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Vanishing of cosmological constant and fully localized gravity in a brane world with extra time(s)

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Abstract

We construct an intersecting brane configuration in six-dimensional space with one extra space-like and one extra time-like dimensions. With a certain additional symmetry imposed on the extra spacetime we have found that effective four-dimensional cosmological constant vanishes automatically, providing the static solution with gravity fully localized at the intersection region as there are no propagating massive modes of graviton. In this way, the same symmetry allows us to eliminate tachyonic states of graviton from the spectrum of the effective four-dimensional theory, thus avoiding phenomenological difficulties coming from the matter instability usually induced in theories with extra time-like dimensions.

1. Introduction

Recently the idea that our world is confined on the four-dimensional subspace of higher-dimensional spacetime [1] attracts considerable attention. Within this scenario, often known as a brane world scenario, several phenomenological, cosmological and astrophysical issues have been revisited. What is particularly exiting within this approach is that the size of extra dimensions can be large [2] or even infinite [3–5], leading to the interesting theoretical alternative to conventional Kaluza–Klein (KK) compactification as well as to the specific predictions that can be tested experimentally in the visible future.

One of the original motivations for the recent versions of brane world models was an explanation of the apparent hierarchy among two fundamental scales, the Planck scale M_{Pl}, and the electroweak scale M_W [2,6] (see also [7]). The scenario of Refs. [2] utilize δ extra compact dimensions with large compactification radii r_n $(n = 1, \ldots, \delta)$ in the factorizable, $M^4 \times N^\delta$, $(4 + \delta)$-dimensional spacetime and thus the apparent weakness of gravity in the visible four-dimensional world (M^4, 3-brane) is explained due to the large volume $V_{N^\delta} \sim \prod_{n=1}^{\delta} r_n$ of the extra-dimensional submanifold N^δ:

$$M_{\mathrm{Pl}}^2 = M_*^{\delta+2} V_{N^\delta}, \tag{1}$$

where M_* is the fundamental high-dimensional scale and M_{Pl} is the ordinary four-dimensional Planck scale.

The scenario of Ref. [6] deals with a 5-dimensional non-factorizable AdS_5 spacetime with two 3-branes

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located at the S^1/Z_2 orbifold fixed points of the fifth compact dimension. Now the weakness of gravity in the visible world 3-brane is explained without recourse to large extra dimensions, but rather as a result of gravity localization on the hidden 3-brane. Gravity localization in such scenario occurs because the five-dimensional Einstein's equations admit the solution for the spacetime metric with a scale factor ("warp factor") which is a falling exponential function of the distance along the extra dimension y perpendicular to the branes: [1]

$$ds^2 = e^{-2k|y|}\, dx_{1+3}^2 + dy^2, \tag{2}$$

when the bulk cosmological constant Λ ($\Lambda < 0$) and the tensions $T_{\rm vis}$ and $T_{\rm hid}$ of the visible and hidden branes, respectively, are related according to:

$$T_{\rm hid} = -T_{\rm vis} = 6M_*^3 k, \quad k = \sqrt{-\frac{\Lambda}{6M_*^3}}. \tag{3}$$

Thus, graviton is essentially localized on the hidden brane with positive tension ($T_{\rm hid} > 0$) which is located at $y = 0$ fixed point of the S^1/Z_2 orbifold, while the standard model particles are assumed to be restricted on the visible brane with negative tension ($T_{\rm vis} < 0$) which is located at $y = \pi r_c$ (r_c is the size of extra dimension) orbifold fixed point. So, a hierarchically small scale factor generated for the metric on the visible brane gives an exponential hierarchy between the mass scales of the visible brane and the fundamental mass scale M_*, after one appropriately rescales the fields on the visible brane. In fact, assuming $M_* \sim M_{\rm Pl}$, TeV-sized electroweak scale can be generated on the visible brane by requiring $r_c \cdot M_* \simeq 12$.

