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Correct Linearization of Einstein's Equations

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Routinely, Einstein's equations are be reduced to a wave form (linearly independent of the second derivatives of the space metric) in the absence of gravitation, the space rotation and Christoffel's symbols. As shown herein, the origin of the problem is the use of the general covariant theory of measurement. Herein the wave form of Einstein's equations is obtained in terms of Zelmanov's chronometric invariants (physically observable projections on the observer's time line and spatial section). The equations so obtained depend solely upon the second derivatives, even for gravitation, the space rotation and Christoffel's symbols. The correct linearization proves that the Einstein equations are completely compatible with weak waves of the metric.

1 Introduction

Gravitational waves are routinely considered as weak waves of the space metric, whereby, one takes a Galilean metric $g^{(0)}_{\alpha\beta}$, whose components are $g^{(0)}_{00}=1$, $g^{(0)}_{0i}=0$, $g^{(0)}_{ik}=-\delta_{ik}$, and says: because gravitating matter is connected to the field of the metric tensor $g_{\alpha\beta}$ by Einstein's equations*

$$R_{\alpha\beta}-\frac{1}{2}\,g_{\alpha\beta}R=-\kappa\,T_{\alpha\beta}+\lambda g_{\alpha\beta}\,,\quad \kappa=\text{const}>0\,,$$

gravitational waves are weak perturbations $\zeta_{\alpha\beta}$ of the Galilean metric. Thus the common metric, consisting of the initially undeformed and wave parts, is $g_{\alpha\beta}=g^{(0)}_{\alpha\beta}+\zeta_{\alpha\beta}$.

According to the theory of partial differential equations, a wave of a field is a Hadamard break [1] in the derivatives of the field function along the hypersurface of the field equations (the wave front). The first derivative of a function at a point determines a direction tangential to it, while the second derivative determines a normal direction. Thus, if a surface in a tensor field is the front of the field wave, the second derivatives of this tensor have breaks there. It is possible to prove in relation to this case in a Riemannian space with the metric $g_{\alpha\beta}$, that d'Alembert's operator $\Box=g^{\alpha\beta}\,\nabla_\alpha\nabla_\beta$ of this field equals zero†. For instance, the wave field of a tensor $Q_{\mu\nu}$ is characterized by the d'Alembert equations $\Box\,Q_{\mu\nu}=0$.

We can apply the d'Alembert operator to any tensor field and equate it to be zero. For this reason any claims that waves of the space metric cannot exist are wrong, even from the purely mathematical viewpoint, independently of those deductions that the authors of those claims adduced.

So, the front of weak wave perturbations $\zeta_{\alpha\beta}$ of a Galilean metric $g^{(0)}_{\alpha\beta}$ is determined by breaks in their second derivatives, while the wave field $\zeta_{\alpha\beta}$ itself is characterized by the d'Alembert equations

$$\Box\,\zeta_{\alpha\beta}=0\,.$$

*We write the Einstein equations in the main form containing the λ-term, because our consideration is outside a discussion of the λ-term.

†Note that the d'Alembert operator consists of the second derivatives.

If the left side of the Einstein equations for the common metric $g_{\alpha\beta}=g^{(0)}_{\alpha\beta}+\zeta_{\alpha\beta}$ reduced to $\Box\,\zeta_{\alpha\beta}$,‡ the equations could be reduced to the form

$$a\,\Box\,\zeta_{\alpha\beta}=-\kappa\,T_{\alpha\beta}+\lambda g_{\alpha\beta}\,,\qquad\text{where}\quad a=\text{const}\,,$$

which, in the absence of matter, become the wave equations $\Box\,\zeta_{\alpha\beta}=0$, meaning that the perturbations $\zeta_{\alpha\beta}$ are waves.

As one calculates the left side of the Einstein equations for the common metric, he obtains a large number of terms where only one is $\Box\,\zeta_{\alpha\beta}$ with a numerical coefficient. Thus one concludes: the Einstein equations are non-linear with respect to the second derivatives of $\zeta_{\alpha\beta}$.

In order to prove gravitational waves, theory should lead to cancellation of all the non-linear terms, as argued by Eddington [2], and Landau and Lifshitz [3]. This process is so-called the *linearization* of the Einstein equations.

2 Problems with the linearization

There is much literature about why the non-linear terms can be cancelled (see Lichnerowicz [4] or Zakharov [5] for details). All the reasons depend upon one initial factor: the theory of measurements we use.

We know two theories of measurements in General Relativity: Einstein's theory of measurements and Zelmanov's theory of physically observable quantities. The first one was built by Einstein in the 1910's. Following him§, we consider the space-time volume of nearby events in order to find a particular reference frame satisfying the properties of our real laboratory. We then express our general covariant equations in terms of the chosen reference frame. Some terms drop out, because of the properties of the chosen reference frame. Briefly, as one calculates the Ricci tensor $R_{\alpha\beta}=g^{\mu\nu}R_{\alpha\mu\nu\beta}$ by the contraction of the Riemann-Christoffel tensor

‡Actually, this problem is to reduce the Ricci tensor for the common metric $g_{\alpha\beta}=g^{(0)}_{\alpha\beta}+\zeta_{\alpha\beta}$ to $\Box\,\zeta_{\alpha\beta}$.

§Einstein gave his theory of measurements partially in many papers. You can see the complete theory in Synge's book [6], for instance.

$$R_{\alpha\mu\nu\beta} = -\Gamma^{\sigma}_{\mu\beta}\,\Gamma_{\alpha\nu,\sigma} + \Gamma^{\sigma}_{\mu\nu}\,\Gamma_{\alpha\beta,\sigma} + \\ + \frac{1}{2}\left(\frac{\partial^2 g_{\mu\nu}}{\partial x^\alpha \partial x^\beta} + \frac{\partial^2 g_{\alpha\beta}}{\partial x^\nu \partial x^\mu} - \frac{\partial^2 g_{\alpha\nu}}{\partial x^\mu \partial x^\beta} - \frac{\partial^2 g_{\mu\beta}}{\partial x^\alpha \partial x^\nu}\right)$$

for $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ (see §105 in [3]), he can reduce it to

$$R_{\alpha\beta} = \frac{1}{2}\, g^{(0)\mu\nu}\, \frac{\partial^2 \zeta_{\alpha\beta}}{\partial x^\mu \partial x^\nu} = \frac{1}{2}\,\Box\,\zeta_{\alpha\beta}$$

and the left side of the Einstein equations to $\Box\,\zeta_{\alpha\beta}$, only if:

1. The reference frame is free of forces of gravity;
2. The reference frame is free of rotation;
3. Christoffel's symbols $\Gamma^{\alpha}_{\mu\nu}$, containing the inhomogeneity of space, are all zero.

Of course, we can find a reference frame where the gravitational potential, the space rotation, and the Christoffel symbols are zero at a given point*. However they cannot be reduced to zero in an area. Moreover, a gravitational wave detector consists of two bodies located far away from each other. In a Weber solid-body detector the distance is several metres, while in a laser interferometer the distance can take even millions of kilometres, as LISA in a solar orbit. It is wrong to interpret any of those as points. So, gravitational forces, the space rotation or the Christoffel symbols cannot be obviated in the equations. This is the main reason why:

> By the methods of Einstein's theory of measurements, the Einstein equations cannot be mathematically correctly linearized with respect to the second derivatives of the weak perturbations $\zeta_{\alpha\beta}$ of the space metric.

Some understand this incompatibility to mean that General Relativity does not permit weak waves of the metric.

This is absolutely wrong, even from the purely mathematical viewpoint: the d'Alembert operator $\Box = g^{\alpha\beta}\,\nabla_\alpha \nabla_\beta$ may be applied to any tensor field, the field of the weak perturbations $\zeta_{\alpha\beta}$ of the metric included, and equated to zero.

This obvious incompatibility can arise for one or both of the following reasons:

1. Einstein's equations in their current form are insufficient to describe our real world;
2. Einstein's theory of measurements is inadequate for the four-dimensional pseudo-Riemannian space.

Einstein's equations were born of his intuition, only the left side thereof is derived from the geometry. However main experimental tests of General Relativity, proceeding from the equations, verify the theory. So, the equations are adequate for describing our real world to within a first approximation.

At the same time, Einstein's theory of measurements has many deficiencies. There are no clear methods for recognition of physically observable components of a tensor field. It set up so that the three-dimensional components of a world-vector field compose its spatially observable part, while the time component is its scalar potential. However this problem becomes confused for a tensor of higher rank, because it has time, spatial, and mixed (space-time) components. There are also other drawbacks (see [8], for instance).

The required mathematical methods have been found by Zelmanov, who, in 1944, fused them into a complete theory of physically observable quantities [9, 10, 11].

*See §7 *Special Reference Frames* in Petrov's book [7].

3 The theory of physically observable quantities

According to Zelmanov, each observer has his own spatial section, set up by a coordinate net spanned over his real reference rest-body and extended far away with its gravitational field. The net is replete with a system of synchronized clocks†. Physically observed by him are projections of world-quantities onto his time line and spatial section, made by the projection operators $b^\alpha = \frac{dx^\alpha}{ds}$ and $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$. Chr.inv.-projections of a world-vector Q^α are $b_\alpha Q^\alpha = \frac{Q_0}{\sqrt{g_{00}}}$ and $h^i_\alpha Q^\alpha = Q^i$, while chr.inv.-projections of a 2nd rank world-tensor $Q^{\alpha\beta}$ are $b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}$, $h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}$, $h^i_\alpha h^k_\beta Q^{\alpha\beta} = Q^{ik}$. Physically observable properties of the space are determined by the non-commutativity of the chr.inv.-operators $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\, v_i \frac{{}^*\partial}{\partial t}$, and the fact that the chr.inv.-metric tensor $h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k$ may not be stationary. They are the chr.inv.-quantities: the gravitational inertial force F_i, the space rotation tensor A_{ik}, and the space deformational rates D_{ik}

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \qquad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,,$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right), \quad v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}\,,$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \quad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \quad D = D^k_k = \frac{{}^*\partial \ln\sqrt{h}}{\partial t}\,,$$

where w is gravitational potential, v_i is the linear velocity of the space rotation, $h = \det\|h_{ik}\|$, and $\sqrt{-g} = \sqrt{h}\,\sqrt{g_{00}}$. The chr.inv.-Christoffel symbols $\Delta^i_{jk} = h^{im}\Delta_{jk,m}$ are built like the usual $\Gamma^\alpha_{\mu\nu} = g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$, using h_{ik} instead of $g_{\alpha\beta}$.

By analogy with the Riemann-Christoffel curvature tensor, Zelmanov derived the chr.inv.-curvature tensor‡

$$C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}\right),$$

from which the contraction $C_{kj} = C_{kij\cdot}^{\cdots i} = h^{im} C_{kimj}$ gives the chr.inv.-scalar observable curvature $C = C^j_j = h^{lj} C_{lj}$.

†Projections onto such a spatial section are independent of transformations of the time coordinate — they are *chronometric invariants*.

‡Here $H_{lki\cdot}^{\cdots j} = \frac{{}^*\partial \Delta^j_{il}}{\partial x^k} - \frac{{}^*\partial \Delta^j_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^j_{km} - \Delta^m_{kl}\Delta^j_{im}$.

4 Correct linearization of Einstein's equations

We now show that Einstein's equations expressed with physically observable quantities may be linearized without problems; proof that waves of weak perturbations of the space metric are fully compatible with the Einstein equations.

Zelmanov already deduced [9] the Einstein equations in chr.inv.-components (the chr.inv.-Einstein equations) in the absence of matter: —

$$\frac{^*\partial D}{\partial t} + D_{jl}D^{jl} + A_{jl}A^{lj} + \left({}^*\nabla_j - \frac{1}{c^2}F_j\right)F^j = 0\,,$$

$$^*\nabla_j\left(h^{ij}D - D^{ij} - A^{ij}\right) + \frac{2}{c^2}F_jA^{ij} = 0\,,$$

$$\frac{^*\partial D_{ik}}{\partial t} - \left(D_{ij}+A_{ij}\right)\left(D^j_k + A^{\cdot j}_{k\cdot}\right) + DD_{ik} + 3A_{ij}A^{\cdot j}_{k\cdot} + \\ + \frac{1}{2}\left({}^*\nabla_iF_k + {}^*\nabla_kF_i\right) - \frac{1}{c^2}F_iF_k - c^2C_{ik} = 0\,,$$

where Zelmanov's $^*\nabla_k$ denotes the chr.inv.-derivative*.

The components of the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$, consisting of a Galilean metric and its weak perturbations, are†

$$g_{00} = 1 + \zeta_{00}\,,\qquad g_{0i} = \zeta_{0i}\,,\qquad g_{ik} = -\delta_{ik} + \zeta_{ik}\,,$$
$$g^{00} = 1 - \zeta^{00},\qquad g^{0i} = -\zeta^{0i},\qquad g^{ik} = -\delta^{ik} - \zeta^{ik},$$
$$h_{ik} = \delta_{ik} - \zeta_{ik},\qquad h^i_k = \delta^i_k\,,\qquad h^{ik} = \delta^{ik} + \zeta^{ik}.$$

Because $\zeta_{\alpha\beta}$ are weak, the products of their components or derivatives vanish. In such a case,

$$F_i = \frac{c}{1+\zeta_{00}}\left(\frac{\partial\zeta_{0i}}{\partial t} - \frac{c}{2}\frac{\partial\zeta_{00}}{\partial x^i}\right),$$

$$A_{ik} = \frac{c}{\sqrt{1+\zeta_{00}}}\left(\frac{\partial\zeta_{0i}}{\partial x^k} - \frac{\partial\zeta_{0k}}{\partial x^i}\right),$$

$$D_{ik} = -\frac{1}{2\sqrt{1+\zeta_{00}}}\frac{\partial\zeta_{ik}}{\partial t}\,,\qquad D = h^{ik}D_{ik} = \delta^{ik}D_{ik}\,,$$

$$C_{imnk} = \frac{\partial^2\zeta_{mk}}{\partial x^i\partial x^n} + \frac{\partial^2\zeta_{in}}{\partial x^m\partial x^k} - \frac{\partial^2\zeta_{mn}}{\partial x^i\partial x^k} - \frac{\partial^2\zeta_{ik}}{\partial x^m\partial x^n}\,.$$

After some algebra, we obtain the chr.inv.-Einstein equations for the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$:

$$\frac{1}{c^2\left(1+\zeta_{00}\right)}\frac{\partial^2\zeta}{\partial t^2} + \frac{\delta^{km}}{\left(1+\zeta_{00}\right)}\left(\frac{\partial^2\zeta_{00}}{\partial x^k\partial x^m} - \frac{2}{c}\frac{\partial^2\zeta_{0m}}{\partial x^k\partial t}\right) = 0\,,$$

$$\frac{\delta^{ij}}{c^2\sqrt{1+\zeta_{00}}}\frac{\partial^2\zeta}{\partial x^j\partial t} - \frac{1}{c^2\sqrt{1+\zeta_{00}}}\frac{\partial^2\zeta^{ij}}{\partial x^j\partial t} + \\ + \frac{2\delta^{im}\delta^{jn}}{c\sqrt{1+\zeta_{00}}}\left(\frac{\partial^2\zeta_{0m}}{\partial x^j\partial x^n} - \frac{\partial^2\zeta_{0n}}{\partial x^j\partial x^m}\right) = 0\,,$$

$$\frac{1}{c^2\left(1+\zeta_{00}\right)}\frac{\partial^2\zeta_{ik}}{\partial t^2} - \frac{1}{c\left(1+\zeta_{00}\right)}\left(\frac{\partial^2\zeta_{0k}}{\partial x^i\partial t} - \frac{\partial^2\zeta_{0i}}{\partial x^k\partial t}\right) + \\ + 2\delta^{mn}\left(\frac{\partial^2\zeta_{mk}}{\partial x^i\partial x^n} + \frac{\partial^2\zeta_{in}}{\partial x^m\partial x^k} - \frac{\partial^2\zeta_{mn}}{\partial x^i\partial x^k} - \frac{\partial^2\zeta_{ik}}{\partial x^m\partial x^m}\right) = 0\,.$$

Note that the obtained equations are functions of only the second derivatives of the weak perturbations of the space metric. So, the Einstein equations have been linearized, even in the presence of gravitational inertial forces and the space rotation. This implies: —

> By the methods of Zelmanov's mathematical theory of chronometric invariants (physically observable quantities), **the Einstein equations are linearized in a mathematically correct way**, i. e. without the assumption of a specific reference frame where there are no gravitational forces or the space rotation.

This is the mathematical proof to the statement: —

> Waves of the weak perturbations of the space metric are fully compatible with the Einstein equations.

*So $^*\nabla_kQ^i = \frac{^*\partial Q^i}{\partial x^k} + \Delta^i_{mk}Q^m$ and $^*\nabla_kQ_i = \frac{^*\partial Q_i}{\partial x^k} - \Delta^m_{ik}Q_m$ are the chr.inv.-derivatives of a chr.inv.-vector Q^i.

†The contravariant tensor $g^{\alpha\beta}$, determined by the main property $g_{\alpha\sigma}g^{\sigma\beta} = \delta^\beta_\alpha$ of the fundamental metric tensor as $(g^{(0)}_{\alpha\sigma} + \zeta_{\alpha\sigma})\,g^{\sigma\beta} = \delta^\beta_\alpha$, is $g^{\alpha\beta} = g^{(0)\alpha\beta} - \zeta^{\alpha\beta}$, while its determinant is $g = g^{(0)}(1+\zeta)$. This is easy to check, taking into account that, because the values of the weak corrections $\zeta_{\alpha\beta}$ are infinitesimal, their products vanish; while we may move indices in $\zeta_{\alpha\beta}$ by the Galilean metric tensor $g^{(0)}_{\alpha\beta}$.

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Einstein's Equivalence Principle and Invalidity of Thorne's Theory for LIGO

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The theoretical foundation of LIGO's design is based on the equation of motion derived by Thorne. His formula, motivated by Einstein's theory of measurement, shows that the gravitational wave-induced displacement of a mass with respect to an object is proportional to the distance from the object. On the other hand, based on the observed bending of light and Einstein's equivalence principle, it is concluded that such induced displacement has nothing to do with the distance from another object. It is shown that the derivation of Thorne's formula has invalid assumptions that make it inapplicable to LIGO. This is a good counter example for those who claimed that Einstein's equivalence principle is not important or even irrelevant.

1 Introduction

Since the behavior of binary pulsars has been interpreted successfully as due to gravitational radiation [1, 2], the existence of gravitational waves is generally accepted. Moreover, the Maxwell Newton Approximation,(1) which generates gravitational waves, has been established independent of the Einstein equation [3]. However, a direct observation of the gravitational waves has not been successful because a gravitational wave is very weak in nature [4].

To obtain the required sensitivity of detection for gravitational waves, two gigantic laser interferometer gravitational wave observatories (LIGO) have been built.(2) Currently they represent the hope of detecting the gravitational waves directly. The confidence on these new apparatus is based on the perceived high sensitivity [5] that is designed according to Thorne's equation, which is motivated on Einstein's theory of measurement [6, 7].

Thorne's [8] equation of motion is as follows [9]:

$$m\frac{d^2\delta x^j}{dt^2} = \frac{1}{2}\, m\frac{\partial^2 h^{\mathrm{TT}}_{jk}}{\partial t^2}\, x^k, \tag{1}$$

where δx^k is the displacement of the test particle with mass m from a fixed object, x^k is the Euclidean-like distance (or the particle's Cartesian coordinate position) of the test particle from the fixed object (at the original the space coordinates), and h^{TT}_{jk} is the first order of the dimensionless "gravitational wave field" that induces the displacement. Then the integration of equation (1) gives,

$$\delta x^j = \frac{1}{2}\, h^{\mathrm{TT}}_{jk} x^k. \tag{2}$$

The superscript TT on the gravitational field is to remind us that the field is "transverse and traceless".

On the other hand, according to Einstein's equivalence principle [10], the Euclidean-like structure [11, 12] that determines the distance between two points is independent of gravity, and this is supported the observed bending of light. Thus, the displacement from a fixed object induced by gravitational wave, according the geodesic equation, has nothing to do with the distance between them (see Section 2). In this paper, it will be shown the errors related to eqs. (1) and (2).

2 Problems in the theory of Thorne

Now let us first derive, according the theory of Thorne [8], the induced phase delay in the interferometer. Since the sources of the gravitational waves are very far away, the waves look very nearly planar as they pass through the observer's proper reference frame.(3) If we orient the x, y, z spatial axes, so the propagation in the z direction, then the transversality of the waves and traceless mean that the non-zero components of the wave field are $h^{\mathrm{TT}}_{xx} = -h^{\mathrm{TT}}_{yy}$, $h^{\mathrm{TT}}_{xy} = h^{\mathrm{TT}}_{yx}$, called respectively the $+$ and $\times$-polarization. For a ($+$)-polarization, if the arm length of the interferometer is L, we have

$$\begin{aligned} \delta x(t) &= \frac{1}{2}\, Lh_+(t) \qquad \text{for mass on } x \text{ axis},\\ \delta y(t) &= -\frac{1}{2}\, Lh_+(t) \quad \text{for mass on } y \text{ axis}. \end{aligned} \tag{3}$$

For a light wavelength λ, if B is the number of bounce back and forth in the arms, the total phase delay is

$$\triangle\phi_{\mathrm{T}} = 8\pi B\,\frac{\delta x}{\lambda} = 4\pi B\,\frac{L}{\lambda}\, h_+\,. \tag{4}$$

Thus, the sensitivity of the interferometer would be increased with longer arms. If Einstein's theory of measurement was valid, then eq. (3) would be an expected result. This explains that eq. (1) was accepted. To show the errors, some detailed analysis is needed.

In a Local frame of free fall, Manasse and Misner [12] claimed that the metric have approximately,

$$-ds^2 = (1+R_{0l0m}x^l x^m)dt^2 + \left(\frac{4}{3}R_{0ljm}x^l x^m\right)dx^j dt - \\ - \left(\delta_{ij} - \frac{1}{3}R_{iljm}x^l x^m\right)dx^i dx^j - \mathrm{O}(|x^j|^3)dx^\alpha dx^\beta \qquad (5)$$

accurate to the second order in small $|x^j|$. The observer in the free fall is located at the origin of the local frame. Eq. (5) is the equation (13.73) in Misner et al. [9]. In the next step (35.12), they claimed to have the equation,

$$\frac{D^2 n^j}{d\tau^2} = -R^j_{0k0}\, n^k = -R_{j0k0}\, n^k, \\ \text{where} \quad n^j = x^j_{\mathrm{B}} - x^j_{\mathrm{A}} = x^j_{\mathrm{B}} \qquad (6)$$

since $x^j_{\mathrm{A}} = 0$. In eq. (5), $|x^j|$ is restricted to be small. However, a problem in this derivation is that R_{j0k0} may not be the same at points A and B. Nevertheless, one may argue that $\Gamma^\mu_{\alpha\beta} = 0$ at A, and (6) is reduced to

$$\frac{d^2 x^j_{\mathrm{B}}}{d\tau^2} = -R_{j0k0}\, x^k_{\mathrm{B}} = \frac{1}{2}\frac{\partial h^{\mathrm{TT}}_{jk}}{\partial \tau^2}\, x^k_{\mathrm{B}}\,. \qquad (7)$$

If it is applied to the case of LIGO, one must show at least a miles long x^j_{B} could be regarded as very small as (5) requires. From the geodesic equation, clearly it is impossible to justify (7) for any frame of reference.

More important, since LIGO is built on the Earth, its frame of reference is not at free fall when gravitational waves are considered. The radius of the Earth is 6.3×10^3 km, but the expected gravitational wave length is only about 15 km [9]. Thus, the Earth can no longer be considered as a test particle when only the gravity of the Sun is considered. In other words, (5) and (7) are inapplicable to LIGO.

Note that Misner et al. [9] have mistaken Pauli's version(4) as Einstein's equivalence principle [10], it is natural that they made related mistakes. For instance, Thorne [15] incorrectly criticized Einstein's equivalence principle as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems. For instance, the problem of tidal forces was answered in Einstein's letter of 12 July 1953 to Rehtz [16] as follows:

"The equivalence principle does not assert that every gravitational field (e. g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Clearly, his principle is for a space where physical requirements are sufficiently satisfied.

In fact, Misner et al. [9] do not understand Einstein's equivalence principle and related theorems in Riemannian space [14, 17]. A simple and clear evidence is in their eq. (40.14) [9; p. 1107], and they got a physically incorrect conclusion on the local time of the Earth in the solar system. Moreover, Ohanian and Ruffini [5; p. 198] also ignored the Einstein-Minkowski condition and had the same problems as shown in their eq. (50). However, Liu [18], Straumann [19], Wald [20], and Weinberg [4] did not make the same mistake. Note that Ohanian, Ruffini, and Wheeler have proclaimed that they are non-believers of Einstein's principles [5].

3 Remarks

In the theory of Thorne, there are major errors because his understanding of Einstein's equivalence principle is inadequate. His equation was motivated by Einstein's theory of measurement, and the superficial consistency with such a theory makes many theorists had confidence on his equation. Now, it is clear that such a support from an invalid theory is proven to be useless. Because Misner et al. [9] do not understand Einstein's equivalence principle, they cannot see that Einstein's theory of measurement is not self-consistent [21, 22].

In addition, since LIGO is built on the Earth, the frame is not at free fall. The radius of the Earth is 6.3×10^3 km, but the expected gravitational wave length is only about 15 km [9]. Thus, the Earth cannot be regarded as a test particle for gravitational waves. Moreover, Thorne was not aware that the Einstein equation has no wave solution [1, 2]. Although Misner, Thorne, and Wheeler [9] claimed plane wave solutions exist, their derivation has been found to be invalid [2, 23]. The second problem has been resolved by a modified Einstein equation, and it has the Maxwell-Newton Approximation as the first order equation [1].

In short, the current theory on the detection of gravitational waves for LIGO is incorrect. The root of these problems is due to that they do not understand Einstein's equivalence principle.(5) Consequently, they also failed to see the Euclidean-like structure is necessary(6) in a physical space [12]. This is a very good counter example for those who believed the Einstein's equivalence principle is not important or even irrelevant [2]. The sensitivity of LIGO will be addressed in a separate paper [24].

Acknowledgments

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Endnotes

(1) The Maxwell-Newton Approximation, whose sources are massive matter, could be identified as a special case of the so-called linearized approximation that has been found to be incompatible with Einstein equation for a dynamic situation [1].

(2) M. Bartusiak [25] has written an interesting book on the great efforts to build LIGO.

(3) Einstein equation has no physically valid wave solution because there is no term in Einstein's equation to accommodate the energy-stress tensor of a gravitational wave that must move with the wave [23]. Thus, a wave solution must come from the modified equation of 1995.

(4) Pauli's [26] version of the principle of equivalence was commonly but mistakenly regarded as Einstein's principle, although Einstein strongly objected to this version as a misinterpretation [15]. In fact, Misner, Thorne, and Wheeler [9; p. 386] falsely claimed that Einstein's equivalence principle is as follows:

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitestimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any other local Lorentz frame in the same or any other region."

However, this is only an alternative version of Pauli's because the Einstein-Minkowski condition,(7) which requires that the local space in a free fall must have a local Lorentz frame, is missing.

(5) There are other surprises. In spite of Einstein's clarification, many theorists, including the editors of Nature, Physical Review, and Science, still do not fully understand special relativity, in particular $E = mc^2$ [27–30].

(6) An existence of the Euclidean-like structure (that Einstein [6] called as "in the sense of Euclidean geometry") is necessary for a physical space [11, 12]. The Euclidean-like structure is operationally defined in terms of spatial measurements essentially the same as Einstein defined the frame of reference for special relativity [31]. Since the attached measuring instruments and the coordinates being measured are under the influence of the same gravity, a Euclidean-like structure emerges from such measurements as if gravity did not exist.

(7) For the Einstein-Minkowski condition, Einstein [10] addressed only the metrics without a crossing space-time element. This creates a false impression that the Einstein-Minkowski condition is trivial.

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3-Space In-Flow Theory of Gravity: Boreholes, Blackholes and the Fine Structure Constant

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A theory of 3-space explains the phenomenon of gravity as arising from the time-dependence and inhomogeneity of the differential flow of this 3-space. The emergent theory of gravity has two gravitational constants: G_N — Newton's constant, and a dimensionless constant α. Various experiments and astronomical observations have shown that α is the fine structure constant $\approx$ 1/137. Here we analyse the Greenland Ice Shelf and Nevada Test Site borehole g anomalies, and confirm with increased precision this value of α. This and other successful tests of this theory of gravity, including the supermassive black holes in globular clusters and galaxies, and the "dark-matter" effect in spiral galaxies, shows the validity of this theory of gravity. This success implies that the non-relativistic Newtonian gravity was fundamentally flawed from the beginning, and that this flaw was inherited by the relativistic General Relativity theory of gravity.

1 Introduction

In the Newtonian theory of gravity [1] the Newtonian gravitational constant G_N determining the strength of this phenomenon is difficult to measure because of the extreme weakness of gravity. Originally determined in laboratory experiments by Cavendish [2] in 1798 using a torsion balance, Airy [3] in 1865 presented a different method which compared the gravity gradients above and below the surface of the Earth. Then if the matter density within the neighbourhood of the measurements is sufficiently uniform, or at most is horizontally layered and known, then such measurements then permitted G_N to be determined, as discussed below, if Newtonian gravity was indeed correct. Then the mass of the Earth can be computed from the value of g at the Earth's surface. However two anomalies have emerged for these two methods: (i) the Airy method has given gravity gradients that are inconsistent with Newtonian gravity, and (ii) the laboratory measurements of G_N using various geometries for the test masses have not converged despite ever increasing experimental sophistication and precision. There are other anomalies involving gravity such as the so-called "dark-matter" effect in spiral galaxies, the systematic effects related to the supermassive blackholes in globular clusters and elliptical galaxies, the Pioneer 10/11 deceleration anomaly, the so-called galactic 'dark-matter' networks, and others, all suggest that the phenomenon of gravity has not been understood even in the non-relativistic regime, and that a significant dynamical process has been overlooked in the Newtonian theory of gravity, and which is also missing from General Relativity.

The discovery of this missing dynamical process arose from experimental evidence [4, 8, 9] that a complex dynamical 3-space underlies reality. The evidence involves the repeated detection of the motion of the Earth relative to that 3-space using Michelson interferometers operating in gas mode [8], particularly the experiment by Miller in 1925/26 at Mt. Wilson, and the coaxial cable RF travel time measurements by Torr and Kolen in Utah in 1985, and the DeWitte experiment in 1991 in Brussels [8]. In all 7 such experiments are consistent with respect to speed and direction. It has been shown that effects caused by motion relative to this 3-space can mimic the formalism of spacetime, but that it is the 3-space that is "real", simply because it is directly observable [4].

