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Inside charged black holes. II. Baryons plus dark matter

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This is the second of two companion papers on the interior structure of self-similar accreting charged black holes. In the first paper, the black hole was allowed to accrete only a single fluid of charged baryons. In this second paper, the black hole is allowed to accrete in addition a neutral fluid of almost non-interacting dark matter. Relativistic streaming between outgoing baryons and ingoing dark matter leads to mass inflation near the inner horizon. When enough dark matter has been accreted that the center-of-mass frame near the inner horizon is ingoing, then mass inflation ceases and the fluid collapses to a central singularity. A null singularity does not form on the Cauchy horizon. Although the simultaneous presence of ingoing and outgoing fluids near the inner horizon is essential to mass inflation, reducing one or the other of the ingoing dark matter or outgoing baryonic streams to a trace relative to the other stream makes mass inflation more extreme, not the other way around as one might naively have expected. Consequently, if the dark matter has a finite cross section for being absorbed into the baryonic fluid, then the reduction of the amount of ingoing dark matter merely makes inflation more extreme, the interior mass exponentiating more rapidly and to a larger value before mass inflation ceases. However, if the dark matter absorption cross section is effectively infinite at high collision energy, so that the ingoing dark matter stream disappears completely, then the outgoing baryonic fluid can drop through the Cauchy horizon. In all cases, as the baryons and the dark matter voyage to their diverse fates inside the black hole, they only ever see a finite amount of time pass by in the outside universe. Thus the solutions do not depend on what happens in the infinite past or future. We discuss in some detail the physical mechanism that drives mass inflation. Although the gravitational force is inward, inward means opposite direction for ingoing and outgoing fluids near the inner horizon. Mass inflation is driven by a feedback loop in which the general relativistic contribution to the gravitational force sourced by the radial pressure accelerates the ingoing and outgoing fluids through each other, which increases the radial pressure, which increases the gravitational force.

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I. INTRODUCTION

This is the second of two companion papers on similarity solutions for the interior structure of spherically symmetric charged black holes.

In a seminal paper, Poisson and Israel [1] showed that if ingoing and outgoing fluids are allowed to pass through each other inside a charged black hole, then the generic consequence is ‘‘mass inflation’’ as the counterstreaming fluids approach the inner horizon. During mass inflation, the interior mass, the Misner-Sharp mass [2], a gauge-invariant scalar quantity, exponentiates to an enormous value. The phenomenon of mass inflation has been confirmed analytically and numerically in many papers [3–9].

In the first paper of this pair, hereafter Paper 1 [10], the black hole was allowed to accrete a relativistic fluid of charged baryons. The baryons did not undergo mass inflation, precisely because, by construction, the single component baryonic fluid considered there was either ingoing or outgoing, not both.

In the present paper we allow the black hole to accrete, in addition to a charged baryonic fluid, a pressureless neutral ‘‘dark matter’’ fluid, which one can imagine as being cold dark matter, or hot dark matter (neutrinos), or even high frequency gravitational waves. The dark matter particles may be either massive or massless; it does not change the character of the solutions. The important thing is that the dark matter passes freely through the baryons, for the most part interacting with the baryons only by gravity, although we do consider what happens if the dark matter has a finite cross section for absorption by the baryons. As expected, relativistic counterstreaming between baryons and dark matter leads to mass inflation near the inner horizon.

Analytic and numerical work on spherically symmetric collapse and mass inflation has commonly modeled the fluid accreted by a black hole as a massless scalar field, usually uncharged [4–8,11–19], but also charged [20–25]. A key property of a massless scalar field is that it allows waves to counterstream relativistically through each other, which allows mass inflation to occur.

In the present paper we choose to adopt a somewhat different approach. A driving motivation for this paper and its companion, Paper 1, was to enquire what happens inside an astronomically realistic black hole, and it has seemed to us that a mix of baryons and dark matter might offer a more realistic description than a massless scalar field. Besides this, we wanted the freedom to explore what happens if the parameters of the model are changed: the electrical conductivity of the baryons (considered in Paper 1), massive versus massless dark matter, the ratio of accreted dark

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matter to baryonic density, or the interaction cross section between dark matter and baryons.

As in Paper 1, our goal is not so much to study the formation of a black hole by gravitational collapse, but rather to explore the interior structure of black holes after their formation. We have in mind the situation of a realistic astronomical black hole, perhaps stellar sized, perhaps supermassive, which is being fed by accretion of matter.

In a supermassive black hole especially, the mass that the black hole acquires by gradual accretion over the course of millions or billions of years can greatly exceed the mass of the seed black hole, formed perhaps from the collapse of the core of a massive star. Under such circumstances it is reasonable to expect that the bulk of the interior structure of the black hole is determined by the accretion history, rather than by the details of the initial collapse event.

The situation considered in the present paper is in a sense the opposite to that considered by Poisson and Israel [1] and others [3,4,7–9,20], who supposed that ingoing radiation or scalar field falling into a preexisting charged black hole would encounter a Price tail [8,20,26,27] of outgoing radiation generated during gravitational collapse. Because the black holes considered in the present paper are assumed to accrete self-similarly into the indefinite future, the theorems recently proven by Dafermos [20] concerning the collapse of isolated self-gravitating systems and the decay of Price tails do not apply. In the present paper, accreted charged baryons are repelled by the charge of the black hole self-consistently produced by previously accreted baryons, and naturally become outgoing. Any outgoing Price tail of radiation generated by gravitational collapse would soon be overwhelmed by the accreted charged baryons. For example, a black hole destined to become a quasar might radiate a fraction of a solar mass in a Price tail at the time it forms from the core collapse of a massive star, but may subsequently accrete 10^8 solar masses or more of baryons.

Since the bulk of the fluid inside the black hole is naturally outgoing, to produce mass inflation the black hole must be allowed to accrete a (small amount of) fluid which remains ingoing, and which streams more or less freely through the outgoing baryons. Dark matter fits the description nicely.

Many previous papers have found that the collapse of a massless scalar field into a charged black hole produces not only a strong spacelike singularity at zero radius but also a weak null singularity at finite radius along the Cauchy horizon [3–5,7,9,17,22,23,28], as already anticipated by Poisson and Israel [1]. A null singularity does not form in the similarity solutions. Why this should be is discussed in Sec. V E.

An important question considered in the present paper is what happens if dark matter and baryons have a finite cross section for interaction at high energy. For mass inflation to persist, and, in particular, for a null singularity to form on the Cauchy horizon, it is necessary that counterstreaming ingoing and outgoing fluids accelerate to arbitrarily close to the speed of light (or become arbitrarily highly blueshifted, for massless streams) relative to each other. This raises the question of whether it is physically realistic to allow counterstreaming at arbitrarily large Lorentz factors. To address this question, we explore the consequences of allowing the dark matter to have a nonzero cross section for absorption by baryons at high collision energy.

The structure of this paper is as follows. Section II, which is a follow-on to Sec. II of Paper 1, presents the general relativistic equations governing the interior and exterior structure of a spherically symmetric black hole that accretes dark matter in addition to charged baryons. Section III brings in the hypothesis of self-similarity, and sets out the equations that follow from that hypothesis, generalizing Sec. III of Paper 1. Section IV gives results for self-similar black holes accreting baryons and dark matter. Section V discusses the physical question of why mass inflation occurs. Section VI addresses the question of what it would actually look like if you fell into one of the black holes described herein. Finally, Sec. VII summarizes the findings of this paper.

II. EQUATIONS

This section presents the general relativistic equations governing a spherically symmetric black hole accreting almost noninteracting dark matter in addition to charged, electrically conducting baryons. The required formalism has already been developed for the most part in Sec. II of Paper 1 [10], to which the reader is referred for notation and further detail.

A. Frames

It proves convenient to work in the rest frame of the baryons, with the dark matter streaming at 4-velocity u_d^m relative to the tetrad frame of the baryons. To be consistent with spherical symmetry, the dark matter must stream radially, so the nonvanishing components of the dark matter 4-velocity u_d^m are the time and radial components

$$u_d^0 = u_d^t, \qquad u_d^i = u_d^r \hat{x}_i. \tag{1}$$

Without loss of generality, the rest mass μ of the dark matter particle can be taken to be either 0 or 1, depending on whether the particle is massless or massive. Thus

$$(u_d^t)^2 - (u_d^r)^2 = \mu^2, \qquad \mu = \begin{cases} 0 & \text{massless} \\ 1 & \text{massive.} \end{cases} \tag{2}$$

Below we will generally present arguments as if the dark matter particle were massive. The case of a massless particle follows from letting the massive particle approach the speed of light and simultaneously letting its rest mass go to zero. Appropriate factors of rest mass μ are included in the formulas below.

Adapted to the dark matter frame is an associated tetrad frame, with inertial axes $\boldsymbol{\gamma}_{d,m}$, corresponding vierbein coefficients α_d, β_d, and γ_d, Eq. (6) of Paper 1, and corresponding time coordinate t_d. The dark matter time coordinate t_d differs from the baryonic time coordinate t because the gauge of time is being chosen, Eq. (5) of Paper 1, so that the proper radial derivative of time is zero; that is, the dark matter time coordinate t_d is arranged to satisfy $\partial_{d,r} t_d = 0$, whereas the baryonic time coordinate t satisfies $\partial_r t = 0$.

The locally inertial axes $\boldsymbol{\gamma}_{d,m}$ in the rest frame of the dark matter are related to the locally inertial axes $\boldsymbol{\gamma}_m$ in the baryonic frame by a Lorentz boost at 4-velocity u_d^m. It follows that the directed derivatives in the dark matter frame, $\partial_{d,t} \equiv \boldsymbol{\gamma}_{d,t} \cdot \boldsymbol{\partial}$ and $\partial_{d,r} \equiv \boldsymbol{\gamma}_{d,r} \cdot \boldsymbol{\partial}$, are equal to the directed derivatives in the baryonic frame Lorentz boosted by 4-velocity u_d^m:

$$\partial_{d,t} = u_d^t \partial_t + u_d^r \partial_r, \qquad \partial_{d,r} = u_d^r \partial_t + u_d^t \partial_r \tag{3}$$

or more explicitly

$$\begin{aligned} \alpha_d \frac{\partial}{\partial t_d}\bigg|_r + \beta_d \frac{\partial}{\partial r}\bigg|_{t_d} &= u_d^t\bigg(\alpha \frac{\partial}{\partial t}\bigg|_r + \beta \frac{\partial}{\partial r}\bigg|_t\bigg) + u_d^r \gamma \frac{\partial}{\partial r}\bigg|_t \\ \gamma_d \frac{\partial}{\partial r}\bigg|_{t_d} &= u_d^t \gamma \frac{\partial}{\partial r}\bigg|_t + u_d^r\bigg(\alpha \frac{\partial}{\partial t}\bigg|_r + \beta \frac{\partial}{\partial r}\bigg|_t\bigg). \end{aligned} \tag{4}$$

Applying Eqs. (4) to the radial coordinate r shows that β_d and γ_d are equal to β and γ Lorentz boosted by 4-velocity u_d^m

$$\beta_d = \beta u_d^t + \gamma u_d^r, \qquad \gamma_d = \gamma u_d^t + \beta u_d^r. \tag{5}$$

Equations (5) reflect the fact that $(\beta, \gamma) = (\partial_t r, \partial_r r)$ form the time and radial components of a covariant 4-vector, the radial 4-gradient, as already remarked in Eq. (12) of Paper 1. The magnitude squared of the covariant 4-vector (β_d, γ_d) is

$$\beta_d^2 - \gamma_d^2 = \mu^2(\beta^2 - \gamma^2) \tag{6}$$

which is null for massless dark matter, $\mu^2 = 0$, or the same as the magnitude squared of (β, γ) for massive dark matter, $\mu^2 = 1$.

