

# Natural Wormholes as Gravitational Lenses

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Visser has suggested traversable 3-dimensional wormholes that could plausibly form naturally during Big Bang inflation. A wormhole mouth embedded in high mass density might accrete mass, giving the other mouth a net *negative* mass of unusual gravitational properties. The lensing of such a gravitationally negative anomalous compact halo object (GNACHO) will enhance background stars with a time profile that is observable and qualitatively different from that recently observed for massive compact halo objects (MACHOs) of positive mass. We recommend that MACHO search data be analyzed for GNACHOs.

## I. INTRODUCTION: WORMHOLES AND NEGATIVE MASS

The work of Morris and Thorne [1,2] has led to a great deal of interest in the formation and properties of three-dimensional wormholes (topological connections between separated regions of space-time) that are solutions of the Einstein's equations of general relativity. Subsequently Visser [3] suggested a wormhole configuration, a flat-space wormhole that is framed by "struts" of an exotic material, a variant of the cosmic string solutions of Einstein's equations [4,5]. To satisfy the Einstein field equations the cosmic string framing Visser wormholes must have a *negative* string tension [3] of  $-1/4G$  and therefore a negative mass density. However, for the total mass of the wormhole system, the negative mass density of the struts should be combined with the effective positive mass density of the wormhole's gravitational field. The overall object could, depending on the details of the model, have positive, zero, or negative net external mass. Note that in hypothesizing the existence of such a wormhole, one has to abandon the averaged null energy condition [1,2]. Therefore, the hypotheses underlying the positive mass theorem no longer apply and there is nothing, in principle, to prevent the occurrence of negative total mass [7]. Some of the Visser wormhole configurations have the shape of a cube or other geometrical solid, but one particularly simple configuration is a flat-space wormhole mouth framed by a single continuous loop of exotic cosmic string.

It has been suggested [4,5] that the inflationary phase of the early universe might produce closed loops of cosmic string. It is therefore at least plausible that a similar mechanism might produce negative-mass string loops framing stable Visser wormholes.

If a particle with positive electric charge passes through such a wormhole, its lines of force, threading through the wormhole aperture, give the entrance mouth an effective positive charge (flux lines radiating outward) and give the exit mouth an effective negative charge (flux lines converging inward.) Similarly, when a massive object passes through the wormhole, the same back-reaction mechanism might cause the entrance mouth to gain mass and the exit mouth to lose mass [6]. Now let us consider a stable Visser wormhole with near-zero mass residing in the mass-energy rich environment of the early universe. The expected density fluctuations of the early universe suggest that the separated wormhole mouths will reside in regions of differing mass density, leading to a mass flow between the regions they connect. As this mass passes through the wormhole, the entrance mouth will gain mass while the exit mouth will lose mass by the same amount. Soon, if the mass flow continues, the exit wormhole mouth will acquire a net negative mass.

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This will lead to a gravitational instability, since the positive mass mouth will attract more mass through its aperture while the negative mass mouth will gravitationally repel nearby mass [8]. Thus the positive-negative imbalance will be fed by gravity and will continue to grow. If this process proceeds without interruption, the exit wormhole mouth might develop a stellar-scale negative mass. Visser wormholes, therefore, provide at least one motivation for seriously considering the possible existence of naturally occurring astronomical objects of sizable negative mass.

We have tried to sketch only one scenario for the possible existence of negative mass objects. We must, however, point out that the non-existence of such objects would not rule out the existence of natural wormholes, since their properties are highly model-dependent. Also, observations providing evidence of negative mass objects would not require the existence of natural wormholes, since other negative-mass objects could conceivably exist, but would indicate a direct violation of the averaged null energy condition mentioned above.

Negative-mass objects, while repelling all nearby mass (positive or negative), would themselves be attracted [8] by the mass of a nearby galaxy and might form part of a galactic halo. They would have unusual gravitational properties that could produce detectable gravitational lensing effects. These lensing effects, for the same absolute mass, are of the same magnitude as those recently detected for massive cosmic halo objects (MACHOs) [9,10], but, as we will show below, are qualitatively different in shape. We here examine in detail the lensing effects of gravitationally negative anomalous compact halo objects (GNACHOs).

