ESCHATOLOGY OF THE BLACK HOLE INTERIOR ☆

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It is argued that the internal evolution of a generic black hole leads to a "fat cigar" configuration near the inner (Cauchy) horizon, inside which curvatures are planckian The mechanism responsible is an enormous inflation of the effective internal mass of the hole, caused by blueshift of the radiative tail of the collapse at the Cauchy horizon Within the cigar the evolutionary history is governed by unknown quantum effects, but may be as short as a Planck time

In its capacity to probe levels of Nature inaccessible to observational techniques of the day, theoretical physics has traditionally scored some of its most conspicuous successes A well-known astrophysical example is the "no hair" theorem [1] for generic gravitational collapse into a black hole, which predicts that the field exterior to the event horizon should settle radiatively into an unexpectedly simple (Kerr–Newman) form, characterized by just three parameters mass, angular momentum and charge

Human curiosity does not recognize practical limitations The next step inevitably beckons: to extend the exploratory reach of theory inward But it has been known for twenty years that even a theoretical expedition will generally not progress far beneath the surface before meeting an impediment at the black hole's inner horizon [2] The ingoing lightlike sheet of this horizon (corresponding to infinite advanced time) is a "Cauchy horizon", an absolute limit to future predictability from generic initial data prescribed at the onset of collapse. It is also violently unstable to ingoing wavelike perturbations, which are blueshifted and amplified (classically, without limit) in its vicinity

It thus appears that attempts to probe a black hole's interior must generally be called off long before approaching the nuclear region $r \sim (10^{-20}\,\mathrm{cm})\ (m/m_\theta)^{1/3}$ where the ambient curvature m/r^3 becomes and remains planckian [3,4]. We shall argue that this impression is incorrect. that the trip to the inner horizon already embraces all but the last 10^{-43} s of the black hole's entire classical history The Cauchy horizon is not a curtain around an unknowable spacious interior but the ultimate brick wall where a generic collapse terminates Here, the geometry approaches a universal form in which charge and angular momentum, the last vestiges of black hole individuality, themselves becomes irrelevant

The mechanism responsible for this remarkable outcome is a combination of two effects Transverse irradiation of the Cauchy horizon by outward emission from the collapsing star focusses its generators, causing it to contract The resulting time-dependence has the crucial effect of separating the Cauchy horizon from the inner apparent horizon, which contracts faster (fig. 1) In these circumstances the second effect is catastrophic inflowing gravitational radiation from the radiative tail of the collapse, blueshifted at the Cauchy horizon, precipitates a total collapse of the inner apparent horizon The result may be described as an inflation of the effective internal mass-parameter of the hole as one approaches the Cauchy horizon at fixed retarded time

Because charge and angular momentum are conserved (the latter at least approximately, in an axisymmetric scenario it will be conserved exactly), these parameters become relatively insignificant in the final state There appears to be nothing able to

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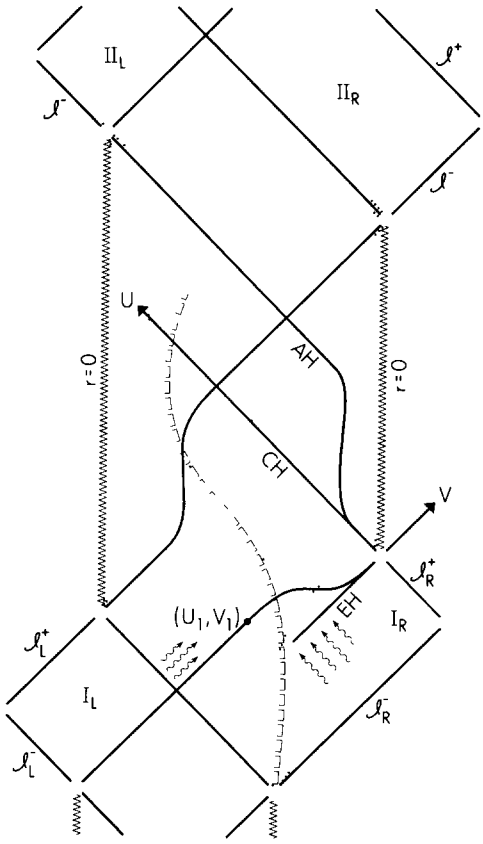