Similarly, applying Eqs. (4) to the time coordinate t_d yields the result that

$$\frac{\partial t_d}{\partial t}\bigg|_r = \frac{\alpha_d \gamma_d}{\alpha \gamma}, \qquad \frac{\partial t_d}{\partial r}\bigg|_t = -\frac{\alpha_d u_d^r}{\gamma}. \tag{7}$$

For self-similar solutions, Eqs. (7) translate into a second explicit relation [the first being Eqs. (5)] between the dark matter and baryon vierbein coefficients. The second relation, Eq. (8) below, is most transparently derived not directly from Eqs. (7), but rather from the transformation of the homothetic 4-vector introduced in Sec. III B of Paper 1 [10]. The components ξ_d^m of the homothetic vector in the tetrad frame of the dark matter are equal to those in the baryonic frame Lorentz boosted by the 4-velocity u_d^m of the dark matter relative to the baryons

$$\xi_d^t = \frac{1}{\eta_d} = \xi^t u_d^t - \xi^r u_d^r, \qquad \xi_d^r = \frac{V_d}{\eta_d} = \xi^r u_d^t - \xi^t u_d^r. \tag{8}$$

The magnitude squared of the homothetic vector in the dark matter frame is

$$(\xi_d^t)^2 - (\xi_d^r)^2 = \mu^2 H \tag{9}$$

which is null for massless dark matter, $\mu^2 = 0$, or equal to the homothetic scalar H, Eq. (58) of Paper 1, for massive dark matter, $\mu^2 = 1$.

B. Einstein equations

In Sec. II A of Paper 1, the tetrad formalism for spherically symmetric geometry was set up and the Einstein and Weyl tensors derived, and in Sec. II B of Paper 1, the resulting Einstein equations were written down for the case where the tetrad frame was taken to be the center-of-mass frame, defined to the frame where the momentum density vanishes. In the present subsection the Einstein equations are given for a radially moving tetrad frame, such as the baryonic frame, which is not necessarily the center-of-mass frame. The Einstein equations (12) in the moving frame are not as physically transparent as those, Eqs. (34) of Paper 1, in the center-of-mass frame.

The most general form of the energy-momentum tensor T_{mn} consistent with spherical symmetry is

$$\begin{aligned} T_{00} &= T^{tt}, \qquad T_{0i} = -T^{tr}\hat{x}_i, \\ T_{ij} &= T^{rr}\hat{x}_i\hat{x}_j + T^{\perp\perp}(\delta_{ij} - \hat{x}_i\hat{x}_j). \end{aligned} \tag{10}$$

Substituting the Einstein tensor, Eq. (26) of Paper 1, into Einstein's equations $G_{mn} = 8\pi T_{mn}$ implies that the quantities F, R, P, and S defined by Eqs. (25) of Paper 1 are

$$\begin{aligned} R &= 4\pi T^{tt}, \qquad F = 4\pi T^{tr}, \\ P &= 4\pi T^{rr}, \qquad S = 4\pi(T^{\perp\perp} - T^{rr}). \end{aligned} \tag{11}$$

The Einstein equations thus become

$$\partial_r M - 4\pi r^2(\gamma T^{tt} + \beta T^{tr}) = 0, \tag{12a}$$

$$\partial_t \gamma - \beta g - 4\pi r T^{tr} = 0, \tag{12b}$$

$$\partial_t \beta - \gamma g + \frac{M}{r^2} + 4\pi r T^{rr} = 0, \tag{12c}$$

$$\begin{aligned} &\partial_t T^{tr} + \partial_r T^{rr} - \frac{2\gamma}{r}(T^{\perp\perp} - T^{rr}) \\ &\quad + 2\bigg(\frac{\beta}{r} + h\bigg)T^{tr} + g(T^{tt} + T^{rr}) = 0. \end{aligned} \tag{12d}$$

Equation (12b) implies that the quantity h, defined by Eq. (22) of Paper 1, satisfies

$$h = \frac{\partial \beta}{\partial r} - \frac{4\pi r T^{tr}}{\gamma}. \tag{13}$$

The result of taking β times Eq. (12c) minus γ times Eq. (12b) is the first law of thermodynamics for the combined baryonic and dark matter fluid

$$\partial_t M + 4\pi r^2(\gamma T^{tr} + \beta T^{rr}) = 0. \tag{14}$$

Einstein's equations automatically incorporate conservation of energy momentum, as expressed by the vanishing of the covariant derivative of the energy-momentum tensor, $D_m T^{mn} = 0$. The time ($n = 0$) component of the energy-momentum conservation equation gives the energy conservation equation

$$\partial_t T^{tt} + \partial_r T^{tr} + \frac{2\beta}{r}(T^{\perp\perp} - T^{rr}) + 2\left(\frac{\gamma}{r} + g\right)T^{tr} + \left(\frac{2\beta}{r} + h\right)(T^{tt} + T^{rr}) = 0 \tag{15}$$

while the spatial components ($n = 1, 2, 3$) of the energy-momentum conservation equation give a momentum conservation equation which reduces precisely to the Euler equation (12d).

C. Dark matter

We take the dark matter (subscripted d) to be a pressureless fluid (dust, either massive or massless) that free falls radially into the black hole. The energy-momentum tensor of the dark matter is thus

$$T_d^{mn} = \rho_d u_d^m u_d^n \tag{16}$$

whose nonzero components are

$$T_d^{tt} = \rho_d (u_d^t)^2, \qquad T_d^{tr} = \rho_d u_d^t u_d^r, \qquad T_d^{rr} = \rho_d (u_d^r)^2. \tag{17}$$

We assume that the dark matter is almost noninteracting, for the most part streaming freely through the baryons, but we wish to consider the possibility, Sec. II E, that dark matter particles interact with baryons when they pass through each other at sufficiently high energy. For simplicity, we assume that dark matter particles that interact with the baryons are simply absorbed into the baryonic fluid, adding their energy and momentum into the baryons, which retain their isotropic pressure and relativistic equation of state. If the rate, the mass per unit volume per unit time, at which dark matter is absorbed into the baryonic fluid is denoted $\dot{\rho}_d$, in the frame of reference of the dark matter, then the equations of energy-momentum conservation for the dark matter are

$$D_m T_d^{mn} = -\dot{\rho}_d u_d^n. \tag{18}$$

For the energy-momentum tensor of Eq. (16), the energy-momentum conservation equations (18) are equivalent to two equations, first, the equation describing unaccelerated free fall of the dark matter,

$$\frac{D u_d^n}{D\tau_d} = 0 \tag{19}$$

where $D/D\tau_d = u_d^m D_m$, and second, the equation for conservation of dark matter rest mass

$$D_m(\rho_d u_d^m) = -\dot{\rho}_d. \tag{20}$$

The free-fall equation (19) gives

$$\partial_{d,t} u_d^r + u_d^t (g u_d^t + h u_d^r) = 0 \tag{21}$$

while Eq. (20) for conservation of dark matter rest mass is

$$\left(\partial_t + \frac{2\beta}{r} + h\right)(\rho_d u_d^t) + \left(\partial_r + \frac{2\gamma}{r} + g\right)(\rho_d u_d^r) = -\dot{\rho}_d. \tag{22}$$

D. Baryons and electric field

The baryons (subscripted b) and the electric field (subscripted e) are coupled. As remarked in Sec. II A, it is simplest to work in the rest frame of the baryons, where the momentum density of baryons is zero $T_b^{tr} = 0$. The energy-momentum tensor of the baryons in the tetrad frame of the baryons is diagonal, with nonzero components (cf. Sec. II C of Paper 1)

$$T_b^{tt} = \rho_b, \qquad T_b^{rr} = T_b^{\perp\perp} = p_b \tag{23}$$

in which the density ρ_b and pressure p_b are assumed to be related by a relativistic equation of state

$$p_b = w\rho_b, \qquad w = \frac{1}{3}. \tag{24}$$

The energy-momentum tensor of the electric field is similarly diagonal, with nonzero components (cf. Sec. II D of Paper 1)

$$T_e^{tt} = -T_e^{rr} = T_e^{\perp\perp} = \rho_e \tag{25}$$

where the electric energy density is $\rho_e = Q^2/(8\pi r^4)$, with Q the charge interior to radius r.

Together, the coupled baryons and electric field satisfy the energy-momentum conservation equation

$$D_m(T_b^{mn} + T_e^{mn}) = \dot{\rho}_d u_d^n, \tag{26}$$

the right-hand side of which represents the energy momentum dumped into the baryonic fluid as the result of dark matter being absorbed by baryons. The time ($n = 0$) and space ($n = 1, 2, 3$) components of Eq. (26) yield the energy and momentum conservation equations for the baryons coupled to the electric field

$$\partial_t(\rho_b + \rho_e) + \frac{4\beta}{r}\rho_e + (\rho_b + p_b)\left(\frac{2\beta}{r} + h\right) = \dot{\rho}_d u_d^t \tag{27}$$

$$\partial_r(p_b - \rho_e) - \frac{4\gamma}{r}\rho_e + (\rho_b + p_b)g = \dot{\rho}_d u_d^r. \tag{28}$$

E. Interaction between dark matter and baryons

We assume that the rate $\dot{\rho}_d$ at which dark matter is absorbed into baryons is proportional to the density ρ_d of dark matter particles, multiplied by an absorption rate per particle σ_d

$$\dot{\rho}_d = \rho_d \sigma_d. \tag{29}$$

In general the rate σ_d will be an integral over collision energy (in the dark matter frame) of the number density of baryons times the absorption cross section times the collision velocity. We treat the absorption rate σ_d as a phenomenological quantity, the aim being to explore what happens as the properties of the absorption rate are varied. If the absorption rate σ_d is assumed to be some function of the baryonic density ρ_b and of the 4-velocity u_d^m between the dark matter and the baryons, then self-similarity requires that the absorption rate be proportional to the square root of the baryonic density, similarly to the electrical conductivity, Eq. (48) of Paper 1, and otherwise to be some arbitrary function of the 4-velocity. Thus the absorption rate σ_d is taken to be

$$\sigma_d = \kappa_d (4\pi\rho_b)^{1/2} \tag{30}$$

where κ_d is a phenomenological dimensionless rate coefficient, which in general could be some arbitrary function of the 4-velocity u_d^m between dark matter and baryons. The factor of 4π in Eq. (30) is introduced to simplify the corresponding self-similar equation (34).

