THROUGH A BLACK HOLE INTO A NEW UNIVERSE?

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The internal structure of spacetime inside a black hole is investigated on the assumption that some limiting curvature exists. It is shown that the Schwarzschild metric inside a black hole can be attached to the de Sitter one at some spacelike junction surface which may represent a short transition layer. The possible fate of the de Sitter space which arises in the interior of a black hole in this model is discussed.

One of the fundamental problems in classical general relativity is the problem of singularities which inevitably arise in the theoretical description of the collapse of a massive body (or the total universe) (see e.g. refs. [1,2]). It is generally believed that the arising of these singularities is usually accompanied by an unlimited increase of the spacetime curvature. Under these conditions the classical Einstein equations are not applicable and one may hope that the proper account of quantum effects may avoid the singularities and hence may cure the disease of the classical theory. It means that quantum corrections or other reasons may drastically modify the properties of the solutions in the region where the curvature becomes large enough and hence change the global structure of spacetime. It is natural to assume that the curvature for the solutions of these modified equations is limited by some universal value $\sim l^{-2}$, where l plays the role of fundamental length. In what follows we suppose that $l \sim l_{\mathrm{Pl}} = (\hbar G/c^3)^{1/2} \sim 10^{-33}$ cm.

Unfortunately we do not know the exact modified equations yet and therefore cannot verify this assumption. But we can accept this assumption as a hypothesis and investigate its possible consequences.

Such an approach was suggested in ref. [3] for studying the collapse of the homogeneous isotropic universe. The aim of this paper is to study the possible structure of the spacetime inside a black hole in the framework of the hypothesis about the existence of a limiting curvature (see also ref. [4]).

We restrict ourselves by considering spherically symmetric black holes. It is instructive to first discuss the case of an eternal black hole and later to consider the more realistic situation of a black hole arising as a result of a gravitational collapse. We assume that the mass m of the black hole is large $[m \gg m_{\mathrm{Pl}} = (\hbar c/G)^{1/2} \sim 10^{-5}\ g]$ and that it is invariable in time. In order to suppress the change of the mass due to the Hawking radiation one may assume that the black hole is surrounded by a thermal bath the temperature of which coincides with the black-hole temperature.

The metric of a static (with the Killing vector $\xi = \xi^\mu \partial_\mu = \partial_t$) spherically symmetric spacetime can be written as follows:

$$\begin{aligned} \mathrm{d}s^2 &= -\frac{\mathrm{d}r^2}{g(r)} + f(r)\,\mathrm{d}t^2 + r^2\,\mathrm{d}\omega^2 \\ &= \epsilon[-\mathrm{d}\tau^2 + F(\tau)\,\mathrm{d}t^2] + r^2(\tau)\,\mathrm{d}\omega^2\,, \end{aligned} \tag{1}$$

where $\mathrm{d}\omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$ is the line element on a unit sphere, $g(r) \equiv \epsilon|g(r)| = -\nabla r \cdot \nabla r$, $f = \xi^2$ and

$$\mathrm{d}\tau = -|g(r)|^{-1/2}\,\mathrm{d}r, \quad F(\tau) = f(r(\tau))\,. \tag{2}$$

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(Here and later we use the units in which $G=c=\hbar=1$.) One can verify that the Ricci tensor R^{ν}_{μ} for this metric obeys the inequality

$$4R^{\nu}_{\mu}R^{\mu}_{\nu}-R^2\geqslant 0\,. \tag{3}$$

The left-hand side of eq. (3) vanishes if and only if

$$R^{\nu}_{\mu}=\Lambda\delta^{\nu}_{\mu}\,. \tag{4}$$

For the particular case of a spherically symmetric spacetime our main hypothesis about the existence of a limiting curvature may be presented in the form

$$\mathscr{R}^2\equiv R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\leqslant\alpha/l^4\,, \tag{5}$$

where l is the characteristic (planckian) length and α is a dimensionless parameter of order one. One can verify that eq. (5) implies that the other possible invariants quadratic in curvature, $C^2=C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$, $R_{\mu\nu}R^{\mu\nu}$ and R^2, are also positive and limited.

As a second hypothesis we assume that when the curvature reaches its maximum value the equation of state becomes of the vacuum-like type (4) or equivalently $R^2=4R^{\nu}_{\mu}R^{\mu}_{\nu}$.