The 3-space is in differential motion, that is one part has a velocity relative to other parts, and so involves a velocity field $\mathbf{v}(\mathbf{r},t)$ description. To be specific this velocity field must be described relative to a frame of observers, but the formalism is such that the dynamical equations for this velocity field must transform covariantly under a change of observer. It has been shown [4, 6] that the phenomenon of gravity is a consequence of the time-dependence and inhomogeneities of $\mathbf{v}(\mathbf{r},t)$. So the dynamical equations for $\mathbf{v}(\mathbf{r},t)$ give rise to a new theory of gravity when combined with the generalised Schrödinger equation, and the generalised Maxwell and Dirac equations [10]. The equations for $\mathbf{v}(\mathbf{r},t)$ involve the gravitational constant* G and a dimensionless constant that determines the strength of a new 3-space self-interaction effect, which is missing from both Newtonian Gravity and General Relativity. Experimental data has revealed [4, 5, 6] the remarkable discovery that this constant is the fine structure constant $\alpha \approx e^2/\hbar c \approx$ 1/137. This dynamics then explains numerous gravitational anomalies, such as the borehole g anomaly, the so-called "dark matter" anomaly in the rotation speeds of spiral galaxies, and that the effective

*This is different from the Newtonian effective gravitational constant G_N defined later.

mass of the necessary black holes at the centre of spherical matter systems, such as globular clusters and spherical galaxies, is $\alpha/2$ times the total mass of these systems. This prediction has been confirmed by astronomical observations [7].

Here we analyse the Greenland and Nevada Test Site borehole g anomalies, and confirm with increased precision this value of α.

The occurrence of α suggests that space is itself a quantum system undergoing on-going classicalisation. Just such a proposal has arisen in *Process Physics* [4] which is an information-theoretic modelling of reality. There quantum space and matter arise in terms of the Quantum Homotopic Field Theory (QHFT) which, in turn, may be related to the standard model of matter. In the QHFT space at this quantum level is best described as a "quantum foam". So we interpret the observed fractal* 3-space as a classical approximation to this "quantum foam" [10].

2 Dynamical 3-space

Relative to some observer 3-space is described by a velocity field $\mathbf{v}(\mathbf{r},t)$. It is important to note that the coordinate $\mathbf{r}$ is not itself 3-space, rather it is merely a label for an element of 3-space that has velocity $\mathbf{v}$, relative to some observer. Also it is important to appreciate that this "moving" 3-space is not itself embedded in a "space"; the 3-space is all there is, although as noted above its deeper structure is that of a "quantum foam".

In the case of zero vorticity $\nabla\times\mathbf{v}=\mathbf{0}$ the 3-space dynamics is given by [4, 6], in the non-relativistic limit,

$$\nabla\cdot\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)+\frac{\alpha}{8}\left((\mathrm{tr}D)^2-\mathrm{tr}(D^2)\right)=-4\pi G\rho, \quad (1)$$

where ρ is the matter density, and where

$$D_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right). \quad (2)$$

The acceleration of an element of space is given by the Euler form

$$\mathbf{g}(\mathbf{r},t)\equiv\lim_{\Delta t\to 0}\frac{\mathbf{v}\left(\mathbf{r}+\mathbf{v}(\mathbf{r},t)\Delta t,t+\Delta t\right)-\mathbf{v}(\mathbf{r},t)}{\Delta t}= \\ =\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\,. \quad (3)$$

It was shown in [10] that matter has the same acceleration† as (3), which gave a derivation of the equivalence principle as a quantum effect in the Schrödinger equation when uniquely generalised to include the interaction of the quantum system with the 3-space. These forms are mandated by Galilean covariance under change of observer‡. This minimalist non-relativistic modelling of the dynamics for the velocity field

*The fractal property of 3-space was found [10] from the DeWitte data.

†Except for the acceleration component induced by vorticity.

‡However this does not exclude so-called relativistic effects, such as the length contraction of moving rods or the time dilations of moving clocks.

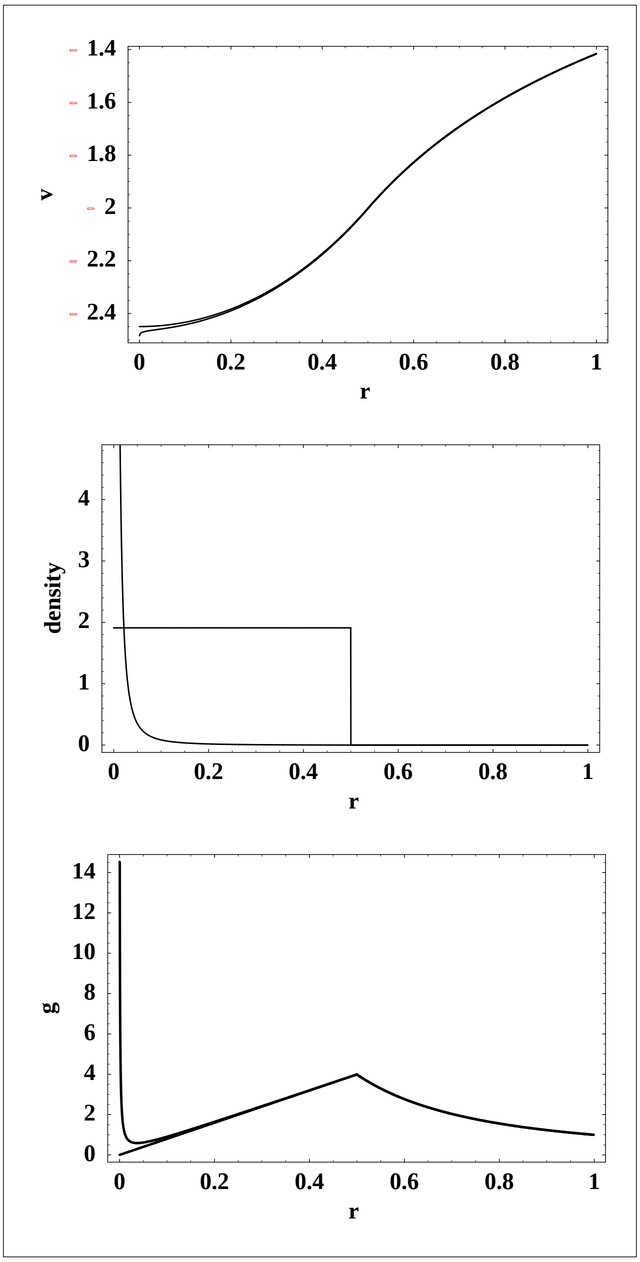

Fig. 1: Upper plot shows speeds from numerical iterative solution of (7) for a solid sphere with uniform density and radius $r=0.5$ for (i) upper curve the case $\alpha=0$ corresponding to Newtonian gravity, and (ii) lower curve with $\alpha=1/137$. These solutions only differ significantly near $r=0$. Middle plot shows matter density and "dark matter" density ρ_{DM}, from (5), with arbitrary scales. Lower plot shows the acceleration from (3) for (i) the Newtonian in-flow from the upper plot, and (ii) from the $\alpha=1/137$ case. The difference is only significant near $r=0$. The accelerations begin to differ just inside the surface of the sphere at $r=0.5$, according to (15). This difference is the origin of the borehole g anomaly, and permits the determination of the value of α from observational data. This generic singular-g behaviour, at $r=0$, is seen in the Earth, in globular clusters and in galaxies.

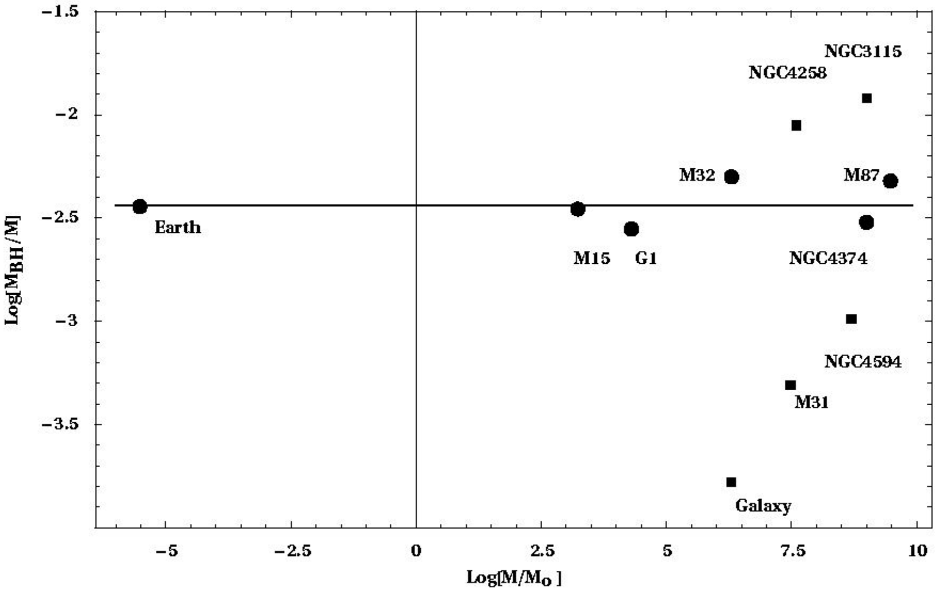

Fig. 2: The data shows $\mathrm{Log}_{10}[M_{BH}/M]$ for the "blackhole" or "dark matter" masses M_{BH} for a variety of spherical matter systems with masses M, shown by solid circles, plotted against $\mathrm{Log}_{10}[M/M_0]$, where M_0 is the solar mass, showing agreement with the "$\alpha/2$-line" ($\mathrm{Log}_{10}[\alpha/2] = -2.44$) predicted by (10), and ranging over 15 orders of magnitude. The "blackhole" effect is the same phenomenon as the "dark matter" effect. The data ranges from the Earth, as observed by the bore hole g anomaly, to globular cluster M15 and G1, and then to spherical "elliptical" galaxies M32 (E2), NGC 4374 (E1) and M87 (E0). Best fit to the data from these star systems gives $\alpha = 1/134$, while for the Earth data in Figs.3,4,5 give $\alpha = 1/137$. In these systems the "dark matter" or "black hole" spatial self-interaction effect is induced by the matter. For the spiral galaxies, shown by the filled boxes, where here M is the bulge mass, the blackhole masses do not correlate with the "$\alpha/2$-line". This is because these systems form by matter in-falling to a primordial blackhole, and so these systems are more contingent. For spiral galaxies this dynamical effect manifests most clearly via the non-Keplerian rotation-velocity curve, which decrease asymptotically very slowly. See [7] for references to the data.

gives a direct account of the various phenomena noted above. A generalisation to include relativistic effects of the motion of matter through this 3-space is given in [4]. From (1) and (3) we obtain that

$$\nabla \cdot \mathbf{g} = -4\pi G\rho - 4\pi G\rho_{DM}, \tag{4}$$

where

$$\rho_{DM}(\mathbf{r}) = \frac{\alpha}{32\pi G}\big((\mathrm{tr}D)^2 - \mathrm{tr}(D^2)\big). \tag{5}$$

In this form we see that if $\alpha \to 0$, then the acceleration of the 3-space elements is given by Newton's *Universal Law of Gravitation*, in differential form. But for a non-zero α we see that the 3-space acceleration has an additional effect, from the ρ_{DM} term, which is an effective "matter density" that mimics the new self-interaction dynamics. This has been shown to be the origin of the so-called "dark matter" effect in spiral galaxies. It is important to note that (4) does not determine $\mathbf{g}$ directly; rather the velocity dynamics in (1) must be solved, and then with $\mathbf{g}$ subsequently determined from (3). Eqn. (4) merely indicates that the resultant non-Newtonian $\mathbf{g}$ could be mistaken as the result of a new form of matter, whose density is given by ρ_{DM}. Of course the saga of "dark matter" shows that this actually happened, and that there has been a misguided and fruitless search for such "matter".

3 Airy method for determining α

We now show that the Airy method actually gives a technique for determining the value of α from Earth based borehole gravity measurements. For a time-independent velocity field (1) may be written in the integral form

$$|\mathbf{v}(\mathbf{r})|^2 = 2G\int d^3r'\,\frac{\rho(\mathbf{r}') + \rho_{DM}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{6}$$

When the matter density of the Earth is assumed to be spherically symmetric, and that the velocity field is now radial* (6) becomes

$$\begin{aligned} v(r)^2 = \frac{8\pi G}{r}\int_0^r s^2\left[\rho(s) + \rho_{DM}(s)\right] ds\, + \\ + 8\pi G\int_r^\infty s\left[\rho(s) + \rho_{DM}(s)\right] ds\,, \end{aligned} \tag{7}$$

*This in-flow is additional to the observed velocity of the Earth through 3-space.

where, with $v' = dv(r)/dr$,

$$\rho_{DM}(r) = \frac{\alpha}{32\pi G}\left(\frac{v^2}{2r^2} + \frac{vv'}{r}\right). \tag{8}$$

Iterating (7) once we find to 1st order in α that

$$\rho_{DM}(r) = \frac{\alpha}{2r^2}\int_r^\infty s\,\rho(s)\,ds + \mathrm{O}(\alpha^2)\,, \tag{9}$$

so that in spherical systems the "dark matter" effect is concentrated near the centre, and we find that the total "dark matter" is

$$\begin{aligned} M_{DM} &\equiv 4\pi\int_0^\infty r^2\rho_{DM}(r)\,dr = \\ &= \frac{4\pi\alpha}{2}\int_0^\infty r^2\rho(r)\,dr + \mathrm{O}(\alpha^2) = \frac{\alpha}{2}M + \mathrm{O}(\alpha^2)\,, \end{aligned} \tag{10}$$

where M is the total amount of (actual) matter. Hence to $\mathrm{O}(\alpha)$ $M_{DM}/M = \alpha/2$ independently of the matter density profile. This turns out to be a very useful property as complete knowledge of the density profile is then not required in order to analyse observational data. As seen in Fig. 1 the singular behaviour of both v and g means that there is a *blackhole** singularity at $r = 0$. Interpreting M_{DM} in (10) as the mass of the blackholes observed in the globular clusters M15 and G1 and in the highly spherical "elliptical" galaxies M32, M87 and NGC 4374, we obtained [7] $\alpha \approx 1/134$, as shown in Fig. 2.

From (3), which is also the acceleration of matter [10], the gravity acceleration† is found to be, to 1st order in α, and using that $\rho(r) = 0$ for $r > R$, where R is the radius of the Earth,

$$g(r) = \begin{cases} \dfrac{\left(1+\frac{\alpha}{2}\right)GM}{r^2}\,, & r > R\,; \\ \dfrac{4\pi G}{r^2}\displaystyle\int_0^r s^2\rho(s)\,ds + \\ +\dfrac{2\pi\alpha G}{r^2}\displaystyle\int_0^r\left(\int_s^R s'\rho(s')ds'\right)ds\,, & r < R\,. \end{cases} \tag{11}$$

This gives Newton's "inverse square law" for $r > R$, even when $\alpha \neq 0$, which explains why the 3-space self-interaction dynamics did not overtly manifest in the analysis of planetary orbits by Kepler and then Newton. However inside the Earth (11) shows that $g(r)$ differs from the Newtonian theory, corresponding to $\alpha = 0$, as Fig. 1, and it is this effect that allows the determination of the value of α from the Airy method.

Expanding (11) in r about the surface, $r = R$, we obtain, to 1st order in α and for an arbitrary density profile,

$$g(r) = \begin{cases} \dfrac{G_N M}{R^2} - \dfrac{2G_N M}{R^3}(r-R)\,, & r > R\,; \\ \dfrac{G_N M}{R^2} - \left(\dfrac{2G_N M}{R^3} - 4\pi\left(1-\dfrac{\alpha}{2}\right)G_N\rho\right)\times \\ \qquad\qquad \times\,(r-R)\,, & r < R\,. \end{cases} \tag{12}$$

where ρ is the matter density at the surface, M is the total matter mass of the Earth, and where we have defined

$$G_N \equiv \left(1+\frac{\alpha}{2}\right)G\,. \tag{13}$$

The corresponding Newtonian gravity expression is obtained by taking the limit $\alpha \to 0$,

$$g_N(r) = \begin{cases} \dfrac{G_N M}{R^2} - \dfrac{2G_N M}{R^3}(r-R)\,, & r > R\,; \\ \dfrac{G_N M}{R^2} - \left(\dfrac{2G_N M}{R^3} - 4\pi G_N\rho\right)(r-R)\,, & r < R\,. \end{cases} \tag{14}$$

Assuming Newtonian gravity (14) then means that from the measurement of difference between the above-ground and below-ground gravity gradients, namely $4\pi G_N\rho$, and also measurement of the matter density, permit the determination of G_N. This is the basis of the Airy method for determining G_N [3].

When analysing the borehole data it has been found [11, 12] that the observed difference of the density gradients was inconsistent with $4\pi G_N\rho$ in (14), in that it was not given by the laboratory value of G_N and the matter density. This is known as the borehole g anomaly and which attracted much interest in the 1980's. The key point in understanding this anomaly is that even allowing for the dynamical rescaling of G, expressions (12) and (14) have a different dependence on $r - R$ beneath the surface. The borehole data papers [11, 12] report the discrepancy, i. e. the anomaly or the gravity residual as it is called, between the Newtonian prediction and the measured below-earth gravity gradient. Taking the difference between (12) and (14), assuming the same unknown value of G_N in both, we obtain an expression for the gravity residual

$$\Delta g(r) \equiv g_N(r) - g(r) = \begin{cases} 0\,, & r > R\,; \\ 2\pi\alpha G_N\rho(r-R)\,, & r < R\,. \end{cases} \tag{15}$$

When $\alpha \neq 0$ we have a two-parameter theory of gravity, and from (11) we see that measurement of the difference between the above ground and below ground gravity gradients is $4\pi\left(1-\frac{\alpha}{2}\right)G_N\rho$, and this is not sufficient to determine both G_N and α, given ρ, and so the Airy method is now understood not to be a complete measurement by itself, i. e. we need to combine it with other measurements. If we now use laboratory Cavendish experiments to determine G_N, then from the borehole gravity residuals we can determine the value of α, as already indicated in [5, 6]. As discussed in Sect. 7 these Cavendish experiments can only determine G_N

*These are called *blackholes* because there is an event horizon, but in all other aspects differ from the *blackholes* of General Relativity.

†We now use the convention that $g(r)$ is positive if it is radially inward.

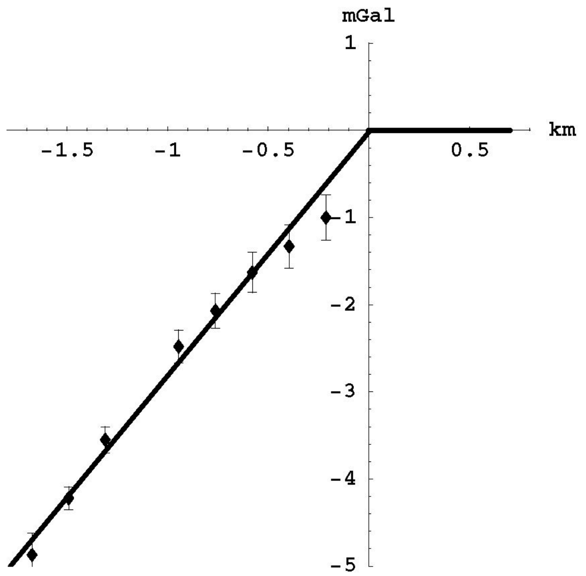

Fig. 3: The data shows the gravity residuals for the Greenland Ice Shelf [11] Airy measurements of the $g(r)$ profile, defined as $\Delta g(r) = g_{\text{Newton}} - g_{\text{observed}}$, and measured in mGal (1mGal $= 10^{-3}$ cm/s^2) and plotted against depth in km. The gravity residuals have been offset. The borehole effect is that Newtonian gravity and the new theory differ only beneath the surface, provided that the measured above surface gravity gradient is used in both theories. This then gives the horizontal line above the surface. Using (15) we obtain $\alpha^{-1} = 137.9 \pm 5$ from fitting the slope of the data, as shown. The non-linearity in the data arises from modelling corrections for the gravity effects of the irregular sub ice-shelf rock topography.

up to corrections of order $\alpha/4$, simply because the analysis of the data from these experiments assumed the validity of Newtonian gravity. So the analysis of the borehole residuals will give the value of α up to $O(\alpha^2)$ corrections, which is consistent with the $O(\alpha)$ analysis reported above.

4 Greenland Ice Shelf borehole data

Gravity residuals from a bore hole into the Greenland Ice Shelf were determined down to a depth of 1.5 km by Ander *et al.* [11] in 1989. The observations were made at the Dye 3 2033 m deep borehole, which reached the basement rock. This borehole is 60 km south of the Arctic Circle and 125 km inland from the Greenland east coast at an elevation of 2530 m. It was believed that the ice provided an opportunity to use the Airy method to determine G_N, but now it is understood that in fact the borehole residuals permit the determination of α, given a laboratory value for G_N. Various steps were taken to remove unwanted effects, such as imperfect knowledge of the ice density and, most dominantly, the terrain effects which arises from ignorance of the profile and density inhomogeneities of the underlying rock. The borehole gravity meter was calibrated by comparison with an absolute gravity meter. The ice density depends on pressure, temperature and air content, with the density rising to its average value of $\rho = 920$ kg/m^3 within some 200 m of the surface, due to compression of the trapped air bubbles. This surface gradient in the density has been modelled by the author, and is not large enough the affect the results. The leading source of uncertainty was from the gravitational effect of the bedrock topography, and this was corrected for using Newtonian gravity. The correction from this is actually the cause of the non-linearity of the data points in Fig. 3. A complete analysis would require that the effect of this rock terrain be also computed using the new theory of gravity, but this was not done. Using $G_N = 6.6742 \times 10^{-11}$ m^{3}s^{-2}kg^{-1}, which is the current CODATA value, see Sect. 7, we obtain from a least-squares fit of the linear term in (15) to the data points in Fig. 3 that $\alpha^{-1} = 137.9 \pm 5$, which equals the value of the fine structure constant $\alpha^{-1} = 137.036$ to within the errors, and for this reason we identify the constant α in (1) as being the fine structure constant. The first analysis [5, 6] of the Greenland Ice Shelf data incorrectly assumed that the ice density was 930 kg/m^3 which gave $\alpha^{-1} = 139 \pm 5$. However trapped air reduces the standard ice density to the ice shelf density of 920 kg/m^3, which brings the value of α immediately into better agreement with the value of $\alpha = e^2/\hbar c$ known from quantum theory.

5 Nevada Test Site borehole data

Thomas and Vogel [12] performed another borehole experiment at the Nevada Test Site in 1989 in which they measured the gravity gradient as a function of depth, the local average matter density, and the above ground gradient, also known as the free-air gradient. Their intention was to test the extracted G_{local} and compare with other values of G_N, but of course using the Newtonian theory. The Nevada boreholes, with typically 3 m diameter, were drilled as a part of the U.S. Government tests of its nuclear weapons. The density of the rock is measured with a $\gamma-\gamma$ logging tool, which is essentially a γ-ray attenuation measurement, while in some holes the rock density was measured with a coreing tool. The rock density was found to be 2000 kg/m^3, and is dry. This is the density used in the analysis herein. The topography for 1 to 2 km beneath the surface is dominated by a series of overlapping horizontal lava flows and alluvial layers. Gravity residuals from three of the bore holes are shown in Figs.4, 5 and 6. All gravity measurements were corrected for the Earth's tide, the terrain on the surface out to 168 km distance, and the evacuation of the holes. The gravity residuals arise after allowing for, using Newtonian theory, the local lateral mass anomalies but assumed that the matter beneath the holes occurs in homogeneous ellipsoidal layers. Here we now report a detailed analysis of the Nevada data. First we note that the gravity residuals from borehole U20AO, Fig. 6,

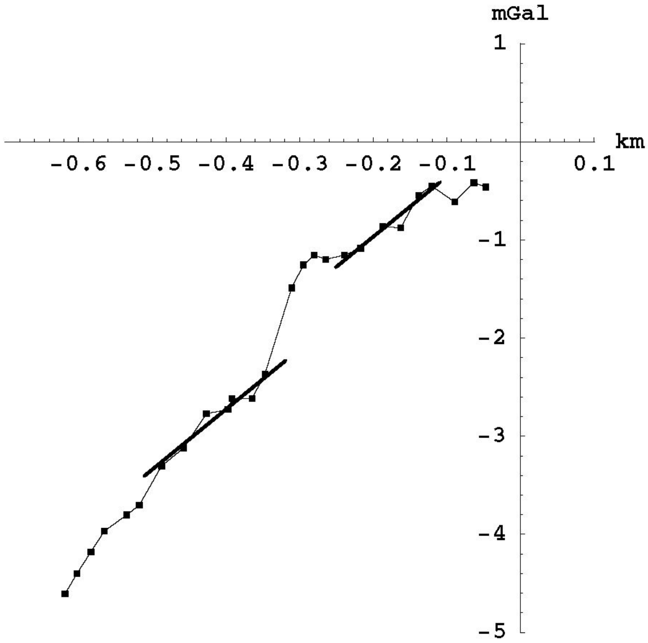

Fig. 4: The data shows the gravity residuals for the Nevada U20AK borehole Airy measurements of the $g(r)$ profile [12], defined as $\Delta g(r) = g_{\text{Newton}} - g_{\text{observed}}$, and measured in mGal, plotted against depth in km. This residual shows regions of linearity interspersed with regions of non-linearity, presumably arising from layers with a density different from the main density of 2000 kg/m^3. Density changes generate a change in the (arbitrary) residual offset. From a least-squares simultaneous fit of the linear form in (15) to the four linear regions in this data and that in Fig. 5 for the data from borehole U20AL, we obtain $\alpha^{-1} = 136.8 \pm 3$. The two fitted regions of data are shown by the two straight lines here and in Fig. 5.

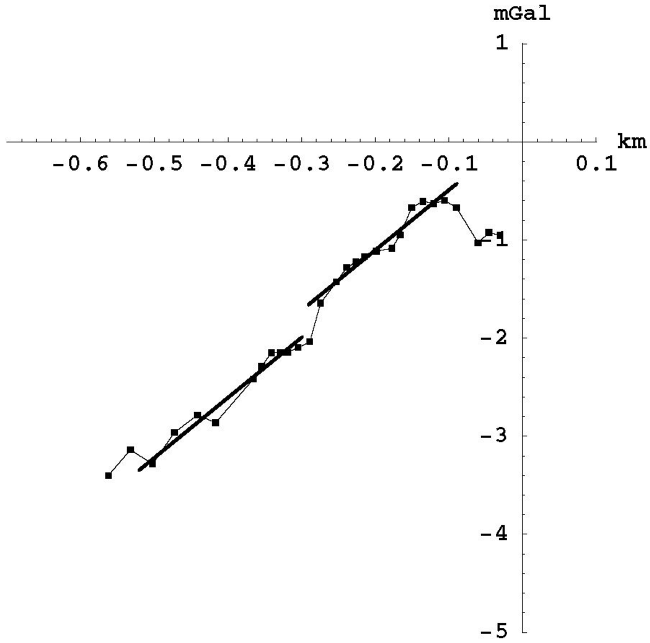

Fig. 5: The data shows the gravity residuals for the Nevada U20AL borehole Airy measurements of the $g(r)$ profile [12], defined as $\Delta g(r) = g_{\text{Newton}} - g_{\text{observed}}$, and measured in mGal, plotted against depth in km. This residual shows regions of linearity interspersed with regions of non-linearity, presumably arising from layers with a density different from the main density of 2000 kg/m^3. Density changes generate a change in the (arbitrary) residual offset. From a least-squares simultaneous fit of the linear form in (15) to the four linear regions in this data and that in Fig. 4 for the data from borehole U20AK in Fig. 4, we obtain $\alpha^{-1} = 136.8 \pm 3$. The two fitted regions of data are shown by the two straight lines here and in Fig. 4.

are not sufficiently linear to be useful. This presumably arises from density variations caused by the layering effect. For boreholes UA20AK, Fig. 4, and UA20AL, Fig. 5, we see segments where the gravity residuals are linear with depth, where the density is the average value of 2000 kg/m^3, but interspersed by layers where the residuals show non-linear changes with depth. It is assumed here that these non-linear regions are caused by variable density layers. So in analysing this data we have only used the linear regions, and a simultaneous least-squares fit to (15), with again $G_{\text{N}} = 6.6742 \times 10^{-11}$ m^3 s^{-2}kg^{-1} as for the Greenland data analysis, of these four linear regions gives $\alpha^{-1} = 136.8 \pm 3$, which again is in extraordinary agreement with the value of 137.04 from quantum theory.

6 Ocean measurements

The ideal Airy experiment would be one using the ocean, as all relevant physical aspects are accessible. Such an experiment was carried out by Zumberge *et al.* in 1991 [13] using submersibles. Corrections for sea floor topography, seismic profiles and sea surface undulations were carried out. However a true Airy experiment appears not to have been performed. That would have required the measurement of the above and below sea-surface gravity gradients. Rather only the below sea-surface gradients were measured, and compared with a predicted gravity gradient using the density of the water and a laboratory value of G_{N} from only one such experiment and, as shown in Fig. 7, these have a large uncertainty. Hence this experiment does not permit an analysis of the data of the form applied to the Greenland and Nevada observations. The value of G_{N} from this ocean experiment is shown in Fig. 7 as experiment #12.

7 G experiments

The new theory of gravity, given in (1) for the case of zero vorticity and in the non-relativistic limit, is a two-parameter theory; G and α. Hence in experiments to determine G (or G_{N}) we expect to see systematic discrepancies if the

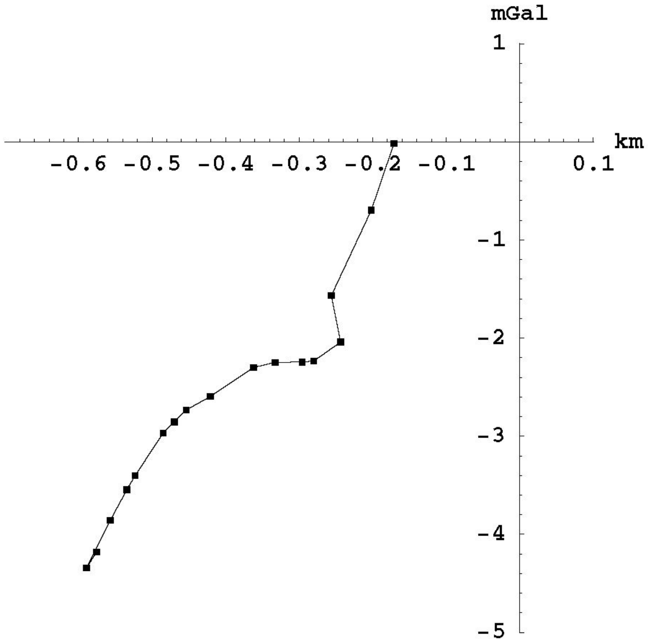

Fig. 6: The data shows the gravity residuals from the third Nevada U20AO borehole Airy measurements of the $g(r)$ profile [12]. This data is not of sufficient linearity, presumably due to non-uniformity of density, to permit a fit to the linear form in (15), but is included here for completeness. There is an arbitrary offset in the residual.