III. SIMILARITY SOLUTIONS

A. Similarity hypothesis

As noted in Paper 1, dimensional analysis reveals the following quantities to be dimensionless [the following equation repeats Eqs. (49) and (50) of Paper 1]:

$$\begin{gathered} \eta \equiv \frac{\alpha r}{t}, \qquad \beta, \qquad \gamma, \qquad \frac{M}{r}, \qquad \frac{Q}{r}, \qquad y \equiv gr, \\ z \equiv 4\pi r^2 \rho_b, \qquad z_q \equiv 4\pi r^2 q, \qquad s \equiv 4\pi r \sigma, \\ z_e \equiv 4\pi r^2 \rho_e = \frac{Q^2}{2r^2}, \end{gathered} \tag{31}$$

where the dimensionless conductivity s is

$$s = \kappa z^{1/2}. \tag{32}$$

With dark matter adjoined, dimensional analysis of the previous equations, combined with Eq. (20) for conservation of dark matter energy momentum, and Eq. (29) for the dark matter absorption rate, shows that the following dark matter quantities are likewise dimensionless:

$$\begin{gathered} \eta_d \equiv \frac{\alpha_d r}{t_d}, \qquad \beta_d, \qquad \gamma_d, \qquad u_d^m, \\ z_d \equiv 4\pi r^2 \rho_d, \qquad s_d \equiv r\sigma_d, \end{gathered} \tag{33}$$

where the dimensionless dark matter absorption rate s_d is, for the phenomenological dark matter absorption rate σ_d given by Eq. (30),

$$s_d = \kappa_d z^{1/2}. \tag{34}$$

The dimensionless variables of Eqs. (31) and (33) form a (more than) complete set for the problem at hand, and it follows [29] that the spherically symmetric Einstein-Maxwell and subsidiary equations admit similarity solutions in which the dimensionless variables are all functions of a single dimensionless variable.

As shown in Paper 1, Eq. (53), the proper radial velocity V of the similarity frame relative to the baryonic tetrad frame is

$$V = \frac{\eta - \beta}{\gamma}. \tag{35}$$

The radial velocity V_d of the similarity frame relative to the dark matter tetrad frame is similarly

$$V_d = \frac{\eta_d - \beta_d}{\gamma_d}. \tag{36}$$

B. Integrals of the similarity equations

The ordinary differential equations determining the self-similar evolution of the baryonic and dark matter fluids admit four integrals, of which three are generalizations of the three integrals given in Sec. III F of Paper 1, and the fourth is an integral for the dark matter.

The first integral follows from Eq. (87) of Paper 1, which here implies

$$\frac{M}{r} = z(\gamma\xi^r - w\beta\xi^t) + z_e + z_d\gamma_d\xi_d^r. \tag{37}$$

Equation (37) differs from the corresponding Eq. (88) of Paper 1 by the addition of the last, dark matter, term on the right-hand side. As in Paper 1, we use Eq. (37) not as one of the evolutionary equations, but rather as a check on the accuracy of the integration.

The second integral of the similarity equation is unchanged from Paper 1. The integral follows from Eq. (89) of Paper 1, which yields an equation for the dimensionless charge density $z_q \equiv 4\pi r^2 q$ [the following repeats Eq. (90) of Paper 1]

$$z_q = \frac{Q(1 + s\xi^t)}{r\xi^r}. \tag{38}$$

The third integral of the similarity equations follows from Eq. (91) of Paper 1, which here yields a revised equation for the dimensionless proper acceleration $y \equiv gr$

$$y = \frac{\{2w\xi^r M/r + 2z_e\xi^r[(1-w)+(1+w)s\xi^t] + z_d\xi^r[-2w\gamma_d\xi^r_d + s_d(w\xi^t u^t_d + \xi^r u^r_d)] - w(1+w)z\xi^t[(1+w)z\xi^t\xi^r + z_d\xi^t_d\xi^r_d]}{(1+w)z[(\xi^r)^2 - w(\xi^t)^2]}. \tag{39}$$

Equation (39) differs from the corresponding Eq. (92) of Paper 1 by the addition of various dark matter terms in the numerator on the right-hand side.

A fourth integral follows from the geodesic integral of motion (62) of Paper 1, which applies to the freely falling dark matter. The homothetic momentum, Eq. (63) of Paper 1, of the freely falling dark matter is $v_{\ln t} = r\xi_m u^m_d = -r\xi^t_d = -t_d/\alpha_d$, and the integral of motion (62) of Paper 1 then shows that

$$\frac{t_d}{\alpha_d} = \tau_d. \tag{40}$$

Equation (40) implies that

$$\xi^t_d = \frac{\tau_d}{r}. \tag{41}$$

C. Similarity differential equations

As in Paper 1, we adopt as a suitable dimensionless integration variable the dimensionless baryonic time parameter x defined by Eq. (93) of Paper 1. The baryonic proper time τ, time coordinate t, and radial coordinate r evolve along the path of the baryonic fluid according to the same equations as before, Eqs. (94) of Paper 1, which we repeat here for completeness:

$$\frac{d\tau}{dx} = r, \tag{42a}$$

$$\frac{d\ln t}{dx} = \eta, \tag{42b}$$

$$\frac{d\ln r}{dx} = \beta. \tag{42c}$$

Equation (42b) presumes that the gauge of baryonic time t is chosen in the natural way, such that the units of time are the same as the units of radius, so that r/t is a dimensionless variable.

Similarly, the dark matter proper time τ_d, time coordinate t_d, and radial coordinate r_d evolve along the path of the dark matter fluid as

$$\frac{d\tau_d}{dx} = \frac{\mu^2 r_d}{k}, \tag{43a}$$

$$\frac{d\ln t_d}{dx} = \frac{\mu^2\eta_d}{k}, \tag{43b}$$

$$\frac{d\ln r_d}{dx} = \frac{\beta_d}{k} \tag{43c}$$

where

$$k \equiv \frac{\xi^r_d}{\xi^r}. \tag{44}$$

Note that the radial coordinate r_d in Eq. (43c) is distinguished from the radial coordinate r in Eq. (42c), because the integration is along the path of the dark matter in Eq. (43c) as opposed to the path of the baryons in Eq. (42c). With no charge to repel their infall, dark matter particles fall in faster than baryons, so that the dark matter radius r_d is less than the baryonic radius r at any given self-similar point inside the outer boundary. For example, if r_d is half of r at a given point, it means that the black hole was twice as large when it accreted the dark matter as it was when it accreted the baryons.

The differential equation (43b) for the dark matter time coordinate t_d again presumes that the gauge of t_d is chosen such that r_d/t_d is a dimensionless variable. Equations (43a) and (43b), along with $\eta_d = 1/\xi^t_d = r_d/\tau_d$ from Eq. (41), imply that $t_d \propto \tau_d$, which together with Eq. (40) implies that α_d is a constant. The constancy of α_d can also be regarded as following from the fact that the acceleration $g_d \equiv -\partial_{d,r}\ln\alpha_d$, cf. Eq. (21) of Paper 1, vanishes for freely falling dark matter, $g_d = 0$; the vanishing of the radial derivative of α_d, coupled with self-similarity, implies that the total derivative $d\alpha_d/dx$ vanishes. It is natural to adopt the gauge choice $\alpha_d = 1$, so that the dark matter time coincides with dark matter proper time, $t_d = \tau_d$. However, the dark matter time t_d (as distinct from dark matter proper time τ_d) is not actually used in this paper.

An overcomplete set of equations [only three of the four equations (45) below are independent, the four variables ξ^t, ξ^r, γ, and β being related by $\beta\xi^t + \gamma\xi^r = 1$, Eq. (47a) below] governing the self-similar evolution of the remaining variables is [the following generalize Eqs. (95) of Paper 1]

$$\frac{d\xi^t}{dx} = -y\xi^r + \gamma\xi^r \tag{45a}$$

$$\frac{d\xi^r}{dx} = -y\xi^t - \beta\xi^r + (1+w)z\xi^t\xi^r + z_d\xi^t_d\xi^r_d \tag{45b}$$

$$\frac{d\gamma}{dx} = \beta y + z_d u^t_d u^r_d \tag{45c}$$

$$\frac{d\beta}{dx} = \gamma y - (1+w)z\gamma\xi^r - z_d[\gamma_d\xi^r_d + (u^r_d)^2] \tag{45d}$$

together with [the following generalize Eqs. (96) of Paper 1]

$$\frac{d\ln[r^{1+3w}z(\xi^r)^{1+w}]}{dx} = \frac{2z_e s}{z} + \frac{z_d s_d u^t_d}{z} \tag{46a}$$

$$\frac{d\ln Q}{dx} = -s \tag{46b}$$

$$\frac{d\ln(r_d z_d\xi^r_d)}{dx} = -\frac{s_d}{k}. \tag{46c}$$

To maintain numerical precision, it is important to avoid expressing small quantities as differences of large quantities. For example, $\xi^t + \xi^r = (1+V)/\eta$ can be tiny near the Cauchy horizon, $V \approx -1$, where mass inflation occurs, though ξ^t and ξ^r are individually substantial. A suitable choice of variables to integrate is $\xi^t + \xi^r$, $\beta - \gamma$, γ, τ_d and r_d, the last two giving ξ^t_d according to Eq. (41). Starting from these variables, the following chain of equations yields the remaining variables in a fashion that ensures numerical precision [the following equations generalize Eqs. (97) of Paper 1]:

$$\xi^t - \xi^r = \frac{2 - (\beta+\gamma)(\xi^t + \xi^r)}{\beta - \gamma} \tag{47a}$$

$$H = (\xi^t + \xi^r)(\xi^t - \xi^r) \tag{47b}$$

$$\frac{2M}{r} = 1 + (\beta+\gamma)(\beta-\gamma) \tag{47c}$$

$$\xi^r_d = [(\xi^t_d)^2 - \mu^2 H]^{1/2} \tag{47d}$$

$$u^t_d - u^r_d = \frac{\xi^t_d + \xi^r_d}{\xi^t + \xi^r} \tag{47e}$$

$$u^t_d + u^r_d = \frac{\mu^2}{u^t_d - u^r_d} \tag{47f}$$

$$\beta_d \pm \gamma_d = (\beta \pm \gamma)(u^t_d \pm u^r_d). \tag{47g}$$

The differential equation for the variable X used in ray tracing, Sec. III D of Paper 1, remains unchanged from Paper 1 [the following repeats Eq. (98) of Paper 1]

$$\frac{dX}{dx} = -\xi^r. \tag{48}$$

During mass inflation, the interior mass M and (the absolute value of) the radial streaming 4-velocity u^r_d increase exponentially, while (the absolute value of) the homothetic scalar H decreases exponentially. The following differential equations, which are consequences of the equations above, are useful for characterizing mass inflation. The interior mass M, which satisfies $2M/r - 1 = \beta^2 - \gamma^2$, Eq. (24) of Paper 1, evolves as

$$\frac{d\ln|2M/r - 1|}{dx} = -8\pi r^2 T^{tr}_\gamma \xi^r \tag{49}$$

where T^{tr}_γ is the proper momentum density relative to the $\gamma = 0$ frame

$$4\pi r^2 T^{tr}_\gamma = \frac{(1+w)z\beta\gamma + z_d \beta_d \gamma_d}{2M/r - 1}. \tag{50}$$

The homothetic scalar $H \equiv (\xi^t)^2 - (\xi^r)^2$, Eq. (58) of Paper 1, evolves as

$$\frac{d\ln|H|}{dx} = \left(\frac{2(\gamma\xi^t + \beta\xi^r)}{H} + 8\pi r^2 T^{tr}_\xi\right)\xi^r \tag{51}$$

where T^{tr}_ξ is the momentum density relative to the no-going $\xi^t = 0$ frame, the frame of reference at the border between ingoing (positive ξ^t) and outgoing (negative ξ^t),

$$4\pi r^2 T^{tr}_\xi = \frac{(1+w)z\xi^t\xi^r + z_d \xi^t_d \xi^r_d}{-H}. \tag{52}$$

The 4-velocity u^r_d of the dark matter through the baryons evolves as

$$\frac{d\ln|u^r_d|}{dx} = \left(-\frac{y}{u^r_d} + \frac{4\pi r^2 H T^{tr}_\xi}{\xi^r_d}\right)u^t_d. \tag{53}$$

D. Boundary conditions at the outer sonic point

As in Paper 1, the boundary conditions of the calculation are set at an outer boundary, taken to be a regular sonic point, outside the outer horizon of the black hole, where the infalling baryonic fluid transitions smoothly from subsonic to supersonic.