Fig 1 Conformal map of the analytically extended exterior geometry of a collapsing spherical star, with radiative inflow and outflow The section to the left of the stellar boundary (stapled curve) bears no resemblance to the physical interior space Shown are the event horizon (EH), Cauchy horizon (CH), inner apparent horizon (AH) and curves of constant r (dotted) To make the figure legible, the degree of deflation of AH has been toned down, and the outflux turned for just a brief interval

stop the inflation until the ambient curvature reaches planckian values, when unknown quantum effects come into play At this point the internal mass-parameter will have reached a value $m \sim m_0^3/m_{\mathrm{Pl}}^2$ at the Cauchy horizon (m_0 is the mass of the stellar progenitor) Inside the black hole formed from a star of ten solar masses, this is about 10^{60} times the mass of the observable universe! Since the Cauchy horizon is divorced from causal contact with the exterior, no trace of this inflation is perceptible to an outside observer, who sees the black hole as quiescent and records the mass m_0

The two qualitative features responsible for mass-inflation that we have isolated above are characteristic of a generic collapse However, the phenomenon is most easily illustrated for the simple example of a spherisymmetric charged (Reissner–Nordstrom) black hole Because of the similarity of the charged and rotational inner-horizon structures, this idealized case should be sufficiently representative

The metric of an arbitrary spherisymmetric spacetime can be written

$$ds^2 = g_{ab}\, dx^a\, dx^b + r^2\, d\Omega^2$$

$$(a, b, \quad = 0, 1)\,,$$

in which (x^0, x^1) are arbitrary co-ordinates in the "radial" (timelike) 2-space $(\theta, \varphi) = \mathrm{const}$, and g_{ab}, r depend on x^0, x^1 only We define scalar fields $f(x^a)$, $m(x^a)$ by

$$1 - \frac{2m}{r} + \frac{e^2}{r^2} = f = g^{ab}(\partial_a r)(\partial_b r)\,, \tag{1}$$

where e may be interpreted as a charge parameter, here taken for simplicity to be a constant

The Einstein field equations are then contained in

$$\partial_a m = 4\pi r^2 T_a{}^b \partial_b t\,, \quad r_{,a}{}^b + \kappa \delta_a{}^b = -4\pi r T_a{}^b\,,$$

$$\kappa \equiv -\tfrac{1}{2}\partial_r f(r, m) = -\frac{m - e^2/r}{r^2}\,.$$

These equations, which are covariant under two-dimensional co-ordinate transformations [5], are easily verified, for example by specializing to standard Schwarzschild co-ordinates r, t They imply the conservation laws $(r^2 T_a{}^b)_{;b} = 0$ The semi-colon denotes two-dimensional covariant differentiation, and the (separately conserved) electrostatic stress–energy contribution has been prededucted from $T_a{}^b$

Two-dimensional wave equations for the key variables follow from the field equations·

$$\Box r \equiv r_{,a}{}^a = -2\kappa\,, \quad \Box m = -r^{-1}(4\pi r^2)^2 T^{ab} T_{ab}\,, \tag{2}$$

$$\Box \ln f = 16\pi f^{-2}\left(1 - \frac{e^2}{r^2}\right) T^{ab} r_{,a} r_{,b} - {}^{(2)}R \tag{3}$$

with ${}^{(2)}R = 2\partial_r \kappa(r, m)$ the curvature scalar of the radial 2-space. These equations are easy to integrate (formally), using the general prescription that $\Box \Psi = \varphi$

gives the value of Ψ at the point A in radial 2-space as

$$\Psi_A = -\tfrac{1}{2} \int_{ACBD} \varphi \, \mathrm{d}S + \Psi_C + \Psi_D - \Psi_B \,, \tag{4}$$

with integration over the interior of the lightlike rhombus whose vertices are A, C, B, D, labelled clockwise from noon