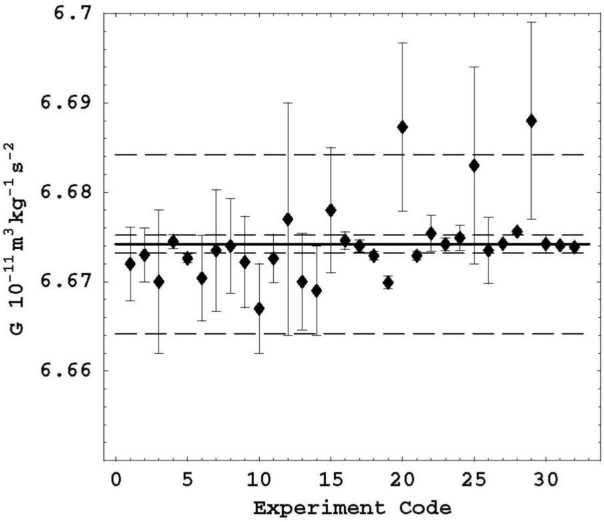

Fig. 7: Results of precision measurements of G_N published in the last sixty years in which the Newtonian theory was used to analyse the data. These results show the presence of a systematic effect, not in the Newtonian theory, of fractional size up to $\Delta G_N/G_N \approx \alpha/4$, which corresponded with the 1998 error bars on G_N (outer dashed lines), with the full line being the current CODATA value of $G_N = 6.6742(10)\times 10^{-11}$ m^{2}s^{-2}kg^{-1}. In 2005 CODATA [20] reduced the error bars by a factor of 10 (inner dashed lines) on the basis of some recent experiments, and so neglecting the presence of the systematic effect.

Newtonian theory is used to analyse the data. This is clearly the case as shown in Fig. 7 which shows the results of such analyses over the last 60 years. The fundamental problem is that non-Newtonian effects of size $\Delta G_N/G_N \approx \alpha/4$ are clearly evident, and effects of this size are expected from (1). To correctly analyse data from these experiments the full theory in (1) must be used, and this would involve (i) computing the velocity field for each configuration of the test masses, and then (ii) computing the forces by using (3) to compute the acceleration field. These computations are far from simple, especially when the complicated matter geometries of recent experiments need to be used. Essentially the flow of space results in a non-Newtonian effective "dark matter" density in (5). This results in deviations from Newtonian gravity which are of order $\alpha/4$. The prediction is that when laboratory Cavendish-type experiments are correctly analysed the data will permit the determination of both G_N and α, and the large uncertainties in the determination of G_N will no longer occur. Until then the value of G_N will continue to be the least accurately known of all the fundamental constants. Despite this emerging insight CODATA* in 2005 [20] reduced the apparent uncertainties in G_N by a factor of 10, and so ignoring the manifest presence of a systematic effect. The occurrence of the fine structure constant α, in giving the magnitude of the spatial self-interaction effect in (1), is a fundamental development in our understanding of 3-space and the phenomenon of gravity. Indeed the implication is that α arises here as a manifestation of quantum processes inherent in 3-space.

*CODATA is the Task Group on Fundamental Constants of the Committee on Data for Science and Technology, established in 1969.

8 Some history

Here we have simply applied the new two-parameter theory of 3-space, and hence of gravity, to the existing data from borehole experiments. However the history of these experiments shows that, of course, the nature of the gravitational anomaly had not been understood, and so the implications for fundamental physics that are now evident could not have been made. The first indications that some non-Newtonian effect was being observed arose from Yellin [14] and Hinze *et al.* [15]. It was Stacey *et al.* in 1981 [17, 16, 18] who undertook systematic studies at the Mt. Isa mine in Queensland, Australia. In the end a mine site is very unsuited for such a gravitational anomaly experiment as by their very nature mines have non-uniform poorly-known density and usually, as well, irregular surface topography. In the end it was acknowledged that the Mt. Isa mine data was unreliable. Nevertheless those reports motivated the Greenland, Nevada and Ocean experiments, as well as above-ground tower experiments [19], all with the assumption that the non-Newtonian effects were being caused by a modification to Newton's *inverse square law* by an additional short-range force — which also involved the notion of a possible "5th-force" [21]. However these interpretations were not supported by the data, and eventually the whole phenomenon of these gravitational borehole anomalies was forgotten.

9 Conclusions

We have extended the results from an earlier analysis [5, 6] of the Greenland Ice Shelf borehole g anomaly data by correcting the density of ice from the assumed value to the actual value. This brought the extracted value of α from approximately 1/139 to approximately 1/137, and so into even closer agreement with the quantum theory value. As well the analysis was extended to the Nevada borehole anomaly data, again giving $\alpha \approx 1/137$. This is significant as the rock density is more than twice the ice density. As well we have included the previous results [7] from analysis of the blackhole masses in globular clusters and elliptical "spherical" galaxies, which gave $\alpha \approx 1/134$, but with larger uncertainty. So the conclusion that α is actually the fine structure constant from quantum theory is now extremely strong. These results, together with the successful explanation for the so-called spiral galaxy "dark-matter" effect afforded by the new theory of gravity, implies that the Newtonian theory of gravity [1] is fundamentally flawed, even at the non-relativistic level, and that the disagreement with experiment and observation can be of fractional order α, or in the case of spiral galaxies and blackholes, extremely large. This failure implies that General Relativity, which reduces to the Newtonian theory in the non-relativistic limit, must also be considered as flawed and disproven.

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The Solar Photosphere: Evidence for Condensed Matter

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The stellar equations of state treat the Sun much like an ideal gas, wherein the photosphere is viewed as a sparse gaseous plasma. The temperatures inferred in the solar interior give some credence to these models, especially since it is counterintuitive that an object with internal temperatures in excess of 1 MK could be existing in the liquid state. Nonetheless, extreme temperatures, by themselves, are insufficient evidence for the states of matter. The presence of magnetic fields and gravity also impact the expected phase. In the end, it is the physical expression of a state that is required in establishing the proper phase of an object. The photosphere does not lend itself easily to treatment as a gaseous plasma. The physical evidence can be more simply reconciled with a solar body and a photosphere in the condensed state. A discussion of each physical feature follows: (1) the thermal spectrum, (2) limb darkening, (3) solar collapse, (4) the solar density, (5) seismic activity, (6) mass displacement, (7) the chromosphere and critical opalescence, (8) shape, (9) surface activity, (10) photospheric/coronal flows, (11) photospheric imaging, (12) the solar dynamo, and (13) the presence of Sun spots. The explanation of these findings by the gaseous models often requires an improbable combination of events, such as found in the stellar opacity problem. In sharp contrast, each can be explained with simplicity by the condensed state. This work is an invitation to reconsider the phase of the Sun.

Introduction

The stellar phase has important consequences, not only for modeling the Sun, but indeed, for the proper treatment of nearly every aspect of astrophysics. Recently, the accepted temperature of the photosphere has been questioned [1]. This hinges on the proper understanding of both blackbody radiation [2] and the liquid state [3]. In modern theory, stars can be essentially infinitely compressed without ever becoming liquid. Outside the Earth's oceans, the liquid state appears all but non-existent in the universe. By invoking the gaseous equations of state [i. e. 4] without the possibility of condensation to the liquid and solid state, the accepted models continue to ignore laboratory findings relative to the existence of these transformations. These issues are not simple. However, sufficient evidence exists to bring into question the gaseous models of the Sun.

The physical evidence

1. The thermal spectrum:

It is hard to imagine that, after more than 100 years, our understanding of blackbody radiation could be questioned. If this is the case, it is because of shortcomings in the work of Gustav Kirchhoff [5, 6] which have previously been overlooked [7]. The arguments hinged on whether or not blackbody radiation is in fact universal as initially advanced by Kirchhoff [5, 6], echoed by Planck [2] and theoretically confirmed by Einstein [8]. In order to dissect the problem, Kirchhoff and Planck are treated together, along with the experimental proof [7]. Einstein's work [8] can then be examined from a conceptual viewpoint [9] without bringing into question any of Einstein's mathematics. Thus, arguments against the universality of blackbody radiation have already been made both on an experimental basis [7] and on a theoretical one [9]. In reality, the entire foundation for the liquid model of the Sun rests on the soundness of these arguments [7, 9]. The belief is that claims of universality are not only overstated, they are incorrect [9]. As such, it is improper to assign any astrophysical temperature based on the existence of a thermal spectrum in the absence of a known isothermal (not adiabatic) and perfectly absorbing enclosure [1, 7, 9].

The Sun possesses a thermal signature as reported early on by Langley [10, 11]. The fact that this spectrum is continuous in nature leads to difficulties for the gaseous models [1]. This is because gases are known to emit radiation only in discrete bands [12]. Consequently, in order to produce the thermal spectrum of the Sun, theoretical astrophysics must currently invoke the summation of numerous spectroscopic processes. Furthermore, this must occur in a slightly shifted manner within each internal layer of the Sun. Many distinct physical processes (bound-bound, bound-free, and free-free) are used to arrive at a single spectrum [i. e. 4]. This constitutes the stellar opacity problem: the summation of many distinct spectroscopic processes to yield a single spectroscopic signature.

In reality, each spectroscopic signature, including the thermal spectrum, must arise from a single spectroscopic process [1]. Just as an NMR spectrum arises from an NMR process, so must a thermal spectrum arise from a thermal process. Whatever process takes place with graphite on Earth must be taking place on the surface of the Sun. That the gaseous models require many spectroscopic processes along with gradually and systematically changing stellar opacities [i. e. 4] is perhaps their greatest obstacle. Gases simply cannot generate thermal spectra in the absence of a rigid body (condensed matter) enclosure. They are restricted to emission in bands.

In contrast, condensed matter can easily generate continuous spectra [13, 14, 15] as a manifestation of its inherent lattice structure. Thus, relative to the existence of a continuous solar spectrum, a condensed matter model of the Sun has distinct advantages.

2. Limb darkening:

The Sun is also characterized by limb darkening. The solar spectrum becomes less bright when viewing the Sun from the center to the limb. Since a change in the thermal spectrum is involved, the gaseous models must once again invoke the stellar opacity problem. Limb darkening is explained by inferring the sampling of varying optical depths. The Sun must be able to slowly and gradually change its thermal spectrum from one temperature to another based on depth using a perfect combination of bound-bound, bound-free and free-free processes at every location inside the Sun. Gaseous theory therefore places a tremendous constraint on nature relative to limb darkening. As stated above, it is not reasonable to expect that a single spectrum is actually resultant from the infinite sum of many distinct and unrelated spectroscopic processes. If a thermal spectrum is produced by the Sun, it must invoke the same mechanism present in the piece of graphite on Earth. That the gaseous models rely on varying optical depths in order to explain limb darkening might appear elegant, but lacks both clarity and support in experimental physics.

In sharp contrast, angle dependence in thermal emission is extremely well documented for condensed matter [14, 15]. Changes in optical depth are not required. Rather, a subtle change in the angle of observation is sufficient. This is precisely what is observed when we monitor the Sun. For instance, even the oceans of the Earth are known to have angle dependent emission intensities at microwave frequencies [16]. Thus, in the condensed matter scenario, limb darkening is an expression of angle of observation without having to make any arguments based on optical depth.

3. Solar collapse:

One of the key requirements of the gaseous models is the need to prevent solar collapse as a result of gravitational forces. Currently, it is advocated that solar collapse is prevented by electron gas pressure in the solar interior and, for larger stars, by radiation pressure. However, the existence of gas pressure relies on the presence of a rigid surface [i. e. 4]. The atmosphere of the Earth does not collapse due to the relatively rigid oceanic and continental surfaces. Within the gaseous models of the stars however, there is no mechanism to introduce the rigid surface required to maintain gas pressure. Theoretical arguments are made [i. e. 4] without experimental foundation. The same holds for internal radiation pressure. There is no experimental basis on Earth for radiation pressure internal to a single object [13, 14, 15]. It is well-established that for the gaseous models of the Sun, complete solar collapse would take place in a matter of seconds should electron gas pressure and internal radiation pressure cease [i. e. 4]. In sharp contrast, relative incompressibility is a characteristic of the liquid state. A liquid Sun is by definition essentially incompressible, and experimental evidence for such behavior in liquids is abundant. Stellar collapse is excluded by the very nature of the phase invoked.

4. Solar density:

The Sun has an average density of 1.4 g/cm^3. The gaseous models distribute this density with radial dependence with the core of the Sun typically approaching a density of 150 g/cm^3 and the photosphere 10^{-7} g/cm^3. If the Sun were truly a gaseous plasma, it would have been much more convenient if the average density did not so well approximate the density of the condensed state (>1 g/cm^3). The gaseous models would be in a much stronger position if the average solar density, for instance, was 10^{-4} g/cm^3. Such a density would clearly not lend itself to the condensed state. In contrast, the known density of the Sun is ideal for a condensed model whose primary constituents are hydrogen and helium. Moreover, for the condensed models [1], the radial dependence of density is not critical to the solution and a uniform distribution of mass may be totally acceptable.

The density of the Sun very closely approaches that of all the Jovian planets. Nonetheless, a great disparity in mass exists between the Sun and these planets. As such, it is probably best not to enter into schemes which involve great changes in internal solar densities. The liquid model maintains simplicity in this area and such a conclusion is viewed as important.

5. Seismology:

The Sun is a laboratory of seismology [17]. Yet, on Earth, seismology is a science of the condensed state. It is interesting to highlight how the gaseous models of the Sun fail to properly fit seismological data. In the work by Bahcall et. al. [18] for instance, experimental and theoretical siesmological findings are compared as a function of Solar radius. Precise fits are obtained for most of the solar sphere. In fact, it is surprising how the interior of the Sun can be so accurately fitted, given that all the data is being acquired from the solar surface. At the same time, this work is unable to fit the data in the exterior 5% of the Sun [18]. Yet, this is precisely the point from which all the data is being collected. The reason that this region cannot be fitted is that the gaseous models are claiming that the photosphere has a density on the order of 10^{-7} g/cm^3. This is lower than practical vacuums on Earth. Thus, the gaseous models are trying to conduct seismology in a vacuum by insisting on a photospheric density unable to sustain seismic activity. For the condensed models of the Sun, this complication is eliminated.

6. Mass displacement:

On July 9, 1996, the SOHO satellite obtained Doppler images of the solar surface in association with the eruption of a flare [19, 20]. These images reveal the clear propagation of transverse waves on the solar surface. The authors of the scientific paper refer to the mass displacement exactly like the action resulting from a pebble thrown in a pond. This is extremely difficult to explain for the gaseous models, yet trivial for the condensed model. The Doppler images show the presence of transverse waves. This is something unique to the condensed state. Gases propagate energy longitudinally. It can be theoretically argued perhaps that gases can sustain transverse waves. These however would be on the order of a few atomic radii at best. In sharp contrast, the waves seen on the Sun extend over thousands of kilometers. Once again, the condensed state provides a greatly superior alternative to the study of transverse waves on the solar surface.

7. The chromosphere and critical opalescence:

Critical opalescence occurs when a material is placed at the critical point, that combination of temperature, pressure, magnetic field and gravity wherein the gas/liquid interface disappears. At the critical point, a transparent liquid becomes cloudy due to light scattering, hence the term critical opalescence. The gas is regaining order, as it becomes ready to enter the condensed phase. It would appear that the Sun, through the chromosphere, is revealing to us behavior at the solar critical point. Under this scenario, the chromosphere is best viewed as the transition phase between the condensed photosphere and the gaseous corona.

In order to shed light on this problem, consider that in the lower region of the corona, the gaseous material exists at a temperature just beyond the critical temperature. The temperature is sufficiently elevated, that it is impossible for condensation to occur, given the gravity present. However, as one moves towards the Sun, the critical temperature increases as a result of increased gravity. Consequently, a point will eventually be reached where the temperature of the region of interest is in fact below the critical temperature. Condensation can begin to occur. As the surface of the Sun is increasingly approached, the critical temperature increases further. This is a manifestation of increased gravity and magnetic forces. By the time the photosphere is reached, the region of interest is now well below the critical temperature and the liquid state becomes stable. The surface at this point is visualized.

Therefore, in the liquid model, the chromosphere represents that region where matter projected into the corona is now in the process of re-condensing in order to enter the liquid state of the photosphere. Such an elegant explanation of the chromosphere is lacking for the gaseous models. Indeed, for these models, the understanding of the chromosphere requires much more than elementary chemical principles.

8. Shape:

The Sun is not a perfect sphere. It is oblate. Solar oblateness [21] is a direct manifestation of solar rotation and can best be understood by examining the rotation of liquid masses [22]. The oblateness of the solar disk has recently come under re-evaluation. While exact measurements have differed in the extent of solar oblateness, it appears that the most reliable studies currently place solar oblateness at 8.77×10^{-6} [21]. In order to understand solar oblateness, astrophysics is currently invoking a relative constant solar density as a function of radial position [21]. This is in keeping with our understanding of liquid body rotations [22], but is in direct opposition to the densities calculated using the gaseous equations of state [i. e. 4]. Interestingly, a relatively constant density is precisely what is invoked in the condensed matter model of the Sun [1]. The question becomes even more important when one considers stars like Achanar whose oblateness approaches 1.5 [23]. Such an observation would be difficult to rationalize were the Sun truly gaseous.

9. Surface activity:

The Sun has extensive surface activity and appears to be boiling. Indeed, several undergraduate texts actually refer to the Sun as a boiling gas. In addition to the boiling action, the Sun is characterized by numerous solar eruptions. Both of these phenomena (boiling and solar eruptions) are extremely difficult to rationalized for the gaseous models. Gases do not boil. They are the result of such action. It is an established fact that liquids boil giving rise to gases. There is no evidence on Earth that superheating a gas can give rise to a region of different density capable of erupting from the gaseous mass. These are extremely complex issues for the gaseous models since actions resembling both boiling and superheating must be generated without having recourse to the liquid state.

In contrast, the presence of superheated liquids within the solar interior could easily explain the production of solar eruptions. The existence of boiling action is well documented for the liquid. Nothing further need be added. Phenomena easily explained in the liquid model, become exceedingly difficult for the gaseous equations of state.

10. Photospheric/coronal flow:

It has been well established that the Sun displays pronounced flows at the surface. Matter can be seen rising from, and descending into, the solar interior. However, matter is also traversing the solar surface in a manner perpendicular to established flows in the corona. The photosphere is characterized not simply by a change in opacity as the gaseous models theorize, but by drastically altered directions of material flow relative to the corona. In the liquid model, the interface delineated by flow directions can be explained based on the existence of a phase transition between the photosphere and the corona. In fact, the orthogonality of mass displacement at the solar surface relative to the corona is reminiscent of the orthogonality observed on Earth between the currents in the oceans and the upward and downwards drafts sometimes observed in the overlying air. It is not trivial for the gaseous models to account for the orthogonality of flow between the photosphere and the corona. By contrast, this is a natural extension of current knowledge relative to liquid/gaseous interfaces for the liquid model.

11. Photospheric imaging:

The solar surface has recently been imaged in high resolution using the Swedish Solar Telescope [24, 25]. These images reveal a clear solar surface in 3D with valleys, canyons, and walls. Relative to these findings, the authors insist that a true surface is not being seen. Such statements are prompted by belief in the gaseous models of the Sun. The gaseous models cannot provide an adequate means for generating a real surface. Solar opacity arguments are advanced to caution the reader against interpretation that a real surface is being imaged. Nonetheless, a real surface is required by the liquid model. It appears that a real surface is being seen. Only our theoretical arguments seem to support our disbelief that a surface is present.

12. Dynamo action:

The Sun is characterized by strong magnetic fields. These magnetic fields can undergo complex winding and protrusions. On Earth however, strong magnetic fields are always produced from condensed matter. The study of dynamos relies on the use of molten sodium [26], not gaseous sodium. It is much more realistic to generate powerful magnetic fields in condensed matter than in sparse gaseous plasmas. Consequently, the liquid model and its condensed phase lends itself much more readily to the requirements that the Sun possesses strong magnetic fields.

13. Sun spots:

The presence of Sun spots have long been noted on the solar sphere. Sun spots are often associated with strong magnetic activity. The gaseous models explain the existence of Sun spots with difficulty. The problem lies in the requirement that different types of order (disorder) can coexist in stellar gases, based on the presence of a magnetic field. While there is ample room here for theoretical arguments justifying the existence of Sun spots in a gaseous model, the situation is less complex in the liquid model. Thus, if one considers that the bulk of the solar photosphere exists with hydrogen and helium adhering to a certain lattice structure, all that is required is a concentration of magnetic fields within a region to produce a change in the lattice. The surface of the Sun is changed from a hypothetical "Type I lattice" to a "Type II lattice". The requirement that a strong magnetic field alters the structure of condensed matter in an ordered lattice from one form to another, is much less than would be required to alter the structure of a gaseous plasma (something which has no inherent lattice).

Conclusion

The evidence in favor of a condensed matter model of the Sun is overwhelming. For every avenue explored, the condensed model holds clear advantages in simplicity of understanding. In fact, it remains surprising that the gaseous models have been able to survive for so long. This is partially due to the elegance with which the theoretical framework is established. Moreover, the gaseous equations of state have such profound implications for astrophysics.

Consequently, it is recognized that the acceptance of any condensed matter model will require such dramatic changes in astrophysics that such adoption cannot be swift. In the meantime, it is important to set out the physical evidence for a liquid model both in manuscript [1] and abstract form [27–30]. Eventually, astrophysics may well be forced to abandon the gaseous models and their equations of state. It is likely that this will occur when the field more fully appreciates the lack of universality in blackbody radiation [7, 9, 31]. At this time, gases will no longer be hypothesized as suitable candidates for the emission of thermal radiation. The need for condensed matter will be self-evident.

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An Analysis of Universality in Blackbody Radiation

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Through the formulation of his law of thermal emission, Kirchhoff conferred upon blackbody radiation the quality of universality [G. Kirchhoff, *Annalen der Physik*, 1860, v. 109, 275]. Consequently, modern physics holds that such radiation is independent of the nature and shape of the emitting object. Recently, Kirchhoff's experimental work and theoretical conclusions have been reconsidered [P. M. L. Robitaille. *IEEE Transactions on Plasma Science*, 2003, v. 31(6), 1263]. In this work, Einstein's derivation of the Planckian relation is reexamined. It is demonstrated that claims of universality in blackbody radiation are invalid.

From the onset, blackbody radiation was unique in possessing the virtue of universality [1, 2]. The nature of the emitting object was irrelevant to emission. Planck [3], as a student of Kirchhoff, adopted and promoted this concept [4, 5]. Nonetheless, he warned that objects sustaining convection currents should not be treated as blackbodies [5].

As previously discussed in detail [6], when Kirchhoff formulated his law of thermal emission [1, 2], he utilized two extremes: the perfect absorber and the perfect reflector. He had initially observed that all materials in his laboratory displayed distinct emission spectra. Generally, these were not blackbody in appearance and were not simply related to temperature changes. Graphite, however, was an anomaly, both for the smoothness of its spectrum and for its ability to simply disclose its temperature. Eventually, graphite's behavior became the basis of the laws of Stefan [7], Wien [8] and Planck [3].

For completeness, the experimental basis for universality is recalled [1, 2, 5, 6]. Kirchhoff first set forth to manufacture a box from graphite plates. This enclosure was a near perfect absorber of light ($\varepsilon = 1$, $\kappa = 1$). The box had a small hole through which radiation escaped. Kirchhoff placed various objects in this device. The box would act as a transformer of light [6]. From the graphitic light emitted, Kirchhoff was able to gather the temperature of the enclosed object once thermal equilibrium had been achieved. A powerful device had been constructed to ascertain the temperature of any object. However, this scenario was strictly dependent on the use of graphite.

Kirchhoff then sought to extend his findings [1, 2, 5]. He constructed a second box from metal, but this time the enclosure had perfectly reflecting walls ($\varepsilon = 0$, $\kappa = 0$). Under this second scenario, Kirchhoff was never able to reproduce the results he had obtained with the graphite box. No matter how long he waited, the emitted spectrum was always dominated by the object enclosed in the metallic box. The second condition was unable to produce the desired spectrum.

As a result, Kirchhoff resorted to inserting a small piece of graphite into the perfectly reflecting enclosure [5]. Once the graphite particle was added, the spectrum changed to that of the classic blackbody. Kirchhoff believed he had achieved universality. Both he, and later, Planck, viewed the piece of graphite as a "catalyst" which acted only to increase the speed at which equilibrium was achieved [5]. If only time was being compressed, it would be mathematically appropriate to remove the graphite particle and to assume that the perfect reflector was indeed a valid condition for the generation of blackbody radiation.

However, given the nature of graphite, it is clear that the graphite particle was in fact acting as a perfect absorber. Universality was based on the validity of the experiment with the perfect reflector. Yet, in retrospect, and given a modern day understanding of catalysis and of the speed of light, the position that the graphite particle acted as a catalyst is untenable. In fact, by adding a perfect absorber to his perfectly reflecting box, it was as if Kirchhoff lined the entire box with graphite. He had unknowingly returned to the first case. Consequently, universality remains without any experimental basis.

Nonetheless, physics has long since dismissed the importance of Kirchhoff's work [9]. The basis for universality no longer rests on the experimental proof [i. e. 9], but rather on Einstein's theoretical formulation of the Planckian relation [10, 11]. It has been held [i. e. 9] that with Einstein's derivation, universality was established beyond doubt based strictly on a theoretical platform. Consequently, there appears to no longer be any use for the experimental proof formulated by Kirchhoff [1, 2, 5]. Physics has argued [9] that Einstein's derivation of the Planckian equations had moved the community beyond the limited confines of Kirchhoff's enclosure. Einstein's derivation, at least on the surface, appeared totally independent of the nature of the emitting compound. Blackbody radiation was finally free of the constraints of enclosure.

In his derivation of the Planckian relation, Einstein has recourse to his well-known coefficients [10, 11]. Thermal equilibrium and the quantized nature of light ($E = \hbar\nu$) are also used. All that is required appears to be (1) transitions within two states, (2) absorption, (3) spontaneous emission, and (4) stimulated emission. However, Einstein also requires that gaseous atoms act as perfect absorbers and emitters or radiation. In practice, of course, isolated atoms can never act in this manner. In all laboratories, isolated groups of atoms act to absorb and emit radiation in narrow bands and this only if they possess a dipole moment. This is well-established in the study of gaseous emissions [12]. As such, Einstein's requirement for a perfectly absorbing atom, knows no physical analogue on earth. In fact, the only perfectly absorbing materials known, exist in the condensed state. Nonetheless, for the sake of theoretical discussion, Einstein's perfectly absorbing atoms could be permitted.

In his derivation, Einstein also invokes the requirement of thermal equilibrium with a Wien radiation field [8], which of course, required enclosure [1, 2]. However, such a field is uniquely the product of the solid state. To be even more specific, a Wien's radiation field is currently produced with blackbodies typically made either from graphite itself or from objects lined with soot. In fact, it is interesting that graphite (or soot) maintain a prominent role in the creation of blackbodies currently used at the National Bureau of Standards [13–17].

Consequently, through his inclusion of a Wien's radiation field [8], Einstein has recourse to a physical phenomenon which is known to be created exclusively by a solid. Furthermore, a Wien's field, directly involves Kirchhoff's enclosure. As a result, claims of universality can no longer be supported on the basis of Einstein's derivation of the Planckian relation. A solid is required. Therefore, blackbody radiation remains exclusively a property of the solid state. The application of the laws of Planck [3], Stefan [7] and Wien [8] to non-solids is without both experimental and theoretical justification.

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Exotic Material as Interactions Between Scalar Fields

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Many theoretical papers refer to the need to create exotic materials with average negative energies for the formation of space propulsion anomalies such as "wormholes" and "warp drives". However, little hope is given for the existence of such material to resolve its creation for such use. From the standpoint that non-minimally coupled scalar fields to gravity appear to be the current direction mathematically. It is proposed that exotic material is really scalar field interactions. Within this paper the Ginzburg-Landau (GL) scalar fields associated with superconductor junctions is investigated as a source for negative vacuum energy fluctuations, which could be used to study the interactions among energy fluctuations, cosmological scalar (i. e., Higgs) fields, and gravity.

1 Introduction

Theoretically, exotic material can be used to establish wormholes by gravitationally pushing the walls apart [1] and for the formation of a warp bubble [2] by providing the negative energy necessary to warp spacetime. Exotic material in combination with gravitation might also produce a net acceleration force for highly advanced propellant-less space propulsion engine cycles.

Negative energy is encountered in models of elementary particles. For example, Jackson [3] invokes Poincare stress, to suppress the TeV/c^2 contribution of electromagnetic field energy to the MeV/c^2 mass of an electron. Also, the Reissner-Nordstrom metric [4], devised 50 years before the development of scalar fields, predicts effects which are negligible more than a few *femtometers* $[10^{-15}\,\mathrm{m}]$ from a charged particle.

Exotic material has the requirement of a "negative average energy density", which violates several energy conditions and breaks Lorentz symmetries. Pospelov and Romalis [5] tell us that the breaking of Lorentz symmetry enables the CPT symmetry, which combines charge conjugation (C), parity (P), and time-reversal (T) symmetries, to be violated. In conventional field theories, the Lorentz and CPT symmetries are automatically preserved. But in quantum gravity, certain restrictive conditions such as locality may no longer hold, and symmetries may be broken. They also suggest that quintessence, a very low-energy 5 keV/cm^3 scalar field ψ with wavelength comparable to the size of the observable universe, is a candidate for dark energy. For in addition to its effect on the expansion of the universe, quintessence might also manifest itself through its possible interactions with matter and radiation [6, 7]. This scalar interaction could lead to a modification of a mass as a function of coordinates and violates the equivalence principle: The mass feels an extra force in the direction of $\nabla\phi$ (ϕ is the phase of the scalar field ψ).

