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Traversable wormholes: Some simple examples

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Building on the work of Morris, Thorne, and Yurtsever, some particularly simple examples of traversable wormholes are exhibited. These examples are notable both because the analysis is not limited to spherically symmetric cases and because it is possible to, in some sense, minimize the use of exotic matter. In particular, it is possible for a traveler to traverse such a wormhole without passing through a region of exotic matter. As in previous analyses, the weak-energy condition is violated in these traversable wormholes.

I. INTRODUCTION

There has recently occurred a major renaissance in wormhole physics. Most energy is being focused on wormholes as possibly significant contributions to quantum gravity, either in the Euclidean or Minkowski formalisms. More interesting, I feel, is the analysis of classical traversable wormholes performed by Morris and Thorne,[1] and by Morris, Thorne, and Yurtsever.[2] These authors have seriously studied the question of what the properties of a classical wormhole would have to be in order for it to be traversable by a human without fatal effects on the traveler. A major result of their investigation was that exotic matter (matter violating the weak-energy condition) was guaranteed to occur at the throat of a traversable wormhole.

Because of the assumed spherical symmetry of their class of models, this meant that any traveler transiting the wormhole necessarily had to pass through a region of exotic matter. The question of how a human body would interact with such exotic matter was left open. In this note I shall construct a particular class of traversable wormholes that are, in general, not spherically symmetric. Though exotic matter is still present in the region of the throat, it is then possible for a traveler to avoid regions of exotic material in his/her/its traversal of the wormhole. Indeed, it is possible to obtain wormholes and geodesics such that the traveler feels no forces, tidal or otherwise, during the trip.

The major technical change in the analysis is this: Whereas Morris and Thorne[1] assumed a spherically symmetric static wormhole, I shall assume an ultrastatic wormhole (i.e., $g_{00} \equiv 1$) with the exotic matter confined to a thin layer. I shall dispense with the assumption of spherical symmetry.

II. THE MODELS

The models I have in mind can be very easily described. Take two copies of flat Minkowski space, and remove from each identical regions of the form $\Omega \times \mathcal{R}$, where Ω is a three-dimensional compact spacelike hypersurface, and $\mathcal{R}$ is a timelike straight line (e.g., the time axis). Then identify these two incomplete spacetimes along the timelike boundaries $\partial\Omega \times \mathcal{R}$. The resulting spacetime is geodesically complete, and possesses two asymptotically flat regions (two universes) connected by a wormhole. The throat of the wormhole is just the junction $\partial\Omega$ at which the two original Minkowski spaces are identified. By construction it is clear that the resulting spacetime is everywhere Riemann flat except possibly at the throat. Consequently we know that the stress-energy tensor in this spacetime is concentrated at the throat, with a δ-function singularity there.

The situation described above is ready made for the application of the junction-condition formalism (also known as the surface-layer formalism). Discussions of this formalism can be found in Misner, Thorne, and Wheeler (Ref. 3, Sec. 21.13), and in a recent article by Blau, Guendelman, and Guth.[4] The basic principle is that since the metric at the throat is continuous but not differentiable, the connection is discontinuous, and so the Riemann curvature possesses a δ-function singularity. The strength of this δ-function singularity can be calculated in terms of the second fundamental form on both sides of the throat. Because of the particularly simple form of the geometry, the second fundamental form at the throat is easily calculated.[5] In suitable coordinates,

$$K^i{}_j = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\rho_1} & 0 \\ 0 & 0 & \frac{1}{\rho_2} \end{bmatrix} . \quad (2.1)$$

Here ρ_1 and ρ_2 are the two principal radii of curvature of the two-dimensional surface $\partial\Omega$. (A convex surface has positive radii of curvature, a concave surface has negative radii of curvature.) Using the symmetry of the wormhole with respect to interchange of the two flat regions, it is a standard result of the junction formalism that the Einstein field equations may be cast in terms of the surface stress-energy tensor $S^i{}_j$ as

$$S^i{}_j = -\frac{1}{4\pi G}(K^i{}_j - \delta^i{}_j K^k{}_k) . \quad (2.2)$$

(Factors of c have been suppressed.) The surface stress-energy tensor may be interpreted in terms of the surface energy density σ and principal surface tensions $\theta_{1,2}$:

$$S^i{}_j = \begin{bmatrix} -\sigma & 0 & 0 \\ 0 & -\theta_1 & 0 \\ 0 & 0 & -\theta_2 \end{bmatrix} . \quad (2.3)$$