Two boundary conditions at the outer sonic point are carried over from Sec. III H of Paper 1, namely, the accretion rate η_s, and the charge-to-mass ratio Q/M_c of the black hole.

Dark matter adds two more boundary conditions, which set the velocity and density of dark matter at the outer sonic point. If there were no mass or charge outside the outer sonic point, then dark matter that free falls from zero velocity at infinity would have $\gamma_d = 1$, and we adopt this value as a natural choice for setting the velocity of the dark matter at the outer boundary:

$$\gamma_d = 1 \quad \text{at the outer sonic point.} \tag{54}$$

The second dark matter boundary condition is the value of the ratio ρ_d/ρ_b of dark matter to baryonic proper mass densities at the outer sonic point, which we vary as discussed in Sec. IV A below.

IV. RESULTS

Section IV of Paper 1 presented results for black holes which accrete a single fluid of charged baryons. The baryons either plunged to the singularity, or else they dropped through the Cauchy horizon, but mass inflation did not occur. This section presents results for black holes which accrete dark matter in addition to baryons, with the aim of exploring the phenomenon of mass inflation.

In Sec. IV A, the dark matter will be assumed to have zero cross section for absorption by baryons. In Sec. IV B, the dark matter will be given a nonzero cross section for absorption by baryons.

As in Paper 1, geometric units $G = c = M_c = 1$ are used, where M_c, Eq. (100) of Paper 1, is the charge-augmented interior mass of the black hole evaluated at the outer boundary, the outer sonic point.

A. Noninteracting dark matter

Figure 1 shows results for a black hole which accretes both charged, nonconducting baryons, and neutral, pressureless, noninteracting dark matter. The dark matter particles here are assumed to be massive, but the results for massless dark matter particles are quite similar. With these assumptions, there are three free parameters set at the outer boundary, the outer sonic point. The accretion rate η_s is set equal to 0.1, the same as adopted in the models of Paper 1, and the charge-to-mass ratio Q/M_c is set equal to 0.8, the same as adopted in the charged models of Paper 1. The third parameter, a new parameter, is the ratio ρ_d/ρ_b of dark matter to baryonic proper mass densities, which we set equal to 0.1 at the outer sonic point

$$\frac{\rho_d}{\rho_b} = 0.1. \tag{55}$$

In an astronomically realistic black hole, the density of accreted dark matter is expected to be only a small fraction of the density of accreted baryons, because whereas baryons can dissipate energy and angular momentum, which allows them to funnel on to a black hole, non- or weakly interacting dark matter cannot so dissipate. However, as with the other two free parameters η_s and Q/M_c, we deliberately choose a large value of ρ_d/ρ_b to make it easier to discern its effects (and to avoid the risk of numerical problems). At fixed $\eta_s = 0.1$ and $Q/M_c = 0.8$, the dark matter to baryonic density ratio is limited to $\rho_d/\rho_b \lesssim 0.4362$ (see Fig. 2), otherwise there is too much neutral dark matter diluting the baryons, and the desired charge-to-mass $Q/M_c = 0.8$ cannot be achieved.

As expected, Fig. 1 shows that mass inflation occurs just above the inner horizon. During mass inflation, the interior mass M increases by many orders of magnitude over a modest range of radii, $r \approx 0.372$–0.1. As the figure shows, mass inflation in due course ceases, for reasons discussed in Sec. V D below. Before mass inflation sets in, the solution with dark matter resembles the solution without dark matter, Fig. 4 of Paper 1. As discussed in Sec. IV B of Paper 1, the geometry of the solution without dark matter in turn resembles, outside the inner horizon, that of a vacuum charged black hole, the Reissner-Nordström geometry.

After mass inflation has stalled, the outgoing baryonic and ingoing dark matter fluids collapse to a spacelike singularity at zero radius. The Penrose diagram of the black hole is the same as that shown in Fig. 3 of Paper 1.

Figure 1 shows that the end of mass inflation coincides roughly with the homothetic scalar H reaching a minimum in absolute value. The homothetic scalar $H = (1 - V^2)/\eta$ is zero at the inner horizon, where $V = -1$, and can be interpreted as offering a gauge-invariant measure of how close to the inner horizon the mass-inflating fluid has reached.

During mass inflation, the interior mass M increases approximately exponentially while the radius r decreases

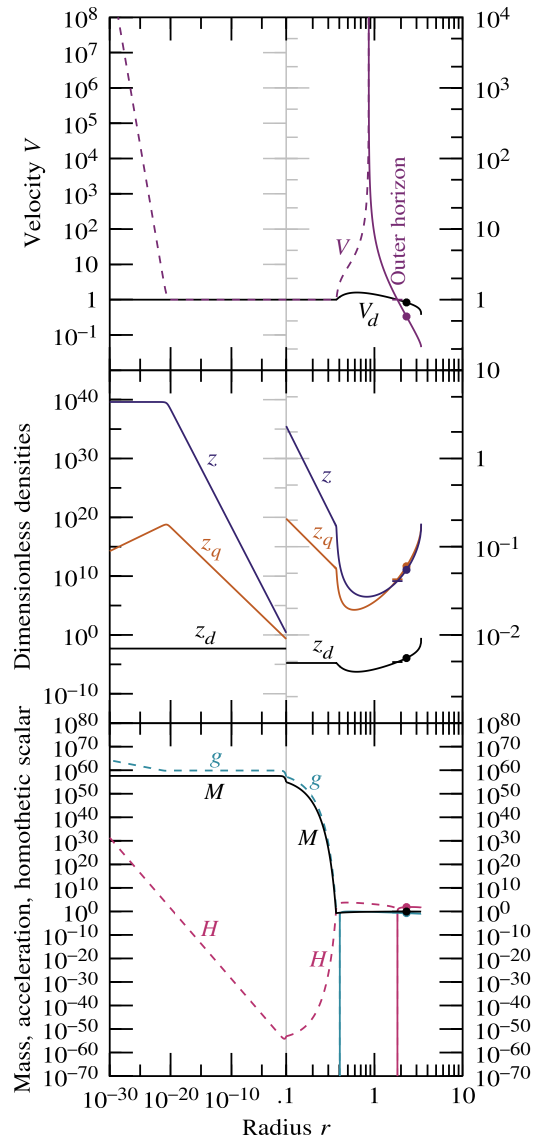

FIG. 1 (color online). A black hole with the same parameters, an accretion rate $\eta_s = 0.1$ and a charge-to-mass $Q/M_c = 0.8$, as that shown in Fig. 4 of Paper 1, except that here the black hole accretes dark matter in addition to baryons, with density ratio $\rho_d/\rho_b = 0.1$ at the outer sonic point. Quantities are plotted against radius r in units where the charge-augmented mass at the outer sonic point is unity, $M_c = 1$. To reveal more detail, the radial axis is split into two regimes with different horizontal and vertical scales. Lines are dashed where quantities are negative. Disks mark the outer sonic point, where $V = \sqrt{w} \approx 0.577$, at which the boundary conditions are set. Short horizontal bars mark the horizon, where $V = 1$. (Upper panel) Proper velocity V of the similarity frame relative to the baryonic frame. (Middle panel) Dimensionless proper baryonic mass and charge densities $z \equiv 4\pi r^2 \rho_b$ and $z_q \equiv 4\pi r^2 q$, and dimensionless proper dark matter mass density $z_d \equiv 4\pi r^2 \rho_d$. (Bottom panel) Interior mass M, proper acceleration g experienced by the baryonic fluid, and the homothetic scalar H. The exponential increase of mass M over a modest range of radii above the inner horizon, $r \sim 0.4$–0.1, is the signature of mass inflation.

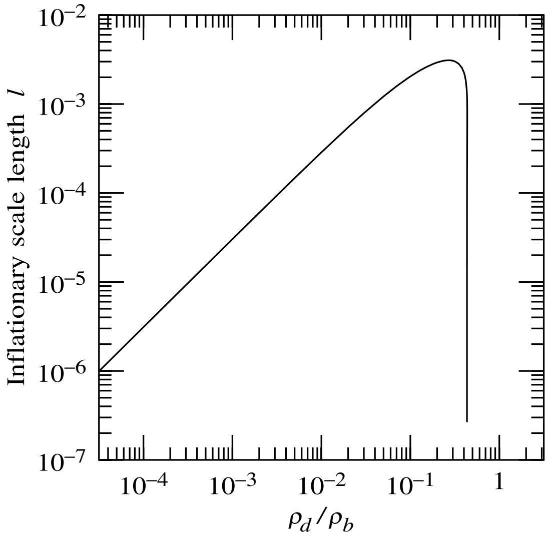

FIG. 2. Dimensionless exponential inflationary scale length l as a function of the ratio ρ_d/ρ_b of dark matter to baryonic proper mass densities at the outer sonic point, for black holes accreting at rate $\eta_s = 0.1$ and with charge-to-mass $Q/M_c = 0.8$. Shorter scale lengths l signify more extreme mass inflation.

only modestly:

$$M \sim \exp[-(\ln r)/l] \tag{56}$$

where l is a dimensionless exponential scale length. The inflationary scale length l is approximately constant during the main part of mass inflation, and can be characterized quantitatively by the reciprocal of the maximum logarithmic derivative of $2M/r - 1$, Eqs. (42c) and (49), during inflation

$$\frac{1}{l} \equiv \max\left[-\frac{d\ln(2M/r-1)}{d\ln r}\right]. \tag{57}$$

Figure 2 shows the inflationary scale length l defined by Eq. (57) as a function of the ratio ρ_d/ρ_b of dark matter to baryonic proper mass density at the outer sonic point, for black holes with the same accretion rate $\eta_s = 0.1$ and charge-to-mass $Q/M_c = 0.8$ as that illustrated in Fig. 1. Shorter scale lengths l signify more extreme mass inflation. The scale length increases approximately linearly at small ratios, $l \approx 0.032\rho_d/\rho_b$, goes through a maximum at $\rho_d/\rho_b \approx 0.27$, and then decreases. As l declines, the curve passes through a mild maximum in ρ_d/ρ_b, at $\rho_d/\rho_b \approx 0.4362$, and terminates at $\rho_d/\rho_b \approx 0.433$, constrained by $\gamma^2 - \beta^2 > 0$ at the outer sonic point.