However, there is even more severe hierarchy problem afflicting fundamental physics which is related to the observed smallness of the 4-dimensional cosmological constant (for reviews see [11]). In the original proposals with large [2] or warped [3,6] extra dimensions this problem remains untouched. Say, the fine-tuned condition (3) which provides the solution (2) is nothing but the condition for the vanishing of the effective 4-dimensional cosmological constant. Several more or less successful attempts have been made recently to attack this problem within the brane world models [12–18]. One solution is to ensure dynamical self-adjustment of the relation (3) by introducing an extra bulk scalar field ϕ with an appropriate bulk potential [12]. In this scenario, however, the scalar fields ϕ becomes singular at a finite distance along the extra dimension [2] and the warp factor in the metric (2) vanishes at singularity. As a result the whole background solution becomes unstable under the bulk perturbations and any procedure which regularize the singularity reintroduces the fine-tuning back [14]. It was shown also in [15] that non-static (cosmological) solutions of [12] might be unstable as well, thus leading to the energy non-conservation as brane world expands (contracts).

In this Letter we would like to suggest an alternative possibility to overcome an unnatural fine-tuning (3) by introducing an extra time-like dimensions. Particularly, we construct the configuration of intersecting branes in a 6-dimensional bulk spacetime with signature $(4, 2)$. With a certain additional symmetry imposed on the extra spacetime, we find that effective 4-dimensional cosmological constant automatically vanishes, providing a static solution with gravity localized at the intersection region. Remarkable, the symmetry which forced the cosmological constant to vanish prevents, at the same time, propagation of a massive Kaluza–Klein (KK) modes of graviton (including those of tachyonic) and thus gives the full localization of gravity at the intersection of branes, while non-trivial warp factor of the background metric allows us to solve Planck/TeV scale hierarchy problem by placing visible 3-brane at distances $\mathcal{O}(10^2 M_{\rm Pl})$ away from the intersection.

2. Extra time-like dimensions and null space dimensional reduction

Except of a very few recent works [19–25] (see also [26–28] for some earlier works on extra time-like dimensions), most of the brane world models considered so far deal with extra space-like dimensions. The

[1] Generalizations to higher-dimensional spacetimes is possible within the intersecting branes scenarios [8], models with string-like defects (in the case of 2 extra dimensions) [9] or along the lines discussed in [10].

[2] For a non-singular solution, see [13].

reason is some pathological features of field theories with extra time-like dimensions associated with appearance of ghost and tachyonic states in the effective four-dimensional theory and related with them instabilities of various kind [19,20,23,24,27]. However, there are no firm theoretical reasons why extra time-like dimensions cannot exist and, moreover, they indeed appear at the fundamental level within the various versions of string theory (see, e.g., [28] for some recent approaches). The tachyonic instabilities, being the subject of experimental verification, can be accommodated within the theoretical models if they do not contradict the existing experimental date. Indeed, in the case of conventional KK compactification with factorized geometry of spacetime one can satisfy experimental upper limits on various processes leading to the instability of matter by choosing sufficiently small radii for the compact extra time-like dimensions [19,22]. [3] The same is actually true for the case of warped compactification [3,6] if our visible world is confined on the brane which is sufficiently close to the so-called Planck scale brane where the graviton zero mode is localized. Otherwise, demanding that the visible brane is displaced from the Planck brane at distances which provides the solution to the Planck/TeV hierarchy problem, one gets for instance an unacceptable rapid gravitational decay of neutron [20,24]. Thus, it seems that generation of Planck/TeV hierarchy is incompatible with extra time-like dimensions in both scenarios with factorizable [19] and non-factorizable geometry [20].

Of course, it is more desirable to overcome appearance of ghost and tachyons in the four-dimensional effective theory. One obvious way to do this is to choose a certain geometry of the extra space preventing the appearance of ghost and tachyons. Indeed, for example, one can start with a topological gravity in higher dimensions generating usual Einstein's gravity in four-dimensional subspace [21]. Then, since the bulk spacetime is topological the propagation of gravitons in the extra space does not take place at all and thus tachyonic KK gravitons do not appear in the effective four-dimensional theory. In such a case, however extra time-like dimensions are non-dynamical and certainly metaphysical. Another example is an extra space with no Killing vectors (see Aref'eva, Volovich in [26]) which prevents the appearance of ghost states in four dimensions.

Here we would like to suggest different possibility to eliminate tachyons and ghosts from an effective four-dimensional theory. Suppose that bulk spacetime contains both time-like and space-like dimensions, i.e., the metric of extra space is Lorentzian. Now we can make so called null space reduction to lower dimensions demanding an extra subspace $M^{(p,q)}$ with p space-like and q time-like dimensions is actually a null space. [4] If so, then the physical states carry no non-zero momentum along $M^{(p,q)}$ subspace and so all KK excitations will be confined on the light cone. Thus the physical states (perhaps except of some additional zero mass modes) are the same as in four-dimensional theory and the effective four-dimensional theory will be tachyon-free. Another important outcome from the null reduction is that since the physical states are essentially the same as in four-dimensional theory the physical laws will be also four-dimensional for all energy scales. Say, $1/r^2$ 4-dimensional Newton's law will remain the same in the null reduced higher-dimensional theory at any distances.