The schematic essence of mass-inflation can be understood from a generalization of the Dray–'t Hooft–Redmount (DTR) relations [6] These pertain to the collision or crossing of two thin, concentric spherical shells, one expanding, the other contracting, with the speed of light (In the generalized form of these relations, the background can be an arbitrary spherisymmetric field) Their intersection, at event $q(U_0, V_0)$ (say), divides the radial 2-space into four sectors A, C, B, D in which $f(x^a)$, defined by (1), has distinct forms f_A, , f_D, corresponding (for example) to different mass functions m_A, , m_D Then the generalized DTR relations require that

$$f_A(q) f_B(q) = f_C(q) f_D(q)$$

The proof can be read off from (4) applied to $\Psi = \ln f$, with integration over an infinitesimal lightlike rhombus enclosing q Since the integrand (3) is linear in the distribution-like stress–energy, with no bilinear term $\delta(U-U_0)\delta(V-V_0)$ (we are using lightlike coordinates), the double integral gives zero

From the DTR relations it follows that if two concentrated streams of energy converge near a horizon of the intervening space B, so that $f_B(q)$ is very small, then $|f_A(q)|$ must be very large, which means that the mass parameter m_A in this space after collision has been inflated by a large factor (This argument has been applied by Blau [7] to elaborate Eardley's scenario for the "death" of a white hole by accretion and blueshift on the past horizon)

In sector B, the apparent horizon (where $f=0$) and the Cauchy horizon (infinite blue shift surface) coincide Arguments based solely on the DTR relations cannot allay doubts that mass-inflation may be merely an artifact of this coincidence To confirm that inflation is associated with the Cauchy – not the apparent – horizon, and that, far from depending on their coincidence, it hinges crucially on their separation under transverse irradiation, we turn to a continuum analysis

The large blueshift near the Cauchy horizon permits use of a short-wavelength ("optical" or "graviton") approximation, in the spirit of Isaacson [8], for the gravitational wave-tail propagating inwards Our idealized continuous model (fig 1) accordingly considers noninteracting lightlike streams $T_{\mathrm{L}}^{ab} = \rho_{\mathrm{L}} l^a l^b$ and $T_{\mathrm{R}}^{ab} = \rho_{\mathrm{R}} n^a n^b$ flowing leftwards and rightwards in a spherisymmetric black hole of fixed charge e As a result of the inflow, the mass $m_{\mathrm{L}}(v)$ measured on $\mathscr{I}^-$ increases with advanced time v from its initial value m_1, tending asymptotically to a maximum m_0 $m_0 - m_{\mathrm{L}}(v) \sim v^{-p}$ $(p \geqslant 11)$ [9].

We introduce Kruskal co-ordinates U, V adapted to the Cauchy horizon of the static black hole which the external field approaches, so that

$$V = -\exp(-\kappa_0 v) \,,$$

$$\kappa_0 = \frac{(m_0^2 - e^2)^{1/2}}{r_0^2} \,, \quad r_0 = m_0 - (m_0^2 - e^2)^{1/2}$$

The radial lightlike vectors can be normalized so that $l_a = -\partial_a V$, $n_a = -\partial_a U$ The conservation laws require that $r^2\rho_{\mathrm{L}}$ and $r^2\rho_{\mathrm{R}}$ be independent of U and V respectively, so that

$$4\pi r^2 \rho_{\mathrm{L}} = L_{\mathrm{L}}(V) = (\kappa_0 V)^{-2} \frac{\mathrm{d}m_{\mathrm{L}}(v)}{\mathrm{d}v} \,,$$

$$4\pi r^2 \rho_{\mathrm{R}} = L_{\mathrm{R}}(U) \tag{5}$$

The factor V^{-2} exhibits the two blueshift factors, of graviton energy and numerical flux, at the Cauchy horizon

The effect of these cross-flowing streams on the effective mass $m(U, V)$ can now be inferred from (2) and (4) Assuming the leftward and rightward fluxes turned on at times V_1 and U_1 respectively, one can easily obtain

$$m(U, V) = \int_{U_1}^{U} \int_{V_1}^{V} (-r' g'_{UV})^{-1} L_{\mathrm{R}}(U') L_{\mathrm{L}}(V') \, \mathrm{d}U' \, \mathrm{d}V' + m_{\mathrm{L}}(v) + m_{\mathrm{R}}(u) - m_1 \tag{6}$$