Since the simultaneous presence of outgoing (baryonic) and ingoing (dark matter) fluids is essential to mass inflation, one might have thought that mass inflation would be most extreme when the densities of baryonic and dark matter were comparable. Figure 2 shows that the opposite is true: mass inflation is most extreme (the inflationary scale length l is smallest) when either the ingoing dark matter stream is reduced to a trace (small ρ_d/ρ_b), or the outgoing baryonic stream is reduced to a trace ($\rho_d/\rho_b \approx 0.433$). In the latter case, the baryonic density decreases by many orders of magnitude inside the black hole, so that at the onset of mass inflation the proper density of baryons is indeed only a trace compared to the proper density of dark matter.

When mass inflation is more extreme in the sense that the inflationary scale length l is small, it is also more extreme in the sense that the mass M exponentiates to a larger value before mass inflation ends.

The conundrum that mass inflation is most extreme when one of the ingoing or outgoing streams is reduced to a trace is considered in Sec. V C below.

The exponential increase of the interior mass M is paralleled by an exponential increase in (the absolute value of) the streaming 4-velocity u_d^r of the dark matter through the baryons, and an exponential decrease in (the absolute value of) the homothetic scalar H

$$u_d^r \sim \exp[-(\ln r)/l], \qquad H \sim \exp[(\ln r)/l]. \tag{58}$$

Figure 2 is practically unchanged if the inflationary scale length l is defined either by the exponential scale length of the 4-velocity u_d^r [see Eq. (53)]

$$\frac{1}{l} \equiv \max\left[-\frac{d\ln(u_d^r)}{d\ln r}\right] \tag{59}$$

or by the exponential scale length of the homothetic scalar H [see Eq. (51)]

$$\frac{1}{l} \equiv \max\left[\frac{d\ln(H)}{d\ln r}\right] \tag{60}$$

in place of Eq. (57).

B. Interacting dark matter

A feature of mass inflation is that ingoing and outgoing fluids stream through each other at ever closer to the speed of light (or become ever more blueshifted, for massless streams). This raises the physical question of what happens if there is a finite cross section for interaction between the ingoing and outgoing fluids at sufficiently high collision energies.

We have carried out a number of numerical experiments in which we have adjusted both the size and dependence on collision energy of the dimensionless rate coefficient κ_d in the absorption rate σ_d, Eq. (30), of dark matter by baryons. In all cases we find that, as long as the rate σ_d is finite (not infinite) at all energies, then the results are similar to those found in Sec. IV A: mass inflation occurs, then comes to an end, whereupon the fluids plunge to the central singularity.

As the absorption rate coefficient κ_d is varied, the degree of mass inflation changes, in a manner consistent with what was found in Sec. IV A. For example, if the parameters are set to those of Fig. 1, namely $\eta_s = 0.1$, $Q/M_c = 0.8$, and $\rho_d/\rho_b = 0.1$, then mass inflation becomes more extreme as the absorption rate coefficient κ_d is increased, with the inflationary scale length l shortening, and the mass M exponentiating to a higher value before inflation ends. Increasing the absorption rate reduces the density of dark matter, which has a similar effect to sliding ρ_d/ρ_b in Fig. 2 to values less than 0.1, thereby reducing the inflationary scale length l.

Is it possible to adjust the interaction between dark matter and baryons so that the baryons drop through the Cauchy horizon, as in Paper 1? The only way to achieve this is to let the absorption rate become infinite at a finite collision energy. If the absorption rate is infinite, then all the dark matter is absorbed and only outgoing baryons remain. As soon as all the dark matter is gone, then the baryons can drop through the Cauchy horizon. This is consistent with the fact, proven in Sec. V A below, that as long as ingoing and outgoing fluids are simultaneously present, then it is impossible for either fluid to drop through the inner horizon.

Figure 3 shows results for a black hole accreting baryons and dark matter with a dimensionless absorption rate coefficient κ_d that becomes numerically infinite when the 4-velocity u_d^m of the dark matter through the baryons is large

$$\kappa_d = 10^{-20}(u_d^t)^2. \tag{61}$$

The rate coefficient κ_d given by Eq. (61) is of course analytically finite at all collision energies, but numerically it is large enough at high collision energy ($u_d^t \gg 10^{10}$) that the dimensionless dark matter density $z_d \equiv 4\pi r^2 \rho_d$ falls below about 10^{-300}, at which point z_d underflows and is set to zero by the numerics. Aside from the fact that κ_d becomes large at large u_d^t, there is nothing magic about the constant of proportionality 10^{-20} or the exponent 2 in Eq. (61). The values are chosen simply to yield an interesting amount of mass inflation, as shown in Fig. 3, before the outgoing baryons drop through the Cauchy horizon.

As Fig. 3 illustrates, the structure of the black hole accreting dark matter with an effectively infinite high-energy absorption rate is similar to that of noninteracting dark matter, Fig. 1, up to the point where the dark matter is completely absorbed. At that point, the outgoing baryons drop through the Cauchy horizon, similar to the situation illustrated in Fig. 4 of Paper 1, where there was no dark matter present.

The similarity solution for the infinitely interacting dark matter model of Fig. 3 does not continue consistently to zero radius inside the Cauchy horizon, but rather terminates at an irregular sonic point at finite radius. This is the same phenomenon as happened in Paper 1 whenever the baryons dropped through the Cauchy horizon. As in Paper 1, the similarity solution of Fig. 3 terminates whether or not a shock is introduced inside the Cauchy horizon. The situation is similar to that illustrated in Fig. 5 of Paper 1, and the reader is referred to Secs. IV B and IV D of Paper 1 for further discussion of this issue.

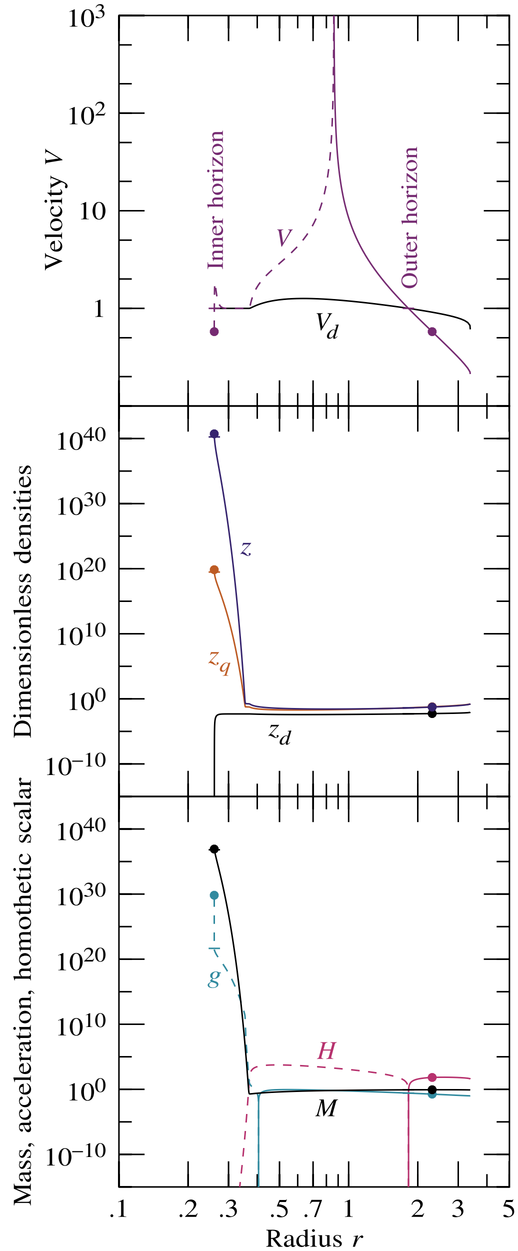

FIG. 3 (color online). Similar to Fig. 1, but for a black hole in which the absorption rate of dark matter by baryons is effectively infinite above a certain large collision energy. Disks mark sonic points, where $V = \pm\sqrt{w}$. Short horizontal bars mark horizons, where $V = \pm 1$. Mass inflation begins, but is cut short as soon as all the dark matter is absorbed. With the dark matter gone, the baryons almost immediately drop through the Cauchy horizon. The similarity solution terminates inside the Cauchy horizon at an irregular sonic point.

The Penrose diagram of the black hole of Fig. 3 is the same as that shown in Fig. 6 of Paper 1.

V. WHY MASS INFLATION HAPPENS

This section discusses the physical question of why mass inflation occurs. Section VA shows that, as long as ingoing and outgoing streams are simultaneously present, they cannot drop through an inner horizon. Section V B shows how the gravitational force drives the counterstreaming of ingoing and outgoing fluids that produces mass inflation. Section V C answers the question of why reducing one of the ingoing dark matter or outgoing baryonic streams to a trace relative to the other stream makes mass inflation more extreme, not the other way around as one might naively have expected. Section V D considers why mass inflation comes to an end, as found empirically in Sec. IVA. Section V E discusses the conditions for a null singularity to form on the Cauchy horizon, something that does not happen in the similarity solutions.

A. Ingoing and outgoing streams cannot drop through an inner horizon

The simultaneous presence of ingoing and outgoing fluids in the vicinity of the inner horizon is, as first pointed out by Poisson and Israel [1], the essential ingredient for mass inflation to occur. In this subsection we show that, in the context of the similarity solutions considered in this paper, as long as ingoing and outgoing fluids are simultaneously present, then it is impossible for the fluids to drop through the inner horizon, because to do so would require that the ingoing and outgoing fluids stream through each other faster than the speed of light, which is impossible.

Below we will refer to the outgoing fluid as baryons, and the ingoing fluid as dark matter, but the argument applies generally to any combination of ingoing or outgoing fluids.

As described in Sec. III C of Paper 1, in similarity solutions a frame of reference is ingoing or outgoing depending on whether the time component ξ^t of the homothetic 4-vector is positive or negative in that frame. The components of the homothetic vector in the dark matter and baryonic frames are related by a Lorentz transformation, Eq. (8). If the proper velocity of the dark matter relative to the baryons is denoted $v_d \equiv u_d^r/u_d^t$ then the time component ξ_d^t of the homothetic vector in the dark matter frame is related to the time component ξ^t in the baryonic frame by, Eq. (8),

$$\xi_d^t = \xi^t u_d^t (1 - V v_d). \tag{62}$$

Since u_d^t is always positive, it follows that the dark matter will have the opposite in/out sign from baryons (viz. ingoing, $\xi_d^t > 0$, if the baryons are outgoing, $\xi^t < 0$) if and only if

$$V v_d > 1 \tag{63}$$

that is, if $v_d > 1/V$ for positive V, or if $v_d < 1/V$ for negative V. We remind the reader that V is the proper velocity of the similarity frame relative to the baryonic frame, and that the absolute value of the velocity is equal to one, $|V| = 1$, at horizons.