Below we explicitly demonstrate these features by considering a 6-dimensional spacetime with one extra time-like and one extra space-like dimensions, i.e., the spacetime with signature $(4,2)$. The null reduction is realized by imposing additional discrete symmetry that prevents propagation of the massive KK modes of graviton including those of tachyonic. In this way we will avoid phenomenological difficulties coming from

[3] It is interesting that in the case when only gravity feels extra times [19,22] the imaginary part of the gravitational self-energy due to the exchange of tachyonic KK gravitons of the spherically symmetric body is periodic and for some critical radii of the body $R = 2\pi kL$ (where L is a size of extra time-like dimension and $k \in N$) it vanishes, so such self-gravitating body in fact is stable. Another interesting phenomena observed in [19,22] is the screening of the gravitational force due to the contribution of the tachyonic KK gravitons.

[4] Note that null space dimensional reduction is that what happens in F-theory (see Vafa in [28]) where the BRST invariance demand the extra $M^{(1,1)}$ subspace of a $10+2=12$ dimensional type IIB string theory with an $U(1)$ super-Maxwell field on the worldsheet to be a null space, so that F-theory in 12 dimensions with signature $(10,2)$ becomes dual to type IIB string theory in 10 dimensions with signature $(9,1)$.

the matter instability usually induced in theories with extra time-like dimensions. Remarkable enough, that the same symmetry forced the cosmological constant to be zero. [5]

3. Intersecting branes: vacuum solutions

Let us consider 6-dimensional spacetime $M^{(4,2)}$ with one extra time-like dimension τ and one extra dimension y, i.e., spacetime with a signature $(4,2)$. Suppose that there are two branes with a worldvolume signature $(4,1)$ ("time brane") and $(3,2)$ ("space brane") embedded in $M^{(4,2)}$ with tensions T_τ and T_y, respectively. The intersection of these branes, which we take to be at $(y=0, \tau=0)$ point for definiteness, is a 4-dimensional subspace (3-brane) of $M^{(4,2)}$ with signature $(3,1)$ which can be identified with a visible world. The relevant action describing such a set-up is:

$$
\begin{aligned}
S &= \int d^6x\,\sqrt{\det g}\left(\frac{1}{\kappa_6^2}R - \Lambda_b\right) \\
&\quad - \int d^6x\,\sqrt{\det g}\,[T_\tau\delta(\tau) + T_y\delta(y)] \\
&= \int\limits_{M^{(4,2)}} d^4x\,dy\,d\tau\,\sqrt{\det g}\left(\frac{1}{\kappa_6^2}R - \Lambda_b\right) \\
&\quad - \int\limits_{M^{(4,1)}} d^4x\,dy\,\sqrt{-\det g^{\tau=0}}\,T_\tau \\
&\quad - \int\limits_{M^{(3,2)}} d^4x\,d\tau\,\sqrt{\det g^{y=0}}\,T_y.
\end{aligned}
\tag{4}
$$

Here $\kappa_6^2 = 16\pi M_6^{-4}$, where M_6 is the six-dimensional fundamental scale of the theory and Λ_b is a bulk cosmological constant. The induced metrics on the branes, $g_{ab}^{\tau=0}(a,b=\mu,y)$ and $g_{\alpha\beta}^{y=0}(\alpha,\beta=\mu,\tau)$, are defined as:

$$
\begin{aligned}
g_{ab}^{\tau=0} &= g_{ab}\big(x^\mu, y, \tau=0\big), \\
g_{\alpha\beta}^{y=0} &= g_{\alpha\beta}\big(x^\mu, y=0, \tau\big),
\end{aligned}
\tag{5}
$$

where g_{MN}, $M,N = \mu(0,1,2,3), y, \tau$, is a six-dimensional metric. We use metric with mostly positive signature $(-+++++-)$. The field equations followed from the above action (4) are:

$$
R_N^M - \frac{1}{2}\delta_N^M R = \frac{\kappa_6^2}{2}T_N^M,
\tag{6}
$$

where the energy momentum tensor T_N^M is expressed through the bulk cosmological constant Λ_b and brane tensions T_τ and T_y as:

$$
\begin{aligned}
T_N^M &= -\Lambda_b\delta_N^M - \sqrt{\frac{-\det g^{\tau=0}}{\det g}}T_\tau\delta(\tau)\delta_a^M\delta_N^a \\
&\quad - \sqrt{\frac{\det g^{y=0}}{\det g}}T_y\delta(y)\delta_\alpha^M\delta_N^\alpha.
\end{aligned}
\tag{7}
$$

