))$. From these equations we learn what the jumps of f and g have to be:

$$[f'(0)] = -\frac{\kappa^2 b_0}{3},$$
$$[g'(0)] = \kappa^2 b_0 (w_* + \frac{2}{3}). \tag{A.5}$$

The G_{05} equation tells us that $f(y)$ and $g(y)$ have to be proportional to each other (up to a constant):

$$\dot{\rho_*} + 3\frac{\dot{a_0}}{a_0}\rho_*\left(1 + w_*\frac{g'(y)}{f'(y)}\right) = 0, \tag{A.6}$$

which implies that $g'(y)/f'(y) = const.$, and using the values of the jumps of the derivatives together with the fact that we have an S^1/Z_2 orbifold implies that the G_{05} equation just reduces to the usual conservation of energy equation of the form

$$\dot{\rho_*} + 3\frac{\dot{a}_0}{a_0}(\rho_* + p_*) = 0 \ . \tag{A.7}$$

Similarly, at the other brane we have

$$\dot{\rho} + 3\frac{\dot{a}_{1/2}}{a_{1/2}}(\rho + p) = 0 \ . \tag{A.8}$$

Plugging the ansatz into the other equations then implies that

$$\begin{aligned} f(y) &= -\frac{\kappa^2}{12m_0}(e^{2|y|m_0b_0} - 1), \\ g(y) &= \frac{\kappa^2(2+3w_*)}{12m_0}(e^{2|y|m_0b_0} - 1). \end{aligned} \tag{A.9}$$

The constant in f and g have been determined such, that in the limit $m_0 \to 0$ the solution exactly reproduces the bulk solution presented in [33]. In order to satisfy Einstein's equations the function $a_0(t)$ has to satisfy the following Friedmann-type equations:

$$\begin{aligned} \left(\frac{\dot{a}_0}{a_0}\right)^2 &= \frac{1}{3}\kappa^2 m_0 \rho_*, \\ \left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{\ddot{a}_0}{a_0} &= \frac{\kappa^2 m_0}{6}(\rho_* - 3p_*). \end{aligned} \tag{A.10}$$

From (A.9) one can see that the Hubble constant at different points along the extra dimension is given by

$$\left(\frac{\dot{a}(y)}{a(y)}\right)^2 \equiv H^2(y) = H^2(0)\left(1 + \frac{\kappa^2}{2m_0}(e^{2|y|m_0b_0} - 1)(\rho_* + p_*) + \mathcal{O}(\rho_*^2)\right), \tag{A.11}$$

which for $\rho_* \ll M_{Pl}^4$ is a very slowly varying function of y. This can be seen by comparing $H^2(\frac{1}{2})$ and $H^2(0)$:

$$H^2(\frac{1}{2}) - H^2(0) \sim H^2(0)\frac{\kappa^2 e^{m_0b_0}}{2m_0}(\rho_* + p_*). \tag{A.12}$$

If $e^{m_0b_0} \sim 10^{30}$, and $\frac{\kappa^2}{m_0} \sim \frac{1}{M_{pl}^4}$, then $H^2(\frac{1}{2}) - H^2(0) \sim H^2(0)\frac{\rho_*(1+w)}{(10^{10}\ \mathrm{GeV})^4}$. Thus for temperatures below 10^{10} GeV on the Planck brane the two branes will expand together, but the expansion rate is completely fixed by the matter on the Planck brane. This shows, that for these temperatures there should be a sensible effective four dimensional theory describing the evolution of the Universe, since $a(0) \sim a(\frac{1}{2}) \sim \langle a \rangle = \int_{-\frac{1}{2}}^{\frac{1}{2}} a(y)dy$. Below we will show that the effective theory calculations do indeed reproduce (A.10).

From the point of view of the negative tension brane one can understand the previous Hubble Law from the following consideration. The above solution depends only on ρ_* and p_*, and not on ρ or p. This means then that the jump equation at $y = 1/2$ completely determines the energy density on the negative tension brane. One finds that the condition is simply given by

$$\rho_*(t) = -\rho(t)\Omega_0^2, \quad p_*(t) = -p(t)\Omega_0^2. \tag{A.13}$$

This explains the results found in [35, 36]*: the expansion equation has the wrong sign if one assumes $\rho > 0$. In [35] $\rho > 0$ was assumed in order to get a sensible phenomenology on the TeV brane. However, we see that the constraint (A.13) implies that for $\rho > 0$ one needs to have $\rho_* < 0$. But from (A.10) we see that ρ_* cannot be negative. Therefore, in this case ρ on our brane must be negative. This is completely due to the constraint (A.13), and not a consequence of the breakdown of the effective 4D theory.