The question is then "Where do we look for exotic material on the scale of laboratory apparatus?" From the standpoint that non-minimally coupled scalar fields to gravity appear to be the current direction mathematically [8]. It is proposed that exotic material is really scalar field interactions.

Within this paper the Ginzburg-Landau (GL) scalar fields associated with superconductor junctions is investigated as a source for negative vacuum energy fluctuations, which could be used to study the interactions among energy fluctuations, cosmological scalar (i. e., Higgs) fields, and gravity. Such an analogy is not much a stretch as it is not hard to show that the Higgs model is simply a relativistic generalization of the GL theory of superconductivity, and the classical field in the Higgs model is analog of cooper-pair Bose condensate [9]. Here, the mechanisms for scalar field interactions or the production of exotic material from the superconductor are discussed and an analogy to energy radiated in gravitational waves is presented.

2 Background

Theoretical work [1] has shown that vacuum fluctuations near a black hole's horizon are exotic due to curvature distortion of space-time. Vacuum fluctuations come about from the notion that when one tries to remove all electric and magnetic fields from some region of space to create a perfect vacuum, there always remain an excess of random, unpredictable electromagnetic oscillations, which under normal conditions averages to zero. However, curvature distortion of space-time as would occur near black holes causes vacuum energy fluctuations to become negative and therefore are "exotic". In earlier wormhole theories [10, 11], exotic material was generally thought to only occur in quantum systems [1]. It seems that the situation has changed drastically; for it has now been shown that even classical systems, such as those built from scalar fields non-minimally coupled to gravity; violate all energy conditions [8]. Gradually, these energy conditions are losing their status, which theoretically could lead even to a workable "warp drive" [12]. Further, recent mathematical models have shown that the amount of energy needed for producing wormholes (and possibly warp drives) is much less than originally thought [13], which may open the door to laboratory scale experiments.

Given that the answer to exotic material for practical propulsion applications is somewhere in between vacuum fluctuation in curved space-time and scalar fields non-minimally coupled to gravity, Ginzburg-Landau (GL) scalar fields associated with superconductor junctions could present themselves as a medium for studying the interactions among energy fluctuations, cosmological scalar fields, and gravity. As in superconductors, the GL scalar field is known [14] to extend small distances beyond the boundaries of a superconducting material. That is, in describing the operation of a Josephson junction array, two or more superconductors can be entangled over gaps of several micrometers, which is large compared to atomic distances.

The introduction of scalar fields into cosmology has been problematic. For example, the Higgs scalar field [15] of particle physics must have properties much different from the scalar field hypothesized to cause the universe to increase its expansion rate 5G years ago. However, the study of particle physics in conjunction with inflationary cosmology presents a new understanding of present day physics through the notion of symmetry breaking [9]. This suggests that the GL scalar field could possibly bridge the gap between the subatomic energy and distance scale of particle physics and the galactic scale of scalar fields in cosmology?

3 Landau-Ginzburg field in the superconductor

The Landau-Ginzburg (GL) field ψ is described as a scalar function

$$\psi = \sqrt{n}\, e^{j\theta}, \tag{1}$$

where $\sqrt{n}$ infers the degree of electron interactions in the superconductor and θ is the phase factor of these interactions.

Electrons in a room temperature superconductor material or normal conductor with no applied external fields are either confined to an atom or move about the composite molecules with random phases; $\sum \phi \approx 0$ (disorder state). However, they are generally thought to be confined to the vicinity of background ions and are positionally fixed. When the superconductor material is cooled to its critical temperature at which time a phase transition occurs, the electrons suddenly agree on a common phase; $\sum \phi > 0$ (ordered state). Again they are generally thought to be confined to the vicinity of background ions and are localized as opposed to gathering in some region creating a large space charge potential.

In a type I superconductor and as the bulk superconductor material cools down (or warms up), various size domains (depending on the cool-down, or warm up, profile) of superconductive material can form surrounded by normal conductive material. When two or more domains are in close proximity, a superconductor-normal conductor-superconductor Josephson junction is formed. In a typical bulk type I superconductor, composed of small randomly arranged crystals or grains, proximity effects would cause the electrons of a single grain to go superconductive (or normal) as a group.

In the type II YBCO superconductor this is also true with the exception that weakly coupled Josephson junctions [16] can also form between individual molecules across the copper oxide planes and across grain boundaries typically composed of an oxide layer. These are referred to as superconductor-insulator-superconductor junctions. The insulation planes degrade the time for proximity effects to cause the electrons of a single grain to go superconductive (or normal) as a group. Therefore in the type II superconductor, a superconductor domain can be as small as one molecule of superconductor material or composed of a multitude of molecules (i. e., grains).

In both the type I & II superconductor at temperatures below $\sim$ 44 K, coherence encompasses all domains, in effect producing one single domain of phase ϕ.

When multiple domains exist, gradients between domains of differing phase ϕ are accompanied by currents that tunnel between the domains as the spaces between the domains form Josephson junctions [14]. The possible current patterns are restricted by the requirement that the GL scalar function ψ must be single-valued and infers a current flow of density $\vec{J}$ given by

$$\vec{\nabla} \arg \psi = \vec{\nabla}\phi = \frac{m}{2\hbar e|\psi|^2}\; \vec{J}; \tag{2}$$

neglecting contributions from external magnetic fields.

3.1 The Landau-Ginzburg free energy potential

The Landau-Ginzburg free energy potential $V(\psi)$ refers to the energy density in the superconductor, and anywhere the scalar field is non-zero. It can even extend into a region μm outside the superconductor. The potential contribution to the free energy (neglecting contributions from external magnetic fields) is given by the energy density function:

$$V(\psi) = \alpha\, |\psi|^2 + \frac{1}{2}\,\beta\, |\psi|^4, \tag{3}$$

where equation (3) can be viewed as a series expansion in powers of $|\psi|^2$.

Two cases arise as depending whether α is positive or negative. If α is positive, the minimum free energy occurs at $|\psi|^2 = 0$, corresponding to the normal state. On the other hand, if $\alpha < 0$, the minimum occurs when

$$|\psi|^2 = |\psi_\infty|^2 = \frac{1}{2}\sqrt{n_1 n_2} \approx \frac{n_{3D}}{2} \tag{4}$$

for identical domain materials (which is assumed hereon) where the notation ψ_∞ is conventionally used because ψ approaches this value infinitely deep in the interior of the superconductor [17], where it is screened from any surface fields or currents. If the two domains are similar in crystalline structure, the two domains can be viewed as a single bulk superconductor and the Landau-Ginzburg free energy potential case for $\alpha < 0$ applies. For $\alpha < 0$

$$V(\psi_\infty) = \frac{-\alpha^2}{2\beta} = \frac{1}{2}\,\alpha\,|\psi_\infty|^2, \tag{5}$$

which gives

$$\alpha = \frac{2}{|\psi_\infty|^2}\,V(\psi_\infty); \qquad \beta = \frac{2}{|\psi_\infty|^4}\,V(\psi_\infty)\,. \tag{6}$$

4 High power flow during phase transition

Given a uniform superconductor, the average energy E_J in the junction between domains is defined by

$$E_J \leqslant \Delta|\psi|^2\,\overline{V}_g \approx \frac{1}{2}\,\Delta n_{3D}\,\overline{V}_g\,. \tag{7}$$

For the Type II YBCO superconductor $n_{3D} \approx 1.69\times10^{28}$ m^3 and $\Delta = 0.014$ eV [17, 18]. Given that grain size diameters in a typical sinter YBCO superconductor are on the order of 1 micron, then for an average domain volume $\overline{V}_g \approx 10^{-18}$ m^3, the energy in the junction $E_J \approx 10^2$ GeV. If the junction energy dissipates on the superconductor relaxation time τ_{sc}, the powers flow $E_J/\tau_{sc} \approx 10^{24}$ eV/s for the YBCO relaxation time $\tau_{sc} \approx 10^{16}$ s.

Such high energy changes during a normal state transition seems a bit extreme, especially considering that not much (if any) is mentioned of this phenomena in the literature. The main reason is that most of the energy should not be seen external to the superconductor since the initial energy transfer from the state change is an internal process. However radiation is known to accompany processes involving charged particles, such as β decay.

Jackson [3] tells us that the radiation accompanying β decay is a Bremsstrahlung spectrum: It sometimes bears the name "inner bremsstrahlung" to distinguish it from bremsstrahlung emitted by the same beta particle in passing through matter. It appears that the spectrum extends to infinity, thereby violating conservation of energy. Qualitative agreement with conservation of energy can obtain by appealing to the uncertainty principle. That is, the acceleration time τ must be of the order of $\tau = \hbar/E$, thereby satisfying the conservation of energy requirement at least qualitatively.

In the superconductor, the uncertainty principle (at least qualitatively) allows for the violation of energy conservation through rapid state change processes, which can produce vortices in the superconductor when proper phase alignment exists among domains [19]. In such a case, high energy radiation, such as Bremsstrahlung accompanying the rapid magnetic field formation cannot be ruled out as theories of superconductivity are not sufficiently understood. This is especially true with the type II superconductor, which exhibits flux pinning throughout the body of the superconductor and allows for flux motion during phase transition.

Further, energy levels of $E_J \approx 10^2$ GeV in the domain junctions could produce tunneling electrons with critical temperature for a phase transition in the Glashow-Weinberg-Salam theory of weak and electromagnetic interactions [20]. Such high energy phase transitions could then lead to effects similar to cosmology inflation, an anti-gravity force thought responsible for the acceleration of the universe [21].

5 Mechanisms for exotic material in the superconductor

In order to produce exotic material or negative vacuum energy fluctuations from superconductors in terms of curvature distortion of spacetime, asymmetric energy fluctuations must be produced. Since the Landau-Ginzburg free energy density is fixed by the number superconductor electrons, the average time rate of change or phase transition time of the superconductor electrons must be asymmetric. That is the power flow eV/s in the phase transition to the superconductor state must be higher than the power in the phase transition to the normal state or vise versa. This process of creating a time varying GL scalar field might then result in a gradient in the surrounding global vacuum scalar field (Higgs, quintessence, or etc.) in the direction of $\nabla\phi$; being measurable as a gravitation disturbance.

Asymmetric phase transitions would require electrons with group velocities that are higher than their normal relaxation times, which are already relativity short. The combination of two phenomena associated with superconductors could achieve this requirement. They are:

(1) The dissipation of the Landau-Ginzburg free energy potential during a rapid superconductor quench referred to as spontaneous symmetry breaking phase transition [22], which implies state changes on very short time scales;

(2) The Hartman effect [23], which implies that the effective group velocity of the electrons across a superconductor junction can become arbitrarily large.

Both spontaneous symmetry breaking and the Hartman effect illustrates Hawking's [24] point about the elusive definition of time in a quantum mechanical process. That is, uncertainty in the theory allow time intervals to be chosen to illustrate how measurable effects might be produced outside the superconductor without contradicting experiments with conventional solid state physics detectors. One such time interval of notice is that which occurs during a spontaneous symmetry breaking phase transitions.

5.1 Spontaneous symmetry breaking phase transition

Phase transitions between the high and low temperature phases of a superconductor involve spontaneous symmetry breaking between order (superconductor electron pair) and disorder (electron) states when the transition occurs over shorter time periods than depicted by the normal relaxation times. Kibble [25] explains that symmetry-breaking phase transitions are ubiquitous in condensed matter systems and in quantum field theories. There is also good reason to believe that they feature in the very early history of the Universe. At which time many such transitions topological defects of one kind or another are formed. Because of their inherent stability, they can have important effects on the subsequent behavior of the system. Experimental evidence validates this by the presence of a magnetic field [26, 27, 19, 28] during spontaneous symmetry breaking phase transition experiments.

In general, each superconductor domain must be taken to follow normal phase transition symmetry, which follows that the energy within these domains is conserved as anomalous energy effects have not been observed during rapid superconductor quenching of low temperature superconductive systems, which have been around for decades. However, it is conceivable that, a small fraction of the energy could be expended in disturbing nearby vacuum fields, without being noticed by crystal-switching apparatus. Since experimentally, the formation of vortices does occur during spontaneous symmetry breaking phase transitions of coupled domains in the Type II superconductor, Lorentz symmetry is violated. This allows for energy conservation violation, whereby, the assumption can be made that the energy fluctuations in the junction between superconductor domains interacts with the vacuum field on a time scale that approaches that of the Planck scale.

Evidence of this comes from Pospelov and Romalis [5], who point out that Lorentz violation could possibly be due to unknown dynamics at the Planck scale. Further, when dealing with interactions described by massless vector particles (gluons) within a relativistic local quantum field theory, Binder [42] indicates that Planck units are assigned to the background fluctuation level and provide for a common base. The gluon field plays the same role for quarks as Jackson's Poincare stress plays for electrons. Therefore, the choice for the energy fluctuation time during a spontaneous symmetry breaking phase transition of electron pairs is taken to be the Planck time T_{pl} .

However, the Planck time is too fast to be observed, which implies that human units must be artificially imposed when measuring superconductor electron fluctuations (or any other Planck time phenomena). To explain this, it is noted that just before electron phase transition and superconductor pair bonding, each electron had an energy deficit ≈ 1 eV. From the uncertainty principle, the electron can maintain this deficit before pairing for a time t according to

$$t = \frac{\hbar}{E}\,, \tag{8}$$

which for the paired electron $E \approx 2$ eV giving $t \approx 3.3\times10^{-16}$ sec. Then by noting

$$E_p\, T_{pl} = E\, t = \hbar \tag{9}$$

a limitation on the electron power flow exist at $E = E_p$ (the Planck energy) and $t = T_{pl}$. This limitation gives the human artificial units for Planck time events according to

$$t = \frac{E_p}{E}\, T_{pl}\,. \tag{10}$$

For example, during an electron pair transition where $t \approx 3.3\times10^{-16}$ s, a power flow $E/T_{pl} = E_p/t \approx 10^{43}$ eV/s per superconductor electron pair is produced, which is much less than the limit defined by $E_p/T_{pl} \approx 10^{71}$ eV/s.

That is, even though the event could have occurred on the Planck time T_{pl}, it took a time t to observe/measure the energy released. According to the uncertainty principle, the observed/measured value of the energy is then

$$E = \left(\frac{T_{pl}}{t}\right) E_p\,. \tag{11}$$

Equation (11) then tells us that energy events that occur on the Planck time are reduced by the ratio of the Planck time to the observed/measured time.

The question is then, "Can this uncertainly in the energy be captured in such a way as to be useable on the human scale?" Evidence for a yes answer arises in superluminal electron velocities in nature, which have been associated with cosmological events, lasers and electrostatic acceleration [29, 30, 31]. In these events however, the total energy in the system is interpreted from the average group velocity, whereby energy is conserved.

5.2 The Hartman effect

In the superconductor another superluminal electron phenomena exists, the Hartman effect [23, 32, 33], which is associated with the junction tunnelling process. The Hartman effect indicates that for sufficiently large barrier widths, the effective group velocity of the electrons across a superconductor junction can become arbitrarily large, inferring a violation of energy conservation.

Muga [34] tells us that defining "tunnelling times" has produced controversial discussion. Some of the definitions proposed lead to tunnelling conditions with very short times, which can even become negative in some cases. This may seem to contradict simple concepts of causality. The classical causality principle states that the particle cannot exit a region before entering it. Thus the traversal time must be positive. However, when trying to extend this principle to the quantum case, one encounters the difficulty that the traversal time concept does not have a straightforward and unique translation in quantum theory. In fact for some of the definitions proposed, in particular for the so called "extrapolated phase time" [35], the naпve extension of the classical causality principle does not apply for an arbitrary potential, even though it does work in the absence of bound states.

Generally, the Hartman effect occurs when the time of passage of the transmitted wave packet in a tunnelling collision of a quantum particle with an opaque square barrier or junction becomes essentially independent of the barrier width [23, 36] and the velocity may exceed arbitrarily large numbers. This "fast tunnelling" has been frequently interpreted as, or related to, a "superluminal effect", see e. g. [37, 38, 39, 40, 41].

The Hartman effect illustrates Hawking's (1988) discussion about ambiguities in defining time in relation to quantum mechanics and cannot be ruled out during spontaneous symmetry breaking phase transition. Therefore, large power flows across the junctions between the domains or junctions are allowed.

6 Energy radiated in gravitational waves

Arbitrarily large electron group velocities (the Hartman effect) induced by spontaneous symmetry breaking phase transitions could conceivably result in a space-like (gravitational) disturbance in nearby vacuum scalar fields with possible momentum and energy transfer about these disturbances for space propulsion applications. Although, a theory that connects the GL scalar field to gravity has yet to be presented, here the general formulation for calculating gravitational radiation from quadrupolar motion [43] is used to illustrate the possible energy radiated in a gravitational wave from the instantaneous power flow through a type II superconductor.

The power radiated L_{rad} in gravitational waves is roughly approximated from the ratio of the square of the internal power flow $\Delta E/\Delta t$ by

$$L_{rad} = \left(\frac{G}{c^5}\right)\left(\frac{\Delta E}{\Delta t}\right)^2 . \tag{12}$$

The time parameter Δt in equation (11) is ill-defined, since General Relativity cannot incorporate the uncertainties of quantum mechanics. For as previously pointed out, even times as short as the Planck time can be used without violating experimental observations.

Here, the time parameter is determined by noting that at the instant of the release of the GL free energy there is a freeze out time:

$$\hat{t} = \sqrt{T_{pl}\,\tau_{sc}} \tag{13}$$

between the transition from the adiabatic (Planck time fluctuations T_{pl}) and impulse (relaxation time τ_{sc}) regimes [26]. This implies an inherent limitation on the power flow though the superconductor, which from equation (10) implies that

$$\frac{\Delta E}{\Delta t} = \frac{E}{T_{pl}} \rightarrow \frac{E_p}{\eta\,\hat{t}} \tag{14}$$

where $\Delta t = \eta\,\hat{t}$ and where η combines geometric (i. e., size, shape, number of domains, & etc.), I-V junction characteristics [44, 45], and any other influence on the propagation of the electrons through the superconductor.

Given that the observed/measured propagation speed of the GL free energy (i. e., electron motion) through the superconductor is limited to the speed of light c, then

$$\eta \rightarrow \frac{T_h}{c\hat{t}} \tag{15}$$

Combining equation (12) with equations (14 & 15) the instantaneous power radiated in gravitational waves is given by

$$L_{rad} \rightarrow \left(\frac{G}{c^5}\right)\left(\frac{E_p}{\eta\,\hat{t}}\right)^2 \approx \left(\frac{G}{c^5}\right)\left(\frac{E_p\,c}{T_h}\right)^2 \tag{16}$$

and the radiated energy in gravitational waves:

$$E_{rad} \rightarrow L_{rad}\,\hat{t} \tag{17}$$

from the freeze-out motion within the superconductor and noting that the gravitational waves is not effected by the superconductor properties (i. e., η).

Assuming a superconductor of thickness $T_h \approx 0.0254$ m, gives the maximum instantaneous power radiated in gravitational waves $L_{rad} \approx 10^{42}$ eV/s and radiated gravitational waves energy $E_{rad} \approx 10^{13}$ eV or $\approx 10^{-4}$ J; measurable on the laboratory scale.

7 Conclusions

The Ginzburg-Landau scalar field associated with the type II superconductor was discussed as a source of exotic material to produce gravitational forces for highly advanced propulsion related systems. Arbitrarily large electron group velocities (the Hartman effect) induced by spontaneous symmetry breaking phase transitions were discussed as the mechanisms for setting up a time-varying GL scalar field, which could conceivably result in gravitational disturbances in nearby vacuum scalar fields applicable to space propulsion. The short time scale behavior discussed provides a possible signature for an experimentalist to verify that new physics is occurring. Such experiments could provide insight into the laws of scalar fields, which need to be formulated for space propulsion engine cycles.

Nomenclature

$V(\psi)$ = energy density (eV/m^3)
n = electron probability density (electrons/m^3)
ϕ = phase of the scalar field
J = current through a superconductor junction (A/m^2)
$n_{3\mathrm{D}}$ = 3-D electron density (m^3)
2Δ = BCS gap energy (eV)
$\overline{V}_g$ = average domains volume (m^3)
T_{pl} = Planck Time = $\sqrt{\hbar G/c^5} \approx 5{\times}10^{-44}$ (s)
$\hbar$ = Plank's Constant $\approx 1.06{\times}10^{-34}$ (J s)
G = gravitation constant = $6.673{\times}10^{-11}$ (N m^2/kg^2)
c = speed of light = $2.9979{\times}10^8$ (m/s)
E_p = Planck Power $\hbar/T_{pl} \approx 10^{28}$ (eV)
T_h = superconductor thickness (m)

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Exact Theory of a Gravitational Wave Detector. New Experiments Proposed

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We deduce exact solutions to the deviation equation in the cases of both free and spring-connected particles. The solutions show that gravitational waves may displace particles in a two-particle system only if they are in motion with respect to each other or the local space (there is no effect if they are at rest). We therefore propose a new experimental statement for the detection of gravitational waves: use a suspended solid-body detector self-vibrating so that there are relative oscillations of its butt-ends. Or, in another way: use a free-mass detector fitted with suspended, vibrating mirrors. Such systems may have a relative displacement of the butt-ends and a time shift in the butt-ends, produced by a falling gravitational wave.

The authors dedicate this paper to the memory of Joseph Weber, who pioneered the detection of gravitational waves.

1 Introduction

As Borissova recently showed [1] by the Synge equation for deviating geodesic lines and the Synge-Weber equation for deviating non-geodesics, Weber's experimental statement on gravitational waves [2] is inadequate. His conclusions were not based upon an exact solution to the equations, but on an approximate analysis of what could be expected. Weber expected that a plane weak wave of the space metric (gravitational wave) may displace two particles at rest with respect to one another. The Weber equations and their solutions formulated in terms of the physically observable quantities show instead that gravitational waves cannot displace resting particles; some effect may be produced only if the particles are in motion.

Here we deduce exact solutions to both the Synge equation and the Synge-Weber equation (the exact theory to free-mass and solid-body detectors). The exact solutions show that we may alter the construction of both solid-body and free-mass detectors so that they may register oscillations produced by gravitational waves. Weber most probably detected them as claimed in 1968 [3, 4, 5], as his room-temperature solid-body pigs may have their own relative oscillations of the butt-ends, whereas the oscillations are inadvertently suppressed as noise in the detectors developed by his all followers, who have had no positive result in over 35-years.

2 Main equations of the theory

We consider two cases of a simple system consisting of two particles, either free or connected by a spring. A falling gravitational wave as a wave of the space metric deforming the space should produce some effect in such a system. Therefore we call such a system a gravitational wave detector.

We will determine the effect produced by a gravitational wave in both kinds of the two-particle systems.

If the particles are connected by a non-gravitational force Φ^α, they move along neighbouring non-geodesic world-lines, according to the non-geodesic equations of motion*

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{1}$$

while relative oscillations of the world-lines (particles) are described by the so-called Synge-Weber equation† [2]

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = \frac{1}{m_0 c^2}\frac{\mathrm{D}\Phi^\alpha}{dv}dv\,. \tag{2}$$

If two neighbouring particles are free ($\Phi^\alpha = 0$), they move along neighbouring geodesic lines, according to the geodesic equations of motion

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = 0\,, \tag{3}$$

while relative oscillations of the geodesics (particles) are given by the so-called Synge equations [6]

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = 0\,. \tag{4}$$

A solution to the deviation equations (4) or (2) gives the deviation $\eta^\alpha = (\eta^0, \eta^1, \eta^2, \eta^3)$ between the particles in the acting gravitational field. Because the field is unspecified in the equations (it is hidden in the formula for the metric ds), the equations allow the deviation to be described in both regular and wave fields of gravitation. Thus to determine

*Here $U^\alpha = \frac{dx^\alpha}{ds}$ is the four-dimensional velocity vector of the particle, tangential to its world-line. It is a unit world-vector: $U_\alpha U^\alpha = 1$. The space-time interval ds along the world-line is used as a parameter for differentiation, m_0 is the rest-mass of the particle, $\Gamma^\alpha_{\mu\nu}$ are Christoffel's symbols of the 2nd kind.

†Here $\frac{\mathrm{D}}{ds}$ is the absolute (covariant) differentiation operator; $R^{\alpha}_{\cdot\beta\gamma\delta}$ is the Riemann-Christoffel curvature tensor; $\eta^\alpha = \frac{\partial x^\alpha}{\partial v}dv$ is the relative deviation vector of the particles; v is a parameter having the same numerical value along a neighbouring world-line, while dv is the difference between its values in the world-lines.

how a gravitational wave causes two test-particles to deviate from one another, we should use the metric ds for this wave field and obtain exact solutions to the deviation equation.

Currently, two main kinds of gravitational wave detectors are presumed:

1. Weber's solid-body detector — a freely suspended bulky cylindrical pig, approximated by two masses connected by a spring (i. e. non-gravitational force). Oscillations of the butt-ends of the pig in the field of a falling gravitational wave are formulated by the Synge-Weber equation of deviating non-geodesics;
2. A free-mass detector, consisting of two freely suspended mirrors, distantly separated. Each mirror is fitted with a laser range-finder for producing measurements of the distance between them. Oscillations of the mirrors under a falling gravitational wave formulated by the Synge equation of deviating geodesics.

Both detectors have a common theory — the Synge-Weber equation, in comparison to the Synge equation, has just the non-zero right side with a force Φ^α connecting the particles. We may solve them using the same method. Before doing that however, we analyse Weber's approach to the main equations and his simplifications.

3 Weber's approach and criticism thereof

Weber proceeded from the proposition that a falling gravitational wave should deform a solid-body pig, represented by a system of two particles connected by a spring. He proposed the relative displacement of the particles η^α consisting of a "basic" displacement r^α (covariantly constant) and an infinitely small relative displacement ζ^α in the butt-ends of the cylinder caused by a falling gravitational wave

$$\eta^\alpha = r^\alpha + \zeta^\alpha, \qquad \zeta^\alpha \ll r^\alpha, \qquad \frac{\mathrm{D}r^\alpha}{ds} = 0\,. \tag{5}$$

Thus the non-geodesic deviation equation is

$$\frac{\mathrm{D}^2\zeta^\alpha}{ds^2} + R^{\alpha}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \left(r^\gamma + \zeta^\gamma\right) = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{6}$$

which he transformed to*

$$\frac{\mathrm{D}^2\zeta^\alpha}{ds^2} + \frac{d^\alpha_\sigma}{m_0 c^2}\frac{\mathrm{D}\zeta^\sigma}{ds} + \frac{k^\alpha_\sigma}{m_0 c^2}\,\zeta^\sigma = -R^{\alpha}_{\cdot\beta\gamma\delta}\left(r^\gamma + \zeta^\gamma\right). \tag{7}$$

This equation is like the equation of forced oscillations, where the curvature tensor is a forcing factor. Weber then finally transformed the equation to

$$\frac{d^2\zeta^\alpha}{dt^2} + \frac{d^\alpha_\sigma}{m_0}\frac{d\zeta^\sigma}{dt} + \frac{k^\alpha_\sigma}{m_0}\,\zeta^\sigma = -c^2 R^{\alpha}_{\cdot 0\sigma 0}\, r^\sigma\,, \tag{8}$$

which can only be obtained under his assumptions:

1. The length r of the pig to be covariantly constant $r = \sqrt{g_{\mu\nu} r^\mu r^\nu}$, which is a "background" for the infinitesimal displacement of the butt-ends $\zeta^\alpha \ll r^\alpha$ caused by a falling gravitational wave. Note that r isn't the length η of the pig in the "equilibrium state". Weber postulated r^α to be covariantly constant, so r is the "unchanged length". In such a case Weber has actually two detectors at the same time: (1) a pig having the covariantly constant length r, which remains unchanged in the field of a falling gravitational wave, (2) a pig having the length ζ, which, being made from the same material and connected to the first pig, changes its length under the same gravitational wave. In actual experiments a solid-body pig has a monolithic body which reacts as a whole to external influences. In other words, by introducing the splitting term $\eta^\alpha = r^\alpha + \zeta^\alpha$ into the equation of the deviating non-geodesics (2), Weber postulated that a falling gravitational wave is an external entity that forces the particles into resonant oscillations;
2. Because the cylindrical pig is freely suspended, it is in free fall;
3. Christoffel's symbols are all zero, so covariant derivatives became regular derivatives. (Of course, we can choose a specific reference frame where $\Gamma^\alpha_{\mu\nu} = 0$ at each given point. Such a reference frame is known as locally geodesic. However, since the curvature tensor is different from zero, $\Gamma^\alpha_{\mu\nu}$ cannot be reduced to zero in a finite area [7]. Therefore, if we connect one particle to a locally geodesic reference frame, in the neighbouring particle $\Gamma^\alpha_{\mu\nu} \neq 0$);
4. The butt-ends of the pig are at rest with respect to the observer ($U^i = 0$) all the time before a gravitational wave passes. This was assumed because the pig was regularly cooled down to a temperature close to 0 K in order to suppress internal molecular motions. With $U^i = 0$, there can only be resonant oscillations of the butt-ends. Parametric oscillations cannot appear there. Therefore Weber and all his followers have expected registration of a signal if a falling gravitational wave produces resonant oscillations in the detector.

Because the same assumptions were applied to the geodesic deviation equation, all that has been said is applicable to a free-mass detector.