We are looking for a static solution of the above equations (6) that respects 4-dimensional Poincare invariance in the x^μ direction. A 6-dimensional line element satisfying this anzatz can be written as:

$$
\begin{aligned}
ds^2 &= A^2(y,\tau)\eta_{\mu\nu}\,dx^\mu\,dx^\nu + B^2(y,\tau)\,dy^2 \\
&\quad - C^2(y,\tau)\,d\tau^2,
\end{aligned}
\tag{8}
$$

where $\eta_{\mu\nu}$ is a 4-dimensional flat Minkowski metric. It is more convenient, however, to perform the actual calculations within a conformally flat metric anzatz

$$
ds^2 = A^2(z,\theta)\eta_{MN}\,dx^M\,dx^N,
\tag{9}
$$

which can be obtained from (8) by the following coordinate transformations: [6]

$$
\begin{aligned}
dz &= \frac{B}{A}\,dy, \\
d\theta &= \frac{C}{A}\,d\tau.
\end{aligned}
\tag{10}
$$

Now using the well-known conformal transformation formulae for the Einstein tensor $G_N^M = R_N^M - \frac{1}{2}\delta_N^M R$

$$
\begin{aligned}
\widetilde{G}_{MN} &= G_{MN} + 4(\nabla_M \ln A\nabla_N \ln A - \nabla_M\nabla_N \ln A) \\
&\quad + 4\eta_{MN}\left(\nabla^2 \ln A + \frac{3}{2}(\nabla \ln A)^2\right),
\end{aligned}
\tag{11}
$$

[5] For some earlier attempts to solve cosmological constant problem by introducing extra time-like dimensions see Aref'eva, Dragovich, Volovich in [26]; Linde in [21,26].

[6] Generally this transformations do exist for the rather special cases. However, they are actually valid in the case of the background solutions we are interested in (see below).

we easily obtain:

$$G^{\mu}_{\nu} = \frac{2}{A^2}\left[\left(\frac{A'}{A}\right)^2 - \left(\frac{\dot{A}}{A}\right)^2 + 2\left(\frac{A''}{A} - \frac{\ddot{A}}{A}\right)\right]\delta^{\mu}_{\nu}, \tag{12}$$

$$G^{z}_{z} = \frac{2}{A^2}\left[5\left(\frac{A'}{A}\right)^2 - \left(\frac{\dot{A}}{A}\right)^2 - 2\frac{\ddot{A}}{A}\right], \tag{13}$$

$$G^{\theta}_{\theta} = \frac{2}{A^2}\left[-5\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{A'}{A}\right)^2 + 2\frac{A''}{A}\right], \tag{14}$$

$$G^{z}_{\theta} = -G^{\theta}_{z} = \frac{4}{A^2}\left[2\frac{\dot{A}A'}{A^2} - \frac{\dot{A}'}{A}\right], \tag{15}$$

where primes and overdots denote the derivatives with respect to space-like z and time-like θ coordinates, respectively. Taking the conformal factor (warp factor) in (9) as [7]

$$A = \frac{1}{k_y|z| + k_\tau|\theta| + 1}, \tag{16}$$

one can easily check that non-diagonal elements (15) of the Einstein tensor vanish, $G^z_\theta = -G^\theta_z = 0$, and thus $(z\theta)$ Einstein's equations are satisfied identically, while the remaining equations will be satisfied if the following relations are fulfilled: [8]

$$k_y^2 - k_\tau^2 = -\frac{\kappa_6^2 \Lambda_b}{10}, \tag{17}$$

$$k_y = \frac{\kappa_6^2 T_y}{4}, \tag{18}$$

$$k_\tau = -\frac{\kappa_6^2 T_\tau}{4}. \tag{19}$$

Here we will assume that the space brane has a positive tension $T_y > 0$, while the time brane the negative one, $T_\tau < 0$, so that both k_y and k_τ are positive. In the opposite case, the negative (positive) tension space (time) brane is expected to be unstable under the fluctuations since in this case brane fluctuation modes show up as a ghost states in the effective theory on the brane worldvolume.