One can check this result by explicitly calculating the effective 4D action for this setup, using the insight from the solution. The full 5D action is given by

$$-\int d^5x\sqrt{g}(\frac{R}{2\kappa^2}+\Lambda)+\int d^4x\sqrt{g_0^{ind}}\mathcal{L}_{Pl}+\int d^4x\sqrt{g_{1/2}^{ind}}\mathcal{L}_{TeV}. \tag{A.14}$$

To calculate the effective 4D action one needs to integrate over the extra dimension. We do this substituting for the metric the ansatz

$$\begin{aligned} a(t,y) &= a_0(t)(1-\frac{\kappa^2\rho_*}{12m_0}(e^{2|y|m_0b_0}-1)),\\ n(t,y) &= (1+\frac{\kappa^2\rho_*(2+3w_*)}{12m_0}(e^{2|y|m_0b_0}-1)), \end{aligned} \tag{A.15}$$

however without assuming that a_0 and ρ satisfy the above Friedmann equations. The result of integrating over the fifth coordinate y to linear order in ρ and ρ_* is

$$\begin{aligned} S_{eff} = &\int dt a_0(t)^3\frac{3(1-\Omega_0^2)}{\kappa^2 m_0}\left(\left(\frac{\dot{a_0}}{a_0}\right)^2+\frac{\ddot{a_0}}{a_0}\right)+\int dt a_0(t)^3\mathcal{L}_{Pl}+\int dt a_0(t)^3\Omega_0^4\mathcal{L}_{TeV}\\ = &\int dt a_0^3(t)\left(-\frac{1}{2}M_{Pl}^2\mathcal{R}^{(4)}+\mathcal{L}_{Pl}+\Omega_0^4\mathcal{L}_{TeV}\right). \end{aligned} \tag{A.16}$$

This is exactly what we expect: the action for the expanding universe with the correct Planck constant, and with total energy density $\rho_{total} = \rho_* + \rho\Omega_0^4$. As expected, the energy density on the "TeV" brane contributes $\rho\Omega_0^4$ to the expansion rate. The expansion rate obtained from this effective action also agrees with the expansion rate for the induced metric in the five–dimensional theory. In fact, the effective theory gives

$$H^2 = \frac{8\pi G_N}{3}\left(\rho_* + \rho\Omega_0^4\right)\ , \tag{A.17}$$

which after substituting the constraint Eq. (A.13), and the relation between M_{Pl} and the five-dimensional parameters, Eq. (2.7), identically agrees with the expansion rate Eq. (A.10) in the five–dimensional theory.

Thus we can see that the effective theory picture is as expected in agreement with the detailed form of the solutions to the Einstein equations: there is no breakdown of the effective

*Note, that a factor of Ω_0^2 has been erroneously omitted in (14) of [35] and in (5) of [36] in the case of ρ_-. The reason for this extra factor is that from the point of view of the negative tension brane the fundamental scale of gravity is of the order of $M_{Pl}^2\Omega_0^2 \sim 1$ TeV.

theory picture since the two branes expand together. The only puzzle that remains is why one needs to have a relation between the matter fields on the two branes of the sort in Eq. (A.13). As we will see in Sections 2 and 3, this is a consequence of the fact that we have not stabilized the radius. It simply tells us, that if we put in matter that does not obey this constraint the extra dimension will want to expand itself. However, if the extra dimension is stabilized, no such constraint should exist and the cosmological expansion is given by the conventional Friedmann equations in the 4D effective theory with no constraint on the magnitude of the matter one can add to the system.

