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LETTER TO THE EDITOR

Structure of the black hole nucleus

E Poisson† and W Israel‡

Theoretical Physics Institute, Avadh Bhatia Physics Laboratory, University of Alberta, Edmonton, Alberta, Canada T6G 2J1

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Abstract. This letter explores different possibilities for the nuclear structure of a black hole formed by a collapse with zero angular momentum. If the stress induced by vacuum polarisation along the axes of the 3-cylinders r = constant is a tension rather than a pressure, the spacetime geometry could be self-regulatory and describable semiclassically down to radii of a few Planck units. The nucleus would then appear as an open string of roughly constant sub-Planckian density, with a thickness of order $(\hbar G^2 M/c^5)^{1/3}$—about 10^{-20} cm for a solar-mass black hole.

No fundamental issue has excited more speculation in recent years than the true nature of the singularities that signal the breakdown of the classical description of spacetime at the beginning of the universe and at endpoints of gravitational collapse.

Quantum fluctuations are expected to induce terms non-linear in curvature [1] in the effective gravitational Lagrangian, expanded in powers of $\hbar G = l_{\mathrm{Pl}}^2$. In default of a renormalisable theory of quantum gravity, the detailed form of the higher-loop corrections is uncertain. But this has not discouraged some intriguing speculations about their possible role in avoiding singularities [1, 2] and affecting, more or less drastically, the connectivity [3], metric signature [4] and dimensionality [5] of the geometry when curvatures approach the Planck scale.

A recurring theme, which has also come up in the context of minisuperspace models coupled to scalar fields [6, 7], is the possibility of a transition at Planck-scale curvatures to a de Sitter [2, 8] or, ultimately, a Euclidean de Sitter [6] phase.

A variant of this idea is implicit in a recent paper by Shen and Zhu [9]. They propose that the Schwarzschild exterior solution should be joined at the event horizon to a singularity-free de Sitter interior filled with fluid having the 'inflationary' stress tensor $T^{\mu\nu} = -\rho g^{\mu\nu}$, and they argue that this join is continuous. Unfortunately this is not so. De Sitter spacetime can never be joined directly to an exterior vacuum, since the O'Brien–Synge junction conditions [10] $T^{\mu\nu}N_\nu = 0$, expressing continuity of pressure at a boundary with normal N_ν, would be violated. It is necessary to interpose a layer of non-inflationary material at the interface. An elementary calculation shows that the transverse pressure for a static spherically symmetric join at $r = a$ is

$$P_\perp = -\rho\theta(a-r) + \tfrac{1}{2}\rho a\delta(r-a).$$

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Since $\delta(r-a)=(g_{rr})^{1/2}\delta(s)$ in terms of proper radial distance s and g_{rr} is singular if $r=a$ is light-like, the result shows that the surface pressure becomes singular (even when considered as a distribution) if an event horizon is chosen as the boundary. The same result can be deduced from a direct application of the theory of light-like surface layers [11] and confirms a conclusion previously stated by Grøn [12]. (The join made by Shen and Zhu introduces a discontinuity in transverse derivatives of r and hence of the intrinsic metric of the boundary.) Thus, a stationary horizon is not a suitable boundary for a transition to the de Sitter phase. But there is no objection in principle to locating such a transition elsewhere.

We take the opportunity to offer some comments on the general nature of the terminal phase and aftermath of a collapse. To an external observer, what happens within a black hole is strictly of academic interest, and discussions of singularities understandably focus on the big bang, with a more-or-less tacit understanding that the terminal singularities of collapse should not differ significantly. Forceful challenges to this time-symmetric viewpoint stem from Penrose [13] and Zel'dovich [14]. Penrose contrasts the chaos of a generic collapse with the high degree of order which seems to have prevailed in the very early universe. Zel'dovich has stressed that exponential damping of Higgs particle kinetic energies during an expanding phase makes the potential-dominated equation of state $P=-\rho$ natural and stable; during a collapsing phase, on the contrary, kinetic energies grow exponentially and $P=\rho$ is the stable equation of state.