Taking away the "time brane" ($T_\tau = 0$) one gets the 6-dimensional version (with an extra time on the brane worldvolume) of Ref. [3], where the bulk cosmological constant is negative, $\Lambda_b < 0$, while in the case $T_y = 0$ one leads to the 6-dimensional version of the model of Ref. [20] with $\Lambda_b > 0$. Thus, generally, depending on the brane tensions T_y and T_τ, bulk spacetime can be anti-de Sitter ($|T_\tau| < |T_y|$, $\Lambda_b < 0$), de Sitter ($|T_\tau| > |T_y|$, $\Lambda_b > 0$) or Minkowskian ($T_\tau = -T_y$, $\Lambda_b = 0$). Remarkably, that in the latter case one can observe an apparent discrete symmetry of the background solution (16)–(19). Indeed, when one exchanges the extra time-like and space-like dimensions, $\theta \leftrightarrow z$ the background solution (16)–(19) with $\Lambda_b = 0$ remains untouched, while the solution with $\Lambda_b < 0$ goes to the one with $\Lambda_b > 0$ and vise versa. Thus if we demand that the Einstein equations (6) are invariant under the $\theta \leftrightarrow z$ exchange than among the solutions (16)–(19) the one with $\Lambda_b = 0$ survives. The fine tuning problem now is resolved since the above invariance demands $T_\tau = -T_y$ and ensures automatic cancellation of the 4-dimensional cosmological constant.

One can worry that the above result just simply follows from the fact that we have considered the anzatz (9) (or equivalently (8)) where the flatness of the 4-dimensional spacetime of the intersection of branes was already assumed. However, this is not the case. Indeed, one can start from the more general anzatz by taking

$$\tilde{g}_{\mu\nu} = \left(1 - \frac{1}{4}H^2\eta_{\mu\nu}x^\mu x^\nu\right)^{-2}\eta_{\mu\nu} \tag{20}$$

instead of the flat 4-dimensional metric $\eta_{\mu\nu}$ in (9). Here H is a "Hubble constant" on the intersection. Now the anzatz (9) with (20) instead of $\eta_{\mu\nu}$ describes maximally symmetric 4-dimensional spacetimes of the intersection of branes, i.e., de Sitter ($H^2 > 0$) or anti-de Sitter ($H^2 < 0$) (the flat Minkowski case considered above corresponds to $H = 0$). Then the components of the Einstein tensor (12) and (13), (14) will be changed by the additional term $+\frac{3H^2}{A^2}\delta^\mu_\nu$ and $+\frac{6H^2}{A^2}$, respectively, while (15) will remain unchanged. It is easy to see that the corresponding Einstein equations will remain invariant under the discrete symmetry $\theta \leftrightarrow z$ if and only if $H = 0$ and $\Lambda_b = 0$. This can be easily understood from the fact that the origin for the non-zero Hubble constant is a non-zero 4-dimensional cosmological constant on the intersection of branes

[7] Turning back to the original coordinates y and τ we will have: $A = (e^{k_y|y|} + e^{k_\tau|\tau|} - 1)^{-1}$, $B = e^{k_y|y|}A$ and $C = e^{k_\tau|\tau|}A$.

[8] While this Letter was in preparation there appeared the paper [23] in the hep-archives where the same solution was considered. An earlier presentation of the present work was given in [24].

which in turn is indeed forbidden if one demands that the theory is invariant under the discrete symmetry imposed above.

Of course, in general, for an arbitrary metric g_{MN} the invariance we have imposed does not take place. In what follows we require here that such an invariance holds for all perturbations around the background solution as well. In other words, we restrict to consider the manifolds which are isometric under the discrete symmetry transformations $\theta \leftrightarrow z$. So, the above invariance can be viewed as a constraint imposed on the system described by the action (4) which holds for the special class of metrics g_{MN} including the background one given by (9), (16)–(19) with $\Lambda_b = 0$.[9] Notice that the vanishing of the bulk cosmological constant, $\Lambda_b = 0$, and the relation $T_\tau = -T_y$ emerge merely from the discrete symmetry imposed and are not consequence of any fine-tuning. As we will see this symmetry leads at the same time to the resolution of the problem of tachyonic states.