B The Case of Vanishing Cosmological Constants

In order to show that our results are generic for any brane model, we will convince ourselves by using arguments similar to those in Section 4 that the ordinary Friedmann equations are recovered for the case of vanishing background cosmological constants as well (the case considered in [33]), once the radion is stabilized. To see this in detail, we again look for the solutions to the 00, ii and 05 components of the Einstein equations only, and just like in Sec. 2 we do not require that that the 55 component is satisfied, since this will be solved by adjusting the radion, which we once again assume to be very heavy. We assume that we include matter density $\rho(t)$ on "our brane", and matter density $\rho_*(t)$ on the other brane. (Note, that since the radius is stabilized, the presence of a second brane in the case of vanishing background cosmological constants is not even necessary. One can obtain all the relevant formulae for this case by just setting $\rho_* = 0$.) We look for an approximate solution (i.e. valid to $\mathcal{O}(\rho_*, \rho)$) on the S^1/Z_2 orbifold in the form

$$\begin{aligned} a(t,y) &= a_0(t)(1+\alpha\rho_*(t)(y-\frac{1}{2})^2+\beta\rho(t)y^2), \\ n(t,y) &= (1+\gamma\rho_*(t)(y-\frac{1}{2})^2+\lambda\rho(t)y^2), \\ b(t,y) &= b_0(1+\delta b), \end{aligned} \tag{B.1}$$

for $0 \leq y \leq \frac{1}{2}$, and the solutions for the other regions are obtained by reflecting around $y=0$ or $y=\frac{1}{2}$. In this ansatz α, β, γ and λ are constants. From the jump equations we obtain that

$$\alpha = \beta = \frac{\kappa^2 b}{6},\ \gamma = -\frac{(2+3w)\kappa^2 b}{6},\ \lambda = -\frac{(2+3w_*)\kappa^2 b}{6}, \tag{B.2}$$

where $p = w\rho$ and $p_* = w_*\rho_*$. One can easily check that with this choice of constants the relevant components of the Einstein equation are satisfied to first order in $\kappa^2 b_0 \rho$ which is assumed to be small, and a_0 satisfies the following Friedmann equations:

$$\begin{aligned} \left(\frac{\dot{a}_0}{a_0}\right)^2 &= \frac{\kappa^2}{3b_0}(\rho+\rho_*), \\ \left(\frac{\dot{a}_0}{a_0}\right)^2 + 2\frac{\ddot{a}_0}{a_0} &= -\frac{\kappa^2}{b_0}(w\rho+w_*\rho_*). \end{aligned} \tag{B.3}$$

These are the correct 4D FRW equations with the correct normalization, since $H^2 \propto \rho$, rather than $H^2 \propto \rho^2$, and as $M_{Pl}^2 = b_0/\kappa^2$ is the relation between the 4D and 5D parameters. The size of δb is obtained from inspecting the G_{55} equation and is readily seen to be $O(\rho^2, \rho_*^2, \rho\rho_*)$. Once again, we find that after stabilization of the radion the ordinary four dimensional Friedmann equations are recovered, with no constraint on what kind of matter one can include on the branes. Thus the ordinary FRW cosmology is recovered which results in ordinary BBN.

C Radion Mass in the Goldberger-Wise Stabilization Mechanism

Here we compute the radion mass using the Goldberger–Wise (GW) mechanism for radion stabilization [19]. It is found that the radion is naturally of $O(\text{TeV})$.

In their mechanism for generating a stabilizing potential, a bulk scalar field Φ is introduced which has a 5D action

$$S_\Phi = \frac{1}{2}\int d^5x\sqrt{\tilde{G}}\left(\tilde{G}^{AB}\partial_A\Phi\partial_B\Phi - m_S^2\Phi^2\right) \ , \tag{C.1}$$

where $\tilde{G}_{AB}$ is the full 5D metric which includes the RS warp factor. The bulk mass m_S^2 is assumed to be somewhat smaller than the Planck scale. So

$$\epsilon' = \frac{4m_S^2}{m_0^2} \tag{C.2}$$

is a small quantity. The bulk scalar also contains two potentials on the Planck and TeV brane,

$$S_{Pl} = \int d^4x\sqrt{-g}\lambda_h(\Phi^2 - v_h^2)^2 \tag{C.3}$$

and

$$S_{TeV} = \int d^4x\sqrt{-g}\lambda_v(\Phi^2 - v_v^2)^2 \ . \tag{C.4}$$

Note that v_v and v_h have mass dimension 3/2. GW solve the equation of motion for Φ, and find that it has a non–trivial dependence in the bulk. In the large $\lambda_{v,h}$ limit, GW find that $\Phi(0) = v_h$ and $\Phi(1/2) = v_h$, so that in this limit the brane–potentials do not contribute to the 4D energy. However, Φ contains both potential and (gradient) kinetic energy in the bulk, so that the energy density in the bulk is inhomogeneous. The radion potential is then obtained by substituting the classical solution for Φ into the above action and integrating over the extra dimension. GW find that the resulting 4D effective potential for the radion is given by

$$V_r(b) = 4m_0e^{-2m_0b}\left(v_v - v_he^{-\epsilon' m_0b/2}\right)^2(1+\frac{\epsilon'}{4}) - \epsilon' m_0v_he^{-(4+\epsilon')m_0b/2}\left(2v_v - v_he^{-\epsilon' m_0b/2}\right) \tag{C.5}$$