Einstein's field equations now yield

$$\sigma = -\frac{1}{4\pi G}\left[\frac{1}{\rho_1}+\frac{1}{\rho_2}\right] ,$$
$$\theta_1 = -\frac{1}{4\pi G}\frac{1}{\rho_2} , \quad (2.4)$$
$$\theta_2 = -\frac{1}{4\pi G}\frac{1}{\rho_1} .$$

This implies that in general ($\partial\Omega$ convex) we will be dealing with *negative* surface energy density and *negative* surface tensions.

It is now easy to see how to build a wormhole such that a traveler encounters no exotic matter. Simply choose Ω to have one flat face. On that face the two principal radii of curvature are infinite and the stress energy is zero. A traveler encountering such a flat face will feel no tidal forces and see no matter, exotic or otherwise. Such a traveler will simply be shunted into the other universe.

We have just seen that in order for the throat of the wormhole to be convex, the surface energy density must be negative. This behavior may be rephrased as a violation of the weak-energy hypothesis at the throat of the wormhole. The violation of weak energy has been noted before in Refs. 1 and 2, wherein very general arguments for this behavior were given. For the traversable wormholes discussed in this Rapid Communication, the argument simplifies considerably. Consider a bundle of light rays impinging on the throat of the wormhole. A little thought (following some geodesics through the wormhole) will convince one that the throat of the wormhole acts as a "perfect mirror," except that the "reflected" light is shunted into the other universe. This is enough to imply that a convex portion of $\partial\Omega$ will defocus the bundle, whereas a concave portion of $\partial\Omega$ will focus it. The "focusing theorem" for null geodesics then immediately implies that convex portions of $\partial\Omega$ violate both the weak-energy condition and the averaged-weak-energy condition. (See, e.g., Ref. 3, exercise 22.14, p. 582, or Hawking and Ellis.[6]) There are examples, such as the Casimir vacuum, of states of quantum fields that violate the weak-energy condition; but it is an open question as to whether quantum field theory ever permits the averaged-weak-energy condition to be violated.[1,2] Until a concrete example of such violation is found in quantum field theory, the possibility of traversable wormholes such as those studied here must be viewed with caution.

III. POLYHEDRAL WORMHOLES

Now that we have discussed nonsymmetric solutions in general, it becomes useful to consider some more special cases. What I am trying to do here is to minimize the use of exotic matter as much as possible. Let the compact set Ω be a cube whose edges and corners have been smoothed by rounding. Then the throat $\partial\Omega$ consists of six flat planes (the faces), twelve quarter cylinders (the edges), and eight octants of a sphere (the corners). Let the edge of the cube be length L, and let the radius of curvature of the cylinders and spheres be r, with $r \ll L$. The stress-energy tensor on the six faces is zero. On the twelve quarter cylinders comprising the edges the surface stress-energy tensor takes the form

$$S^i{}_j = \frac{1}{4\pi G r}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} . \quad (3.1)$$

This means that each quarter cylinder supports an energy per unit length of $\mu = \sigma 2\pi r/4 = -(1/4\pi G r)(\pi r/2) = -1/8G$, and a tension $T = \theta_1 2\pi r/4 = \mu$. [Note that μ and T are well behaved and finite (though negative) as $r \rightarrow 0$.] The energy concentrated on each of the eight octants at the corners is $E = \sigma 4\pi r^2/8 = -(1/2\pi G r)(\pi r^2/2) = -r/4G$, which tends to zero as $r \rightarrow 0$.