4. Linearized perturbations

To examine the gravity induced by a matter source localized on the 4-dimensional intersection of branes let us consider the linearized perturbations around the background metric (9), (16)–(19). Taking

$$g_{MN} = A^2(z,\theta)(\eta_{MN} + \gamma_{MN}) \tag{21}$$

and keeping linear in γ_{MN} terms only in (6) we get:

$$\begin{aligned}
&-\frac{1}{2}\partial_K\partial^K\gamma_{MN} - \frac{1}{2}\partial_M\partial_N\gamma^K_K + \partial^K\partial_{(M}\gamma_{N)K} \\
&\quad + \frac{1}{2}\eta_{MN}\left(\partial_K\partial^K\gamma^L_L - \partial^K\partial^L\gamma_{KL}\right) \\
&\quad + 2\big[\big(2\partial_{(M}\gamma_{N)K} - \partial_K\gamma_{MN} \\
&\qquad - \eta_{MN}\big(2\partial^L\gamma_{KL} - \partial_K\gamma^L_L \\
&\qquad + 2\gamma_{KL}\partial^L + 3\gamma_{KL}n^L\big)\big)\big]n^K = 0,
\end{aligned} \tag{22}$$

where $n_K \equiv \partial_K \ln A = (0,0,0,0,A'/A,\dot{A}/A)$ and indices are raised and lowered by the 6-dimensional flat metric η_{MN}. We work with so-called Randall–Sundrum (RS) gauge [3,20,29] which is defined as a 4-dimensional transverse traceless gauge

$$\partial^\mu\gamma_{\mu\nu} = 0, \qquad \gamma^\mu_\mu \equiv \gamma = 0 \tag{23}$$

along with an extra conditions

$$\gamma_{M\tau} = \gamma_{My} = 0. \tag{24}$$

Although RS gauge (23), (24) becomes inconsistent with equations of motion when one considers an extra matter sources beyond those given by branes itself [30], for the discussion of the gravity localization on the intersection of branes it is rather convenient and we will keep it here and return to this point later.[10]

In the above gauge (23), (24) one left with 2 physical massless degrees of freedom on the intersection, which corresponds to just 2 polarization states of the 4-dimensional graviton and Eqs. (22) simplify significantly to become:

$$\partial_M\partial^M\gamma_{\mu\nu} + 4\partial_M\gamma_{\mu\nu}n^M = 0. \tag{25}$$

Boundary conditions on $\gamma_{\mu\nu}$ can be deduced by integration of (25) from just below to just above of the time and space branes resulting in the Darmois–Israel matching conditions [32]:[11]

$$\gamma'_{\mu\nu}\big|_{z=0} = 0, \qquad \dot{\gamma}_{\mu\nu}\big|_{\theta=0} = 0. \tag{26}$$

To proceed further, we separate the intersection worldvolume and extra spacetime coordinates in (25) setting

[9] On the language of the initial action (4) the vanishing of cosmological constant can be related to the following symmetry reasons. One can consider a discrete transformation changing the signature of the metric, $g_{MN} \to -g_{MN}$ (and hence $R \to -R$). The first term in the action (4) changes the sign if $\Lambda_b = 0$. (In fact, in theories with even number of spacetime dimensions the bulk cosmological constant can be forbidden by assuming that when the metric changes the signature also the action changes the sign. E.g., in the usual $1+3$ case the change $g_{\mu\nu} \to -g_{\mu\nu}$ is equivalent to the changing the signature $(+,-,-,-)$ by the signature $(-,+,+,+)$.) If along this transformation also the τ and y branes are exchanged, then the whole action changes sign, $S \to -S$, if $T_\tau = -T_y$.

[10] For some alternative gauges as well as discussions on subtleties in the RS gauge choice see [30,31].

[11] Actually from the Darmois–Israel formalism one can get in general another type of boundary conditions, which tell us that the derivatives of the metric are just continuous $\gamma'_{\mu\nu}|_{z=0^+} = \gamma'_{\mu\nu}|_{z=0^-}$, $\dot{\gamma}_{\mu\nu}|_{\theta=0^+} = \dot{\gamma}_{\mu\nu}|_{\theta=0^-}$, but not necessarily zero as in (26). Here, following to [3], we also assume that similar to the background metric the perturbations are also even under the discrete transformations $z \to -z$ and $\theta \to -\theta$. Then the only boundary conditions consistent with these symmetries are those given in (26).