In their computation GW dropped terms of $O(\epsilon'^2)$ in V_r, so the following computation of the radion mass is only accurate to this order. Then

$$\frac{V_r'(b)}{4m_0^2 e^{-2m_0 b}} = -\left(v_v - v_h e^{-\epsilon' m_0 b/2}\right)\left(2(v_v - v_h e^{-\epsilon' m_0 b/2}) + \frac{\epsilon'}{2}(v_v - 3v_h e^{-\epsilon' m_0 b/2})\right) + \frac{\epsilon'}{2} v_h e^{-\epsilon' m_0 b/2}\left(2v_v - v_h e^{-\epsilon' m_0 b/2}\right) + O(\epsilon'^2) \quad \text{(C.6)}$$

There is one trivial solution at $b \to \infty$. In addition, we expect a local maximum and a local minimum. These two are at

$$v_v - v_h e^{-\epsilon' m_0 b/2} = \delta \quad \text{(C.7)}$$

where δ is the solution to

$$\delta^2 - \frac{\epsilon'}{1+\epsilon'}\frac{v_v^2}{4} - \frac{\epsilon'}{1+\epsilon'}\frac{v_v}{2}\delta = 0 \ . \quad \text{(C.8)}$$

That is

$$\frac{\delta}{v_v} = \frac{1}{4}\frac{\epsilon'}{1+\epsilon'} \pm \frac{\sqrt{\epsilon'}}{2(1+\epsilon')}\left(1 + \frac{5}{4}\epsilon'\right)^{1/2} \ . \quad \text{(C.9)}$$

Then at these two extremum

$$V''(b_0) = -8\epsilon' \delta m_0^3 v_v e^{-2m_0 b_0} + O(\epsilon'^2) \ . \quad \text{(C.10)}$$

Since this is $O(\delta\epsilon')$, we keep δ to $O(\sqrt{\epsilon'})$. Using the above solution for δ, we find that to this order

$$\delta^\pm = \pm\frac{\sqrt{\epsilon'}}{2} v_v \ . \quad \text{(C.11)}$$

So δ^- is the solution corresponding to the local minimum. This is expected, since there should be local maximum between this local minimum and the minimum at $b \to \infty$. Then

$$V''(b) = 4\epsilon'^{3/2} m_0^3 v_v^2 e^{-2m_0 b_0} + O(\epsilon'^2) \ . \quad \text{(C.12)}$$

Using Eq. (4.11) and $\Omega_0 = e^{-m_0 b_0/2}$, the radion mass is

$$\begin{aligned} m_r^2 &= \frac{8}{3}\epsilon'^{3/2}\frac{m_0 v_v^2}{M_{Pl}^2}\Omega_0^2 \\ &\sim \epsilon'^{3/2}\Lambda_W^2 \ , \end{aligned} \quad \text{(C.13)}$$

since $m_0 \sim M_{Pl}$ and $v_v \sim M_{Pl}^{3/2}$. So the radion mass is $O(\text{TeV})^2$. For the GW solution, $\epsilon' \sim 1/40$, so the radion is roughly an order of magnitude below Λ_W.

D Newton Force Law Between Particles on the TeV and Planck Branes

We have seen that the expansion in the effective four dimensional theory is given by

$$H^2 = \frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^2 m_0}{3}\frac{1}{1-\Omega_0^2}\left(\rho_* + \rho\Omega_0^4 + \rho_{bulk}\right) \ . \tag{D.1}$$