Under these two hypotheses the metric (1) describing the spacetime of an eternal black hole allows the following specification. The Schwarzschild metric (i.e. eq. (1) with $g=f=-1+2m/r$) can be used to approximate the geometry for $r>r^0$, where

$$r_0=(12/\alpha)^{1/6}(2m/l)^{1/3}l \tag{6}$$

is the value of the radius r at which the invariant $\mathscr{R}^2$ for the Schwarzschild metric reaches its limiting value α/l^4. For $2m\gg l$ one has $l\ll r_0\ll 2m$. This means that the surface Σ_0 where $r=r_0$ is spacelike. It lies inside the event horizon and has the topology $S^2\times R^1$, i.e. it is an infinite (in direction t) "tube" of radius r_0. Strictly speaking in order to describe the geometry of the spacetime outside and inside the event horizon (including the region near Σ_0) one must use a Kruskal-like analytical continuation of the Schwarzschild metric. But if we are interested in a description of the metric only in the vicinity of Σ_0 it is also possible (and much more convenient for our purpose) to use the Schwarzschild-like element (1) with $g=f>0$ so that r is a timelike coordinate. It is worthwhile noting that for the case of a black hole inside a thermal bath the relation $2m\gg l$ implies also that the change of geometry due to the presence of the thermal radiation at $(m/l)^2l\gg r\gg r_0$ can be neglected. As for the future evolution of the geometry for $\tau>\tau_0$ ($r<r_0$) we cannot specify the two unknown functions in eq. (1) until we know the exact field equations. Nevertheless our second hypothesis guarantees that beginning at some time moment $\tau_1>\tau_0$ ($r_1<r_0$) we can approximate these field equations by eq. (4). In the case of a spherically symmetric spacetime this means that the geometry is described by the de Sitter metric which can be written in the form (1) with $g=f=(r/l)^2-1$. Here $l=(\frac{1}{3}\Lambda)^{1/2}$ and if we assume that this length parameter l coincides with the parameter l in (5) then we have $\alpha=24$.

In the general case the global structure of the spacetime under consideration may depend on the details of the transition region $\tau_0<\tau<\tau_1$. But in the particular case when the duration $\Delta\tau/\tau=\tau_1-\tau_0$ of this transition is short ($\Delta\tau/l\sim 1$) only some of its integral characteristics become important. In the latter case one may consider this layer as "a thin massive shell" and sew the Schwarschild metric ($\tau<\tau_0$) with the de Sitter one ($\tau>\tau_1$) using the approach developed by Israel [5]. According to this approach we suppose that $r_0=r_1$ and consider Σ_0 ($r=r_0=r_1$) as a junction surface which separates the Schwarzschild and de Sitter geometries. The junction conditions at this surface require the three-geometries induced by both geometries to be identical while for the jumps of the external curvature $[K^m_n]\equiv(K_{\text{de Sitter}})^m_n-(K_{\text{Schwarzschild}})^m_n$ one has ($m, n=1, 2, 3$)

$$[K^m_n]-\delta^m_n[K^l_l]\equiv -8\pi S^m_n\,, \tag{7}$$

where

$$S^m_n\equiv\int_{\tau_0}^{\tau_1}\mathrm{d}\tau\, T^m_n\,. \tag{8}$$

The tensor T^{ν}_{μ} is the effective energy–momentum tensor which is defined in the transition layer as the right-hand side of the field equations written in a Einstein-like form.

In the case under consideration one has

$$S^t_t=\frac{\lambda}{4\pi}\,,\quad S^\theta_\theta=S^\varphi_\varphi=\frac{\kappa+\lambda}{8\pi}\,,$$

$$\kappa\equiv[K^t_t]=\frac{1}{r_0}\,[x^2(x^2-1)^{-1/2}+\tfrac{1}{2}y(y-1)^{-1/2}]\,, \tag{9}$$

$$\lambda \equiv [K^0_0] = -\frac{1}{r_0}\,[(y-1)^{1/2}-(x^2-1)^{1/2}]\,, \qquad (9\ \text{cont'd})$$

where $x=r_0/l$ and $y=2m/r_0$. For $2m \gg l$ these relations read

$$\kappa \simeq \frac{1}{2l}\,(\beta+2), \quad \lambda \simeq -\frac{1}{l}\,(\beta-1)\,. \qquad (10)$$

It is worth-while noting the "large parameter" $2m/l$ does not enter these relations and hence there is no contradiction with our assumption that the time interval of the transition $\Delta\tau$ is short. Indeed if we suppose that in the transition layer T^μ_ν reaches the planckian value ($T^\mu_\nu \sim l^{-2}$) then the proper time duration $\Delta\tau$ of this layer estimated as $\Delta\tau \sim S^m_n/T^m_n$ is comparable with the planckian time l.