$\gamma_{\mu\nu} = h_{\mu\nu}(x^\sigma)\Psi_m(z,\theta)$. Then the Eqs. (25) are splitted as:

$$\partial_\sigma \partial^\sigma h_{\mu\nu} = m^2 h_{\mu\nu}, \tag{27}$$

$$\big(\partial_y \partial^y - \partial_\tau \partial^\tau + 4n_y \partial_y - 4n_\tau \partial_\tau\big)\Psi_m(z,\theta) = -m^2 \Psi_m(z,\theta), \tag{28}$$

where m^2 is a separation constant. Eqs. (27), (28) describe the propagation of a free massive spin-2 particle in the intersection worldvolume with a mass m^2 and the wave function $\Psi_m(z,\theta)$. Although we are not able to solve Eq. (28) explicitly in the general case, but it is obvious that one has normalizable solutions for the wave function $\Psi_m(z,\theta)$ corresponding to the mass eigenvalues in the whole range $-\infty < m^2 < +\infty$. This readily follow from the fact that the normalize massive KK excitations along the time-like extra dimension are actually tachyonic ($m^2 < 0$) [20] while those of along the space-like extra dimension are bradionic ($m^2 > 0$) [3]. Finally, in general case, one expects also a collection of infinitely many massless gravitons propagating in the 4-dimensional worldvolume. One of them is a "true" zero mode localized on the intersection, while others are the excitations along the light-cone in extra spacetime. Obviously, with such a spectrum of KK-states one can not have any satisfactory phenomenology on the intersection worldvolume. Besides the already mentioned problem with tachyonic KK-states of graviton [20], one could worry about the infinitely degenerate massless gravitons as well, since they also might bring inconsistencies when one goes beyond the linearized approximation [33].

Now let us proceed to the null space dimensional reduction by requiring that Eqs. (27), (28) are invariant under the interchange of extra time-like and extra space-like coordinates, $\theta \leftrightarrow z$, as it was proposed in the previous section. This symmetry demand vanishing of m^2 in (27), (28), $m^2 = 0$, and the Eqs. (27), (28) now become:

$$\partial_\sigma \partial^\sigma h_{\mu\nu} = 0, \tag{29}$$

$$\big(\partial_y \partial^y - \partial_\tau \partial^\tau + 2kA\big(\delta(\theta) - \delta(z)\big)\big)\psi(z,\theta) = 0. \tag{30}$$

Here we set $\psi(z,\theta) = A^2 \Psi_0(z,\theta)$ in order to canonically normalize the kinetic term in (30) and $k_y = k_\tau \equiv k$. Now, Eqs. (29), (30) describe the propagation of massless spin-2 particle localized on the 4-dimensional intersection region. Indeed, the only normalizable solution which is consistent with boundary conditions (26) is:

$$\psi(z,\theta) = \sqrt{\frac{3}{2}}\,\frac{k}{(k(|z| + |\theta|) + 1)^2}, \tag{31}$$

where we have properly normalized the wave function, $\iint dz\, d\theta\, |\psi(z,\theta)|^2 = 1$. Thus the discrete symmetry $\theta \leftrightarrow z$ not only provides automatic cancelation of the effective cosmological constant on the 4-dimensional intersection, but also singles out the zero mode solution (31), for the graviton. This in turn means that gravity is fully localized on the 4-dimensional intersection and the Newton law at all distances there is just ordinary 4-dimensional one. Indeed the Newton potential of the two point-like mass M_1 and M_2 placed on the intersection at a distance r from each other is:

$$V(r) = G_N^{(6)} \frac{M_1 M_2}{r} \big|\psi(0,0)\big|^2 = G_N^{(4)} \frac{M_1 M_2}{r}, \tag{32}$$

where the effective 4-dimensional Newton constant $G_N^{(6)} \equiv \kappa_4^2/2 = 8\pi M_4^{-2}$ is related to the 6-dimensional one $G_N^{(6)} \equiv \kappa_6^2/2 = 8\pi M_6^{-4}$ through the following relation:

$$G_N^{(4)} = \frac{3k^2}{2} G_N^{(6)}. \tag{33}$$

Note also that, while the effects of extra dimensions are essentially hidden for the 4-dimensional observer, they show up in the non-trivial warp factor A (16). Thus the solution to the Planck/TeV scale hierarchy problem is possible in our scheme in the spirit proposed in [6], i.e., by placing the TeV 3-brane with standard model particles localized on it in the bulk at an appropriate distance from the intersection. Finally, the extension of the above scheme to more extra dimensions is also possible, although it requires more sophisticated extra symmetries to remove tachyonic KK modes from the effective 4-dimensional theory and to ensure natural (without fine tuning) vanishing of the cosmological constant.