Since the velocity v_d of the dark matter relative to the baryonic frame is necessarily less than or equal to the speed of light, $|v_d| \leq 1$, it follows from Eq. (63) that dark matter can have the opposite in/out sign from the baryonic frame only in superluminal regions of the geometry, where $|V| > 1$. Ingoing and outgoing fluids cannot coexist in the same contiguous subluminal region, where $|V| < 1$, since to do so they would have to move faster than light relative to each other. If ingoing and outgoing fluids fall through the inner horizon, then they must necessarily pass into separate ingoing and outgoing subluminal regions.

Approaching the inner horizon, where $|V| \to 1$, ingoing and outgoing frames must necessarily approach the speed of light, $|v_d| \to 1$, relative to each other, according to Eq. (63). At the inner horizon, $|V| = 1$, ingoing and outgoing objects must necessarily stream through each other at the speed of light, $|v_d| = 1$, which is problematic if the objects have finite rest mass. Indeed it is problematic even if the objects have zero rest mass, because, at the inner horizon, a light ray which has finite energy in an ingoing frame must appear infinitely blueshifted in an outgoing frame.

The above two paragraphs have demonstrated the claimed assertion, that as long as ingoing and outgoing fluids are simultaneously present, then it is impossible for the fluids to drop through the inner horizon, because to do so they would have to exceed the speed of light. A corollary of the argument is that as soon as one of the streams is exhausted, then the other stream can drop through the inner horizon, an ingoing horizon if only ingoing fluid remains, or an outgoing horizon, the Cauchy horizon, if only outgoing fluid remains.

B. Gravity drives mass inflation

The previous subsection, Sec. VA, showed that for ingoing and outgoing fluids to approach an inner horizon, they must stream ever faster through each other. But what drives such counterstreaming?

The thing that drives ingoing and outgoing fluids to stream ever faster through each other is the inward gravitational force. The trick is that "inward," meaning in the direction of smaller radius r, means opposite directions for the ingoing and outgoing fluids. Ingoing and outgoing fluids are both accelerated inwards, but nevertheless they are accelerated in opposite directions.

In Sec. IV C of Paper 1 we attached a gyroscope to an infalling observer, and, having initialized the gyroscope so that it points towards the black hole, we defined the direction in which the gyroscope points as the immutable direction towards the black hole. In the locally inertial (tetrad) frame of the infalling observer, the direction towards the black hole is not necessarily the direction of smaller circumferential radius r. Rather, the direction of smaller circumferential radius r is determined by the sign of the vierbein coefficient $\gamma \equiv \partial_r r$, Eq. (12) of Paper 1. A positive γ means that the gyroscope points in the direction of smaller proper circumferential radii r. Conversely, a negative γ means that the gyroscope points in the direction of larger proper circumferential radii. Zero γ means that the circumferential radius is an extremum.

The reader who is not yet convinced that accelerating inwards, i.e. making the radial coordinate velocity $\beta \equiv \partial_t r$ more negative, can mean accelerating in two opposite directions is invited to consider what happens to the radial 4-gradient $(\beta, \gamma) \equiv (\partial_t r, \partial_r r)$ under accelerations, i.e. under Lorentz boosts, Eq. (5), in the case at hand, where the radial 4-gradient is timelike, $\beta^2 - \gamma^2 > 0$, and β is negative.

The sign of the vierbein coefficient γ is not in one-to-one correspondence with whether the fluid is ingoing or outgoing (cf. Table II of Paper 1), but the two do correspond just outside the inner horizon: γ is positive if the fluid is ingoing, negative if the fluid is outgoing.

What drives mass inflation is a feedback loop in which the increasing radial pressure of the counterstreaming fluids amplifies the gravitational force, which accelerates the counterstreaming, which in turn increases the radial pressure. To see how this works, consider the gravitational force equation (12c) which governs the acceleration $\partial_t \beta$ of the coordinate radial velocity $\beta \equiv \partial_t r$. Equation (12c) is nominally for the baryonic tetrad frame, but essentially the same equation remains valid in the dark matter tetrad frame, if all quantities in the equation are reinterpreted as relative to the dark matter frame.

The gravitational force equation (12c) expresses the radial acceleration $\partial_t \beta$ as a sum of three terms: the familiar attractive Newtonian force $-M/r^2$, an additional general relativistic gravitational force $-4\pi r T^{rr}$ whose source is the radial pressure T^{rr}, and a force γg which comes from the acceleration generated by pressure balance (which includes the Lorentz force), and whose presence expresses the principle of equivalence. Numerically (Sec. IVA), the dominant term during mass inflation, for both baryons and dark matter, is the inward gravitational force term $-4\pi r T^{rr}$. It is this general relativistic force $-4\pi r T^{rr}$, which increases as the square of the streaming radial 4-velocity u_d^r, that drives mass inflation. By contrast, the Newtonian contribution $-M/r^2$, to the gravitational force increases only linearly with the streaming radial 4-velocity u_d^r. In the baryons the inward gravitational force is partially opposed by the γg force from a strong pressure gradient generated as a backreaction to the gravitational force, but still the primary gravitational force wins. The dark matter is pressureless, so in that case the γg force is zero.

It is curious that the gravitational force term $-4\pi r T^{rr}$ is the key player in two seemingly opposite roles. On the one hand, in a vacuum charged black hole this gravitational force term is repulsive, thanks to the negative radial pressure of the electric field. It is this gravitational repulsion that causes a vacuum black hole to contain an inner horizon, where ingoing and outgoing frames accelerate to the speed of light relative to each other. Without the gravitational repulsion, ingoing and outgoing fluids would not be inclined to accelerate through each other to the speed of light, and mass inflation would not begin. On the other hand, it is this same gravitational force term, with an exponentially growing positive pressure rather than a passive negative pressure, that provides the feedback loop that drives mass inflation.

The electric force behaves differently from the gravitational force. Irrespective of the sign of γ, electrically charged particles can consistently interpret the electric force as being caused either by a positive charge Q located in the direction of the black hole, or by a negative charge $-Q$ located in the direction away from the black hole. Either way, a positively charged particle is always repelled in the direction away from the positively charged black hole.

C. Why is less more?

In Sec. IVA it was found that reducing one of the ingoing dark matter or outgoing baryonic streams to a trace amount relative to the other stream actually resulted in more extreme mass inflation, in the sense that the inflationary scale length l, Eq. (57), became shorter, and the mass M exponentiated to a larger value, before mass inflation came to an end. An analogous result was found in Sec. IV B, where, for example, increasing the absorption rate of dark matter by baryons so that the ingoing dark matter stream was reduced to a trace again led to more extreme mass inflation. Since the simultaneous presence of both ingoing and outgoing streams is a prerequisite for mass inflation to occur, one might have thought that comparable amounts of ingoing and outgoing fluid would produce more inflation. But the opposite is true: less of one stream relative to the other produces more inflation, with comparable amounts of ingoing and outgoing fluid producing the least inflation. Why?

For definiteness, consider the (more realistic) case where the dark matter density is only a small fraction of the baryonic density (an analogous argument applies in the opposite case where the baryonic density is only a small fraction of the dark matter density). Mass inflation begins at around the time that the general relativistic gravitational force $-4\pi r T^{rr}$ becomes dominant in the acceleration equation (12c), which happens when the radial streaming 4-velocity u_d^r has become large enough in absolute value. The smaller the dark matter density, the larger the 4-velocity u_d^r must become in order for $-4\pi r T^{rr}$ to take over as the dominant gravitational force. A larger 4-velocity u_d^r requires that the ingoing and outgoing fluids approach closer to the inner horizon before mass inflation begins. As the fluids approach the inner horizon, the characteristic scale length l over which the 4-velocity u_d^r increases becomes shorter and shorter. When in due course mass inflation sets in, it is this characteristic scale length that determines the scale length over which M and u_d^r then exponentiate together. A smaller dark matter density thus implies a smaller inflationary length scale l, hence more extreme inflation.

D. Why does mass inflation end?

In Sec. IV A it was found that mass inflation eventually ceased, whereafter the ingoing and outgoing fluids plunged to a spacelike singularity at zero radius. Why does mass inflation end?

What happens is that the streaming 4-velocity u_d^r ceases its exponential growth, and indeed starts shrinking instead of growing. The dominant term in the differential equation (53) governing the evolution of the 4-velocity u_d^r is the term proportional to T_ξ^{tr}, which can be interpreted as the direct gravitational force term. The other term, proportional to the acceleration $y \equiv gr$ generated as a result of pressure balance, slightly counteracts the direct T_ξ^{tr} term, but not much. Now T_ξ^{tr} is the momentum density in the no-going $\xi^t = 0$ frame of reference, at the border between ingoing frames (positive ξ^t) and outgoing frames (negative ξ^t). The momentum density T_ξ^{tr}, Eq. (52), is a sum of two opposing terms, one from the outgoing baryons, the other from the ingoing dark matter. At the outset of inflation, the contribution to T_ξ^{tr} from the baryons exceeds that from the dark matter. As mass inflation continues, the relative contribution from dark matter gradually becomes more important. This is because as time goes by, the dark matter streaming through the baryons is accreted later and later in the evolution of the black hole, and in the similarity solutions the mass of accreted dark matter increases linearly with time. Eventually, the contribution to T_ξ^{tr} from the ingoing dark matter exceeds that from the outgoing baryons, and T_ξ^{tr} switches sign. Equivalently, the center-of-mass frame, where $T^{tr} = 0$, switches from outgoing to ingoing. At this point, or actually just before this point thanks to the $y \equiv gr$ term, the streaming 4-velocity u_d^r starts shrinking instead of growing, and mass inflation has come to an end.

We must admit that we find it physically somewhat mysterious that the gravitational force can operate in different directions on the ingoing and outgoing fluids, as argued in Sec. V B, and yet the proper streaming 4-velocity u_d^r can nevertheless shrink. We can only assume that this mystery can be attributed to the difference between coordinate velocities and proper velocities. Whatever the case, the mathematics governing u_d^r, Eq. (53), is clear enough.

The behavior of the homothetic scalar H during inflation is closely related to that of the streaming 4-velocity u_d^r. The Eq. (51) governing the evolution of H contains, like Eq. (53) governing u_d^r, two terms: a term proportional to the momentum density T_ξ^{tr} in the no-going frame, and another term [the one proportional to $(\gamma\xi^t + \beta\xi^r)/H$] which during inflation is subdominant and acts in mild opposition to the principal T_ξ^{tr} term. As with u_d^r, the end of inflation is signaled by T_ξ^{tr} changing sign, and accordingly H, which decreased exponentially in absolute value during inflation, starts rising back up again. This is precisely what was found empirically in Sec. IV A, where inflation stalled at about the time that the homothetic scalar H passed through a minimum in absolute value.