The conformal Penrose diagram for the spacetime under consideration is presented in fig. 1. For convenience we use the freedom in choice of the Kruskal coordinates in order to guarantee the same "coordinate form" of Σ_0 as it has in the de Sitter coordinates. The spacetime in the region lying in the future with respect to any Cauchy surface Σ in the Kruskal region is regular and complete. It should be noted that Σ is not a global Cauchy surface. Since $H^\pm_{DS}$ are Cauchy horizons such a global Cauchy surface does not exist at all. The anisotropic (Kasner-like) contraction of space in the interior of the black hole changes into the de Sitter deflation which in its turn (at the surface Σ_1) changes into the inflationary de Sitter expansion. The surface Σ_1 has topology S^3 and in this sense the diagram presented in fig. 1 describes the closed world formation inside the black hole.

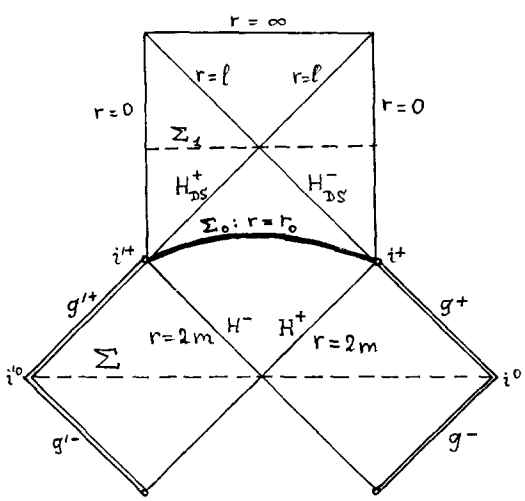

Fig. 1. Conformal Penrose diagram for the spacetime of a spherically symmetric eternal black hole with a de Sitter space in its interior. Σ_0 is a junction surface which represents the thin transition layer. $H^\pm$ are the event horizons of the black hole while $H^\pm_{DS}$ are the Cauchy horizons.

The conformal diagram shown in fig. 1 resembles to some extent the maximal analytical continuation of either Reissner–Nordström or Kerr metric in its structure. One of the main differences is that in our case one may expect the stability of the Cauchy horizons $H^\pm_{DS}$ while the Cauchy horizons in Reissner–Nordström or Kerr spacetime were shown to be unstable [6] (see also ref. [7]). This instability is related to an infinite blue-shift of signals sent into the black hole from external space and registered by an observer crossing the Cauchy horizon. This effect combined with the classical or quantum radiation falling into a black hole will result in the divergence of T^μ_ν near the Cauchy horizon and its instability. In our case if only the hypothesis about the limiting curvature is valid, the backreaction of matter does not allow $\mathscr{R}^2$ and hence T^μ_ν to grow without limit and after the curvature reaches its limit we would have the de Sitter space. Hence we may expect that in our case there is no such instability. (A discussion of the structure of the black-hole interior and the properties of the Cauchy horizons can be also found in ref. [8].)

Now we turn to a more realistic case in which the black hole arises as a result of a gravitational collapse. For simplicity we suppose that the collapsing matter does not possess pressure so that the metric inside it can be written in the form

$$ds^2 = -d\tau^2 + a^2(\tau)(d\chi^2 + \sin^2\chi\, d\omega^2)\,, \qquad (11)$$

where

$$a(\tau) = a_0(1-\cos\eta), \quad \tau = a_0(\eta - \sin\eta)\,, \qquad (12)$$

and $0 \leqslant \chi \leqslant \chi_0 < \frac{1}{2}\pi$. This metric can be used to approximate the geometry of a contracting dust cloud until the moment $\tau = \tau_0$ when

$$a \equiv \tilde{a} = (60/\alpha)^{1/6}(a_0/l)^{1/3} l\,. \qquad (13)$$

At this moment the spacetime curvature inside the cloud $\mathscr{R}^2 = 60 a_0^2/a^2$ reaches its limit α/l^4. According to our hypotheses some time later after the transition layer the geometry in the region occupied by matter would also become de Sitter-like. A more detailed

analysis of the junction conditions at the surface which separate the matter, the vacuum outside the matter and the de Sitter phase allows one to describe the complete structure of the spacetime. The corresponding conformal Penrose diagram is shown in fig. 2. It shows that the gravitational collapse produces a de Sitter-like universe inside the black hole which after the stage of deflation becomes inflationary so that the situation is quantitatively the same as in the case of an eternal black hole.

Let us discuss now what happens when the mass m of the black hole decreases due to the process of evaporation. The radiation of energy to infinity in this process is accompanied by a negative energy flux through the horizon inside the black hole. In order to initiate this situation we use the Vaidya metric [9] which we write in the form

$$ds^2 = f\,dv^2 + 2\,dv\,dr + r^2\,d\omega^2\,,$$
$$f \equiv -\nabla r\cdot\nabla r = 2m(v)/r - 1\,. \qquad (14)$$

This metric is the solution of the Einstein equations for the energy–momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi r^2}\frac{dm}{dv}\,v_{,\mu}v_{,\nu}\,, \qquad (15)$$

which describes the radiation flux into the black hole.