Weber didn't solve his final equation (8). He limited himself by using $R^{\alpha}_{\cdot 0\sigma 0}\, r^\sigma$ as a forcing factor in his calculations of expected oscillations in solid-body detectors. Exact solution of Weber's final equation with all his assumptions was obtained by Borissova in the 1970's [8]. The assumptions actually mean that the solution of the Weber equation (8), with his requirement for r^α and its length $r = \sqrt{g_{\mu\nu} r^\mu r^\nu}$, must be covariantly constant: $\frac{\mathrm{D}r^\alpha}{ds} = 0$. Borissova showed that in the case of a gravitational wave linearly polarized

*Weber takes Φ^α as the sum of the returning (elastic) force $k^\alpha_\sigma \zeta^\sigma$ and the force $d^\alpha_\sigma \frac{\mathrm{D}\zeta^\sigma}{ds}$ setting up the damping factor (tensors k^α_σ and d^α_σ describe the peculiarities of the spring).

in the x^2 direction, and propagating along x^1, the equation $\frac{\mathrm{D}r^\alpha}{ds}=0$ gives $r^2=r^2_{(0)}\left[1-A\sin\frac{\omega}{c}\left(ct+x^1\right)\right]$ (the detector oriented along x^2). From this result, she obtained the Weber equation (8) in the form*

$$\frac{d^2\zeta^2}{dt^2}+2\lambda\frac{d\zeta^2}{dt}+\Omega_0^2\,\zeta^2=-A\,\omega^2 r^2_{(0)}\sin\frac{\omega}{c}\left(ct+x^1\right), \tag{9}$$

i. e. an equation of forced oscillations, where the forcing factor is the relative displacement of the particles caused by the gravitational wave. She then obtained the exact solution: the relative displacement $\eta^2=\eta_y$ of the butt-ends is

$$\begin{aligned}\eta^2 = r^2_{(0)}\Big[1-A\sin\frac{\omega}{c}\left(ct+x^1\right)\Big]+Me^{-\lambda t}\sin(\Omega t+\alpha)\,-\\ -\,\frac{A\,\omega^2 r^2_{(0)}}{(\Omega_0^2-\omega^2)^2}\cos\Big(\omega t+\delta+\frac{\omega}{c}\,x^1\Big)\,,\end{aligned} \tag{10}$$

where $\Omega=\sqrt{\Omega_0^2-\omega^2}$, $\delta=\arctan\frac{2\lambda\omega}{\omega^2-\Omega_0^2}$, while M and α are constants. In this solution the relative oscillations consist of the "basic" harmonic oscillations and relaxing oscillations (first two terms), and the resonant oscillations (third term). As soon as the source's frequency ω coincides with the basic frequency of the detector $\Omega_0=\omega$, resonance occurs: in such a case even weak oscillations may be registered.

Thus, by his equation (6), Weber actually postulated that gravitational waves force rest-particles to undergo relative resonant oscillations. It was amazing that the exact solution showed that! Moreover, his assumptions led to a specific construction of the detectors, where parametric oscillations are obviated. As we show further by the exact solution of the deviation equations, gravitational waves may produce oscillations in only moving particles, in both solid-body and free-mass detectors.

4 Correct solution: a resting detector (Weber's case)

Our solution of the deviation equations depends on a specific formula for the space metric whereby we calculate the Riemann-Christoffel tensor. Because the sources of gravitational waves (double stars, pulsars, etc.) are far away from us, we expect received gravitational waves to be weak and plane. Therefore we consider the well-known metric of weak plane gravitational waves

$$\begin{aligned}ds^2 = c^2dt^2-(dx^1)^2-(1+a)(dx^2)^2+\\ +\;2b\,dx^2dx^3-(1-a)(dx^3)^2,\end{aligned} \tag{11}$$

where a and b are functions of $ct+x^1$ (if propagation is along x^1), while a and b are infinitesimal so that squares and products of their derivatives vanish. The wave field described by this metric has a purely deformational origin, because it is derived from the non-stationarity of the spatial components g_{ik} of the fundamental metric tensor $g_{\alpha\beta}$. This metric is preferred because it satisfies Einstein's equations in vacuum $R_{\alpha\beta}=0$ ($R_{\alpha\beta}$ is Ricci's tensor).

Because we seek solutions applicable to real experiments, we solve the deviation equations in the terms of physically observable quantities†.

The non-geodesic equations of motion (1) have two physically observable projections [11]

$$\begin{aligned}&\frac{dm}{d\tau}-\frac{m}{c^2}\,F_i\mathrm{v}^i+\frac{m}{c^2}\,D_{ik}\mathrm{v}^i\mathrm{v}^k=\frac{\sigma}{c}\,,\\ &\frac{d}{d\tau}(m\mathrm{v}^i)-mF^i+2m\left(D^i_k+A^{\cdot i}_{k\cdot}\right)+m\Delta^i_{kn}\mathrm{v}^k\mathrm{v}^n=f^i,\end{aligned} \tag{12}$$

where m is the relativistic mass of the particle; $\mathrm{v}^i=\frac{dx^i}{d\tau}$ is its three-dimensional observable velocity, the square of which is $\mathrm{v}^2=h_{ik}\mathrm{v}^i\mathrm{v}^k$; $h_{ik}=-g_{ik}+\frac{g_{0i}g_{0k}}{g_{00}}$ is the observable metric tensor; $d\tau=\sqrt{g_{00}}\,dt+\frac{g_{0i}}{c\sqrt{g_{00}}}dx^i$ is the observable time interval, which is different to the coordinate time interval $dt=\frac{1}{c}\,dx^0$; $F_i=\frac{1}{\sqrt{g_{00}}}\left(\frac{\partial\mathrm{w}}{\partial x^i}-\frac{\partial v_i}{\partial t}\right)$ is the observable gravitational inertial force, where w is the gravitational potential, while $\sqrt{g_{00}}=1-\frac{\mathrm{w}}{c^2}$; $v_i=-\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation; $A_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x^i}-\frac{\partial v_i}{\partial x^k}\right)+\frac{1}{2c^2}\left(F_iv_k-F_kv_i\right)$ is the tensor of observable angular velocities of the space rotation; $D_{ik}=\frac{1}{2\sqrt{g_{00}}}\frac{\partial h_{ik}}{\partial t}$ the tensor of observable rates of the space deformations; $\Delta^i_{kn}=h^{im}\Delta_{kn,m}$ are the spatially observable Christoffel symbols, built like Christoffel's usual symbols $\Gamma^\alpha_{\mu\nu}=g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$; $\sigma=\frac{\Phi_0}{\sqrt{g_{00}}}$ is the observable projection of the non-gravitational force Φ^α onto the observer's time line, while $f^i=\Phi^i$ is its observable projection onto his spatial section.

If a particle rests with respect to an observer ($\mathrm{v}^i=0$), its observable equations of motion (12) take the form

$$\frac{dm_0}{d\tau}=\frac{\sigma}{c}=0\,,\qquad m_0\,F^i=-f^i. \tag{13}$$

Clearly, if a two-particle system is in free fall ($F^i=0$) and also rests with respect to an observer (as happens with a solid-body detector in Weber's experimental statement), a non-gravitational force connecting the particles has no effect on their motion: two resting particles connected by a spring have the same behaviour as free ones.

Therefore, to find what effect is produced by a gravitational wave on a resting solid-body detector or a free-mass detector, we should solve the same Synge equations of the deviating geodesics.

If, as Weber assumed, the observer's reference frame is "synchronous" ($F^i=0$, $A_{ik}=0$, $dt=d\tau$), the metric of weak

*Here $2\lambda=\frac{b}{m_0}$ and $\Omega^2_{(0)}=\frac{k}{m_0}$ are derived from the formula for the non-gravitational force $\Phi^2=-k\zeta^2-b\dot{\zeta}^2$, acting along x^2 in this case. The elastic coefficient of the "spring" is k, the friction coefficient is b.

†Physically observable (chronometrically invariant) are the projections of a four-dimensional quantity onto the time line and the spatial section of an observer [9]. See a brief account of that in [10], for instance.

plane gravitational waves (11) has just $D_{ik} = \frac{1}{2\sqrt{g_{00}}} \frac{\partial h_{ik}}{\partial t} \neq 0$. Let the wave propagate along x^1. Then $D_{22} = -D_{33} = \frac{1}{2}\,\dot{a}$ and $D_{23} = \frac{1}{2}\,\dot{b}$, where the dot means differentiation by t. The rest of the components of D_{ik} are zero. In such a case the time observable projection of the Synge equation (4) vanishes, while its spatial observable projection is

$$\frac{d^2\eta^i}{dt^2} + 2D^i_k \frac{d\eta^k}{dt} = 0\,, \tag{14}$$

which is, in component notation,

$$\begin{aligned} &\frac{d^2\eta^1}{dt^2} = 0\,,\\ &\frac{d^2\eta^2}{dt^2} + \frac{da}{dt}\frac{d\eta^2}{dt} + \frac{db}{dt}\frac{d\eta^3}{dt} = 0\,,\\ &\frac{d^2\eta^3}{dt^2} - \frac{da}{dt}\frac{d\eta^3}{dt} + \frac{db}{dt}\frac{d\eta^2}{dt} = 0\,. \end{aligned} \tag{15}$$

The first of these (the deviating acceleration along the wave propagation direction x^1) shows that transverse waves don't produce an effect in the direction of propagation.

We look for exact solutions to the remaining two equations of (15) in the case where a gravitational wave is linearly polarized in the x^2 direction ($b = 0$). First integrals of the equations are $\frac{d\eta^2}{dt} = C_1\, e^{-a}$ and $\frac{d\eta^3}{dt} = C_2\, e^{+a}$. Expanding e^{-a} and e^{+a} into series (high order terms vanish there), we obtain

$$\frac{d\eta^2}{dt} = C_1\,(1-a)\,, \qquad \frac{d\eta^3}{dt} = C_2\,(1+a)\,. \tag{16}$$

Let the gravitational wave be simple harmonic $\omega = \mathrm{const}$ with a constant amplitude $A = \mathrm{const}$: $a = A\sin\frac{\omega}{c}(ct + x^1)$. We then obtain exact solutions to the equations — the non-zero relative displacements produced in the two-particle system by the gravitational wave falling along x^1:

$$\begin{aligned} \eta^2 &= \dot{\eta}^2_{(0)}\Big[t + \frac{A}{\omega}\cos\frac{\omega}{c}\,(ct + x^1)\Big] + \eta^2_{(0)} - \frac{A}{\omega}\,\dot{\eta}^2_{(0)}\,,\\ \eta^3 &= \dot{\eta}^3_{(0)}\Big[t - \frac{A}{\omega}\cos\frac{\omega}{c}\,(ct + x^1)\Big] + \eta^3_{(0)} - \frac{A}{\omega}\,\dot{\eta}^3_{(0)}\,. \end{aligned} \tag{17}$$

These are the exact solutions of the Synge equation in a particular case, realised today in all solid-body and free-mass detectors. Looking at the solutions, we conclude:

> Transverse gravitational waves of a deformational sort may produce an effect in a two-particle system, resting as a whole with respect to the observer, only if the particles initially oscillate with respect to each other. If the particles are at rest in the initial moment of time, a falling gravitational wave cannot produce relative displacement of the particles.

Therefore the correct theory of a gravitational wave detector we have built states:

> Solid-body and free-mass detectors of current construction cannot register gravitational waves in principle; in cooling a solid-body detector and initially placing two distant mirrors at rest in a free-mass detector, inherent free oscillations are suppressed, thereby preventing registration of gravitational waves by the detectors.

In order to make the detectors sensitive to gravitational waves, we propose the following changes to their current construction:

For a free-mass detector: Introduce relative oscillations of the mirrors along their mutual line of sight. Such a modified system may have a reaction to a falling gravitational wave as an add-on to the relative velocity of the mirrors on the background of their basic relative oscillations.

For a solid-body detector: Don't cool the cylindrical pig, or better, apply relative oscillations of the butt-ends. Then the pig may have a reaction to a falling gravitational wave: an add-on to the noise of the self-deforming oscillations regularly detected as a piezoelectric effect*.

By the foregoing modifications to the exact theory of a gravitational wave detector, a solid-body detector, and especially a free-mass detector, may register gravitational waves.

Our theoretical result shows that to detect gravitational waves, the best method would be a detector consisting of two moving "particles". From the purely theoretical perspective, this is a general case of the deviation equations, where both particles move with respect to the observer at the initial moment of time. We obtain therefore, exact solutions for the general case and, as a result, consider detectors built on moving "particles" — a suspended, self-vibrating solid-body pig or suspended, vibrating mirrors in a free-mass detector.

5 Correct solution: a moving detector (general case)

If Weber had solved the deviation equation in conjunction with the equations of motion, he would have come to the same conclusion as us: gravitational waves of the deformational sort may produce an effect in a two-particle system only if the particles are in motion. Therefore we are going to solve the deviation equation in conjunction with the equations of motion in the general case where both particles move initially

*Because of this, it is most probable that Weber really detected gravitational waves in his experiments of 1968–1970 [3, 4, 5] where he used room-temperature detectors ". . . spaced about 2 km. A number of coincident events have been observed, with extremely small probability that they are statistical. It is clear that on rare occasions these instruments respond to a common external excitation which may be gravitational radiation" [3]. "Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small" [4]. "Other experiments involve observations to rule out the possibility that the detectors are being excited electromagnetically. These results are evidence supporting an earlier claim that gravitational radiation is being observed" [5].

We both highly appreciate the work of Joseph Weber (1919–2000). Surely, if he was still alive he would be enthusiastic about our current results, and with us, immediately undertake new experiments for the detection of gravitational waves.

with respect to the observer ($U^i \neq 0$). (We mean that both particles move at the same velocity.)

We do this with the Synge-Weber equation of the deviating non-geodesics, because the Synge equation of the deviating geodesics is actually the same when the right side is zero.

We write the Synge-Weber equation (2) in the expanded form (with similar terms reduced)

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2}+2\Gamma^\alpha_{\mu\nu}\frac{d\eta^\mu}{ds}U^\nu+\frac{\partial\Gamma^\alpha_{\beta\delta}}{\partial x^\gamma}U^\beta U^\delta\eta^\gamma=\frac{1}{m_0c^2}\frac{\partial\Phi^\alpha}{\partial x^\gamma}\eta^\gamma, \quad (18)$$

where ds may be expressed through the observable time interval $d\tau=\sqrt{g_{00}}\,dt+\frac{g_{0i}}{c\sqrt{g_{00}}}dx^i$ as $ds=cd\tau\sqrt{1-\mathrm{v}^2/c^2}$.

According to Zelmanov [9], any vector Q^α has two observable projections $\frac{Q_0}{\sqrt{g_{00}}}$ and Q^i, where the time projection may be calculated as $\frac{Q_0}{\sqrt{g_{00}}}=\sqrt{g_{00}}\,Q^0-\frac{1}{c}\,v_iQ^i$. We denote $\sigma=\frac{\Phi_0}{\sqrt{g_{00}}}$ and $f^i=\Phi^i$ for the connecting force Φ^α, while $\varphi=\frac{\eta_0}{\sqrt{g_{00}}}$ and η^i for the deviation η^α.

We consider the Synge-Weber equation (18) in a non-relativistic case, because the velocity of the particles is obviously small. In such a case, in the metric of weak plane gravitational waves (11), we have*

$$\begin{gathered} d\tau=dt\,, \qquad \eta^0=\eta_0=\varphi\,, \quad \Phi^0=\Phi_0=\sigma\,, \\ \Gamma^0_{kn}=\frac{1}{c}D_{kn}\,, \quad \Gamma^i_{0k}=\frac{1}{c}D^i_k\,, \quad \Gamma^i_{kn}=\Delta^i_{kn}\,, \end{gathered} \quad (19)$$

while all other Christoffel symbols are zero. We obtain the time and spatial observable projections of the Synge-Weber equation (18), which are

$$\begin{gathered} \frac{d^2\varphi}{dt^2}+\frac{2}{c}D_{kn}\frac{d\eta^k}{dt}\mathrm{v}^n+\left(\varphi\frac{\partial D_{kn}}{\partial t}+c\frac{\partial D_{kn}}{\partial x^m}\eta^m\right)\frac{\mathrm{v}^k\mathrm{v}^n}{c^2}= \\ =\frac{1}{m_0}\left(\frac{\varphi}{c}\frac{\partial\sigma}{\partial t}+\frac{\partial\sigma}{\partial x^m}\eta^m\right), \\ \frac{d^2\eta^i}{dt^2}+\frac{2}{c}D^i_k\left(\frac{d\varphi}{dt}\mathrm{v}^k+c\frac{d\eta^k}{dt}\right)+2\Delta^i_{kn}\frac{d\eta^k}{dt}\mathrm{v}^n+ \\ +2\left(\frac{\varphi}{c}\frac{\partial D^i_k}{\partial t}+\frac{\partial D^i_k}{\partial x^m}\eta^m\right)\mathrm{v}^k+\left(\frac{\varphi}{c}\frac{\partial\Delta^i_{kn}}{\partial t}+\frac{\partial\Delta^i_{kn}}{\partial x^m}\eta^m\right)\mathrm{v}^k\mathrm{v}^n= \\ =\frac{1}{m_0}\left(\frac{\varphi}{c}\frac{\partial f^i}{\partial t}+\frac{\partial f^i}{\partial x^m}\eta^m\right). \end{gathered} \quad (20)$$

We solve the deviation equations (20) in the field of a weak plane gravitational wave falling along x^1 and linearly polarized in the x^2 direction ($b=0$). In such a field we have

$$\begin{gathered} D_{22}=-D_{33}=\frac{1}{2}\,\dot{a}\,, \qquad \frac{d}{dx^1}=\frac{1}{c}\frac{d}{dt}\,, \\ \Delta^1_{22}=-\Delta^1_{33}=-\frac{1}{2c}\,\dot{a}\,, \quad \Delta^2_{12}=-\Delta^3_{13}=\frac{1}{2c}\,\dot{a}\,, \end{gathered} \quad (21)$$

so that the deviation equations (20) in component form are

$$\begin{gathered} \frac{d^2\varphi}{dt^2}+\frac{\dot{a}}{c}\left(\frac{d\eta^2}{dt}\mathrm{v}^2-\frac{d\eta^3}{dt}\mathrm{v}^3\right)+ \\ +\frac{\ddot{a}}{2c^2}\left(\varphi+\eta^1\right)\left((\mathrm{v}^2)^2-(\mathrm{v}^3)^2\right)=\frac{1}{m_0}\left(\frac{1}{c}\frac{\partial\sigma}{\partial t}+\frac{\partial\sigma}{\partial x^m}\eta^m\right), \\ \frac{d^2\eta^1}{dt^2}-\frac{\dot{a}}{c}\left(\frac{d\eta^2}{dt}\mathrm{v}^2-\frac{d\eta^3}{dt}\mathrm{v}^3\right)- \\ -\frac{\ddot{a}}{2c^2}\left(\varphi+\eta^1\right)\left((\mathrm{v}^2)^2-(\mathrm{v}^3)^2\right)=\frac{1}{m_0}\left(\frac{1}{c}\frac{\partial f^1}{\partial t}+\frac{\partial f^1}{\partial x^m}\eta^m\right), \\ \frac{d^2\eta^2}{dt^2}+\frac{\dot{a}}{c}\left(\frac{d\varphi}{dt}+\frac{d\eta^1}{dt}\right)\mathrm{v}^2+\dot{a}\,\frac{d\eta^2}{dt}\left(1+\frac{\mathrm{v}^1}{c}\right)+ \\ +\frac{\ddot{a}}{c}\left(\varphi+\eta^1\right)\left(1+\frac{\mathrm{v}^1}{c}\right)\mathrm{v}^2=\frac{1}{m_0}\left(\frac{1}{c}\frac{\partial f^2}{\partial t}+\frac{\partial f^2}{\partial x^m}\eta^m\right), \\ \frac{d^2\eta^3}{dt^2}-\frac{\dot{a}}{c}\left(\frac{d\varphi}{dt}+\frac{d\eta^1}{dt}\right)\mathrm{v}^3-\dot{a}\,\frac{d\eta^3}{dt}\left(1+\frac{\mathrm{v}^1}{c}\right)- \\ -\frac{\ddot{a}}{c}\left(\varphi+\eta^1\right)\left(1+\frac{\mathrm{v}^1}{c}\right)\mathrm{v}^3=\frac{1}{m_0}\left(\frac{1}{c}\frac{\partial f^3}{\partial t}+\frac{\partial f^3}{\partial x^m}\eta^m\right). \end{gathered} \quad (22)$$

This is a system of 2nd order differential equations with respect to φ, η^1, η^2, η^3, where the variable coefficients of the functions are the quantities $\dot{a}$, $\ddot{a}$, v^1, v^2, v^3.

We may find a from the given metric of the gravitational wave field, while v^i are the solutions to the non-geodesic equations of motion (12). By the given non-relativistic case in a field of weak plane linearly polarized gravitational wave, the equations of motion take the form

$$\begin{gathered} \frac{\dot{a}}{2c}\left((\mathrm{v}^2)^2-(\mathrm{v}^3)^2\right)=\frac{\sigma}{m_0}\,, \\ \frac{d\mathrm{v}^1}{dt}-\frac{\dot{a}}{2c}\left((\mathrm{v}^2)^2-(\mathrm{v}^3)^2\right)=\frac{f^1}{m_0}\,, \\ \frac{d\mathrm{v}^2}{dt}+\dot{a}\,\mathrm{v}^2\left(1+\frac{\mathrm{v}^1}{c}\right)=\frac{f^2}{m_0}\,, \\ \frac{d\mathrm{v}^3}{dt}-\dot{a}\,\mathrm{v}^3\left(1+\frac{\mathrm{v}^1}{c}\right)=\frac{f^3}{m_0}\,. \end{gathered} \quad (23)$$

5.1 Solution for a free-mass detector

We first find the solution for a simple case, where two particles don't interact with each other ($\Phi^\alpha=0$) — the right side is zero in the equations. This is a case of a free-mass detector. We find the quantities v^i from the equations of motion (23), which, since $\Phi^\alpha=0$, become geodesic

$$\begin{gathered} (\mathrm{v}^2)^2=(\mathrm{v}^3)^2, \qquad \frac{d\mathrm{v}^1}{dt}=0\,, \\ \frac{d\mathrm{v}^2}{dt}+\dot{a}\,\mathrm{v}^2=0\,, \qquad \frac{d\mathrm{v}^3}{dt}+\dot{a}\,\mathrm{v}^3=0\,. \end{gathered} \quad (24)$$

From this we see that a transverse gravitational wave doesn't produce an effect in the longitudinal direction: $\mathrm{v}^1=\mathrm{v}^1_{(0)}=\mathrm{const}$. Therefore, henceforth, $\mathrm{v}^1_{(0)}=0$.

*By the metric of weak plane gravitational waves (11), there is no difference between upper and lower indices.

The remaining equations of (24) may be integrated without problems. We obtain: $\mathrm{v}^2=\mathrm{v}^2_{(0)}e^{-a}$, $\mathrm{v}^3=\mathrm{v}^3_{(0)}e^{+a}$. Assuming the wave simple harmonic, $\omega=\text{const}$, with a constant amplitude, $A=\text{const}$, i. e. $a=A\sin\frac{\omega}{c}(ct+x^1)$, and expanding the exponent into series, we obtain

$$\begin{aligned} \mathrm{v}^2 &= \mathrm{v}^2_{(0)}\left[1-A\sin\frac{\omega}{c}(ct+x^1)\right],\\ \mathrm{v}^3 &= \mathrm{v}^3_{(0)}\left[1+A\sin\frac{\omega}{c}(ct+x^1)\right], \end{aligned} \tag{25}$$

i. e. a gravitational wave has an effect only in directions orthogonal to its propagation. Clearly, a gravitational wave doesn't affect particles at rest with respect to the local space where the wave propagates.

Substituting the solutions (25) into the equations of the deviating non-geodesics (22) and setting the right side to zero as for geodesics, we obtain

$$\begin{aligned} &\frac{d^2\varphi}{dt^2}+\frac{\dot a}{c}\left(\frac{d\eta^2}{dt}\mathrm{v}^2_{(0)}-\frac{d\eta^3}{dt}\mathrm{v}^3_{(0)}\right)=0\,,\\ &\frac{d^2\eta^1}{dt^2}-\frac{\dot a}{c}\left(\frac{d\eta^2}{dt}\mathrm{v}^2_{(0)}-\frac{d\eta^3}{dt}\mathrm{v}^3_{(0)}\right)=0\,,\\ &\frac{d^2\eta^2}{dt^2}+\dot a\frac{d\eta^2}{dt}+\frac{\dot a}{c}\left(\frac{d\varphi}{dt}+\frac{d\eta^1}{dt}\right)\mathrm{v}^2_{(0)}+\frac{\ddot a}{c}\left(\varphi+\eta^1\right)\mathrm{v}^2_{(0)}=0\,,\\ &\frac{d^2\eta^3}{dt^2}-\dot a\frac{d\eta^3}{dt}-\frac{\dot a}{c}\left(\frac{d\varphi}{dt}+\frac{d\eta^1}{dt}\right)\mathrm{v}^2_{(0)}-\frac{\ddot a}{c}\left(\varphi+\eta^1\right)\mathrm{v}^2_{(0)}=0\,. \end{aligned} \tag{26}$$

Summing the first two equations and integrating the sum, we obtain $\varphi+\eta^1=B_1t+B_2$, where $B_{1,2}$ are integration constants. Substituting these into the other two, we obtain

$$\begin{aligned} &\frac{d^2\eta^2}{dt^2}+\dot a\frac{d\eta^2}{dt}+\frac{\dot a}{c}B_1\mathrm{v}^2_{(0)}+\frac{\ddot a}{c}\big(B_1t+B_2\big)\mathrm{v}^2_{(0)}=0\,,\\ &\frac{d^2\eta^3}{dt^2}-\dot a\frac{d\eta^2}{dt}-\frac{\dot a}{c}B_1\mathrm{v}^3_{(0)}-\frac{\ddot a}{c}\big(B_1t+B_2\big)\mathrm{v}^3_{(0)}=0\,. \end{aligned} \tag{27}$$

The equations differ solely in the sign of a, and can therefore be solved in the same way. We introduce a new variable $y=\frac{d\eta^2}{dt}$. Then we have a linear uniform equation of the 1st order with respect to y

$$\dot y+\dot a y=-\frac{\dot a}{c}B_1\mathrm{v}^2_{(0)}-\frac{\ddot a}{c}\big(B_1t+B_2\big)\mathrm{v}^2_{(0)}\,, \tag{28}$$

which has the solution

$$y=e^{-F}\left(y_0+\int_0^t g(t)\,e^F dt\right),\quad F(t)=\int_0^t f(t)\,dt\,, \tag{29}$$

where $F(t)=\dot a$, $g(t)=-\frac{\dot a}{c}B_1\mathrm{v}^2_{(0)}-(B_1t+B_2)\mathrm{v}^2_{(0)}$. Expanding the exponent into series in the solution, and then integrating, we obtain

$$\begin{aligned} y=\dot\eta^2 &= \dot\eta^2_{(0)}\left[1-A\sin\frac{\omega}{c}(ct+x^1)\right]-\\ &-\frac{A\omega}{c}\mathrm{v}^2_{(0)}\big(B_1t+B_2\big)\cos\frac{\omega}{c}(ct+x^1)+\frac{A\omega}{c}B_2\mathrm{v}^2_{(0)}\,. \end{aligned} \tag{30}$$

Integrating this equation, and applying the same method for η^3, we arrive at the final solutions: the relative displacements η^2 and η^3 in a free-mass detector are

$$\begin{aligned} \eta^2 &= \eta^2_{(0)}+\left(\dot\eta^2_{(0)}+\frac{A\omega B_2\mathrm{v}^2_{(0)}}{c}\right)t+\frac{A}{\omega}\left(\dot\eta^2_{(0)}-\frac{\mathrm{v}^2_{(0)}}{c}B_1\right)\times\\ &\times\left[\cos\frac{\omega}{c}(ct+x^1)-1\right]-\frac{A\mathrm{v}^2_{(0)}}{c}\big(B_1t+B_2\big)\sin\frac{\omega}{c}(ct+x^1)\,, \end{aligned} \tag{31}$$

$$\begin{aligned} \eta^3 &= \eta^3_{(0)}+\left(\dot\eta^3_{(0)}-\frac{A\omega B_2\mathrm{v}^3_{(0)}}{c}\right)t-\frac{A}{\omega}\left(\dot\eta^3_{(0)}-\frac{\mathrm{v}^3_{(0)}}{c}B_1\right)\times\\ &\times\left[\cos\frac{\omega}{c}(ct+x^1)-1\right]+\frac{A\mathrm{v}^3_{(0)}}{c}\big(B_1t+B_2\big)\sin\frac{\omega}{c}(ct+x^1)\,. \end{aligned} \tag{32}$$

With $\dot\eta^2$ and $\dot\eta^3$, we integrate the first two equations of (26). We obtain thereby the relative displacement η^1 in a free-mass detector and the time shift φ at its ends, thus

$$\eta^1=\dot\eta^1_{(0)}t-\frac{A}{\omega c}\left(\mathrm{v}^2_{(0)}\dot\eta^2_{(0)}-\mathrm{v}^3_{(0)}\dot\eta^3_{(0)}\right)\left[1-\cos\frac{\omega}{c}(ct+x^1)\right]+\eta^1_{(0)}, \tag{33}$$

$$\varphi=\dot\varphi_{(0)}t+\frac{A}{\omega c}\left(\mathrm{v}^2_{(0)}\dot\eta^2_{(0)}-\mathrm{v}^3_{(0)}\dot\eta^3_{(0)}\right)\left[1-\cos\frac{\omega}{c}(ct+x^1)\right]+\eta^1_{(0)}. \tag{34}$$

Finally, we substitute φ and η^1 into $\varphi+\eta^1=B_1t+B_2$ to fix the integration constants $B_1=\dot\varphi_{(0)}+\dot\eta^1_{(0)}$ and $B_2=\varphi_{(0)}+\eta^1_{(0)}$.

Thus we have obtained the solutions to the Synge equation of deviating geodesics. We see that relative displacements of two free particles in the directions x^2 and x^3, transverse to that of gravitational wave propagation consist of:

1. Displacements, increasing linearly with time;
2. Harmonic oscillations at the frequency ω of a falling gravitational wave;
3. Oscillations, the amplitude of which increases linearly with time (last term in the solutions).

The first two of the displacements are permitted in the transverse direction x^2 or x^3, only if the particles initially move in this direction with respect to the local space ($\mathrm{v}^2\neq0$ or $\mathrm{v}^3\neq0$) or with respect to each other ($\dot\eta^2\neq0$ or $\dot\eta^3\neq0$). For instance, if they are a rest with respect to x^2, an x^1-directed gravitational wave doesn't displace them in this direction.

The third of the displacements is permitted only if the particles initially move with respect to each other in the longitudinal direction ($\dot\eta^1\neq0$).

We see from the solution for η^1 that gravitational waves may displace the particles even in the same direction of the wave propagation, if the particles initially move in this direction with respect to each other.

The solution φ is the time shift in the clocks located at both particles, caused by a falling gravitational wave*. From (34), this effect is permitted if the particles move both with

*We assume $\varphi_{(0)}=0$: time count starts from zero. We assume as well $\dot\varphi_{(0)}=0$: time flows uniformly in the absence of a wave gravitational field.

respect to the local space and each other in at least one of the transverse directions x^2 and x^3.

In view of these results, we propose a new experimental statement for the detection of gravitational waves, based on a free-mass detector.