That two mass scales, $O(M_{Pl})$ in ρ_*, and $O(\text{TeV})$ in $\rho\Omega_0^4$, appear in the expansion rate for the effective theory is at first surprising. This suggests that observers living on the TeV brane will measure "Planck–massed" particles living on the Planck brane to really have "Planckian" masses, even though the cutoff on the TeV brane is $O(\text{TeV})$. In fact, this is confirmed by a computation of the Newtonian force law between a particle on the Planck brane and a particle on the TeV brane. To see this, the 5D Lagrangian for a point mass particle with bare mass m_1 living on the Planck brane, and a point mass particle with bare mass m_2 living on the TeV brane is

$$-m_1\int d^5x\delta(y)\delta^4(x-z_1(\tau))\sqrt{\bar{g}^{\mu\nu}\dot{x}_\mu\dot{x}_\nu} - m_2\int d^5x\delta(y-1/2)\delta^4(x-z_2(\tau))\sqrt{\bar{g}^{\mu\nu}\dot{x}_\mu\dot{x}_\nu}\ , \tag{D.2}$$

where $\bar{g}_{\mu\nu} = \Omega^2(y)g_{\mu\nu}$ and indices are raised and lowered with respect to $\bar{g}$. The stress-energy tensor (with respect to $\bar{g}$) for one of these particles is

$$\bar{T}_{\mu\nu} = -m_i\frac{1}{\sqrt{\bar{g}}}\delta(y-y_i)\delta^4(x-z_i(\tau))\frac{\dot{x}_\mu\dot{x}_\nu}{\sqrt{\bar{g}^{\mu\nu}\dot{x}_\mu\dot{x}_\nu}} \equiv m_i S_{\mu\nu}(\bar{g}_{\mu\nu}) \ . \tag{D.3}$$

But if we express S in terms of g, and raise and lower indices with g, then

$$\bar{T}_{\mu\nu} = m_i\Omega^{-1}(y_i)S_{\mu\nu}(g_{\mu\nu}). \tag{D.4}$$

But with $\bar{g} = \Omega^2 + \bar{h}$, the (linearized) interaction with gravity is

$$\int dy\sqrt{\bar{g}}\bar{T}_{\mu\nu}\bar{h}^{\mu\nu} = m_i\int dy\Omega^3\sqrt{g}S_{\mu\nu}\bar{h}^{\mu\nu} \ . \tag{D.5}$$

This should be expressed in terms of the graviton zero mode, which is $h_{\mu\nu}(x,y) = h_{\mu\nu}(x)$ where $\bar{h}_{\mu\nu}(x,y) = \Omega^2(y)h_{\mu\nu}(x,y)$. Then the coupling of the particles to the zero mode is

$$-m_i\int dy\Omega\sqrt{g}S_{\mu\nu}(x,y)h^{\mu\nu}(x) \ . \tag{D.6}$$

So we see that each particle couples to h with strength $\Omega(y_i)m_i$. The Newtonian force computed from the exchange of the zero mode only, $h(x,y) = h(x)$, is then

$$F_N = G_N m_1\Omega(y_1)m_2\Omega(y_2)/r \ . \tag{D.7}$$

So the Newtonian force law and Hubble Law for matter living on different branes are consistent. This result is consistent with [13], where the authors found that the physical masses are given by $\Omega(y_i)m_i$. Therefore both mass scales appear in the 4D effective theory.

E Couplings of Bulk Matter

In this Appendix, we show what the natural suppression scale for couplings of bulk fields are to matter on the Planck or the TeV branes. Following [18], we consider a general bulk scalar Φ (not necessarily the GW field of radius stabilization) with bulk action

$$S_\Phi = \frac{1}{2}\int d^5x\sqrt{\tilde{G}}\left(\tilde{G}^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2\right) \ , \tag{E.1}$$

where $\tilde{G}_{AB}$ is the full 5D metric which includes the RS warp factor, and m is the bare 5D mass of this scalar, assumed to be of the order of the 5D Planck mass M. We assume that this field Φ has some generic couplings with matter on a brane of the form

$$\sqrt{g^{ind}}\frac{(\Phi\Phi)^p}{M^{3p}}\frac{\mathcal{O}^{4+q}}{M^q}, \tag{E.2}$$

where $\mathcal{O}^{4+q}$ denotes a composite operator of fields living on one of the branes, and has mass dimension $4+q$. We assume that in the bare Lagrangian every suppression scale is proportional to the M. Performing the usual field redefinition for the fields living on the brane results in $\mathcal{O}^{4+q} \to \Omega^{-q}(y_i)\tilde{\mathcal{O}}^{4+q}$, where $\tilde{\mathcal{O}}^{4+q}$ has canonically normalized fields and physical masses, and $\Omega(y_i)$ is the warp factor at the position of the brane, $\Omega(0)=1$, $\Omega(\frac{1}{2}) = 10^{-15}$. We can see that the effect of this this redefinition is to turn the suppression factor M^q for the operators on the brane into $\sim \Lambda_W^q$ for matter on the TeV brane. For matter on the Planck brane the suppression factor remains unchanged since no rescaling of the fields is needed.