## II. GRAVITATIONAL LENSING BY NEGATIVE MASSES

Naively, if a gravitationally attractive positive mass acts as a converging lens that brightens a background star, one might expect a gravitationally repulsive negative mass to act as a diverging lens that causes a background star to briefly grow dimmer. Actually the lensing of a negative mass is not analogous to a diverging lens. In certain circumstances it can produce more light enhancement than does the lensing of an equivalent positive mass.

Fig. 1 shows the geometry of starlight that is gravitationally lensed by an object of negative mass. Starlight is radiated from the stellar source  $S$  and is detected by the observer at  $D$ . A gravitationally negative object  $N$  lies between source and observer at an impact-parameter distance  $b$  from the source-detector axis  $DS$ . An off-axis ray of light passes near the negative mass object  $N$ , coming within a distance  $a$  of it, and is deflected [11,12] by an angle  $\delta = (4G|M|/c^2)/a$  where  $G$  is Newton's gravitational constant,  $|M|$  is the absolute value of the deflecting mass, and  $c$  is the velocity of light. The angle  $\delta$  is an external angle of the triangle formed by the direct and deflected rays. This triangle has interior angles  $\alpha$  and  $\beta$ , so  $\delta = \alpha + \beta$ . If the detector-source distance is  $L_S$  and the detector-mass distance is  $L_N$ , then we have the following equations:

$$\alpha = (b - a)/L_N \quad (\alpha \ll 1) \quad (1)$$

$$\beta = (b - a)/(L_S - L_N) \quad (\beta \ll 1) \quad (2)$$

$$\delta = \alpha + \beta = \frac{4G|M_N|}{c^2} \frac{1}{a}. \quad (3)$$

These lead to the dimensionless quadratic equation:

$$A^2 - AB + 1 = 0 \quad (B = A + 1/A) \quad (4)$$

where  $A \equiv a/a_0$  is the dimensionless distance of closest approach of the deflected ray,  $B \equiv b/a_0$  is the dimensionless impact parameter distance of the deflecting mass, and

$$a_0 \equiv \sqrt{\frac{4G|M_N|}{c^2} \frac{L_N(L_S - L_N)}{L_S}} \quad (5)$$

is the characteristic gravitational length scale of the problem. For a positive lensing mass,  $a_0$  would be the radius of the Einstein ring produced when the mass is positioned at zero impact parameter ( $b=0$ ). To give some feeling for this length scale, if the stellar source  $S$  is in the Large Magellanic Cloud and a negative lensing mass  $N$  of one solar mass is in the galactic halo, then  $L_S = 2 \times 10^{21}$  m,  $L_N = 5 \times 10^{20}$  m, and  $a_0 = 1.5 \times 10^{12}$  m or about 10 AU.

Solving quadratic (4) for  $A$  gives two solutions:

$$A_{\pm} = \frac{1}{2}[B \pm \sqrt{B^2 - 4}]. \quad (6)$$

When  $B > 2$  there are two real solutions of the quadratic, corresponding to two rays that are deflected to the observer. When  $B < 2$  there are no real solutions, indicating that the deflection is blocking all rays from reaching the observer. At  $B = 2$ ,  $A_+ = A_- = B/2$  and, as will be discussed below, the rainbow-like caustic occurs, allowing many rays to reach the observer and producing a dramatic brightening of the background star. Fig. 2 shows this schematically. The negative mass deflects rays in inverse proportion to their distance of closest approach, creating a shadowed umbra region where light from the source is extinguished. At the edges of the umbra the light rays accumulate to form the caustic and give a very large increase in light intensity. The intensity falls slowly to normal at larger transverse distances.

### III. LIGHT MODULATION PROFILES

If the unmodified intensity of the background star is  $I_0$  and the altered intensity of the background star in the presence of the negative lensing mass for each solution is  $I_{\pm}$ , then the partial amplification factors  $p_{\pm} \equiv I_{\pm}/I_0$  are given by [11]:

$$p_{\pm} = \frac{a_{\pm}}{b} \left| \frac{da_{\pm}}{db} \right| = \frac{A_{\pm}}{B} \left| \frac{dA_{\pm}}{dB} \right| = \frac{(B \pm \sqrt{B^2 - 4})^2}{4B\sqrt{B^2 - 4}} \quad (7)$$