In summary, inflation comes to an end when the center-of-mass frame of the counterstreaming fluids switches from outgoing to ingoing. This condition is true even in the case where the baryonic density is a trace compared to the dark matter when mass inflation begins. This case corresponds to the small inflationary scale lengths l attained at the right edge of Fig. 2, where the accreted dark matter to baryonic density ρ_d/ρ_b is near maximal. As mentioned in the commentary to Fig. 2, even though the dark matter and baryonic densities are comparable at the sonic point boundary in this case, $\rho_d/\rho_b \approx 0.433$ for the parameters of Fig. 2, the baryonic density decreases by many orders of magnitude inside the black hole, so that indeed the baryonic density is driven to a trace compared to the dark matter density by the time mass inflation begins. But while the baryonic density diminishes, the components ξ^m of the homothetic vector in the baryonic frame grow, with the net result that the baryonic contribution $(1 + w)z\xi^t\xi^r$ to the momentum density T_ξ^{tr} exceeds the dark matter contribution $z_d\xi_d^t\xi_d^r$ at the onset of inflation, even though $z \ll z_d$. As mass inflation continues, the dark matter contribution to the momentum density T_ξ^{tr} grows relatively more important, as in the usual case. Eventually, the contribution to T_ξ^{tr} from the ingoing dark matter exceeds that from the outgoing baryons, T_ξ^{tr} switches sign, and mass inflation comes to an end.

E. Null singularity on the Cauchy horizon?

Many previous papers have found that the collapse of a massless scalar field into a charged black hole produces not only a strong spacelike singularity at zero radius but also a weak null singularity at finite radius along the Cauchy horizon [3–5,7,9,17,22,23,28]. Indeed, [17] find that two distinct null singularities may form, one ingoing and one outgoing. However, Burko [6,13] finds numerically that a null singularity forms only if the scalar field set up outside the horizon falls off sufficiently rapidly, the required degree of rapidity depending on the parameters of the problem, such as the charge-to-mass ratio of the black hole. If too much scalar field continues to be accreted, then no null singularity forms, and the field collapses to a central singularity. In the earliest numerical simulation, Gnedin and Gnedin [15] found only a spacelike central singularity, no null singularity, and it seems likely that the initial conditions for their scalar field exceeded the Burko bound (as opposed to there being some flaw [4,5] in the [15] method).

A null singularity does not form in the similarity solutions, and it may be presumed that this is because the assumption of self-similarity precludes it. As described in Sec. V D, mass inflation ceases once sufficient dark matter has been accreted that the center-of-mass frame near the inner horizon becomes ingoing rather than outgoing. At this point, the outgoing baryons do not persist at finite radius, but rather collapse to a spacelike singularity at zero radius. The self-similar hypothesis requires that the mass of accreted dark matter increases linearly with time into the indefinite future, so that outgoing baryonic fluid must inevitably encounter, sooner or later, enough ingoing dark matter to bear it down to the central singularity.

The methods of the present paper, which is restricted to self-similar solutions, are insufficiently powerful to answer definitely the question of what circumstances lead to a null singularity on the Cauchy horizon in the general, non-self-similar case. However, the results do suggest that two criteria may be key. We state these two criteria below in the form of conjectures, couched in somewhat loose language.

The first conjectured key criterion for the formation of a null singularity on the Cauchy horizon is that the amount of ingoing fluid accreted by the black hole should be finite and "sufficiently small." This comes from the empirical finding that the counterstreaming ingoing and outgoing fluids collapse to a singularity once sufficient ingoing fluid has been accreted. Roughly, collapse happens when the mass of accreted ingoing fluid is comparable to the mass of accreted outgoing fluid, although we cannot be sure that this approximate criterion is true in general.

To avoid confusion, we should comment that by ingoing and outgoing accreted fluid, we mean fluid whose properties are such that it becomes ingoing or outgoing near the inner horizon. The term accretion is also intended in a loose sense. For example, ingoing fluid could possibly emit outgoing fluid, or vice versa, so the source of ingoing and outgoing fluids may not necessarily be accretion.

The second conjectured key criterion is that the black hole should accrete ingoing fluid into the indefinite future. This comes from the idea that if the accretion of ingoing fluid is cut short, then the outgoing fluid will run through all the available ingoing fluid, and will then promptly drop through the Cauchy horizon.

It should also be commented that the definition of ingoing versus outgoing adopted in this paper and in Paper 1, that a frame is ingoing or outgoing according to whether the time component ξ^t of the homothetic 4-vector is positive or negative, works only for self-similar solutions, since the homothetic vector exists only in self-similar solutions. An alternative definition that would work in a general spherically symmetric metric would be to define a frame as ingoing or outgoing according to the sign of the vierbein coefficient γ. This definition agrees with our adopted definition in the important mass-inflationary region near the inner horizon. Indeed, it has been seen in Sec. V B that the sign of γ is physically at the heart of mass inflation, because it is the sign of γ that determines which direction is inward (meaning the direction of smaller circumferential radius r), and therefore in which direction the gravitational force operates, towards the black hole for ingoing fluid (positive γ), and away from the black hole for outgoing fluid (negative γ).

VI. APPEARANCE OF THE BLACK HOLE

Section V of Paper 1 considered the question: What does it actually look like if you fall inside one of the black holes described in that paper? This section addresses the same question for the black holes considered in the present paper.

Figure 4 shows, for the two models illustrated in Figs. 1 and 3, the angular size χ_{ph} and blueshift of photons from the edge of the black hole, as observed either in the baryonic rest frame or in the radially free-falling dark matter rest frame. The two points of view are related by a radial Lorentz boost. The observed angular size χ_{ph} of the black hole (the subscript ph signifying photons from the photon sphere equivalent) is given by Eq. (73) of Paper 1, and the observed blueshift of photons at the edge of the black hole is given by Eq. (74) of Paper 1. For the free-fall dark matter frame, Eqs. (73) and (74) of Paper 1 apply with the baryonic homothetic vector ξ^m replaced by its dark matter counterpart ξ_d^m, Eq. (8). The horizontal axis on Fig. 4 is the radius as measured in the corresponding frame, r for the baryons, r_d for the dark matter.

Figure 4 shows that, down to the point where mass inflation begins, the appearance of black holes accreting dark matter in addition to charged baryons is similar to the appearance of black holes accreting only charged baryons, middle panel of Fig. 12 of Paper 1. As discussed in Sec. V of Paper 1, this appearance is in turn similar to that of the corresponding vacuum black hole, the Reissner-Nordström solution.

From the point of view of an outgoing observer, such as one in the baryonic rest frame, the black hole (that is, any of the black holes considered in this paper) increases in angular size until it covers almost the entire sky. The view of the outside universe correspondingly shrinks to a small, intensely bright, blueshifted point above the observer. As long as mass inflation continues, the point gets smaller, brighter, and more blueshifted.

From the point of view of an ingoing observer on the other hand, such as one in the free-fall dark matter rest frame, the black hole (again meaning any of the black holes

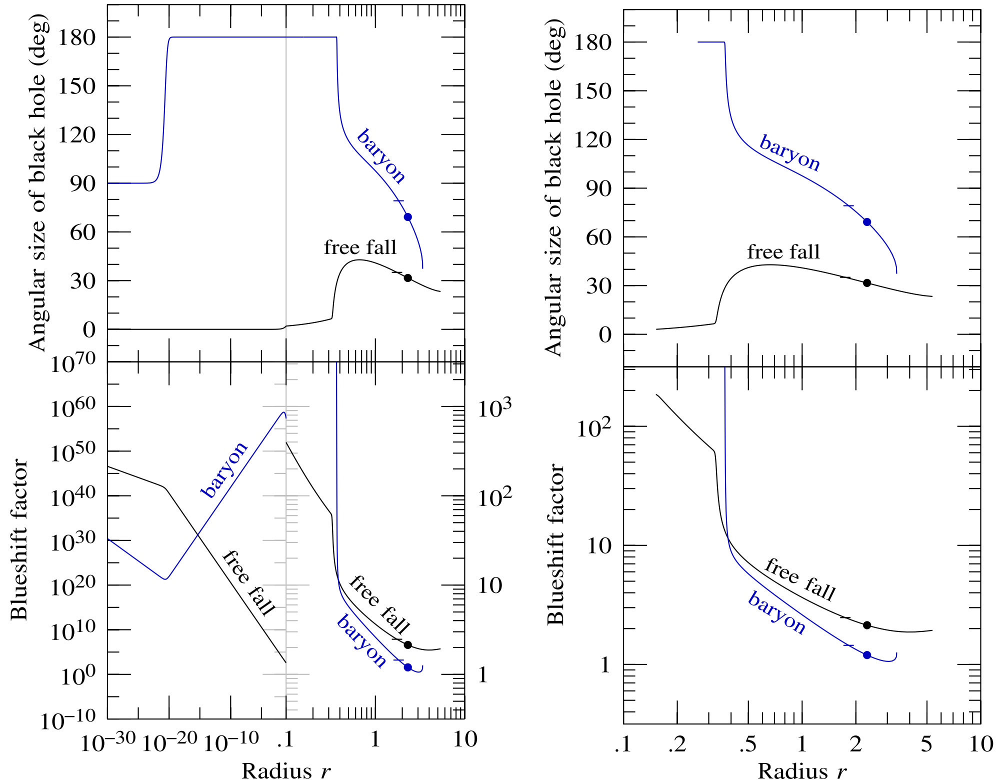

FIG. 4 (color online). Angular size $\chi_{\rm ph}$ of the black hole and the blueshift of photons at the edge of the black hole perceived by observers in either the baryonic frame or the free-fall dark matter frame, for (left) the model of Fig. 1, where the black hole accretes noninteracting dark matter, and (right) the model of Fig. 3, where the black hole accretes dark matter whose cross section for absorption by baryons is effectively infinite at high energy. Light from the outside universe is visible only from outside the Cauchy horizon, so lines terminate infinitesimally outside the Cauchy horizon even in the model at right, in which the baryons drop inside the Cauchy horizon.

considered in this paper) first increases in angular size, but then shrinks as the observer approaches the inner horizon. In contrast to the outgoing observer who sees the outside universe concentrate to a small point, the ingoing observer sees the outside universe cover almost the whole sky. To the ingoing observer, the sky near the edge of the black hole appears blueshifted, but the sky away from the edge is mostly redshifted. During mass inflation, the black hole continues to shrink, and to be surrounded by a concentrating, brightening halo.