It follows from (5) that $m(U, V) \to \infty$ as $V \to 0$ from below, if $L_{\mathrm{R}}(U)$ is positive [The possibility that rg_{UV} may diverge in this limit is easily excluded by applying (4) to the equation $\Box \ln(-rg_{UV}) = r^{-4}(3e^2 - r^2)$]

Although this spherical model is highly idealized, the qualitative features responsible for mass-inflation remain operative in a generic, rotating collapse One may expect quite generally that the geometry becomes increasingly Schwarzschild-like, with a huge and growing mass parameter m, as the Cauchy horizon is approached. Asymmetries, whose angular scale is still of order unity at the Cauchy horizon, become locally insignificant compared with the extremely small (ultimately planckian) length-scale associated with m

Before the ambient curvature m/r^3 actually reaches Planck values, and breakup of the spacetime continuum invalidates the conventional picture of wave propagation, no mechanism for curbing the inflation appears able to function. For instance, depletion of the graviton inflow through scattering off bumps of the inflating background curvature never becomes effective, because size of the bumps – proportional to $|V|^{1/2}$ – never catches up to the blueshifted graviton wavelength, proportional to $|V|$

Transition from the classical to the quantum phase of evolution is marked by a hypersurface Σ_Q on which $m(U, V)/r^3 \approx 1$ (in Planck units), where $r(U, V) \approx r_0(U)$, the radius of the Cauchy horizon. On Planck time-scales, $r_0(U)$ is nearly constant, transverse irradiation by the (unblueshifted) stellar outflux generates contraction on a time-scale comparable with r_0 According to (6), $m(U, V)$ is an increasing function of both U and V, with $\partial m/\partial U \sim |V|^{-1}$, $\partial m/\partial V \sim V^{-2}$ Thus, Σ_Q lies within Planck distance from the Cauchy horizon and is spacelike. Unlike the situation in cosmology, the quantum phase of internal black hole evolution develops from a known and relatively well-defined initial state on Σ_Q

With the dynamics of the quantum evolution one enters a realm of speculation. The correct effective action for quantum gravity at planckian scales is unknown, for one thing But some of the possibilities can be enumerated

Inflow from the radiative tail of the collapse will stop, certainly beyond the Cauchy horizon and possibly just before it The later history of the geometry may reasonably be assumed to depend solely on r (here a timelike co-ordinate) The Kantowski–Sachs models then provide the simplest minisuperspace description

Quantum analyses of such models [10], based on the Einstein–Hilbert action for pure gravity, elicit nothing to ameliorate the classical inevitability of singularity formation within a proper time

$$r = \int_0^{r_0} \left(\frac{r}{2M}\right)^{1/2} \mathrm{d}r \approx 1 ,$$

where $M \approx r_0^3$ is the inflated mass They show that the wave function is exponentially damped for histories (e g with spacelike trajectories) not terminating in a singularity, illustrating Wheeler's "law of unanimity" [11] for the correspondence between classical and quantum singularities Nor does inclusion of scalar and vector fields [12,10] appear to offer realistic hope of escape from some form of singularity.

While use of the Einstein–Hilbert action under these extreme conditions is certainly unfounded, cosmological studies based on more general gravitational lagrangians has not yet progressed beyond the simplest (usually Robertson–Walker) models Heterotic string theory singles out the Gauss–Bonnet class of actions in ten dimensions [13] It would certainly be of interest to examine their implications for Kaluza–Klein cosmologies with four-dimensional Kantowski–Sachs sections

In the above scenarios, duration of the quantum phase is of the order of a Planck time. Taking account of vacuum polarization may conceivably prolong this to macroscopic scales If the axial component of vacuum stress along the cylindrical 3-spaces r=const. happens to be a tension, one can adduce arguments [4] suggesting that curvature within $r = r_0$ may be self-regulator, never overshooting the Planck threshold In that case the singularity is deferred to a time at least comparable with (and perhaps greatly exceeding) r_0.

The many uncertainties surrounding the quantum evolutionary phase will not be resolved soon But a definite picture of the evolution down to the Cauchy horizon appears to be emerging To the extent that the region of planckian curvature that fills the interior of this surface may be considered "singular" for ordinary physical purposes, it can be said that we have a complete picture of the internal history of a generic black hole up to the formation of a "fat cigar singularity" near the Cauchy horizon

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