New experiment for a free-mass detector: A free-mass detector, where two mirrors are suspended and vibrating so that they have free oscillations with respect to each other or along parallel (vertical or horizontal) lines. With the mirrors oscillating along parallel lines, such a system moves with respect to the local space ($\mathrm{v}^i_{(0)} \neq 0$), while with the mirrors oscillating with respect to each other the system has non-stationary relative displacements of the butt-ends ($\eta^i_{(0)} \neq 0$, $\dot{\eta}^i_{(0)} \neq 0$). According to the exact theory of a free-mass detector given above, a falling gravitational wave produces a relative displacement of the mirrors, that may be registered with a laser range-finder (or similar system). Moreover, as the theory predicts, a time shift is produced in the mirrors, that may be registered by synchronized clocks located with each of the mirrors: their asynchronization implies a gravitational wave detection.

5.2 Solution for a solid-body detector

We assume an elastic force connecting two particles in a solid-body detector to be $\Phi^\alpha = -k^\alpha_\sigma x^\sigma$, where k^α_σ is the elastic coefficient. We assume the force Φ^α to be independent of time, i. e. $k^0_\sigma = 0$. In such a case the equations of motion of the particles (23) take the form

$$\left.\begin{aligned}
&\frac{\dot{a}}{2c}\Big((\mathrm{v}^2)^2 - (\mathrm{v}^3)^2\Big) = 0\,,\\
&\frac{d\mathrm{v}^1}{dt} - \frac{\dot{a}}{2c}\Big((\mathrm{v}^2)^2 - (\mathrm{v}^3)^2\Big) = -\frac{k^1_\sigma}{m_0}\,x^\sigma,\\
&\frac{d\mathrm{v}^2}{dt} + \dot{a}\,\mathrm{v}^2\left(1+\frac{\mathrm{v}^1}{c}\right) = -\frac{k^2_\sigma}{m_0}\,x^\sigma,\\
&\frac{d\mathrm{v}^3}{dt} - \dot{a}\,\mathrm{v}^3\left(1+\frac{\mathrm{v}^1}{c}\right) = -\frac{k^3_\sigma}{m_0}\,x^\sigma.
\end{aligned}\right\} \tag{35}$$

Thus a transverse gravitational wave doesn't produce an effect in the longitudinal direction x^1: $\mathrm{v}^1 = \mathrm{v}^1_{(0)} = \mathrm{const}$. Therefore, henceforth, $\mathrm{v}^1 = 0$ and $k^1_\sigma = 0$. In such a case the equations of motion take the form

$$\frac{d\mathrm{v}^2}{dt} + \dot{a}\,\mathrm{v}^2 = -\frac{k^2_\sigma}{m_0}\,x^\sigma, \qquad \frac{d\mathrm{v}^3}{dt} - \dot{a}\,\mathrm{v}^3 = -\frac{k^3_\sigma}{m_0}\,x^\sigma. \tag{36}$$

The equations differ solely by the sign of $\dot{a}$. Therefore we solve only the first of them. The second equation may be solved following the same method.

Let $k^2_\sigma = k^3_\sigma = k = \mathrm{const}$, i. e. the solid-body pig is elastic in only two directions transverse to the direction x^1 of the gravitational wave propagation. With that, the equation of motion in the x^2 direction is

$$\frac{d^2x^2}{dt^2} + \frac{k}{m_0}\,x^2 = -A\,\omega\cos\frac{\omega}{c}\,(ct+x^1)\,\frac{dx^2}{dt}\,. \tag{37}$$

Denoting $x^2 \equiv x$, $\frac{k}{m_0} = \Omega^2$, $A\omega = -\mu$, we reduce this equation to the form

$$\ddot{x} + \Omega^2 x = \mu\cos\frac{\omega}{c}\,(ct+x^1)\,\dot{x}\,, \tag{38}$$

where μ is the so-called "small parameter". This is a "quasi-harmonic" equation: with $\mu = 0$, such an equation is a harmonic oscillation equation; while if $\mu \neq 0$ the right side plays the rôle of an forcing factor — we obtain a forced oscillation equation.

We solve this equation using the small parameter method of Poincaré, known also as the perturbation method: we consider the right side as a perturbation of a harmonic oscillation described by the left side. The Poincaré method is related to exact solution methods, because a solution produced with the method is a power series expanded by the small parameter μ (see Lefschetz, Chapter XII, §2 [12]).

Before we solve (38) we introduce a new variable $t' = \Omega t$ in order to make it dimensionless as in [12], and $\mu' = \frac{\mu}{\Omega}$

$$\ddot{x} + x = \mu'\cos\frac{\omega}{\Omega c}\,(ct'+\Omega x^1)\,\dot{x}\,, \tag{39}$$

where we differentiate by t'. A general solution of this equation, representable as the equivalent system

$$\dot{x} = y\,, \qquad \dot{y} = -x + \mu'\cos\frac{\omega}{\Omega c}\,(ct'+\Omega x^1)\,y \tag{40}$$

with the initial data $x_{(0)}$ and $y_{(0)}$ at $t' = 0$, is determined by the series pair (Lefschetz)

$$\left.\begin{aligned}
x &= P_0\,(x_{(0)}, y_{(0)}, t') + \mu' P_1\,(x_{(0)}, y_{(0)}, t') + \ldots\\
y &= \dot{P}_0\,(x_{(0)}, y_{(0)}, t') + \mu' \dot{P}_1\,(x_{(0)}, y_{(0)}, t') + \ldots
\end{aligned}\right\}. \tag{41}$$

We substitute these into (40) and, equating coefficients in the same orders of μ', obtain the recurrent system

$$\left.\begin{aligned}
&\ddot{P}_0 + P_0 = 0\\
&\ddot{P}_1 + P_1 = \dot{P}_0\cos\frac{\omega}{\Omega c}\,(ct'+\Omega x^1)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}\right\} \tag{42}$$

with the initial data $P_0(0) = \xi$, $\dot{P}_0(0) = \vartheta$, $P_0(0) = \dot{P}_0(0) = 0$ $(n > 0)$ at $t' = 0$. Because the amplitude A (we have it in the variable $\mu' = -\frac{\omega}{\Omega}A$) is small, this problem takes only the first two equations into account. The first of them is a harmonic oscillation equation, with the solution

$$P_0 = \xi\cos t' + \vartheta\sin t', \tag{43}$$

while the second equation, with this solution, is

$$\ddot{P}_1 + P_1 = (-\xi\sin t' + \vartheta\cos t')\cos\frac{\omega}{\Omega c}\,(ct'+\Omega x^1)\,. \tag{44}$$

This is a linear uniform equation. We solve it following Kamke (Part III, Chapter II, §2.5 in [13]). The solution is*

*Here we go back to the initial variables.

$$P_1 = \frac{\vartheta\Omega^2}{2}\left\{\frac{\cos\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\cos\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\} - \\ -\frac{i\xi\Omega^2}{2}\left\{\frac{\sin\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\sin\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\}, \tag{45}$$

where the brackets contain the real and imaginary parts of the formula $e^{i(\Omega-\omega)t-\frac{\omega}{c}x^1}+e^{i(\Omega+\omega)t+\frac{\omega}{c}x^1}$. Going back to $x^2 = x$, we obtain the final solution in the reals

$$x^2 = \xi\cos\Omega t + \vartheta\sin\Omega t - \\ -\frac{A\omega\Omega\vartheta}{2}\left\{\frac{\cos\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\cos\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\}, \tag{46}$$

while the solution for x^3 will differ solely in the sign of the amplitude A.

With this result we solve the equations of the deviating non-geodesics (22). Because a solid-body detector has a freedom for motion less than a free-mass detector, we assume $\mathrm{v}^1 = 0$, $\mathrm{v}^2 = \mathrm{v}^3$, $\Phi^1 = 0$, $\Phi^2 = -\frac{k}{m_0}\eta^2$, $\Phi^3 = -\frac{k}{m_0}\eta^3$. Note that $\mathrm{v}^2 = \mathrm{v}^3$ means that the initial conditions ξ and ϑ are the same in both the directions x^2 and x^3. Therefore we obtain

$$\frac{d^2\varphi}{dt^2} = 0\,, \qquad \frac{d^2\eta^1}{dt^2} = 0\,, \tag{47}$$

i. e. a gravitational wave doesn't change both the vertical size of the pig and the time shift φ at its butt-ends: we may put $\varphi = 0$ and $\eta^1 = 0$. With all these, the deviation equation along x^2 takes the form*

$$\frac{d^2\eta^2}{dt^2} + \frac{k}{m_0}\eta^2 = -A\,\omega\cos\frac{\omega}{c}\left(ct + x^1\right)\frac{d\eta^2}{dt}\,, \tag{48}$$

having the same form as equation (37). So the solution η^2 is like (46), but with the difference that the initial constants ξ and ϑ depend on $\eta^2_{(0)}$, $\eta^3_{(0)}$ and $\dot{\eta}^2_{(0)}$, $\dot{\eta}^3_{(0)}$. It is

$$\eta^2 = \xi\cos\Omega t + \vartheta\sin\Omega t - \\ -\frac{A\omega\Omega\vartheta}{2}\left\{\frac{\cos\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\cos\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\}. \tag{49}$$

Thus two spring-connected particles in the field of a gravitational wave may experience the following effects:

1. Free relative oscillations at a frequency Ω;
2. Forced relative oscillations, caused by the gravitational wave of frequency ω; they occur in the directions transverse to the wave propagation;
3. Resonant oscillations, which occur as soon as the gravitational wave's frequency becomes double the frequency of the particle's free oscillation ($\omega = 2\Omega$); in such a case even weak oscillations caused by the gravitational wave may be detected;

*We write and solve only the equation for η^2, because it differs to that for η^3 solely by the sign of the amplitude A. See (22).

The second and third effects are permitted only if the particles have an initial relative oscillation. If there is no initial oscillation, gravitational waves cannot produce an effect in such a system. Owing to this result, we propose a new experimental statement for the detection of gravitational waves by a solid-body detector.

New experiment for a solid-body detector: Use a solid-body cylindrical pig, horizontally suspended and self-vibrating so that there are relative oscillations of its butt-ends ($\eta^2_{(0)} \neq 0$, $\dot{\eta}^2_{(0)} \neq 0$). Such an oscillation may be induced by alternating electromagnetic current or something like this. Or, alternatively, use a similarly suspended, vibrating pig so that it has an oscillation in the horizontal plane. Such a system has a non-zero velocity with respect to the observer's local space ($\mathrm{v}^2_{(0)} \neq 0$, $\mathrm{v}^3_{(0)} \neq 0$). Both systems, according to the exact theory of a solid-body detector, may have a reaction to gravitational waves (up to resonance) that may be measured as a piezo-effect in the pig.

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Changes in the Fine Structure of Stochastic Distributions as a Consequence of Space-Time Fluctuations

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This is a survey of the fine structure stochastic distributions in measurements obtained by me over 50 years. It is shown: (1) The forms of the histograms obtained at each geographic point (at each given moment of time) are similar with high probability, even if we register phenomena of completely different nature — from biochemical reactions to the noise in a gravitational antenna, or α-decay. (2) The forms of the histograms change with time. The iterations of the same form have the periods of the stellar day (1.436 min), the solar day (1.440 min), the calendar year (365 solar days), and the sidereal year (365 solar days plus 6 hours and 9 min). (3) At the same instants of the local time, at different geographic points, the forms of the histograms are the same, with high probability. (4) The forms of the histograms depend on the locations of the Moon and the Sun with respect to the horizon. (5) All the facts are proof of the dependance of the form of the histograms on the location of the measured objects with respect to stars, the Sun, and the Moon. (6) At the instants of New Moon and the maxima of solar eclipses there are specific forms of the histograms. (7) It is probable that the observed correlations are not connected to flow power changes (the changes of the gravity force) — we did not find the appropriate periods in changes in histogram form. (8) A sharp anisotropy of space was discovered, registered by α-decay detectors armed with collimators. Observations at 54° North (the collimator was pointed at the Pole Star) showed no day-long periods, as was also the case for observations at 82° North, near the Pole. Histograms obtained by observations with an Easterly-directed collimator were determined every 718 minutes (half stellar day) and with observations using a Westerly-directed collimator. (9) Collimators rotating counter-clockwise, in parallel with the celestial equator, gave the probability of changes in histograms as the number of the collimator rotations. (10) Collimators rotating clockwise once a day, show no day-long periods, and similarly, collimators pointed at the Pole Star, and measurements taken near the North Pole. All the above lead us to the conclusion (proposition) that the fine structure of the histograms should be a result of the interference of gravitational waves derived from orbital motions of space masses (the planets and stars).

Introduction

Earlier we showed that the fine structure of the spectrum of amplitude variations in the results of measurements of processes of different nature (in other words, the fine structure of the dispersion of results or the pattern of the corresponding histograms) is subject to "*macroscopic fluctuations*", changing regularly with time. These changes indicate that the "dispersion of results" that remains after all artifacts are excluded inevitably accompanies any measurements and reflects very basic features of our world. In our research, we have come to the conclusion that this dispersion of results is the effect of space-time fluctuations, which, in their turn, are caused by the movement of the measured object in an anisotropic gravitational field. Among other things, this conclusion means that the examination of the detailed pattern of distributions obtained from the results of measurement of the dynamics of processes of different nature uncovers laws which cannot be revealed using traditional methods for the analysis of time series.

These assertions are based on the results of long-term experimental investigations conducted for many decades. The major part of these results, begun in 1958, is published in Russian. The goal of this paper is to give a brief review of those results and provide corresponding references.

The most general conclusion of our research is that there is evidence that the fine structure of stochastic distributions is not accidental. In other words, noncasual is the pattern of histograms plotted from a rather small number of the results of measurement of the dynamics of processes of different nature, from biochemical reactions and noise in gravitational antennae, to α-decay [1–24].

1 The "effect of near zone"

The first element of evidence of the histogram pattern changing regularly in time is the "effect of near zone". This effect means that similar histograms are significantly more likely to appear in the nearby (neighbouring) intervals of the time series of the results of measurements. The similarity of the pattern of histograms plotted from independent intervals of a time series implies the presence of an external (towards the process studied) factor, which determines the pattern of the histogram. The independence of the "near zone" effect of the nature of the process indicates that this factor has a quite general nature.

2 Measurements of processes of different nature

The second element of evidence comes from the similarity of the pattern of histograms plotted from the results of simultaneous independent measurements of processes of different nature at the same geographical point. In view of the fundamental difference in the nature of those processes and methods of their measurement, such a similarity also means that the factor, determining the histogram pattern, has a quite general nature. The similarity of histograms when under study are the processes, in which the ranges of transduced energy differ by dozens of orders (40 orders if the matter concerns the noise in a gravitational antenna, and the phenomenon of α-decay), implies that this factor has no relation to energy.

3 Regular changes in the histogram patterns

The third element of evidence for noncasuality of the histogram patterns is their regular changing with time. The regularities are revealed in the existence of the following periods in the change of the probability of similar histograms to appear.

3.1. Near-daily periods; these are well-resolvable "sidereal" (1436 min) and "solar" (1440 min) daily periods. These periods imply dependence of the histogram pattern on the rotation of the Earth around its axis. The pattern is determined by two independent factors: the position relative to the starry sky and that relative to the Sun.

3.2. Approximately 27-day periods. These periods can be considered as an indication of the dependence of the histogram pattern on the position relative to the nearby celestial bodies: the Sun, the Moon and, probably, the planets.

3.3. Yearly periods; these are well-resolvable "calendar" (365 solar days) and "sidereal" (365 solar days plus 6 h and 9 min) yearly periods.

All these periods imply the dependence of the obtained histogram pattern on two factors of rotation — (1) rotation of the Earth around its axis, and (2) movement of the Earth along its circumsolar orbit.

4 The observed local-time synchronism

The dependence of the histogram pattern on the Earth rotation around its axis is clearly revealed in the phenomenon of *synchronization at the local time*, when similar histograms are very likely to appear at different geographical points (from Arctic to Antarctic, in the Western and Eastern hemispheres) at the same *local* time. It is astonishing that the local-time synchronism with the precision of 1 min is observed independently of the regional latitude at the most extreme distances — as extreme as possible on the Earth (about 15,000 km).

5 The synchronism observed at different latitudes

The dependence of the histogram pattern on the Earth rotation around its axis is also revealed in the disappearance of the near-daily periods close to the North Pole. Such measurements were conducted at the latitude of 82° North in 2000. The analysis of histograms from the 15-min and 60-min segments showed no near-daily periods, but these periods remain in the sets of histograms plotted from the 1-min segments. Also remaining was the local-time synchronism in the appearance of similar histograms.

Following these results, it would be very interesting to conduct measurements as close as possible to the North Pole. That was unfeasible, and so we performed measurements with collimators, which channel α-particles emitted in a certain direction from a sample of ^{239}Pu. The results of those experiments made us change our views fundamentally.

6 The collimator directed at the Pole Star

Measurements were taken with the collimator directed at the Pole Star. In the analysis of histograms plotted from the results of counting α-particles that were travelling North (in the direction of the Pole Star), the near-daily periods were not observed, nor was the near-zone effect. The measurements were made in Pushchino (54° latitude North), but the effect is as would be expected at 90° North, i. e. at the North Pole. This means that the histogram pattern depends on the spatial direction of the process measured. Such a dependence, in its turn, implies a sharp anisotropy of space. Additionally, it becomes clear that the matter does not concern any "effect" or "influence" on the object under examination. The case in point is changes, fluctuations of the space-time emerging from the rotation of the Earth around its axis and the movement of the planet along its circumsolar orbit [9, 13, 14, 15, 19, 20, 21].

7 The East and West-directed collimators

This effect was confirmed in experiments with two collimators, directed East and West correspondingly. In those experiments, two important effects were discovered.

7.1. The histograms registered in the experiments with the East-directed collimator ("east histograms") are similar to those "west histograms" that are delayed by 718 min, i. e. by half of the sidereal day.

7.2. No similar histograms were observed in the simultaneous measurements with the "east" and "west" collimators. Without collimators, it is highly probable for similar histograms to appear at the same place and time. This space-time synchronism disappears when α-particles streaming in the opposing directions are counted.

These results are in agreement with the concept that the histogram pattern depends on the vector of the α-particle emission relative to a certain point at the coelosphere [20].

8 The experiments with the rotating collimators

These investigations were naturally followed by experiments with rotating collimators [22, 24].

8.1. With the collimator rotating counter-clockwise (i. e., together with the Earth), the coelosphere was scanned with a period equal to the number of the collimator rotations per day plus one rotation made by the Earth itself. We examined the dependence of the probability of similar histograms to appear on the number of collimator rotations per day. Just as expected, the probability turned out to jump with periods equal to 1440 min divided by the number of collimator rotations per day plus 1. We evaluated data at 1, 2, 3, 4, 5, 6, 7, 11 and 23 collimator rotations per day and found periods equal to 12, 8, 6 etc. hours. The analysis of highly resolved data (with a resolution of 1 min) revealed that each of these periods had two extrema: "sidereal" and "solar". These results indicate that the histogram pattern is indeed determined by how the direction of the α-particle emission relates to the "picture of the heaven" [24].

8.2. When the collimator made 1 clockwise rotation per day, the rotation of the Earth was compensated for (α-particles always undergo emission in the direction of the same region of the coelosphere) and, correspondingly, the daily periods dissappeared. This result was completely analogous to the results of measurements near the North Pole and measurements with the immobile collimator directed towards the Pole Star [20].

8.3. With the collimator placed in the ecliptic plane, directed at the Sun and making 1 clockwise rotation per day, α-particles are constantly emitted in the direction of the Sun. As was expected, the near-daily periods, both solar and sidereal, disappeared under such conditions.

9 The 718-min period

The pattern of histograms is determined by a complex set of cosmo-physical factors. It follows from the existence of the near-27-day periods, that amongst these factors may be the relative positions and states of the Sun, the Moon and the Earth. We repeatedly observed similar histograms during the risings and settings of the Sun and the Moon. A very large volume of work has been carried out. Yet we have not found a histogram pattern which would be characteristic for those instants. A review and analysis of the corresponding results will be given in a special paper. Here, I shall note one quite paradoxical result: on the days of equinox one can see a clear period in the appearance of similar histograms, which is equal to 718 min (i. e. half of the sidereal day). There is no such period on the days of solstice. This phenomenon indicates that the histogram pattern depends on the ecliptic position of the Sun. If that is indeed so, we can expect that on the equator the period of 718 min will be observed year-round.

10 The observations during eclipses

All the results presented above were obtained by the evaluation of tens of thousands of histogram pairs in every experiment, so these results have a stochastic character. A completely different approach is used in the search for characteristic histogram patterns in the periods of the New Moon and solar eclipses. In these cases, we go right to the analysis of the histogram patterns at a certain predetermined moment. Doing so, we have discovered an amazing phenomenon. At the moment of the New Moon, a certain characteristic histogram appears practically simultaneously at different longitudes and latitudes — all over the Earth. This characteristic histogram corresponds to a time segment of 0.5–1.0 min [21]. When the solar eclipse reaches maximum (as a rule, this moment does not coincide with the time of the New Moon), a specific histogram also appears; however, it has a different pattern. Such specific patterns emerge not only in the moments of the New Moon or solar eclipses. But the probability of their appearance at these very moments at different places and on different dates (months, years) being accidental is extremely low. These specific patterns do not relate to tidal effects. Nor do they depend on position on the Earth's surface, where the Moon's shadow falls during the eclipse or the New Moon.

11 The possible nature of "macroscopic fluctuations"

I have presented above a brief review of the main phenomena that are united by the notion of "macroscopic fluctuations". A number of works suggested different hypotheses on the nature of those phenomena [3, 9, 10, 13–15, 19, 27–31], concerning some general categories such as discreteness and continuity, symmetry, the nature of numbers, stochasticity. In this section of the paper I draw attention to the question of how some of the discovered phenomena can be considered in relation to these general categories.

11.1. The non-energetic nature of the phenomena. Fluctuations of space-time [14, 19].

It is clear that we deal with non-energetic phenomena. As mentioned above, the ranges of energies in biochemical reactions, noise in gravitational antennae, and α-decay, differ by many orders. At the same time, the corresponding histogram patterns are similar with a high probability at the same local time at different geographical points. The only thing common to such different processes is the space-time in which they occur. Therefore, the characteristics of space-time change every successive moment.

It is important to note that the "macroscopic fluctuations" do not result from the effect of any factors on the object under examination. They just reflect the state of the space-time.

The changes in space-time can follow the alterations of the gravitational field. These alterations are determined by the movement of the examined object in a heterogeneous gravitational field. The heterogeneity results from the existence of "mass thicknesses", i. e. heavenly bodies. The movement includes the daily rotation of the Earth, its translocation along its circumsolar orbit and, probably, the drift of the solar system in the galaxy. All these forms of movement seem to be reflected in the corresponding periods of variation of histogram patterns. How the fluctuations of space-time transform into the pattern of histograms is unclear.

11.2. Fractality [14, 19].

We suppose that the histogram pattern varies due to the change of the cosmo-physical conditions in the process of the Earth movement around its axis and along its circumsolar orbit. Then we might expect that the shorter are the intervals for which histograms are plotted, the more similar would be the histogram patterns. This corresponds to the concept of "lifetime" of a certain idea of form. This concept is an obvious consequence of the "effect of near zone", when the probability of histogram patterns to be similar is higher for the histograms from the neighbouring intervals.

However, we failed to find such a short interval for which the histogram pattern "would not have time to change". The maximum probability for histograms to be similar only in the first, the nearest interval, does not change upon variation of this interval from several hours to milliseconds. This phenomenon corresponds to the notion of "fractality"; however, the physical meaning of this fractality needs to be clarified.

Following the dependence of the histogram pattern on direction obtained in the experiments with collimators, we deal with a spatial heterogeneity on the scale of the order of 10^{-13} cm: the dependence of the histogram pattern should be determined before the emission of α-particles from the nucleus. Therefore, to "stop the instant", stop the histogram changing, we should have worked with correspondingly small time intervals. Perhaps this will be possible someday soon.

11.3. The mirror symmetry, chirality of histograms [7].

Quite often (up to 30% of cases), the patterns of the successive histograms are reflection symmetric. There are right and left forms, and they may be very complex. This phenomenon possibly means that chirality is an inherent feature of space-time.

11.4. "Stochasticity along abscissa and regularity along ordinate".

Our main result — evidence of non-stochasticity of the fine structure of sampling distributions, i. e. the fine structure of the spectrum of amplitude fluctuations in processes of any nature, i. e. the fine structure of the corresponding histograms — implies the existence of a particular class of macroscopic stochastic processes.

Among such processes is radioactive decay. This is an "*a priori* stochastic" (i. e. stochastic according to the accepted criteria) process. However, the pattern of histograms (i. e. the fine structure of the amplitudes of fluctuations of the decay rate) changes regularly with time.

The point is that in the majority of cases, stochasticity is treated as an irregular succession of events — succession in time, just one after another. This is "stochasticity along the axis of abscisses".

For macroscopic processes, the distributions of the amplitudes of fluctuations of measured quantities are considered to correspond to smooth distributions of Gauss-Poisson type. The available fitting criteria are integral, they are based on averaging, smoothing of those fluctuations. Such fitting criteria cannot "sense" the fine structure of distributions. According to these criteria, the processes we study, such as radioactive decay, correspond well to traditional views.

However, known for more then a hundred years is a noticeable exception — atomic spectra. While the transitions of electrons from one level to another are "*a priori* stochastic", the energies of the levels are sharply discrete. The "stochastic along the abscissa" process of transition is "regular along the ordinate".

The result of our work is the discovery of analogous macroscopic processes. In the process of fluctuating, the measured quantities take values, some of which are observed more often than the others; there are "forbidden" and "allowed" values of the measured quantities. This is what we see in the fine structure of histograms, with all its "peaks and troughs". The "macroscopic quantization" differs from the quantization in the microworld. Here only the "idea of histogram form" remains invariant, whereas the concrete values, corresponding to extrema, can change. This is the main difference between the spectra of amplitude fluctuations of macroscopic processes and the atomic spectra.

11.5. The fine structure of histograms. The presence of "peaks and troughs" in histogram patterns is a consequence of two causes: arithmetic (algorithmic) and physical [7, 14, 19].

11.5.1. The arithmetic or algorithmic cause of discreteness [7, 14, 19] lies in a very unequal number of factors (divisors) corresponding to the natural sequence. If the measured value is a result of operations based on the algorithms of division, multiplication, exponentiation, then discreteness will be unavoidable. Correspondingly, the histogram patterns will be determined by these algorithms. This can be seen, for example, in the computer simulation of the process of radioactive decay (Poisson statistics). The pattern of some histograms obtained in such a simulation is indistinguishable from the pattern of histograms plotted for the radioactive decay data. However, the sequence of "computer" histogram patterns, in contrast to that of "physical" ones, does not depend on time and can be reproduced over and over again by launching the simulation program with the same parameters. This sequence is determined by the nature of numbers and the algorithms used. In our work we experienced an unusual incident, when the sequence of histogram patterns created by a random number generator was similar, with high probability, to the sequence obtained from the radioactive decay data. If studied systematically, this case might give a clue to the nature of those "physical algorithms" that determine the time changes of the patterns of physical histograms [19].

11.5.2. The physical cause of discreteness is the interference of wave fluxes [19].

The fine structure of histograms, the presence of narrow extrema, cannot have a probabilistic nature. According to Poisson statistics, with which radioactive decay roughly accords, the width of such extrema should be of order $N^{1/2}$.

Therefore, if neighbouring extrema in the histogram pattern have similar values of N, they should overlap, but they do not. Such narrow extrema can arise only as a result of interference. Hence, the fine structure of histograms plotted from the results of measurements of any nature would be a result of an interference of some waves. As follows from all the material presented above, the issue concerns processes caused by the movement of the Earth (and objects on its surface) relative to the "mass thicknesses". So it would be logical to define the waves whose interference is reflected in the histogram patterns as "gravitational".

The results of experiments with collimators, producing narrow beams of α-particles, lead us to conclude for a sharp anisotropy of our world. The corresponding wave fluxes should be very narrow.

Collimators are not necessary to reveal this anisotropy. We observe highly resolved daily and yearly periods in the changing of the probability of a certain histogram pattern to appear repeatedly (the resolution is 1 min). The histogram patterns specific for the New Moon and solar eclipses can appear at different geographical points synchronously, with an accuracy of 0.5 min. The local-time synchronism at different geographical points (almost 15,000 km apart) is also determined by a sharp extremum on the curve of distribution over intervals with a resolution of 1 min. In the experiments with the rotation of collimators, the "sidereal" and "solar" periods are also observed with one-minute resolution.

Taken together, all these facts can mean that we deal with narrowly directed wave fluxes, "beams". The narrowness of these putative fluxes or beams is smaller than the aperture of collimators. Collimators with the diameter of 0.9 mm and length of 10 mm isolate in the coelosphere a window of about 5°, corresponding to approximately 20 min of the Earth's daily rotation rate. This fact, noted by Kharakoz, could be explained if we admit that the "beams" are more narrow than the aperture of our collimators.

Even with the fact that the matter concerns the changes of the histogram pattern and the movement of the Earth relative to the sphere of fixed stars, the Moon and the Sun, it is not necessary to consider anisotropy as being only due to the heterogeneous distribution of masses (presence of celestial bodies) in space. It is possible that this anisotropy is caused by a preferential direction, which, for example, is due to the drift of the solar system towards the constellation of Hercules. The existence of such a direction is an old problem of physics. In this connection, of great value for us are the results of the interference experiments of Dayton Miller [43], the experiments and conception of Alais [42], de Witte's measurements [47] and Cahill's conception [44, 46]. It is necessary to mention that several years ago, Lyapidevsky [29] and Dmitrievsky [30] considered the preferential direction in space as the cause of the effects we observed.

In this case, we can say that for many years, we have studied phenomena indicating the existence of gravitational waves. Then the problem of detection of gravitational waves can be approached differently: instead of using bulky and expensive devices, such as Weber's antennae, one could register the changes of the fine structure of histograms plotted from the results of measurements of certain chosen processes.

The fine structure of the histogram pattern we registered while solar eclipses manifests a resonance in an interference picture, built by a bulky space masses. Most probable this is a gravitational wave pattern. The histogram patterns specific for solar eclipses recall to Crother's analysis of Kepler's laws in General Relativity, wherein he showed that space-time is locally anisotropic for a rotating spherical body [49]. In this situation, we suppose that of principal importance are works by Borissova [50] and Rabounski [51] on the theory and methods of detection of gravitational waves and the concept of "global scaling" advanced by Hartmut Muller [52].