5. Gravity on the intersection in the Minkowski bulk

Here we would like to discuss the null space dimensional reduction in the case of the flat Minkowski bulk spacetime. From the preceding sections, at first glance, it seems that KK modes are confined on the light cone even in the case of a Minkowski bulk spacetime and thus 4-dimensional observer will still obtain $1/r^2$ Newton's law on the intersection. Then, leaving aside the solution to the hierarchy problem via warped compactification, one can ask: do we really need to warp up or compactify the extra dimensions in order to have consistent 4-dimensional physics on the intersection region? The answer seems would be positive for all interactions except of gravity. Indeed, to have a consistent 4-dimensional gravity on the intersection it is not sufficient to correctly reproduce the Newton law, but one should ensure that the gravity on the intersection is just a tensor-type, i.e., an extra polarization states are actually decoupled from the 4-dimensional massless graviton. If it is not a case then one fails to explain some familiar gravitational experiments such as the experiments on the bending of light in the gravitational field of the Sun and the experiments measured precession rate of the Mercury orbit.

These extra polarization states show up in the tensor structure of the propagator of massless graviton. Basically this is related to the well-known fact that the massless limit of the massive graviton is actually discontinuous in the flat background spacetime [34]. So one should evaluate full propagator including the tensor structure as well. Here we come to the point the discussion on which we have postponed in the previous section. As we have mentioned there, the RS gauge (21), (24) becomes inconsistent when one consider an extra matter sources say localized on the intersection of the branes. One can relax the traceless condition in (21), but now extra scalar polarization state appear in the spectrum of the massless states on the 4-dimensional intersection. Thus one can worry that this scalar polarization state can not be removed from the physical spectrum and the gravity on the 4-dimensional intersection is scalar-tensor type. As it is shown in [30], actually this is not the case in the original RS model [3]. The extra scalar can indeed be gauged away. The physical reason for this is that in the RS model one has a zero mode graviton localized on the brane for which one expect to have the usual 4-dimensional massless propagator. Since in the case considered in the previous section the graviton zero mode is also localized on the intersection, the same procedure used in [30] can be applied to show that the gravity on the intersection is just the Einstein-type one.

Now turning to the case of the Minkowski bulk, it is obvious that the extra polarization states can not be gauged away, since gravity is allowed to freely propagate in the infinite Minkowski bulk spacetime. Although we can hide all massive KK-states through the null space dimensional reduction, so that the scalar part of the propagator of graviton will be just as for the massless 4-dimensional one, but the tensor structure of the propagator is expected to be higher-dimensional, due to the contributions of the massless scalar modes. Note once again that this point is peculiar to the gravitational interactions. Say gauge fields, living in the null reduced Minkowski bulk, can correctly reproduce 4-dimensional physics on the intersection.

6. Conclusions

We discussed the cosmological constant problem within the brane world scenario with extra time-like dimension. Particularly, we considered the intersecting brane configuration in the 6-dimensional spacetime with one extra time-like and one extra space-like dimensions. Among the possible warped background solutions to the Einstein equations we have found one with vanishing cosmological constant which is invariant under the discrete interchange of extra time-like and extra space-like dimensions. This simple symmetry can be suitably generalized (see footnote 9) to ensure vanishing of the bulk cosmological constant, $\Lambda_b = 0$, and the relation $T_\tau = -T_y$ in the original action (4) and thus to single out the desired background solution with flat 4-dimensional intersection of branes. On the other hand, it is remarkable that the same symmetry leads to the fully localized gravity on the 4-dimensional intersection worldvolume. Thus the problem of natural (without fine tuning) vanishing of the cosmological constant and the stability problem related with extra time-like dimension are solved simultaneously.

Clearly, several questions have to be answered before the above proposal can be considered as a candidate solution to the cosmological constant problem. Among them are the localization of standard model fields on the 4-dimensional intersection and the origin of the discrete symmetry imposed, which in the given set-up perhaps looks rather artificial. One can hope to find an answers to these questions at a more fundamental level. In this respect it is interesting to investigate whether the above or similar constructions can be obtained as a low energy limit of more fundamental string theory.

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