The overall relative intensity  $\mathcal{I}_{neg} = p_+ + p_-$  is the modulation in brightness of the background star as detected by the observer, and is given by:

$$\mathcal{I}_{neg} = p_+ + p_- = \frac{B^2 - 2}{B\sqrt{B^2 - 4}}. \quad (8)$$

It is interesting to compare this with the similar expression for relative intensity  $\mathcal{I}_{pos}$  of a background source that is lensed by an object of *positive* mass:

$$\mathcal{I}_{pos} = \frac{B^2 + 2}{B\sqrt{B^2 + 4}}. \quad (9)$$

For the same dimensionless impact parameter  $B$  with  $B > 2$ , it is always true that  $\mathcal{I}_{neg} > \mathcal{I}_{pos}$ , so for large impact parameters a negative mass actually provides *more* light enhancement through lensing than an equivalent positive mass. When  $B \rightarrow 2$ , the overall intensity  $\mathcal{I}_{neg} \rightarrow \infty$ , a condition indicating a caustic at which light rays from many trajectories are deflected toward the observer. It provides a distinctive and unusual signature for negative-mass lensing.

The intensity modulation that is actually observed occurs when the lensing mass, which is assumed to be moving with transverse velocity  $V$ , crosses near the source-detector axis  $DS$  with a minimum impact parameter  $b_0$  and a minimum dimensionless impact parameter  $B_0 = b_0/a_0$ . The time-dependent impact parameter is therefore  $b(t) = \sqrt{b_0^2 + (Vt)^2}$ , and

$$B(t) = B_0 \sqrt{1 + \left(\frac{t}{T_0}\right)^2} \quad (10)$$

where  $T_0 = b_0/V$  is the transit time across the distance of the minimum impact parameter and is the characteristic time scale of the problem.

Eqns. 8 and 10 are used to calculate the light enhancement profile of the process, taking  $\mathcal{I}_{neg} = 0$  when  $|B| < 2$ , and there are no real solutions to Eqn. 4. Fig. 3 shows these light enhancement profiles, plotting  $\mathcal{I}_{neg}$  vs.  $t/T_0$  for a range of minimum dimensionless impact parameters ranging from  $B_0 = 0.50$  to 2.20. Fig. 4 shows similar light enhancement profiles for a positive mass of the same size and range of  $B_0$  values. As can be seen, the light enhancement profiles are qualitatively different for positive and negative lensing masses of the same magnitude and geometry. In particular, the negative mass curves are much sharper, show stronger but briefer light enhancements, and for  $B_0 < 2$  show a precipitous drop to zero intensity, i. e., extinction of the light from the source when the lensing mass deflects all light from  $S$  away from the observer.

Such a signature might possibly be confused with that of occultation by a dark foreground object. However, an occultation would not be preceded by a dramatic rise in intensity and might, if the object had a significant atmosphere, have different light profiles at different wavelengths. Therefore, the negative gravitational lensing presented here, if observed, would provide distinctive and unambiguous evidence for the existence of a foreground object of negative mass.

## IV. CONCLUSION

The calculations presented above show that objects of negative gravitational mass, if they exist, can provide a very distinctive light enhancement profile. Since three groups are presently conducting searches for the gravitational lensing of more normal positive mass objects, we suggest that these searches be slightly broadened so that the signatures of the objects discussed above are not overlooked by over-specific data selection criteria and software cuts. We recommend that MACHO search data be analyzed for evidence of GNACHOs.

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FIG. 1. Geometry for gravitational lensing by a negative mass object. Off axis light rays from stellar source  $S$  are deflected to detector  $D$  by the gravitational repulsion of the negative lensing mass (GNACHO)  $N$ .

FIG. 2. Light deflection by a negative mass object (horizontal scale highly compressed). Light is swept out of the central region, creating an umbra region of zero intensity. At the edges of the umbra the rays accumulate, creating a rainbow-like caustic and enhanced light intensity.

FIG. 3. Intensity profile of a gravitationally negative anomalous compact halo object (GNACHO) as it passes near the source-detector axis  $DS$ . The several curves correspond to minimum dimensionless impact parameter values  $B_0 = 0.50$  (at edge of plot), 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.10, and 2.20 (small central bump). (See text for definitions of the variables.)

FIG. 4. Intensity profile of an object of equivalent positive mass in the same geometries. Here  $B_0 = 0.50$  is the highest curve, and  $B_0 = 2.20$  is the lowest.

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