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On the Coupling Constants, Geometric Probability and Complex Domains*

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By recurring to Geometric Probability methods it is shown that the coupling constants, $\alpha_{EM}, \alpha_W, \alpha_C$, associated with the electromagnetic, weak and strong (color) force are given by the *ratios* of measures of the sphere S^2 and the Shilov boundaries $Q_3 = S^2 \times RP^1$, *squashed* S^5, respectively, with respect to the Wyler measure $\Omega_{Wyler}[Q_4]$ of the Shilov boundary $Q_4 = S^3 \times RP^1$ of the poly-disc D_4 (8 real dimensions). The latter measure $\Omega_{Wyler}[Q_4]$ is linked to the geometric coupling strength α_G associated to the gravitational force. In the conclusion we discuss briefly other approaches to the determination of the physical constants, in particular, a program based on the Mersenne primes p-adic hierarchy. The most important conclusion of this work is the role played by higher dimensions in the determination of the coupling constants from pure geometry and topology alone and which does *not* require to invoke the anthropic principle.

1 Geometric probability

Geometric Probability [1] is the study of the probabilities involved in geometric problems — the distributions of length, area, volume, etc. for geometric objects under stated conditions. One of the most famous problem is the Buffon's Needle Problem of finding the probability that a needle of length l will land on a line, given a floor with equally spaced parallel lines a distance d apart. The problem was posed by the French naturalist Buffon in 1733. For $l < d$ the probability is

$$P = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, \frac{l\,|\cos\theta|}{d} = \frac{4l}{2\pi d}\int_0^{\frac{\pi}{2}} \cos\theta = \frac{2l}{\pi d} = \frac{2ld}{\pi d^2}. \quad (1.1)$$

Hence, the Geometric Probability is essentially the *ratio* of the areas of a rectangle of length $2d$, and width l and the area of a circle of radius d. For $l > d$, the solution is slightly more complicated [1]. The Buffon needle problem provides with a numerical experiment that determines the value of π empirically. Geometric Probability is a vast field with profound connections to Stochastic Geometry.

Feynman long ago speculated that the fine structure constant may be related to π. This is the case as Wyler found long ago [2]. We will take the fine structure constant based on Feynman's physical interpretation of the electron's charge as the probability amplitude that an electron emits/absorbs a photon. The clue to evaluate this probability within the context of Geometric Probability theory is provided by the electron self-energy diagram. Using Feynman's rules, the self-energy $\Sigma(p)$ as a function of the electron's incoming/outgoing energy-momentum p_μ is given by the integral involving the photon and electron propagator along the internal lines

$$-i\,\Sigma(p) = (-ie)^2 \int \frac{d^4k}{(2\pi)^4}\,\gamma^\mu \frac{i}{\gamma^\rho(p_\rho - k_\rho) - m}\,\frac{-ig_{\mu\nu}}{k^2}\,\gamma^\nu. \quad (1.2)$$

The integral is taken with respect to the values of the photon's energy-momentum k^μ. By inspection one can see that the electron self-energy is proportional to the fine structure constant $\alpha_{EM} \sim e^2$, the square of the probability amplitude (in natural units of $\hbar = c = 1$) and physically represents the electron's emission of a virtual photon (off-shell, $k^2 \neq 0$) of energy-momentum k_ρ at a given moment, followed by an absorption of this virtual photon at a later moment.

Based on this physical picture of the electron self-energy graph, we will evaluate the Geometric Probability that an electron emits a photon at $t = -\infty$ (infinite past) and re-absorbs it at a much later time $t = +\infty$ (infinite future). The off-shell (virtual) photon associated with the electron self-energy diagram *asymptotically* behaves on-shell at the very moment of emission ($t = -\infty$) and absorption ($t = +\infty$). However, the photon can remain off-shell in the intermediate region between the moments of emission and absorption by the electron. The fact that Geometric Probability is a classical theory does not mean that one cannot derive the fine structure constant (which involves the Planck constant) because the electron self-energy diagram is itself a quantum (one-loop) Feynman process; i. e. one can recur to Geometric Probability to assign proper geometrical measures to Feynman diagrams, not unlike the Twistor-diagrammatic version of the Feynman rules of QFT.

The topology of the boundaries (at conformal infinity) of the past and future light-cones are spheres S^2 (the celestial sphere). This explains why the (Shilov) boundaries are essential mathematical features to understand the geometric derivation of all the coupling constants. In order to describe the physics at infinity we will recur to Penrose's ideas [12] of conformal compactifications of Minkowski spacetime by attaching the light-cones at conformal infinity. Not unlike the one-point compactification of the complex plane by adding the points at infinity leading to the Gauss-Riemann sphere.

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The conformal group leaves the light-cone fixed and it does not alter the causal properties of spacetime despite the rescalings of the metric. The topology of the conformal compactification of real Minkowski spacetime $\bar{M}_4 = S^3 \times S^1/Z_2 = S^3 \times RP^1$ is precisely the same as the topology of the Shilov boundary Q_4 of the 4 complex-dimensional poly-disc D_4. The action of the discrete group Z_2 amounts to an antipodal identification of the future null infinity $\mathcal{I}^+$ with the past null infinity $\mathcal{I}^-$; and the antipodal identification of the past timelike infinity i^- with the future timelike infinity, i^+, where the electron emits, and absorbs the photon, respectively.

Shilov boundaries of homogeneous (symmetric spaces) complex domains, G/K [9]–[11] are not the same as the ordinary topological boundaries (except in some special cases). The reason being that the action of the isotropy group K of the origin is not necesarily *transitive* on the ordinary topological boundary. Shilov boundaries are the minimal subspaces of the ordinary topological boundaries which implement the Maldacena-'T Hooft-Susskind holographic principle [15] in the sense that the holomorphic data in the interior (bulk) of the domain is fully determined by the holomorphic data on the Shilov boundary. The latter has the property that the maximum modulus of any holomorphic function defined on a domain is attained at the Shilov boundary.

For example, the poly-disc D_4 of 4 complex dimensions is an 8 real-dim Hyperboloid of constant negative scalar curvature that can be identified with the conformal relativistic *curved* phase space associated with the electron (a particle) moving in a $4D$ Anti de Sitter space AdS_4. The poly-disc is a Hermitian symmetric homogeneous coset space associated with the $4D$ conformal group $SO(4,2)$ since $D_4 = SO(4,2)/SO(4) \times SO(2)$. Its Shilov boundary *Shilov* $(D_4) = Q_4$ has precisely the *same* topology as the $4D$ conformally compactified real Minkowski spacetime $Q_4 = \bar{M}_4 = S^3 \times S^1/Z_2 = S^3 \times RP^1$. For more details about Shilov boundaries, the conformal group, future tubes and holography we refer to the article by Gibbons [14] and [9], [18].

The role of the conformal group in gravity in these expressions (besides the holographic bulk/boundary *AdS/CFT* duality correspondence [15]) stems from the MacDowell Mansouri-Chamseddine-West formulation of gravity based on the conformal group $SO(3,2)$ which has the same number of 10 generators as the $4D$ Poincaré group. The $4D$ vielbein e^a_μ which gauges the spacetime translations is identified with the $SO(3,2)$ generator $A^{[a5]}_\mu$, up to a crucial scale factor R, given by the size of the Anti de Sitter space (de Sitter space) throat. It is known that the Poincaré group is the Wigner-Inonu group contraction of the de Sitter Group $SO(4,1)$ after taking the throat size $R = \infty$. The spin-connection ω^{ab}_μ that gauges the Lorentz transformations is identified with the $SO(3,2)$ generator $A^{[ab]}_\mu$. In this fashion, the e^a_μ, ω^{ab}_μ are encoded into the $A^{[mn]}_\mu$ $SO(3,2)$ gauge fields, where m, n run over the group indices 1, 2, 3, 4, 5. A word of caution, gravity is a gauge theory of the full diffeomorphisms group which is infinite-dimensional and which includes the translations. Therefore, strictly speaking gravity is not a gauge theory of the Poincaré group. The Ogiovetsky theorem shows that the diffeomorphisms algebra in $4D$ can be generated by an infinity of *nested* commutators involving the $GL(4,R)$ and the $4D$ Conformal Group $SO(4,2)$ generators.

In [19] we have shown why the MacDowell-Mansouri-Chamseddine-West formulation of gravity, with a cosmological constant and a topological Gauss-Bonnet invariant term, can be obtained from an action inspired from a BF-Chern-Simons-Higgs theory based on the conformal $SO(3,2)$ group. The AdS_4 space is a natural vacuum of the theory. The vacuum energy density was *derived* to be precisely the geometric-mean between the UV Planck scale and the IR throat size of de Sitter (Anti de Sitter) space. Setting the throat size to coincide with the future horizon scale (of an accelerated de Sitter Universe) given by the Hubble scale (today) R_H, the geometric mean relationship yields the observed value of the vacuum energy density $\rho \sim (L_P R_H)^{-2} = (L_P)^{-4}(L_P^2/R_H^2) \sim 10^{-120} M^4_{Planck}$. Nottale [24] gave a different argument to explain the small value of ρ based on Scale Relativistic arguments. It was also shown in [19] why the Euclideanized AdS_{2n} spaces are $SO(2n-1, 2)$ instantons solutions of a non-linear sigma model obeying a double self duality condition.

A typical objection to the possibility of being able to derive the values of the coupling constants, from pure thought alone, is that there are an infinite number of possible analytical expressions that accurately reproduce the values of the couplings within the experimental error bounds. However, this is not our case because once the gauge groups $U(1)$, $SU(2)$, $SU(3)$ are known there are *unique* expressions stemming from Geometric Probability which furnish the values of the couplings. Another objection is that it is a meaningless task to try to derive these couplings because these are not constants per se but vary with respect to the energy scale. The running of the coupling constants is an *artifact* of the perturbative Renormalization Group program. We will see that the values of the couplings derived from Geometric Probability are precisely those values that correspond to the natural physical scales associated with the EM, Weak and Strong forces.

Another objection is that physical measurements of irrational numbers are impossible because there are always experimental limitations which rule out the possibility of actually measuring the *infinite* number of digits of an irrational number. This experimental constraint does not exclude the possibility of deriving exact expressions based on π as we shall see. We should not worry also about obtaining numerical values within the error bars in the table of the coupling constants since these numbers are based on the values of *other* physical constants; i. e. they are based on the particular *consensus* chosen for all of the other physical constants.

In our conventions, $\alpha_{EM} = e^2/4\pi = 1/137.036\ldots$ in the natural units of $\hbar = c = 1$, and the quantities $\alpha_{weak}, \alpha_{color}$ are the Geometric Probabilities $\tilde{g}_w^2, \tilde{g}_c^2$, after *absorbing* the factors of 4π of the conventional $\alpha_W = (g_w^2/4\pi)$, $\alpha_C = (g_c^2/4\pi)$ definitions used in the Renormalization Group (RG) program.

2 The fine structure constant

In order to define the Geometric Probability associated with this process of the electron's emission of a photon at i^- $(t = -\infty)$, followed by an absorption at i^+ $(t = +\infty)$, we must take into account the important fact that the photon is on-shell $k^2 = 0$ *asymptotically* (at $t = \pm\infty$), but it can move off-shell $k^2 \neq 0$ in the intermediate region which is represented by the *interior* of the $4D$ conformally compactified real Minkowski spacetime which agrees with the Shilov boundary of D_4 (the four-complex-dimensional poly-disc) $Q_4 = \bar{M}_4 = S^3 \times S^1/Z_2 = S^3 \times RP^1$. The Q_4 has four-real-dimensions which is half the real-dimensions of D_4 (2×4=8).

The measure associated with the celestial spheres S^2 (associated with the future/past light-cones) at timelike infinity i^+, i^-, respectively, is $V(S^2) = 4\pi r^2 = 4\pi$ $(r = 1)$. Thus, the *net* measure corresponding to the two celestial spheres S^2 at timelike infinity $i^\pm$ requires an overall factor of 2 giving $2V(S^2) = 8\pi$ $(r = 1)$. The factor of $8\pi = 2\times 4\pi$ can also be interpreted in terms of the two-helicity degrees of freedom, corresponding to a spin 1 massless photon, assigned to the area of the celestial sphere. The Geometric Probability is defined by the ratio of the (dimensionless volumes) measures associated with the celestial spheres S^2 at i^+, i^- timelike infinity, where the photon moves on-shell, relative to the Wyler measure $\Omega_{Wyler}[Q_4]$ associated with the full *interior* region of the conformally compactified $4D$ Minkwoski space $Q_4 = \bar{M}_4 = S^3 \times S^1/Z_2 = S^3 \times RP^1$, where the massive electron is confined to move, as it propagates from i^- to i^+, (and *off-shell* photons can also live in):

$$\alpha_{EM} = \frac{2V(S^2)}{\Omega_{Wyler}[Q_4]} = \frac{8\pi}{\Omega_{Wyler}[Q_4]} = \frac{1}{137.03608\ldots} \qquad (2.1a)$$

after inserting the Wyler measure

$$\Omega_{Wyler}[Q_4] = \frac{V(S^4)V(Q_5)}{[V(D_5)]^{\frac{1}{4}}} = \frac{8\pi^2}{3}\frac{8\pi^3}{3}\left(\frac{\pi^5}{2^4\times 5!}\right)^{-\frac{1}{4}}. \qquad (2.1b)$$

The Wyler measure $\Omega_{Wyler}[Q_4]$ [2] is *not* the standard measure (dimensionless volume) $V(Q_4) = 2\pi^3$ calculated by Hua [10] but requires some elaborate procedure.

It was realized by Smith [5] that the presence of the Wyler measure in the expression for α_{EM} given by eq.-(2.1) was consistent with Wheeler ideas that the observed values of the coupling constants of the Electromagnetic, Weak and Strong Force can be obtained if the geometric force strengths (measures related to volumes of complex homogenous domains associated with the $U(1)$, $SU(2)$, $SU(3)$ groups, respectively) are all *divided* by the geometric force strength of gravity α_G (related to the $SO(3, 2)$ MMCW Gauge Theory of Gravity) and which is not the same as the $4D$ Newton's gravitational constant $G_N \sim m_{Planck}^{-2}$. Hence, upon dividing these geometric force strengths by the geometric force strength of gravity α_G one is dividing by the Wyler measure factor because (see below) $\alpha_G \equiv \Omega_{Wyler}[Q_4]$.

Furthermore, the expression for $\Omega_{Wyler}[Q_4]$ is also consistent with the Kaluza-Klein compactification procedure of obtaining Maxwell's EM in $4D$ from *pure* gravity in $5D$ since Wyler's expression involves a $5D$ domain D_5 from the very start; i. e. in order to evaluate the Wyler measure Ω_{Wyler} $[Q_4]$ one requires to embed D_4 into D_5 because the Shilov boundary space $Q_4 = S^3 \times RP^1$ is *not* adequate enough to implement the action of the $SO(5)$ group, the compact version of the Anti de Sitter Group $SO(3, 2)$ that is required in the MacDowell-Mansouri-Chamseddine-West (MMCW) $SO(3, 2)$ gauge formulation of gravity. However, the Shilov boundary of D_5 given by $Q_5 = S^4 \times RP^1$ is adequate enough to implement the action of $SO(5)$ via isometries (rotations) on the internal symmetry space $S^4 = SO(5)/SO(4)$. This justifies the embedding procedure of $D_4 \to D_5$

The 5 complex-dimensional poly-disc $D_5 = SO(5, 2)/SO(5) \times SO(2)$ is the 10 real-dim Hyperboloid $\mathcal{H}^{10}$ corresponding to the relativistic curved phase space of a particle moving in $5D$ Anti de Sitter Space AdS_5. The Shilov boundary Q_5 of D_5 has 5 real dimensions (half of the 10-real-dim of D_5). One cannot fail to notice that the hyperboloid $\mathcal{H}^{10}$ can be embedded in the 11-dim pseudo-Euclidean $R^{9,2}$ space, with two-time like directions. This is where 11-dim lurks into our construction.

Having displayed Wyler's expression of the fine structure constant α_{EM} in terms of the ratio of dimensionless measures, we shall present a Fiber Bundle (a sphere bundle fibration over a complex homogeneous domain) derivation of the Wyler expression based on the bundle $S^4 \to E \to D_5$, and explain below why the propagation (via the determinant of the Feynman propagator) of the electron through the *interior* of the domain D_5 is what accounts for the "obscure" factor $V(D_5)^{1/4}$ in Wyler's formula for α_{EM}.

We begin by explaining why Wyler's measure $\Omega_{Wyler}[Q_4]$ in eq.-(2.1) corresponds to the measure of a S^4 bundle fibered over the base curved-space $D_5 = SO(5, 2)/SO(5) \times SO(2)$ and *weighted* by a factor of $V(D_5)^{-1/4}$. This $S^4 \to E \to D_5$ bundle is linked to the MMCW $SO(3, 2)$ Gauge Theory formulation of gravity and explains the essential role of the gravitational interaction of the electron in Wyler's formula [5] corroborating Wheeler's ideas that one must normalize the geometric force strengths with respect to gravity in order to obtain the coupling constants. The subgroup $H = SO(5)$ of the isotropy group (at the origin) $K = SO(5) \times SO(2)$ acts naturally on the Fibers $F = S^4 = SO(5)/SO(4)$, the internal symmetric space, via isometries (rotations). Locally, and only locally, the Fiber bundle E is the product $D_5 \times S^4$.

The restriction of the Fiber bundle E to the Shilov boundary Q_5 is written as $E|_{Q_5}$ and *locally* is the product of $Q_5 \times S^4$, but this is *not* true globally unless the fiber bundle admits a global section (the bundle is trivial). So, the volume $V(E|_{Q_5})$ does not necessary always factorize as $V(Q_5) \times V(S^4)$. Setting aside this subtlety, we shall pursue a more physical route, suggested by Wyler in unpublished work [3]*, to explain the origin of the "obscure normalization" factor $V(D_5)^{1/4}$ in Wyler's measure $\Omega_{Wyler}[Q_4] = (V(S^4)\times V(Q_5)/V(D_5)^{1/4})$, which suggests that the volumes may not factorize.

The relevant physical feature of this measure factor $V(D_5)^{1/4}$ is that it encodes the *spinorial* degrees of freedom of the electron, like the factor of 8π encodes the two-helicity states of the massless photon. The Feynman propagator of a massive scalar particle (inverse of the Klein-Gordon operator) $(D_\mu D^\mu - m^2)^{-1}$ corresponds to the *kernel* in the Feynman path integral that in turn is associated with the Bergman kernel $K_n(z,z')$ of the complex homogenous domain D_n, proportional to the Bergman constant $k_n \equiv 1/V(D_n)$, i. e.

$$\begin{aligned}(D_\mu D^\mu - m^2)^{-1}(x^\mu) = \frac{1}{(2\pi\mu)^D}\int d^D p\, \frac{e^{-ip_\mu x^\mu}}{p^2 - m^2 + i\epsilon} \leftrightarrow \\ \leftrightarrow K_n(\mathbf{z}, \bar{\mathbf{z}}') = \frac{1}{V(D_n)}(1 - \mathbf{z}\bar{\mathbf{z}}')^{-2n},\end{aligned} \tag{2.2}$$

where we have introduced a momentum scale μ to match units in the Feynman propagator expression, and the Bergman Kernel $K_n(\mathbf{z}, \bar{\mathbf{z}}')$ of D_n whose dimensionless entries are $\mathbf{z} = (z_1, z_2, \ldots, z_n)$, $\mathbf{z}' = (z'_1, z'_2, \ldots, z'_n)$ is given as

$$K_n(\mathbf{z}, \bar{\mathbf{z}}') = \frac{1}{V(D_n)}(1 - \mathbf{z}\bar{\mathbf{z}}')^{-2n} \tag{2.3a}$$

$V(D_n)$ is the dimensionless Euclidean volume found by Hua $V(D_n) = (\pi^n/2^{n-1}n!)$ and satisfies the reproducing and normalization properties

$$f(z) = \int_{D_n} f(\xi)\, K_n(z,\xi)\, d^n\xi\, d^n\bar{\xi}\,, \quad \int_{D_n} K_n(z,\bar{z})\, d^n z\, d^n \bar{z} = 1. \tag{2.3b}$$

The *key* result that can be inferred from the Feynman propagator (kernel) $\leftrightarrow$ Bergman kernel K_n correspondence, when $\mu = 1$, is the $(2\pi)^{-D} \leftrightarrow (V(D_n))^{-1}$ correspondence; i. e. the fundamental hyper-cell in momentum space $(2\pi)^D$ (when μ=1) corresponds to the dimensionless volume $V(D_n)$ of the domain, where $D = 2n$ real dimensions. The regularized vacuum-to-vacuum amplitude of a free *real* scalar field is given in terms of the zeta function $\zeta(s) = \sum_i \lambda_i^{-s}$ associated with the eigenvalues of the Klein-Gordon operator by

$$Z = <0|0> = \sqrt{\det(D_\mu D^\mu - m^2)^{-1}} \sim \exp\left[\frac{1}{2}\frac{d\zeta}{ds}(s=0)\right]. \tag{2.4}$$

In case of a *complex* scalar field we have to *double* the number of degrees of freedom, the amplitude then factorizes into a product and becomes $Z = \det(D_\mu D^\mu - m^2)^{-1}$.

Since the Dirac operator $\mathcal{D} = \gamma^\mu D_\mu + m$ is the "square-root" of the Klein-Gordon operator $\mathcal{D}^\dagger\mathcal{D} = D_\mu D^\mu - m^2 + \mathcal{R}$ ($\mathcal{R}$ is the scalar curvature of spacetime that is zero in Minkowski space) we have the numerical correspondence

$$\begin{aligned}\sqrt{\det(\mathcal{D})^{-1}} = \sqrt{\det\left(D_\mu D^\mu - m^2\right)^{-1/2}} = \\ = \sqrt{\sqrt{\det\left(D_\mu D^\mu - m^2\right)^{-1}}} \leftrightarrow k_n^{1/4} = \left(\frac{1}{V(D_n)}\right)^{1/4},\end{aligned} \tag{2.5}$$

because $\det \mathcal{D}^\dagger = \det \mathcal{D}$, and

$$\begin{aligned}\det \mathcal{D} = e^{\operatorname{tr}\ln \mathcal{D}} = e^{\operatorname{tr}\ln (D_\mu D^\mu - m^2)^{1/2}} = \\ = e^{\frac{1}{2}\operatorname{tr}\ln(D_\mu D^\mu - m^2)} = \sqrt{\det(D_\mu D^\mu - m^2)}\,.\end{aligned} \tag{2.6}$$

The vacuum-to-vacuum amplitude of a *complex* Dirac field Ψ (a fermion, the electron) is $Z = \det(\gamma^\mu D_\mu + m) = \det \mathcal{D} \sim \exp[-(d\zeta/ds)(s=0)]$. Notice the $\det(\mathcal{D})$ behavior of the fermion versus the $\det(D_\mu D^\mu - m^2)^{-1}$ behavior of a complex scalar field due to the Grassmanian nature of the Gaussian path integral of the fermions. The vacuum-to-vacuum amplitude of a Majorana (real) spinor (half of the number of degrees of freedom of a complex Dirac spinor) is $Z = \sqrt{\det(\gamma^\mu D_\mu + m)}$. Because the complex Dirac spinor encodes both the dynamics of the electron and its antiparticle, the positron (the negative energy solutions), the vacuum-to-vacuum amplitude corresponding to the electron (positive energy solutions, propagating forward in time) must be then $Z = \sqrt{\det(\gamma^\mu D_\mu + m)}$.

Therefore, to sum up, the origin of the "obscure" factor $V(D_5)^{1/4}$ in Wyler's formula is the *normalization* condition of $V(S^4) \times V(Q_5)$ by a factor of $V(D_5)^{1/4}$ stemming from the correspondence $V(D_5)^{1/4} \leftrightarrow Z = \sqrt{\det(\gamma^\mu D_\mu + m)}$ and which originates from the vacuum-to-vacuum amplitude of the fermion (electron) as it propagates forward in time in the domain D_5. These last relations emerge from the correspondence between the Feynman fermion (electron) propagator in Minkowski spacetime and the Bergman Kernel of the complex homogenous domain after performing the Wyler map between an unbounded domain (the interior of the future lightcone of spacetime) to a bounded one. In general, the Bergman Kernel gives rise to a Kahler potential $F(z,\bar{z}) = \log K(z,\bar{z})$ in terms of which the Bergman metric on D_n is given by

$$g_{i\bar{j}} = \frac{\partial^2 F}{\partial z^i \partial \bar{z}^i}\,. \tag{2.7}$$

We must emphasize that this Geometric probability explanation *is very different* from the interpretations provided in [2, 5, 6, 7] and properly accounts for all the numerical factors. Concluding, the Geometric Probability that an electron emits a photon at $t = -\infty$ and absorbs it at $t = +\infty$, is given by the *ratio* of the dimensionless measures (volumes):

$$\begin{aligned}\alpha_{EM} = \frac{2V(S^2)}{\Omega_{Wyler}[Q_4]} = 8\pi \frac{1}{V(S^4)}\frac{1}{V(Q_5)}[V(D_5)]^{\frac{1}{4}} = \\ = \frac{9}{8\pi^4}\left(\frac{\pi^5}{2^4 \times 5!}\right)^{1/4} = \frac{1}{137.03608\ldots}\end{aligned} \tag{2.8}$$

*I thank Frank (Tony) Smith for this information and many discussions.

in very good agreement with the experimental value. This is easily verified after one inserts the values of the Euclideanized *regularized* volumes found by Hua [10]

$$V(D_5) = \frac{\pi^5}{2^4 \times 5!}, \quad V(Q_5) = \frac{8\pi^3}{3}, \quad V(S^4) = \frac{8\pi^2}{3}. \qquad (2.19)$$

In general

$$V(D_n) = \frac{\pi^n}{2^{n-1} n!}, \qquad V(S^{n-1}) = \frac{2\pi^{n/2}}{\Gamma(n/2)}, \qquad (2.10)$$

$$V(Q_n) = V(S^{n-1} \times RP^1) = V(S^{n-1}) \times V(RP^1) = \\ = \frac{2\pi^{n/2}}{\Gamma(n/2)} \times \pi = \frac{2\pi^{(n+2)/2}}{\Gamma(n/2)}. \qquad (2.11)$$

Objections were raised to Wyler's original expression by Robertson [4]. One of them was that the hyperboloids (discs) are *not* compact and whose volumes diverge because the Lobachevsky metric diverges on the boundaries of the poly-discs. Gilmore explained [4] why one requires to use the Euclideanized regularized volumes because Wyler had shown that it is possible to map an unbounded physical domain (the interior of the future light cone) onto the interior of a homogenous bounded domain without losing the causal structure and on which there exist also a complex structure. A study of Shilov boundaries, holography and the future tube can be found in [14].

Furthermore, in order to resolve the scaling problems of Wyler's expression raised by Robertson, Gilmore showed why it is essential to use *dimensionless* volumes by setting the throat sizes of the Anti de Sitter hyperboloids to $r = 1$, because this is the only choice for r where all elements in the bounded domains are also coset representatives, and therefore, amount to honest group operations. Hence the scaling objections against Wyler raised by Robertson were satisfactory solved by Gilmore [4]. Thus, all the volumes in this section and forth, are based on setting the scaling factor $r = 1$.

The question as to *why* the value of α_{EM} obtained in Wyler's formula is precisely the value of α_{EM} observed at the *scale* of the Bohr radius a_B, has not been solved, to our knowledge. The Bohr radius is associated with the ground (most stable) state of the Hydrogen atom [5]. The spectrum generating group of the Hydrogen atom is well known to be the conformal group $SO(4, 2)$ due to the fact that there are two conserved vectors, the angular momentum and the Runge-Lentz vector. After quantization, one has two commuting $SU(2)$ copies $SO(4) = SU(2) \times SU(2)$. Thus, it makes physical sense why the Bohr-scale should appear in this construction. Bars [16] has studied the many physical applications and relationships of many seemingly distinct models of particles, strings, branes and twistors, based on the (super) conformal groups in diverse dimensions. In particular, the relevance of two-time physics in the formulation of M, F, S theory has been advanced by Bars for some time. The Bohr radius corresponds to an energy of $137.036 \times 2 \times 13.6$ eV $\sim 3.72 \times 10^3$eV. It is well known that the Rydberg scale, the Bohr radius, the Compton wavelength of electron, and the classical electron radius are all related to each other by a successive scaling in products of α_{EM}.

To finalize this section and based on the MMCW $SO(3, 2)$ Gauge Theory formulation of gravity, with a Gauss-Bonnet topological term plus a cosmological constant, the (dimensionless) Wyler measure was *defined* as the geometric coupling strength of gravity [5]:

$$\Omega_{Wyler}[Q_4] = \frac{V(S^4)V(Q_5)}{[V(D_5)]^{\frac{1}{4}}} \equiv \alpha_G\,. \qquad (2.12)$$

The relationship between α_G and the Newtonian gravitational G constant is based on the value of the coupling $(1/16\pi G)$ appearing in the Einstein-Hilbert Lagrangian $(R/16\pi G)$, and goes as follows:

$$\begin{aligned} &(16\pi G)(m^2_{Planck}) = \alpha_{EM}\alpha_G = 8\pi \Rightarrow \\ &\Rightarrow G = \frac{1}{16\pi}\frac{8\pi}{m^2_{Planck}} = \frac{1}{2m^2_{Planck}} \Rightarrow \\ &\Rightarrow G m^2_{proton} = \frac{1}{2}\left(\frac{m_{proton}}{m_{Planck}}\right)^2 \sim 5.9 \times 10^{-39} \end{aligned} \qquad (2.13)$$

and in natural units $\hbar = c = 1$ yields the physical force strength of gravity at the Planck Energy scale 1.22×10^{19} GeV. The Planck mass is obtained by equating the Schwarzschild radius $2G\,m_{Planck}$ to the Compton wavelength $1/m_{Planck}$ associated with the mass; where $m_{Planck}\sqrt{2} = 1.22 \times 10^{19}$ GeV and the proton mass is 0.938 GeV. Some authors define the Planck mass by absorbing the factor of $\sqrt{2}$ inside the definition of $m_{Planck} = 1.22 \times 10^{19}$ GeV.

3 The weak and strong couplings

We turn now to the derivation of the other coupling constants. The Fiber Bundle picture of the previous section is essential in our construction. The Weak and the Strong geometric coupling constant strength, defined as the probability for a particle to emit and later absorb a $SU(2)$, $SU(3)$ gauge boson, can both be obtained by using the main formula derived from Geometric Probability (as ratios of dimensionless measures/volumes) after one identifies the suitable homogeneous domains and their Shilov boundaries to work with.

Since massless gauge bosons live on the lightcone, a null boundary in Minkowski spacetime, upon performing the Wyler map, the gauge bosons are confined to live on the Shilov boundary. Because the $SU(2)$ bosons $W^{\pm}$, Z^0 and the eight $SU(3)$ gluons have *internal* degrees of freedom (they carry weak and color charges) one must also include the measure associated with the their respective internal spaces; namely, the measures relevant to Geometric Probability calculations are the measures corresponding to the appropriate sphere bundles fibrations defined over the complex bounded homogenous domains $S^m \to E \to \mathcal{D}_n$.