For its part, the singularity of a black hole has a property that one would not expect to be shared by its cosmological counterpart. There appears to be a meaningful sense in which one can speak of its 'evolution' (with increasing advanced time) and relaxation to a 'final' state which, for zero angular momentum, is, on the average, isotropic. The strong cosmic censorship hypothesis [13] requires the singularity to be generically spacelike. It is therefore conventional to view the nucleus as a 'big crunch', a fracture in the classical geometry that appears at one stroke. However, the interior of the black hole inherits a natural time-ordering from exterior advanced time, and in this sense one may consider the fracture as developing progressively. At sufficiently late times this development is controlled almost exclusively by the evolution of the external field of the black hole (figure 1). (A disturbance cannot, of course, be conducted along the fracture itself since different points are not in causal contact.) Thus, as the external field relaxes to a Schwarzschild form, the decay of external asymmetries should be reflected near the singularity as a spatial damping with increasing distance t along the axis of the spacelike 3-cylinders $r=$ constant (where r, t are the usual Schwarzschild coordinates which have now exchanged their roles as space and time).

Decay of non-spherical perturbations near the centre of the black hole at late advanced times $v>v_1\gg 2M$ can be understood as a coalition of two effects. Infalling radiation could directly influence the tail segment $v>v_1$ of the central nucleus only for $v>v_1$, but by then the external field is already quiescent. Radiation flowing out of the collapsing star after it crossed the horizon, and outward scattering by the curvature of initially inflowing radiation while the field was settling down at early times, will also affect the nucleus. However, as the caption to figure 1 explains, the only part of this radiation that reaches the tail segment is the negligibly small amount beamed into a shell whose radial thickness at early times was exponentially small, of order $\Delta r\sim 2M\exp(-v_1/4M)$.

One thus arrives at a remarkably simple picture of an asymptotically stationary state for a zero angular momentum collapse in which spherical symmetry holds, not

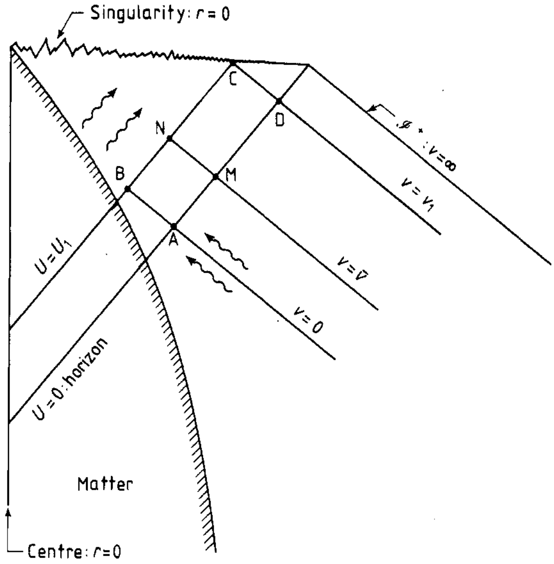

Figure 1. Collapse of a star with small aspherical perturbations to form a non-rotating black hole. Select a late retarded time v_1 and any intermediate time $\bar{v}$ such that $2M \ll \bar{v} \ll v_1$. Consider the sector ABCD within the horizon, given by $0 < v < v_1$, $0 < U < U_1$ (U and V are Kruskal coordinates, and $U_1 = V_1^{-1} = \exp(-v_1/4M)$). The value of a field perturbation δg at any interior point of this sector is given by the Green formula as an integral over the pair of characteristic initial 3-slices, AB and AMD, of the product of its initial values with a causal Green function. The integrand is regular and uniformly bounded when expressed as a function of r and v.