In models where the dark matter is noninteracting, such as the model shown in the left panel of Fig. 4, or where the dark matter has a finite (not infinite) cross section for absorption by baryons, mass inflation eventually ceases, as discussed in Secs. IV A and V D. As mass inflation comes to an end, an outgoing observer, such as one in the baryonic frame, sees the view of the outside universe start to reexpand, and to become less bright and less blueshifted. As the outgoing baryons plunge to the singularity at zero radius, their view of the outside universe expands to a radius of 90°. The 90° view near the singularity is similar to that seen by an observer who falls into a Schwarzschild black hole, and can be attributed to the same enormous tidal force that stretches the infaller radially and crushes them horizontally. As in the Schwarzschild solution, the blueshift at the edge of the black hole tends to infinity as the outgoing baryonic observer approaches the singularity, but the amount of time that the observer sees pass by in the outside universe, the integral of blueshift over proper time, is finite.

As remarked above, an ingoing observer sees a different view, a tiny black hole surrounded by a bright, blueshifted halo. The ingoing observer sees the halo become more concentrated, brighter, and more blueshifted, not only during mass inflation, but also thereafter, all the way down to the singularity at zero radius. Although the blueshift around the black hole tends to infinity at the singularity, the ingoing observer sees, like the outgoing observer, only a finite time pass by in the outside universe.

In models where the dark matter absorption rate is effectively infinite at high energy, such as the model shown in the right panel of Fig. 4, the outgoing baryons drop though the Cauchy horizon as soon as the ingoing dark matter is completely absorbed. If the outgoing baryons could see the outside universe (which they cannot, because by assumption there is no ingoing matter or radiation left to see), then as the outgoing baryons dropped through the Cauchy horizon, their view of the outside universe would disappear in an infinitely bright, blueshifted, concentrated flash, in which the entire future of the outside universe passes by.

The right panel of Fig. 4 shows the view seen by observers in the freely falling ingoing dark matter frame, cut short once the dark matter has been completely absorbed. Cutting the view short seems natural since the model is specifically constructed so that there is no ingoing fluid—no dark matter—beyond a certain point. However, if there were ingoing test particles (with vanishing energy-momentum tensor), then their view would resemble that illustrated in the middle panel of Fig. 12 of Paper 1, in which the ingoing test particles fall to zero radius, encountering outgoing baryons accreted at ever earlier times, while the outgoing particles' view of the outside universe blueshifts to infinity.

As already mentioned, in most cases observers see only a finite time go by in the outside universe as they voyage to their doom inside the black hole. The exception is that if an outgoing observer drops through the Cauchy horizon, then, if the outgoing observer could see the outside universe, the observer would see the entire future of the universe pass by. Of course, the outgoing observer can drop through the Cauchy horizon only if there is no ingoing fluid left, in which case the observer cannot see the outside universe. In effect then, there is no case in which an infalling observer ever sees an infinite future go by.

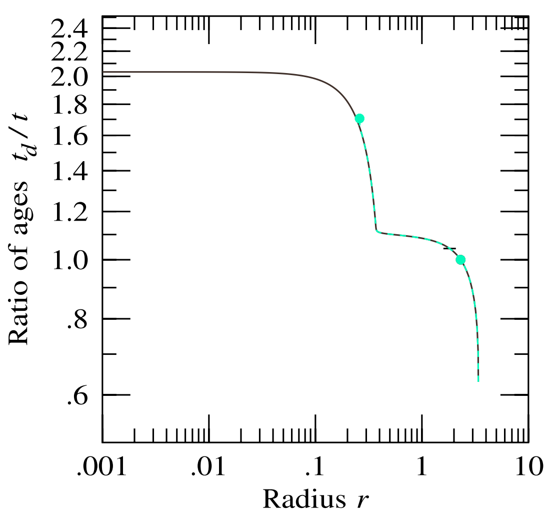

FIG. 5 (color online). Ingoing dark matter and outgoing baryons streaming through each other at radius r inside the black hole were accreted by the black hole at different times. The graph shows the ratio t_d/t of ages of the black hole when the dark matter versus baryons were accreted. For example, a ratio $t_d/t = 2$ means that the black hole was twice as old when it accreted the dark matter as it was when it accreted the baryons. The solid line is for the model with noninteracting dark matter shown in Fig. 1, while the dashed line almost coincident with the solid line is for the model shown in Fig. 1, in which the dark matter has an effectively infinite cross section for absorption by baryons at high energy.

For the black holes illustrated in Fig. 4, the amount of time that an infalling observer sees during their voyage into the black hole is quite modest. The baryons and dark matter that stream through each other inside the black hole were accreted at different ages t and t_d, and Fig. 5 shows, for the two models illustrated in Figs. 4, the ratio t_d/t of these two ages at each point inside the black hole. The ratio of ages equals the reciprocal of the ratio of radii, $t_d/t = r/r_d$, computed from Eqs. (42c) and (43c). For the noninteracting model, the ratio of ages tends to $t_d/t \to 2.034$ as $r \to 0$. The ratio is only slightly different if the dark matter is massless instead of massive, $t_d/t \to 2.007$ as $r \to 0$ (photons that do not scatter off baryons can be regarded as a kind of massless dark matter). The ratio $t_d/t \approx 2$ as $r \to 0$ means that the baryons see a factor of 2 into the future: the baryons see dark matter which was accreted when the black hole was twice as old as when the black hole accreted the baryons. Similarly, the dark matter sees a factor of 2 into the past: the dark matter sees baryons which were accreted when the black hole was half as old as when the black hole accreted the dark matter.

Roughly speaking, mass inflation ceases and the fluids collapse to a singularity when comparable masses of outgoing baryons and ingoing dark matter have been accreted. Thus the ratio t_d/t of ages would be larger if the ratio ρ_d/ρ_b of accreted dark matter to baryonic density were reduced.

Figure 5 shows that the case of the infinitely interacting dark matter is similar to that of the noninteracting dark matter until the dark matter is completely absorbed, beyond which there is no dark matter left to see, or to be seen by, the baryons.

At the end of Sec. V of Paper 1 it was remarked that the function $H(X)$, which plays the essential part in ray tracing, Eq. (70) of Paper 1, was reasonably approximated as a cubic or quartic polynomial in X. While this approximation remains satisfactory before mass inflation starts, it fails completely once mass inflation sets in. During and after mass inflation, X hardly varies at all, while H, as illustrated in Fig. 1, varies by many, many orders of magnitude.

VII. SUMMARY

In this the second of two companion papers, we have investigated self-similar solutions for spherically symmetric charged black holes that accrete a pressureless fluid of neutral dark matter (massive or massless) in addition to a relativistic fluid ($p_b/\rho_b = 1/3$) of charged baryons. The primary aim has been to investigate mass inflation.

As first pointed out by [1], the essential ingredient of mass inflation is the simultaneous presence of ingoing and outgoing fluids near the inner horizon. In the present paper, the accreted charged baryonic fluid, repelled by the charge of the black hole generated self-consistently by previously accreted charged baryons, naturally becomes outgoing. The accreted dark matter, which is neutral, remains ingoing. Relativistic counterstreaming between outgoing baryons and ingoing dark matter then leads to mass inflation near the inner horizon, as expected.

Section V discussed the physical causes underlying mass inflation. In Sec. V A we showed that, in the context of the similarity solutions considered in this paper, as long as ingoing and outgoing fluids are simultaneously present, then it is impossible for the fluids to drop through the inner horizon, because in order to do so the fluids would have to stream through each other faster than light, which is impossible. A corollary of this argument is that, if either of the ingoing or outgoing streams is exhausted, then the other stream can promptly drop through the inner horizon, an ingoing horizon if only ingoing fluid is present, or an outgoing (Cauchy) horizon if only outgoing fluid is present.

As argued in Sec. V B, the thing that drives ingoing and outgoing fluids to stream ever faster through each other during mass inflation is the inward gravitational force. The trick is that, in the region near the inner horizon, inward, meaning in the direction of smaller circumferential radius r, means opposite directions for the ingoing and outgoing fluids. For ingoing fluid, the direction of smaller radius points towards the black hole, whereas for outgoing fluid, the direction of smaller radius points away from the black hole.

In Sec. V B, we remarked on the curious dual role played by the pressure contribution to the gravitational force. On the one hand, it is the negative radial pressure of the electric field that produces the gravitational repulsion that decelerates the inward flow of space into the black hole, and that therefore causes a vacuum black hole to contain an inner horizon. Without this negative pressure, there would be no inner horizon, and no mass inflation. On the other hand, the same pressure contribution to the gravitational force, with an exponentially growing positive pressure rather than a passive negative pressure, provides the feedback loop that drives mass inflation.

Since the simultaneous presence of outgoing (baryonic) and ingoing (dark matter) fluids is essential to mass inflation, one might have thought that mass inflation would be strongest in black holes which accrete comparable amounts of baryonic and dark matter. In the numerical experiments presented in Sec. IV we found that, on the contrary, mass inflation becomes more extreme as one of the ingoing or outgoing streams is reduced to a trace relative to the other. Thus, paradoxically, there is a huge difference between the case of no dark matter (considered in Paper 1) and the case of a tiny trace of dark matter. With no dark matter, the baryons can drop quietly through the Cauchy horizon. With a trace of ingoing dark matter, the outgoing baryons cannot drop through the Cauchy horizon, and instead undergo extravagant mass inflation.

In the similarity solutions considered in the present paper, mass inflation does not continue to arbitrarily large value of the interior mass M, but rather comes to an end. Mass inflation ends at approximately the time that the center-of-mass frame of the counterstreaming fluids switches from outgoing to ingoing, whereafter the fluids collapse to a spacelike singularity at zero radius. It has widely been considered that a generic consequence of mass inflation is a weak null singularity on the Cauchy horizon, and this conclusion is undoubtedly valid in the situation (different from that considered in the present paper) where the outgoing fluid is a power-law tail [8,26,27] of radiation generated when the black hole first collapses. More recently Burko [6,13], reporting numerical experiments on the collapse of a massless scalar field into a charged black hole, found that a null singularity forms only if the amplitude of the scalar field falls off sufficiently rapidly. It is not clear whether Burko's criterion is essentially the same as that found here, but the results are at least consistent.

A feature of mass inflation is that the streaming velocity between ingoing and outgoing fluids can reach huge Lorentz gamma factors, which raises the question of whether it is physically plausible to allow relativistic streaming at immense energies. In Sec. IV B we explored numerically the consequences of allowing the dark matter to have a finite cross section for being absorbed by the baryons. Consistent with the conclusion that mass inflation becomes more extreme as the amount of dark matter is reduced to a trace, we find that increasing the absorption rate merely makes mass inflation more extreme. The only caveat to this conclusion is that if the absorption rate is infinite above some collision energy, then the ingoing dark matter is absorbed completely, whereupon the outgoing baryons can drop promptly through the Cauchy horizon.

In Sec. VI we discussed what an observer who falls inside one of the black holes considered in this paper would see. Among other things, we found that in all cases an infalling observer witnesses only a finite amount of time pass by in the outside universe. This is true even for an observer who drops through the Cauchy horizon, because although such an observer would, if they could see the outside universe, see the entire future of the universe pass by as they dropped through the Cauchy horizon, in fact the observer cannot see the outside universe, because photons that come unscattered from the outside universe are necessarily ingoing, and an outgoing observer cannot pass through the Cauchy horizon as long as there is any trace of ingoing matter or radiation.

ACKNOWLEDGMENTS

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