IFUP-TH 44/93 October 1993 hep-th/9310157

Black Hole Complementarity and the Physical Origin of the Stretched Horizon

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ABSTRACT. We discuss the idea of black hole complementarity, recently suggested by Susskind *et al.*, and the notion of stretched horizon, in the light of the generalized uncertainty principle of quantum gravity. We discuss implications for the no-hair theorem and we show that within this approach quantum hair arises naturally.

PACS categories: 04.60, 12.25, 97.60.

The problems related to the application of quantum mechanics to black holes rank between the most challenging in theoretical physics. Despite great effort, it has not yet been reached a consensus on the validity of Hawking's claim [1] that the evolution of states in the presence of black holes violates unitarity. Recently, an extremely interesting proposal, close in spirit to previous work of 't Hooft [2], has been put forward by Susskind and coworkers [3-6] and termed "black hole complementarity". The basic observation is that physics looks very different to an observer in free fall in a black hole and to a "fiducial observer" at rest with respect to the black hole, outside the horizon. Crossing the horizon of a very massive black hole, the free falling observer should not experience anything out of the ordinary. If the mass of the hole M is much larger than Planck mass $M_{\rm Pl}$ a classical description of the black hole should be adequate, and in classical general relativity the horizon merely represents a coordinate singularity, while physical quantities like the curvature are non-singular. Furthermore, from the point of view of the free falling observer, the flux of Hawking radiation is switched off when he approaches the horizon. This can be shown observing that near the horizon, with an appropriate change of variables, the Schwarzschild metric approaches the Rindler metric, and a free falling observer in Schwarzschild spacetime becomes a free falling observer in flat Minkowski space – and certainly does not detect any radiation. The point of view of a fiducial observer is dramatically different. In Schwarzschild coordinates, a fiducial observer at a distance rfrom a Schwarzschild black hole measures an effective temperature

$$T = (1 - \frac{2GM}{r})^{-1/2} T_H , \qquad (1)$$

where $T_H = \hbar/(8\pi GM)$ is Hawking temperature. Climbing out of the gravitational potential well, the radiation is gravitationally red-shifted by a factor $(1 - \frac{2GM}{r})^{1/2}$ and is seen by an observer at infinity as having temperature T_H . Instead, at the horizon r = 2GM the temperature measured by a fiducial observer diverges. For a fiducial observer, this temperature is certainly a very real effect. If too close to the horizon, he would be killed by the eccessive heat. From this point of view, a fiducial observer regards the black hole horizon as a physical membrane, endowed with real physical properties. More in general, within the membrane paradigm [7] all interactions of a black hole with the external environment, as seen by a fiducial observer, are described in terms of a two-dimensional membrane endowed with properties like electric conductivity, viscosity, entropy and temperature.

The difference between the point of view of free falling and fiducial observers can be of relevance to the information loss problem. There are various approaches to this problem (for reviews see e.g. [8]), and each one has its own difficulties; in particular, if one assumes that the black hole evaporates completely then the core of the information loss problem is that it appears very difficult to reconcile the point of view of the free falling and fiducial observers, without questioning basic postulates of quantum mechanics. In fact, as discussed in [3], the assumption that the evolution of states is unitary, togeter with the superposition principle, forces upon us the conclusion that all distinctions between the infalling states must be obliterated soon after they cross the horizon; this is certainly very difficult to reconcile with the point of view of the free falling observer and with the equivalence principle, since to the free falling observer the horizon is no special place. Of course, after passing the horizon the free falling observer cannot communicate anymore with fiducial observers, so that no immediate logical contradiction arises; for instance, the free falling observer cannot report on the lack of substance of the membrane. More in general, the investigation of various gedanken experiments carried out in [5] indicates that "apparent logical contradictions can always be traced to unsubstantiated assumptions about physics at or beyond the Planck scale" [5, 9]. This implies that, contrarily to the common opinion, the information loss paradox cannot be addressed without a detailed knowledge of a full quantum theory of gravity.

Both in the membrane paradigm and in discussing black hole complementarity a key role is played by the concept of stretched horizon. It has been found [7] that the description of the black hole in terms of a membrane takes a much simpler and elegant form if the horizon is stretched, i.e. if the surface of the black hole is moved at a slightly larger radius, and a set of membrane-like conditions are imposed at the stretched horizon. This allows to get rid of many irrelevant details of the infalling fields and at the same time "regularizes" the infinities coming from the infinite red-shift factor between r = 2GM and $r = \infty$.¹ In particular, the temperature measured by a fiducial observer at the stretched horizon is large but finite. The amount of stretching is however rather arbitrary. Because of this, and because the free falling observer would not agree on its existence, the membrane has been con-

¹A conceptually similar approach is given by the "brick wall" model of 't Hooft [2].