If the Green formula is first applied to the characteristic sector BAM, one infers that δg is of order $\bar{v}$ on the slice MN, given by $v = \bar{v}$. The radial thickness of this slice is exponentially small: $\overline{\Delta r} \approx 2M\bar{V}U_1 = 2M \exp(-v_1 + \bar{v})/4M$. It therefore contributes negligibly when the Green formula is applied a second time to the characteristic sector NMD. This yields the field δg at a point $r = \varepsilon$, $v = v_1$ near C. The contribution of the slice MD is at most of order $v\bar{v}^{-2}$, since an exterior perturbation decays like $v^{-(2l+2)}$ for a multipole of order l [15]. Thus, by choosing $\bar{v} \sim v_1^{2/3}$ for example, one verifies that $\delta g(r = \varepsilon, v_1) \to 0$ as $v_1 \to \infty$.

just externally, but down to the smallest radii at which classical notions of causality retain a meaning. In this picture, the geometry is described by the Schwarzschild vacuum solution down to the quantum barrier at radius $r_Q = M^{1/3}$ (about 10^{13} in Planck units or 10^{-20} cm for a solar-mass black hole), where the curvature M/r^3 grows to order unity. Below this may exist a layer $r_{QG} \lesssim r < r_Q$ of uncertain depth in which the geometry remains effectively classical and governed by field equations of the form

$$G^{\mu\nu} = 8\pi T^{\mu\nu}(\text{vacuum polarisation}) \sim R^2 \ldots \qquad (1)$$

representing one-loop vacuum polarisation effects of the gravitational and other quantised fields.

Attempts to probe the semiclassical layer are blocked above all by current ignorance of the composition of the GUT soup. However, some of the different possibilities that may occur can be evaluated.

Of decisive importance is the sign of T^t_t. Inside the horizon, $(-T^t_t)$ represents a tension along the axes of the spacelike 3-cylinders of constant time, $r = \text{constant}$. It determines the gravitational mass $M(r)$ interior to the radius r and the negative (time) component of the metric, $g^{rr} = -[2M(r) - r]/r$, through the field equation

$$M'(r) = 4\pi r^2(-T_t^t). \tag{2}$$

We set schematically

$$(-T_t^t) = (3a^2/4\pi)M^2(r)/r^6 \tag{3}$$

for $r \gg 1$, where a^2 is a coefficient of order unity (in Planck units), proportional to the effective number of fields, whose value is uncertain even as regards sign. The solution of (2) and (3) is

$$M(r)/r^3 = [a^2 + (r/r_Q)^3]^{-1}. \tag{4}$$

If a^2 is negative, the curvature rises steeply near $r = r_Q$, quickly overshooting the Planck threshold and precipitating a singularity of the classical geometry for $r = |a|^{2/3} r_Q$.

More can be said if a^2 is positive. In this case vacuum polarisation has a self-regulatory effect in accordance with the hundred year old principle of Le Châtelier [16]. According to (4), curvatures do not rise above the Planck limit [2], and the semiclassical layer extends downwards until, at $r \lesssim 1$, the inevitable rise of the contribution $(1 - g^{rr})^2 r^{-4}$ from the transverse curvature of the 3-cylinders $r =$ constant begins to dominate the right-hand side of (1) and the geometry finally becomes quantised.

Within the semiclassical layer the metric takes the de Sitter-like form

$$ds^2 \approx -\tfrac{1}{2}a^2\, d\tau^2 + e^{-2\tau}(d\Omega^2 + e^{2\psi(\tau)}\, dt^2)$$

where $\tau = -\ln r$ and $\psi(\tau)$ satisfies

$$\psi'(\tau) \approx 2\pi a^2(T_r^r - T_t^t).$$

Although valid only for $\tau \lesssim 0$ this metric suggests the interesting possibility of a 'long squeeze' replacing the 'big crunch': for an observer at rest in the spaces $r =$ constant, the encounter with the singularity is deferred to the remote future [17]. (But the singularity is accessible in finite proper time to inertial observers circulating about the 3-cylinders, because of time dilation effects.)

We note in conclusion that (4) leads to the appearance of an inner (Cauchy) horizon $r = 2M(r)$, $g^{rr} = 0$ for $r = 2^{-1/2}a$. If $a \lesssim 1$ this falls outside the domain of our semiclassical considerations, but if a should happen to be one or two orders of magnitude larger than 1, the instabilities associated with Cauchy horizons could lead to interesting new phenomena. However, we shall refrain from piling speculation upon speculation. It seems evident that it would be of considerable interest to know the detailed expression of (1)—or its asymptotic form for $r \ll M$—for the Hartle–Hawking state (the unique regular t-independent vacuum state) on a general spherically symmetric background.

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