We decompose the field Φ as [18]

$$\Phi(x,y) = \frac{1}{\sqrt{b_0}}\sum_n \Psi_n(x)\varphi_n(y), \tag{E.3}$$

where [18]

$$\varphi_n(y) = \frac{\Omega(y)^{-2}}{N_n}\left[J_\nu\left(\frac{m_n}{m_0}\Omega(y)^{-1}\right) + b_{n\nu}Y_\nu\left(\frac{m_n}{m_0}\Omega(y)^{-1}\right)\right], \tag{E.4}$$

J_ν and Y_ν are Bessel functions of order $\nu = \sqrt{4+\frac{m^2}{m_0^2}}$, m_n is the mass of the Kaluza–Klein mode Ψ_n, and N_n and $b_{n\nu}$ are normalization constants. With this substitution for Φ the interaction of a particular KK mode to the brane fields is given by operators of the form

$$\frac{(\Psi_n\Psi_n)^p}{M^{3p}}\frac{\tilde{\mathcal{O}}^{4+q}}{(\Omega(y_i)M)^q}(\varphi_n(y_i))^{2p}. \tag{E.5}$$

Thus we need to find the approximate value of the wave function of the KK modes at the position of the two branes. First we calculate the suppression at the TeV brane. We will use the approximate values $\nu \sim 2$, $N_n \sim \Omega_0^{-1}$. For small KK masses $b_{n\nu}$ is approximately given by $\pi m_n^2/4m_0^2 \sim \Omega_0^2$, thus for arguments of order one in the Bessel functions the contribution of Y_ν can be neglected. Thus the approximate value of φ_n is given by *

$$\varphi_n(1/2) \sim \frac{\Omega_0^{-2}}{\Omega_0^{-1}} = \frac{1}{\Omega_0}, \tag{E.6}$$

which then turns every factor of M in (E.5) into $\sim \Lambda_W$. Thus the interactions with matter on the TeV wall are suppressed by the TeV scale as expected.

To find the value of the wave functions close to the origin, we expand $J_2(x_n) \sim x_n^2$ and $Y_2(x_n) \sim -\frac{1}{x_n^2}$ for small values of $x_n = m_n/m_0$. Thus now the contribution of Y_2 will dominate, and we get the approximate value

$$\varphi_n(0) \sim \frac{1}{\Omega_0^{-1}} x_n^2 \frac{1}{x_n^2} \sim \Omega_0. \tag{E.7}$$

Thus the interactions on the Planck brane are suppressed by a scale higher than M_{Pl}, namely $M_{Pl}10^{15}$. This can be understood by the suppression of the wave functions of the KK modes close to the Planck brane. If there was no suppression of the wave functions, then the couplings would be suppressed by M_{Pl}. The small values of the wave functions yield an additional suppression, thus resulting in this highly suppressed interaction strength.

In fact, this wavefunction suppression at the Planck brane is required to maintain the large hierarchy at the loop level in the effective 4D theory. Consider for example a scalar field living on the Planck wall with mass of $\mathcal{O}(M_{Pl})$, which has renormalizable couplings to scalar fields in the bulk. In the 4D effective theory, the bulk scalar fields appear as a tower of KK modes, with masses starting at $\mathcal{O}(\text{TeV})$, whereas the Planck scalar has $\mathcal{O}(M_{Pl})$ mass. Without any wavefunction suppression of the bulk scalars at the Planck wall, loops of this Planck scalar field generate $\mathcal{O}(M_{Pl}^2)$ quadratic divergences, which would destabilize the hierarchy. With a wavefunction suppression of Ω_0 , however, the quadratic divergences are $\mathcal{O}(\text{TeV})^2$ and thus do not destabilize the hierarchy.

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*Note that the normalization of these wave functions is $\int dy \Omega^2 \phi_{(m)} \phi_{(n)} = \delta_{mn}$.

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