sidered as a useful mathematical construction rather than a physical object: for instance (see [7], pag. 31) "...it is very useful to regard these boundary conditions as arising from physical properties of a *fictitious* membrane residing at the location of the stretched horizon. More specifically, it is useful to *pretend* that the stretched horizon is endowed with a surface density of electric charge ..." (our italics). However, performing a gedanken experiment aimed at measuring the radius of the horizon with the best possible accuracy, we have recently found [10], using only rather general arguments, that the error Δx on the radius of the horizon is subject to a generalized uncertainty principle,

$$\Delta x \ge \frac{\hbar}{\Delta p} + \text{const.} \, G\Delta p \tag{2}$$

which implies the existence of a minimum error on the order of the Planck length times a numerical constant, which is shown in [10] to be larger than one. It is tempting to assume that eq. (2) actually represents a generalized uncertainty principle which governs all measurements in quantum gravity; a similar uncertainty principle has been found in string theory [11-14]. Independently of the correctness of the latter assumption, eq. (2) holds for the measurement of a black hole radius, which is the case in which we are now interested. More exactly, eq. (2) only holds for Δp not large compared with $M_{\rm Pl}$, since Δp is the error on the momentum of a particle emitted by the black hole and detected at infinity, and we do not really know how to describe the particle if its energy is super-planckian. The two terms on the right-hand side can be considered as the first terms in an expansion in powers of $\Delta p/M_{\rm Pl}$. The knowledge of the exact expression would in principle require a full quantum theory of gravity. In the following we will assume that the exact expression valid for arbitrarily large values of $\Delta p/M_{\rm Pl}$ does not spoil the main result which can be inferred from eq. (2), namely the fact that there exists a minimum error on the horizon radius. We can also try to guess the exact form of the generalized uncertainty principle making the assumption that it can be derived from an algebraic structure, in the same sense in which the standard uncertainty principle is a consequence of the Heisenberg algebra. In [15] we have found that there is indeed an appropriate algebraic structure, and it is given by a deformation of the Heisenberg algebra involving a deformation parameter κ with dimensions of mass,

$$[x_i, x_j] = -\frac{\hbar^2}{\kappa^2} i\epsilon_{ijk} J_k \tag{3}$$

$$[x_i, p_j] = i\hbar \delta_{ij} (1 + \frac{E^2}{\kappa^2})^{1/2} .$$
(4)

(*E* is the energy and J_i the angular momentum). In the limit $\kappa \to \infty$ it reduces to the Heisenberg algebra. In the following κ will be identified with $M_{\rm Pl}$, apart from numerical factors (alternatively, we can identify \hbar/κ with the string length times a numerical factor of order one, in order to recover the string uncertainty principle). It is remarkable that, under relatively mild assumptions, this deformed algebra is unique, essentially because the Jacobi identities provide very stringent requirements on the possible deformations of an algebra.

From eq. (4) we immediately derive the generalized uncertainty principle

$$\Delta x_i \Delta p_j \ge \frac{\hbar}{2} \delta_{ij} \left\langle \left(1 + \frac{E^2}{\kappa^2} \right)^{1/2} \right\rangle.$$
(5)

Expanding the square root in powers of $(E/\kappa)^2$ and using $\langle \mathbf{p}^2 \rangle = \mathbf{p}^2 + (\Delta p)^2$, where $(\Delta p)^2 = \langle (\mathbf{p} - \langle \mathbf{p} \rangle)^2 \rangle$, at first order one obtains

$$\Delta x_i \Delta p_j \ge \frac{\hbar}{2} \delta_{ij} \left(1 + \frac{E^2 + (\Delta p)^2}{2\kappa^2} \right) \,. \tag{6}$$

which reproduces eq. (2) in the limit $E \ll \kappa, \Delta p \stackrel{<}{\sim} \kappa$. Instead, in the limit $\langle \mathbf{p} \rangle^2 \sim (\Delta p)^2 \gg \kappa^2$ one obtains

$$\Delta x \ge \operatorname{const} \times \frac{\hbar}{\kappa} \,. \tag{7}$$

These results suggest that the horizon is subject to irreducible quantum fluctuations which provides it with a physical thickness. In this case, we can attempt to promote the membrane from a useful mathematical construction to a real physical entity: in spite of the fact that the nature of its microphysical degrees of freedom is at present quite elusive, still from the point of view of fiducial observers the membrane has definite and real physical properties, and it has a physical thickness and a location in space which are well defined and determined by physics, rather than by our "regularization" procedure. In particular, we see that it extends beyond the nominal horizon by a few Planck lengths (if we choose $\kappa \sim M_{\rm Pl}$). This agrees with the choice suggested in [3] in the case of two-dimensional dilaton gravity, while in [7] the stretched horizon is assumed to extend outward by a finite fraction of the Schwarzschild radius. It is important to observe that the thickness of the membrane is independent of the black hole mass; this implies that even the horizon of a "classical" black hole, with $M \gg M_{\rm Pl}$, acquires a thickness because of quantum effects.

Of course, the membrane does not exists for the free falling observer; however, the principle of black hole complementarity protects us from logical inconsistencies. To ask whether the membrane exists or not is like asking whether a photon went through a specific arm of an interferometer. The answer depends on the setting of the experiment. In our case, on whether the observer is in free fall or not.