It is therefore safe to take the $r \rightarrow 0$ limit. In this limit Ω becomes an ordinary (sharp cornered) cube. The stress-energy tensor for the wormhole is then concentrated entirely on the edges of the cube where $\mu = T = -1/8G = -1.52\times10^{43}$ J/m $= -\frac{1}{8}$ (Planck mass/Planck length). [Note that although the Planck mass and Planck length are quantum concepts (i.e., they depend on $\hbar$), the ratio of a Planck mass to a Planck length is independent of $\hbar$.] Needless to say, energies and tensions of this magnitude (let alone *sign*) are well beyond current technological capabilities.

It is interesting to note that the stress energy present at the edges of the cube is identical to the stress-energy tensor of a *negative tension* classical string. To see this, write the Nambu-Goto action in the form

$$S = T\int d^2\xi\, d^4x\, \delta^4(x^\mu - X^\mu(\xi))\sqrt{-\det(h_{\alpha\beta})} , \quad (3.2)$$

where $h_{\alpha\beta}(\xi)=\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X^\mu(\xi))$. Varying with respect to the spacetime metric yields the classical spacetime stress-energy tensor

$$\Theta^{\mu\nu}(x^\rho) = -T\int d^2\xi\, \delta^4(x^\rho - X^\rho(\xi)) h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu . \tag{3.3}$$

For a classical string stretched along the x axis we may choose the world-sheet coordinates such that $X^\mu(\xi) \equiv X^\mu(\tau,\sigma)=(\tau,\sigma,0,0)$. The stress-energy tensor is then quickly calculated to be

$$\Theta^\mu{}_\nu(t,x,y,z) = -T\delta(y)\delta(z)\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} . \tag{3.4}$$

This is exactly the algebraic form of the stress-energy tensor just obtained for the edges of a cubical wormhole. Note, however, that field-theoretic models of strings lead to positive string tensions. No natural mechanism for generating negative string tension is currently known.

Having dealt with cubical wormholes, generalizations are immediate. First, note that the length of the edge L nowhere enters into the calculation. This implies that any rectangular prism would do just as well. The generalization to Ω being an arbitrary polyhedron is also straightforward. Consider an arbitrary polyhedron with edges and corners smoothed by rounding. At each edge the geometry is locally that of two planes joined by a fraction $(\phi/2\pi)$ of a cylinder. Here ϕ is the "bending angle" at the edge in question. The local geometry at each corner is that of some fraction of a sphere. As before the energy concentrated on the corners tends to zero as $r\rightarrow 0$. The energy per unit length concentrated on each edge is now $\mu=-(1/4\pi Gr)\phi r=-\phi/4\pi G$. As before the limit $r\rightarrow 0$ is well behaved, in which case we obtain $\mu=T=-\phi/4\pi G$. It is instructive to compare this with the known geometry of a single infinite-length classical string. Note that each edge is surrounded by a total of $2(\pi+\phi)$ radians ($\pi+\phi$ radians in each universe). Thus the deficit angle (φ) at each edge is given by $\varphi=-2\phi$. In terms of the deficit angle $\mu=T=\varphi/8\pi G$, which is the usual relationship for classical strings. An edge of the polyhedron is said to be convex if the bending angle is positive. If an edge is convex the tension is negative. Conversely, if an edge is concave, the tension at that edge is positive.

IV. CONCLUSIONS

In this paper I have exhibited some rather simple examples of traversable wormholes. I have been able to avoid the use of spherical symmetry. Although these wormholes require the presence of exotic matter, it is possible to exhibit wormholes and geodesics such that the traveler does not have to encounter the exotic matter directly. On the other hand, the required presence of exotic matter, while it is not a cause for panic, is certainly a cause for concern. I have attempted to obtain the required exotic stress energy by considering the Casimir energy associated with oscillations of a classical string. So far I have been unable to do so.

Unresolved problems include the stability of such wormholes against perturbations and causal constraints on the construction of such wormholes. The big open question, naturally, is whether exotic matter is in fact obtainable in the laboratory. The theoretical problems are daunting, and the technological problems seem completely beyond our reach.

[1]M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).

[2]M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).

[3]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[4]S. K. Blau, E. I. Guendelman, and A. H. Guth, Phys. Rev. D **35**, 1747 (1987).

[5]A. J. McConnell, *Applications of Tensor Analysis* (Dover, New York, 1957).

[6]S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).