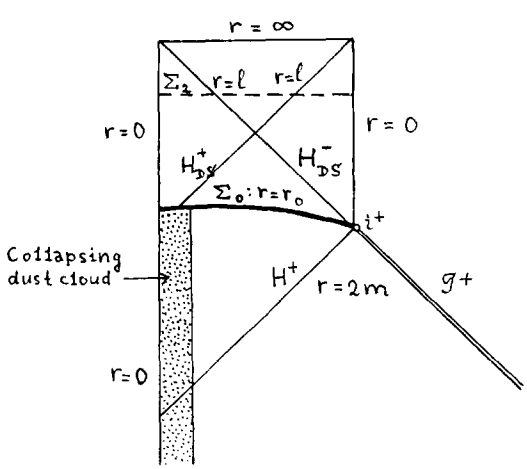

Fig. 2. Conformal Penrose diagram for the spacetime of a spherically symmetry black hole which arises as a result of the gravitational collapse of a dust cloud. Σ_0 is a junction surface which represents the thin transition layer. After this layer lies the de Sitter space at the stage of deflation. H^+ is the event horizon and $H^{\pm}_{DS}$ at the Cauchy horizons.

For $dm/dv^<$ the energy density of this radiation is negative.

One can easily show that the curvature invariant $\mathscr{R}^2$ for the metric (14) reads $\mathscr{R}^2 = 48m^2(v)/r^6$ and hence the equation of the junction surface Σ_0 is $r = r_0(v) = (12/\alpha)^{1/6}[2m(v)/l]^{1/3}l$. This surface lies inside the apparent horizon ($f=0$) until the advanced time reaches the value v_1 defined by the condition $r_0(v_1) = l/\beta$, where $\beta \equiv (\frac{1}{12}\alpha)^{1/4}$. If the evaporation ends before this time the resulting spacetime structure is qualitatively the same as the one presented in fig. 2. In this case the black hole of a minimum possible mass $m_{\min} \geqslant \frac{1}{2}l$ remains ("maximon" [10] or "elementary black hole" [11]). If the final mass $m_{\min}$ is smaller than $\frac{1}{2}l$ then there arises a version of a "semiclosed" world. It is necessary to stress that the "massive thin shell" approach in such a situation becomes questionable and one must treat the results obtained in the framework of this approach with caution. If the stable "maximons" do not exist then one may expect that the remnant of the black hole may just disappear at the final stage of evaporation. This pure quantum effect would change the topology of space and hence it would not allow a regular classical description.

Now we briefly discuss the possible fate of the de Sitter world which according to our model may be present in the interior of a black hole. First of all it should be noted that the de Sitter space is usually unstable [12]. It seems likely that if the hypothesis about the existence of a limiting curvature is valid then such an instablility at the stage of deflation might by suppressed. There is a possibility that at the end of the deflation when the closed world has planckian dimensions it can just disappear in the process of quantum annihilation. If this does not happen then the decay of this world which begins its inflationary expansion may create a new macroscopic universe in the same manner as happens in the usual inflation models [12]. The result of this decay depends on the effective hypersurface on which it occurs and hence on the nature of the Λ-term. In particular one may expect that a new closed Friedmann universe will arise as a result of this process. In this case the de Sitter space decays on some hypersurface Σ_2 (see fig. 2). The spacetime in the future with respect to Σ_2 will coincide with the spacetime of an expanding closed Friedmann universe. Another possibility is the crea-

tion of a white hole in a new asymptotically flat universe which lies in the absolute future with respect to the original asymptotically flat space.

The model considered may be interpreted as "the creation of the universe in a laboratory" via a black hole which may be formed by contraction of matter up to high density. This conclusion contradicts the theorem of ref. [13]. The reason is that in our case the assumptions of this theorem (in particular the existence of the existence of a global Cauchy surface as well as the condition of energodominance) may be violated. (Another possibility of violating this theorem was discussed in ref. [14].)

In conclusion, it should be stressed once again that the consideration in this paper is based on rather restrictive assumptions about the properties of the effective gravitational equations at high curvatures. There exist various possibilities to violate our assumptions. For example at small distances it may become important that the dimensionality of real spacetime is higher than four. Nevertheless we hope that the model described with a closed world in the interior of a black hole may be useful and that this picture or its main features will survive in a future theory. If this happens then the possibility (which was discussed earlier in connection with the Reissner–Nordström or Kerr spacetime) "to travel" from our universe into a new one which is the absolute future with respect to us may still be open.

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