Promoting the membrane to a real, physical object implies a radical revision of some of the common wisdom concerning black holes. In particular, one realizes that there is no reason to expect that the classical no-hair theorem extends in the quantum domain as well. Let us remind the form of the classical no-hair theorem for the simple case of a massive scalar field (see [18] for a discussion of the relevance of the no-hair theorem to the information loss problem and to the possible relation between black holes and elementary particles). Introducing the tortoise coordinate

$$r_* = r + 2GM \log \frac{r - 2GM}{2GM} \tag{8}$$

the wave equation for a scalar field of mass μ in the Schwarzschild background, after expanding in partial waves, reads

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2}\right)\psi_{l,m} = \left(1 - \frac{2GM}{r}\right)\left(\mu^2 + \frac{2GM}{r^3} + \frac{l(l+1)}{r^2}\right)\psi_{l,m}.$$
 (9)

In the zero-frequency limit the second derivatives with respect to r_* is always positive. However, the domain $2GM \leq r < \infty$ corresponds to $-\infty < r_* < \infty$, and therefore a solution which decreases exponentially at $r = \infty$ must blow up at the horizon. Thus there is no physically acceptable static solution: "a black hole has no hair". In the quantum case, of course the right-hand side of eq. (9) receives corrections, which we cannot control. However, it is usually argued that these corrections should not alter the asymptotic behavior at spatial infinity, nor close to the horizon, where the dominant effect is given by the factor 1-2GM/r. Then, the no-hair theorem simply follows from the fact that r_* ranges from $-\infty$ to $+\infty$.

From the membrane point of view, it is not difficult to see where this argument can fail. Physically, we are not allowed to extrapolate the solution inside the membrane. Such an extrapolation would imply to enter a region of super-planckian temperatures and therefore to make assumptions about physics beyond the Planck scale – which is just what the principle of black hole complementarity warns us not to do. As long as we stop at the border of the membrane, as determined physically by the generalized uncertainty principle, r_* only covers a semi-infinite range, and a solution which decays exponentially at spatial infinity is finite on the stretched horizon. Thus, there is no reason to expect that the no-hair theorem goes through even in quantum gravity.

The fact that the membrane paradigm makes possible to violate the nohair theorem at the quantum level is not at all surprising. After all, in the approach of refs. [3-6] the membrane is just the place where the infalling information is stored, before being re-radiated in such a way as to preserve quantum coherence, according to the mechanism suggested by Page [19]. The various states of the membrane correspond to different internal states of the black hole, and the difference between the internal states manifest itself to an observer at infinity through differences in the Hawking radiation; this is nothing but quantum hair.

Another point which deserves attention is that, at least as far as the radius of the black hole horizon is concerned, there exists a minimal *spatial* distance on the order of the Planck length. This has the surprising consequence (already pointed out in this context by Susskind [6]) that at this length scale Lorentz transformations must saturate. This implies a deep revision of kinematics at the Planck scale. A possible example of a different kinematic framework is provided by quantum deformations of the Poincaré algebra [16]. In [17] we found in fact that, in the κ -deformed Poincaré algebra, the κ -deformed Newton-Wigner position operator and the generators of translations and rotations actually obey the algebra (3,4). Another possible kinematic framework is provided by string theory, see below.

The principle of black hole complementarity has also important conse-

quences for the mental image that we have of black holes. The important lesson that we learn is that we should be very careful not to mix up the point of view of free falling observers with that of fiducial observers. Much of the seemingly paradoxical features of the information loss problem come from a confusion between these two points of view and is rooted in the implicit and seemingly undisputable assumption that there exists a notion of invariant event. However, Susskind [6] has made the crucial observation that black hole complementarity implies that even the notion of invariant event cannot be anymore relied upon. He further observes that string theory has just the properties required by black hole complementarity, as far as the notion of event is concerned: if a string falls toward a black hole, an observer at infinity sees the string spreading when it reaches the stretched horizon, until it covers the horizon completely, while a free falling observer sees a string with constant transverse and longitudinal size which crosses the horizon without any peculiar behavior. We wish to point out that the non-invariance of the concept of event in string theory can be seen also at a more fundamental level and is in fact well-known (see [20], pag. 29). In general, events are defined in terms of interactions: in classical physics the collision between two billiard balls constitues a typical event. In quantum field theory, a typical event is the emission of a photon by a source. If we represent it by a Feynman diagram, the vertex of the interaction defines the spacetime location of the event. We can describe this location in different reference frames, but the location itself has an invariant meaning. In string theory, we must instead consider the splitting of strings as defining events. However, the point in spacetime at which a string splits into two strings appears different to different observers (see fig. 1.6 of ref. [20]) and correspondingly there is no Lorentz invariant notion of event. Thus, the non-invariance of the notion of event is not specific to physics in the vicinity of black holes, although black holes act as a sort of magnifier of this effect.

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