Furthermore, the Geometric Probability interpretation for $\alpha_{weak}, \alpha_{strong}$ agrees with Wheeler's ideas [5] that one must normalize these geometric force strengths with respect to the geometric force strength of gravity $\alpha_G = \Omega_{Wyler}[Q_4]$ found in the last section. Hence, after these explanations, we will show below why the weak and strong couplings are given, respectively, by the *ratio* of the measures (dimensionless volumes):

$$\alpha_{weak} = \frac{\Omega[Q_3]}{\Omega_{Wyler}[Q_4]} = \frac{\Omega[Q_3]}{\alpha_G} = \frac{\Omega[Q_3]}{(8\pi/\alpha_{EM})}, \tag{3.1}$$

$$\alpha_{color} = \frac{\Omega[\textit{squashed}\, S^5]}{\Omega_{Wyler}[Q_4]} = \frac{\Omega[sq.S^5]}{\alpha_G} = \frac{\Omega[sq.S^5]}{(8\pi/\alpha_{EM})}. \tag{3.2}$$

As always, one must insert the values of the regularized (Euclideanized) dimensionless volumes provided by Hua [10] (set the scale $r = 1$). We must also clarify and emphasize that we define the quantities $\alpha_{weak}, \alpha_{color}$ as the probabilities $\tilde{g}^2_W$, $\tilde{g}^2_C$, by absorbing the factors of 4π in the conventional $\alpha_W = (g^2_W/4\pi)$, $\alpha_C = (g^2_C/4\pi)$ definitions (based on the Renormalization Group (RG) program) into our definitions of probability $\tilde{g}^2_W$, $\tilde{g}^2_C$.

Let us evaluate the α_{weak}. The internal symmetry space is $CP^1 = SU(2)/U(1)$ (a sphere $S^2 \sim CP^1$) where the isospin group $SU(2)$ acts via isometries on CP^1. The Shilov boundary of D_2 is $Q_2 = S^1 \times RP^1$ but is not adequate enough to accommodate the action of the isospin group $SU(2)$. One requires to have the Shilov boundary of D_3 given by $Q_3 = S^2 \times S^1/Z_2 = S^2 \times RP^1$ that can accommodate the action of the $SU(2)$ group on S^2. A Fiber Bundle over $D_3 = SO(3,2)/SO(3) \times SO(2)$ whose $H = SO(3) \sim SU(2)$ subgroup of the isotropy group (at the origin) $K = SO(3) \times SO(2)$ acts on S^2 by simple rotations. Thus, the relevant measure is related to the fiber bundle E restricted to Q_3 and is written as $V(E|_{Q_3})$.

One must notice that due to the fact that the $SU(2)$ group is a double-cover of $SO(3)$, as one goes from the $SO(3)$ action on S^2 to the $SU(2)$ action on S^2 , one must take into account an extra factor of 2 giving then

$$\begin{aligned} V(CP^1) &= V\big(SU(2)/U(1)\big) = \\ &= 2V\big(SO(3)/U(1)\big) = 2V(S^2) = 8\pi\,. \end{aligned} \tag{3.3}$$

In order to obtain the weak coupling constant due to the exchange of $W^{\pm}Z^0$ bosons in the four-point tree-level processes involving four leptons, like the electron, muon, tau, and their corresponding neutrinos (leptons are fundamental particles that are lighter than mesons and baryons) which are confined to move in the interior of the domain D_3, and can emit (absorb) $SU(2)$ gauge bosons, $W^{\pm}Z^0$, in the respective s, t, u channels, one must take into account a factor of the square root of the determinant of the fermionic propagator, $\sqrt{\det \mathcal{D}^{-1}} = \sqrt{\det\,(\gamma^\mu D_\mu + m)^{-1}}$, for each *pair* of leptons, as we did in the previous section when an electron emitted and absorbed a photon. Since there are *two* pairs of leptons in these four-point tree-level processes involving *four* leptons, one requires *two* factors of $\sqrt{\det\,(\gamma^\mu D_\mu + m)^{-1}}$, giving a net factor of $\det\,(\gamma^\mu D_\mu + m)^{-1}$ and which corresponds now to a net normalization factor of $k_n^{1/2} = (1/V(D_3))^{1/2}$, after implementing the Feynman kernel $\leftrightarrow$ Bergman kenel correspondence. Therefore, after taking into account the result of eq.-(3.3), the measure of the $S^2 \to E \to D_3$ bundle, restricted to the Shilov boundary Q_3, and weighted by the net normalization factor $(1/V(D_3))^{1/2}$, is

$$\Omega(Q^3) = 2V(S^2)\frac{V(Q_3)}{V(D_3)^{1/2}}\,. \tag{3.4}$$

Therefore, the Geometric probability expression is given by the ratio of measures (dimensionless volumes):

$$\begin{aligned} \alpha_{weak} &= \frac{\Omega[Q^3]}{\Omega_{Wyler}[Q_4]} = \frac{\Omega[Q^3]}{\alpha_G} = \frac{2V(S^2)V(Q_3)}{V(D_3)^{1/2}}\frac{\alpha_{EM}}{8\pi} = \\ &= (8\pi)(4\pi^2)\left(\frac{\pi^3}{24}\right)^{-\frac{1}{2}}\frac{\alpha_{EM}}{8\pi} = 0.2536\ldots \end{aligned} \tag{3.5}$$

that corresponds to the weak coupling constant ($g^2/4\pi$ based on the RG convention) at an energy of the order of

$$E = M = 146\ \text{GeV} \sim \sqrt{M^2_{W_+} + M^2_{W_-} + M^2_Z} \tag{3.6}$$

after the expressions inserted (setting the scale $r = 1$)

$$V(S^2) = 4\pi\,,\quad V(Q_3) = 4\pi^2,\quad V(D_3) = \frac{\pi^3}{24} \tag{3.7}$$

into the formula (3-5). The relationship to the Fermi coupling goes as follows (with the energy scale $E = M = 146$ GeV):

$$\begin{aligned} G_F &\equiv \frac{\alpha_W}{M^2} \Rightarrow G_F\, m^2_{proton} = \left(\frac{\alpha_W}{M^2}\right) m^2_{proton} = \\ &= 0.2536 \times \left(\frac{m_{proton}}{146\ \text{GeV}}\right)^2 \sim 1.04 \times 10^{-5} \end{aligned} \tag{3.8}$$

in very good agreement with experimental observations. Once more, it is unknown why the value of α_{weak} obtained from Geometric Probability corresponds to the energy scale related to the W_+, W_-, Z_0 boson mass, after spontaneous symmetry breaking.

Finally, we shall derive the value of α_{color} from eq.-(3.2) after one defines what is the suitable fiber bundle. The calculation is based on the book by L. K. Hua [10, p. 40, 93]. The symmetric space with the $SU(3)$ color force as a local group is $SU(4)/SU(3) \times U(1)$ which corresponds to a bounded symmmetric domain of type $I(1,3)$ and has a Shilov boundary that Hua calls the "characteristic manifold" $CI(1,3)$. The volume $V\big(CI(m,n)\big)$ is:

$$V(CI) = \frac{(2\pi)^{mn - m(m-1)/2}}{(n-m)!\,(n-m+1)!\ldots(n-1)!} \tag{3.9}$$

so that for $m = 1$ and $n = 3$ the relevant volume is then $V(CI) = (2\pi)^3/2! = 4\pi^3$. We must remark at this point that $CI(1,3)$ is *not* the standard round S^5 but is the *squashed* five-dimensional $\tilde{S}^5$.*

The domain of which $CI(1,3)$ is the Shilov boundary is denoted by Hua as $RI(1,3)$ and whose volume is

$$V(RI) = \frac{1!\,2!\dots(m-1)!\,1!\,2!\dots(n-1)!\,\pi^{mn}}{1!\,2!\dots(m+n-1)!} \qquad (3.10)$$

so that for $m=1$ and $n=3$ it gives $V(RI)=1!2!\pi^3/1!2!3!= =\pi^3/6$ and it also agrees with the volume of the standard six-ball.

The internal symmetry space (fibers) is as follows $CP^2 = =SU(3)/U(2)$ whose isometry group is the color $SU(3)$ group. The base space is the $6D$ domain $B_6 = SU(4)/U(3) = =SU(4)/SU(3)\times U(1)$ whose subgroup $SU(3)$ of the isotropy group (at the origin) $K = SU(3)\times U(1)$ acts on the internal symmetry space CP^2 via isometries. In this special case, the Shilov and ordinary topological boundary of B_6 both coincide with the *squashed* S^5 [5].

Since Gilmore, in response to Robertson's objections to Wyler's formula [2], has shown that one must set the scale $r=1$ of the hyperboloids $\mathcal{H}^n$ (and S^n) and use *dimensionles* volumes, if we were to equate the volumes $V(CP^2) = =V(S^4, r=1)$ [5], this would be tantamount of choosing another scale [25] R (the unit of geodesic distance in CP^2) that is *different* from the unit of geodesic distance in S^4 when the radius $r=1$, as required by Gilmore. Hence, a bundle map $E\to E'$ from the bundle $CP^2\to E\to B_6$ to the bundle $S^4\to E'\to B_6$, would be required that would allow us to replace the $V(CP^2)$ for $V(S^4, r=1)$. Unless one decides to *calibrate* the unit of geodesic distance in CP^2 by choosing $V(CP^2)=V(S^4)$.

Using again the same results described after eq.-(2.2), since a quark can emit and absorb later on a $SU(3)$ gluon (in a one-loop process), and is confined to move in the interior of the domain B_6, there is *one* factor only of the square root of the determinant of the Dirac propagator $\sqrt{\det \mathcal{D}^{-1}} = =\sqrt{\sqrt{\det\,(D_\mu D^\mu - m^2)^{-1}}}$ and which is associated with a normalization factor of $k_n^{1/4} = (1/V(B_6))^{1/4}$. Therefore, the measure of the bundle $S^4\to E'\to B_6$ restricted to the *squashed* S^5 (Shilov boundary of B^6), and weighted by the normalization factor $(1/V(B_6))^{1/4}$, is then

$$\Omega[\mathit{squashed}\, S^5] = \frac{V(S^4)\; V(\mathit{squashed}\, S^5)}{V(B_6)^{1/4}} \qquad (3.11)$$

and the ratio of measures

$$\begin{aligned}\alpha_s &= \frac{\Omega[sq.S^5]}{\Omega_{Wyler}[Q_4]} = \frac{\Omega[sq.S^5]}{\alpha_G} = \frac{V(S^4)V(sq.S^5)}{V(B_6)^{1/4}}\frac{\alpha_{EM}}{8\pi} = \\ &= \left(\frac{8\pi^2}{3}\right)(4\pi^3)\left(\frac{\pi^3}{6}\right)^{-1/4}\frac{\alpha_{EM}}{8\pi} = 0.6286\dots\end{aligned} \qquad (3.12)$$

matches, remarkably, the strong coupling value $\alpha = g^2/4\pi$ at an energy E related precisely to the pion masses [5]

*Frank (Tony) Smith, private communication.

$$E = 241 \text{ MeV} \sim \sqrt{m^2_{\pi^+} + m^2_{\pi^-} + m^2_{\pi^0}}. \qquad (3.13)$$

The one-loop Renormalization Group flow of the coupling is given by [28]:

$$\alpha_s(E^2) = \alpha_s(E_0^2)\left[1 + \frac{\left(11 - \frac{2}{3}N_f(E^2)\right)}{4\pi}\alpha_s(E_0^2)\ln\left(\frac{E^2}{E_0^2}\right)\right]^{-1} \qquad (3.14)$$

where $N_f(E^2)$ is the number of quark flavors whose mass $M^2 < E^2$. For the specific numerical details of the evaluation (in energy-intervals given by the diverse quark masses) of the Renormalization Group flow equation (3-14) that yields $\alpha_s(E = 241 \text{ MeV})\sim 0.6286$ we refer to [5]. Once more, it is unknown why the value of α_{color} obtained from Geometric Probability corresponds to the energy scale $E = 241$ MeV related to the masses of the pions. The pions are the known lightest quark-antiquark pairs that feel the strong interaction.

Rigorously speaking, one should include higher-loop corrections to eq.-(3.14) as Weinberg showed [28] to determine the values of the strong coupling at energy scales $E = 241$ MeV. This issue and the subtleties behind the calibration of scales (volumes) by imposing the condition $V(CP^2)=V(S^4)$ need to be investigated. For example, one could calibrate lengths in terms of the units of geodesic distance in CP^2 (based on Gilmore's choice of $r=1$) giving $V(CP^2) = =V(S^5; r=1)/V(S^1; r=1) = \pi^2/2!$ [25], and it leads now to the value of $\alpha_s = 0.1178625$ which is very close to the value of α_s at the energy scale of the Z-boson mass (91.2 GeV) and given by $\alpha_s = 0.118$ [28].

4 Mersenne primes p-adic hierarchy. Other approaches

To conclude, we briefly mention other approaches to the determination of the physical parameters. A hierarchy of coupling constants, including the cosmological constant, based on Seifert-spheres fibrations was undertaken by [26]. The ratios of particle masses, like the proton to electron mass ratio $m_p/m_e \sim 6\pi^5$ has also been calculated using the volumes of homogeneous bounded domains [5, 6]. A charge-mass-spin relationship was investigated in [27]. It is not known whether this procedure should work for Grand Unified Theories (GUT) based on the groups like $SU(5)$, $SO(10)$, E_6, E_7, E_8, meaning whether or not one could obtain, for example, the $SU(5)$ coupling constant consistent with the Grand Unification Models based on the $SU(5)$ group and with the Renormalization Group program at the GUT scale.

Beck [8] has obtained all of the Standard Model parameters by studying the numerical minima (and zeros) of certain potentials associated with the Kaneko coupled two-dim lattices (two-dim non-linear sigma-like models which resemble Feynman's chess-board lattice models) based on Stochastic Quantization methods. The results by Smith [5] (also based on Feynman's chess board models and hyper-diamond lattices) are analytical rather than being numerical [8] and it is not clear if there is any relationship between these latter two approaches. Noyes has proposed an iterated numerical hierarchy based on Mersenne primes $M_p = 2^p - 1$ for *certain* values of $p = primes$ [20], and obtained a quite large number of satisfactory values for the physical parameters. An interesting coincidence is related to the iterated Mersenne prime sequence

$$\begin{gathered} M_2 = 2^2 - 1 = 3\,, \qquad M_3 = 2^3 - 1 = 7\,, \\ M_7 = 2^7 - 1 = 127\,, \quad 3 + 7 + 127 = 137\,, \\ M_{127} = 2^{127} - 1 \sim 1.69{\times}10^{38} \sim \left(\frac{M_{Planck}}{m_{proton}}\right)^2. \end{gathered} \tag{4.1}$$

Pitkanen has also developed methods to calculate physical masses recurring to a p-adic hierarchy of scales based on Mersenne primes [21].

An important connection between anomaly cancellation in string theory and perfect even numbers was found in [23]. These are numbers which can be written in terms of sums of its divisors, including unity, like $6 = 1 + 2 + 3$, and are of the form $P(p) = \frac{1}{2}\, 2^p (2^p - 1)$ if, and only if, $2^p - 1$ is a Mersenne prime. Not all values of $p = prime$ yields primes. The number $2^{11} - 1$ is not a Mersenne prime, for example. The number of generators of the anomaly free groups $SO(32)$, $E_8 \times E_8$ of the 10-dim superstring is 496 which is an even perfect number. Another important group related to the unique tadpole-free bosonic string theory is the $SO(2^{13}) = SO(8192)$ group related to the bosonic string compactified on the $E_8 \times SO(16)$ lattice. The number of generators of $SO(8192)$ is an even perfect number since $2^{13} - 1$ is a Mersenne prime. For an introduction to p-adic numbers in Physics and String theory see [22].

A lot more work needs to be done to be able to answer the question: is all this just a mere numerical coincidence or is it design? However, the results of the previous sections indicate that it is very *unlikely* that these results were just a mere numerical coincidence (senseless numerology) and that indeed the values of the physical constants could be actually calculated from pure thought, rather than invoking the anthropic principle; i. e. namely, based on the interplay of harmonic analysis, geometry, topology, higher dimensions and, ultimately, number theory. The fact that the coupling constants involved the ratio of measures (volumes) may cast some light on the role of the world-sheet areas of strings, and world volumes of p-branes, as they propagate in target spacetime backgrounds of diverse dimensions.

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A Brief History of Black Holes

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Neither the layman nor the specialist, in general, have any knowledge of the historical circumstances underlying the genesis of the idea of the Black Hole. Essentially, almost all and sundry simply take for granted the unsubstantiated allegations of some ostentatious minority of the relativists. Unfortunately, that minority has been rather careless with the truth and is quite averse to having its claims corrected, notwithstanding the documentary evidence on the historical record. Furthermore, not a few of that vainglorious and disingenuous coterie, particularly amongst those of some notoriety, attempt to dismiss the testimony of the literature with contempt, and even deliberate falsehoods, claiming that history is of no importance. The historical record clearly demonstrates that the Black Hole has been conjured up by combination of confusion, superstition and ineptitude, and is sustained by widespread suppression of facts, both physical and theoretical. The following essay provides a brief but accurate account of events, verifiable by reference to the original papers, by which the scandalous manipulation of both scientific and public opinion is revealed.

It has frequently been alleged by theoretical physicists (e. g. [1, 2]) that Newton's theory of gravitation either predicts or adumbrates the black hole. This claim stems from a suggestion originally made by John Michell in 1784 that if a body is sufficiently massive, "all light emitted from such a body would be made to return to it by its own power of gravity". The great French scientist, P. S. de Laplace, made a similar conjecture in the eighteenth century and undertook a mathematical analysis of the matter.

However, contrary to popular and frequent expert opinion, the Michell-Laplace dark body, as it is actually called, is not a black hole at all. The reason why is quite simple.

For a gravitating body we identify an escape velocity. This is a velocity that must be achieved by an object to enable it to leave the surface of the host body and travel out to infinity, where it comes to rest. Therefore, it will not fall back towards the host. It is said to have escaped the host. At velocities lower than the escape velocity, the object will leave the surface of the host, travel out to a finite distance where it momentarily comes to rest, then fall back to the host. Consequently, a suitably located observer will see the travelling object twice, once on its journey outward and once on its return trajectory. If the initial velocity is greater than or equal to the escape velocity, an observer located outside the host, anywhere on the trajectory of the travelling object, will see the object just once, as it passes by on its outward unidirectional journey. It escapes the host. Now, if the escape velocity is the speed of light, this means that light can leave the host and travel out to infinity and come to rest there. It escapes the host. Therefore, all observers located anywhere on the trajectory will see the light once, as it passes by on its outward journey. However, if the escape velocity is greater than the speed of light, then light will travel out to a finite distance, momentarily come to rest, and fall back to the host, in which case a suitably located observer will see the light twice, once as it passes by going out and once upon its return. Furthermore, an observer located at a sufficiently large and finite distance from the host will not see the light, because it does not reach him. To such an observer the host is dark: a Michell-Laplace dark body. But this does not mean that the light cannot leave the surface of the host. It can, as testified by the closer observer. Now, in the case of the black hole, it is claimed by the relativists that no object and no light can even leave the event horizon (the "surface") of the black hole. Therefore, an observer, no matter how close to the event horizon, will see nothing. Contrast this with the escape velocity for the Michell-Laplace dark body where, if the escape velocity is the speed of light, all observers located on the trajectory will see the light as it passes out to infinity where it comes to rest, or when the escape velocity is greater than the speed of light, so that a suitably close observer will see the light twice, once when it goes out and once when it returns. This is completely opposite to the claims for the black hole. Thus, there is no such thing as an escape velocity for a black hole, and so the Michell-Laplace dark body is not a black hole. Those who claim the Michell-Laplace dark body a black hole have not properly understood the meaning of escape velocity and have consequently been misleading as to the nature of the alleged event horizon of a black hole. It should also be noted that nowhere in the argument for the Michell-Laplace dark body is there gravitational collapse to a point-mass, as is required for the black hole.

The next stage in the genesis of the black hole came with Einstein's General Theory of Relativity. Einstein himself never derived the black hole from his theory and never admitted the theoretical possibility of such an object, always maintaining instead that the proposed physical basis for its existence was incorrect. However, he was never able to demonstrate this mathematically because he did not understand the basic geometry of his gravitational field. Other theoreticians obtained the black hole from Einstein's equations by way of arguments that Einstein always objected to. But Einstein was over-ruled by his less cautious colleagues, who also failed to understand the geometry of Einstein's gravitational field.

The solution to Einstein's field equations, from which the black hole has been extracted, is called the "Schwarzschild" solution, after the German astronomer Karl Schwarzschild, who, it is claimed by the experts, first obtained the solution and first predicted black holes, event horizons, and Schwarzschild radii, amongst other things. These credits are so commonplace that it comes as a surprise to learn that the famous "Schwarzschild" solution is not the one actually obtained by Karl Schwarzschild, even though all the supposed experts and all the textbooks say so. Furthermore, Schwarzschild did not breathe a word about black holes, because his true solution does not allow them.

Shortly after Einstein published the penultimate version of his theory of gravitation in November 1915, Karl Schwarzschild [3] obtained an exact solution for what is called the static vacuum field of the point-mass. At that time Schwarzschild was at the Russian Front, where he was serving in the German army, and suffering from a rare skin disease contracted there. On the 13th January 1916, he communicated his solution to Einstein, who was astonished by it. Einstein arranged for the rapid publication of Schwarzschild's paper. Schwarzschild communicated a second paper to Einstein on the 24th February 1916 in which he obtained an exact solution for a sphere of homogeneous and incompressible fluid. Unfortunately, Schwarzschild succumbed to the skin disease, and died about May 1916, at the age of 42.

Working independently, Johannes Droste [4] obtained an exact solution for the vacuum field of the point-mass. He communicated his solution to the great Dutch scientist H. A. Lorentz, who presented the solution to the Dutch Royal Academy in Amsterdam at a meeting on the 27th May 1916. Droste's paper was not published until 1917. By then Droste had learnt of Schwarzschild's solution and therefore included in his paper a footnote in acknowledgement. Droste anticipated the mathematical procedure that would later lead to the black hole, and correctly pointed out that such a procedure is not permissible, because it would lead to a non-static solution to a static problem. Contra-hype!

Next came the famous "Schwarzschild" solution, actually obtained by the great German mathematician David Hilbert [5], in December 1916, a full year after Schwarzschild obtained his solution. It bears a little resemblance to Schwarzschild's solution. Hilbert's solution has the same form as Droste's solution, but differs in the range of values allowed for the incorrectly assumed radius variable describing how far an object is located from the gravitating mass. It is this incorrect range on the incorrectly assumed radius variable by Hilbert that enabled the black hole to be obtained. The variable on the Hilbert metric, called a radius by the relativists, is in fact not a radius at all, being instead a real-valued parameter by which the true radii in the spacetime manifold of the gravitational field are rightly calculated. None of the relativists have understood this, including Einstein himself. Consequently, the relativists have never solved the problem of the gravitational field. It is amazing that such a simple error could produce such a gigantic mistake in its wake, but that is precisely what the black hole is — a mistake for enormous proportions. Of course, the black hole violates the static nature of the problem, as pointed out by Droste, but the black hole theoreticians have ignored this important detail.

The celebrated German mathematician, Hermann Weyl [6], obtained an exact solution for the static vacuum field of the point-mass in 1917, by a very elegant method. He derived the same solution that Droste had obtained.

Immediately after Hilbert's solution was published there was discussion amongst the physicists as to the possibility of gravitational collapse into the nether world of the nascent black hole. During the Easter of 1922, the matter was considered at length at a meeting at the Collège de France, with Einstein in attendance.

In 1923 Marcel Brillouin [7] obtained an exact solution by a valid transformation of Schwarzschild's original solution. He demonstrated quite rigorously, in relation to his particular solution, that the mathematical process, which later spawned the black hole, actually violates the geometry associated with the equation describing the static gravitational field for the point-mass. He also demonstrated rigorously that the procedure leads to a non-static solution to a static problem, just as Droste had pointed out in 1916, contradicting the very statement of the initial problem to be solved — what is the gravitational field associated with a spherically symmetric gravitating body, where the field is unchanging in time (static) and the spacetime outside the body is free of matter (i. e. vacuum), other than for the presence of a test particle of negligible mass?

In mathematical terms, those solutions obtained by Schwarzschild, Droste and Weyl, and Brillouin, are mutually consistent, in that they can be obtained from one another by an admissible transformation of coordinates. However, Hilbert's solution is inconsistent with their solutions because it cannot be obtained from them or be converted to one of them by an admissible transformation of coordinates. This fact alone is enough to raise considerable suspicions about the validity of Hilbert's solution. Nonetheless, the relativists have not recognised this problem either, and have carelessly adopted Hilbert's solution, which they invariably call "Schwarzschild's" solution, which of course, it is certainly not.

In the years following, a number of investigators argued, in one way or another, that the "Schwarzschild" solution, as Hilbert's solution became known and Schwarzschild's real solution neglected and forgotten, leads to the bizarre object now called the black hole. A significant subsequent development in the idea came in 1949, when a detailed but erroneous mathematical study of the question by the Irish mathematical physicist J. L. Synge [8], was read before the Royal Irish Academy on the 25th April 1949, and published on the 20th March 1950. The study by Synge was quite exhaustive but being based upon false premises its conclusions are generally false too. Nonetheless, this paper was hailed as a significant breakthrough in the understanding of the structure of the spacetime of the gravitational field.

It was in 1960 that the mathematical description of the black hole finally congealed, in the work of M. D. Kruskal [9] in the USA, and independently of G. Szekeres [10] in Australia. They allegedly found a way of mathematically extending the "Schwarzschild" solution into the region of the nascent black hole. The mathematical expression, which is supposed to permit this, is called the Kruskal-Szekeres extension. This formulation has become the cornerstone of modern relativists and is the fundamental argument upon which they rely for the theoretical justification of the black hole, which was actually christened during the 1960's by the American theoretical physicist, J. A. Wheeler, who coined the term.

Since about 1970 there has been an explosion in the number of people publishing technical research papers, textbooks and popular science books and articles on various aspects of General Relativity. A large proportion of this includes elements of the theory of black holes. Quite a few are dedicated exclusively to the black hole. Not only is there now a simple black hole with a singularity, but also naked singularities, black holes without hair, supermassive black holes at the centres of galaxies, black hole quasars, black hole binary systems, colliding black holes, black hole x-ray sources, charged black holes, rotating black holes, charged and rotating black holes, primordial black holes, mini black holes, evaporating black holes, wormholes, and other variants, and even white holes! Black holes are now "seen" everywhere by the astronomers, even though no one has ever found an event horizon anywhere. Consequently, public opinion has been persuaded that the black hole is a fact of Nature and that anyone who questions the contention must be a crackpot. It has become a rather lucrative business, this black hole. Quite a few have made fame and fortune peddling the shady story.

Yet it must not be forgotten that all the arguments for the black hole are theoretical, based solely upon the erroneous Hilbert solution and the meaningless Kruskal-Szekeres extension on it. One is therefore lead to wonder what it is that astronomers actually "see" when they claim that they have found yet another black hole here or there.

Besides the purely mathematical errors that mitigate the black hole, there are also considerable physical arguments against it, in addition to the fact that no event horizon has ever been detected.

What does a material point mean? What meaning can there possibly be in the notion of a material object without any spatial extension? The term material point (or point-mass) is an oxymoron. Yet the black hole singularity is supposed to have mass and no extension. Moreover, there is not a single shred of experimental evidence to even remotely suggest that Nature makes material points. Even the electron has spatial extent, according to experiment, and to quantum theory. A "point" is an abstraction, not a physical object. In other words, a point is a purely mathematical object. Points and physical objects are mutually exclusive by definition. No one has ever observed a point, and no one ever will because it is unobservable, not being physical. Therefore, Nature does not make material points. Consequently, the theoretical singularity of the black hole cannot be a point-mass.

It takes an infinite amount of observer time for an object, or light, to reach the event horizon, irrespective of how far that observer is located from the horizon. Similarly, light leaving the surface of a body undergoing gravitational collapse, at the instant that it passes its event horizon, takes an infinite amount of observer time to reach an observer, however far that observer is from the event horizon. Therefore, the black hole is undetectable to the observer since he must wait an infinite amount of time to confirm the existence of an event horizon. Such an object has no physical meaning for the observer. Furthermore, according to the very same theoreticians, the Universe started with a Big Bang, and that theory gives an alleged age of 14 billion years for the Universe. This is hardly enough time for the black hole to form from the perspective of an external observer. Consequently, if black holes exist they must have been created at the instant of the Bang. They must be primordial black holes. But that is inconsistent with the Bang itself, because matter at that "time", according to the Big Bang theoreticians, could not form lumps. Even so, they cannot be detected by an external observer owing to the infinite time needed for confirmation of the event horizon. This now raises serious suspicions as to the validity of the Big Bang, which is just another outlandish theory, essentially based upon Friedmann's expanding Universe solution, not an established physical reality as the astronomers would have us believe, despite the now commonplace alleged observations they adduce to support it.

At first sight it appears that the idea of a binary system consisting of two black holes, or a hole and a star, and the claim that black holes can collide, are physical issues. However, this is not quite right, notwithstanding that the theoreticians take them as well-defined physical problems. Here are the reasons why these ideas are faulty. First, the black hole is allegedly predicted by General Relativity. What the theoreticians routinely fail to state clearly is that the black hole comes from a solution to Einstein's field equations when treating of the problem of the motion of a test particle of negligible mass in the vicinity of a single gravitating body. The gravitational field of the test particle is considered too small to affect the overall field and is therefore neglected. Therefore, Hilbert's solution is a solution for one gravitating body interacting with a test particle. It is not a solution for the interaction of two or more comparable masses. Indeed, there is no known solution to Einstein's field equations for more than one gravitating body. In fact, it is not even known if Einstein's field equations actually admit of solutions for multi-body configurations. Therefore, there can be no meaningful theoretical discussion of black hole binaries or colliding black holes, unless it can be shown that Einstein's field equations contain, hidden within them, solutions for such configurations of matter. Without at least an existence theorem for multi-body configurations, all talk of black hole binaries and black hole collisions is twaddle (see also [11]). The theoreticians have never provided an existence theorem.

It has been recently proved that the black hole and the expanding Universe are not predicted by General Relativity at all [12, 13], in any circumstances. Since the Michell-Laplace dark body is not a black hole either, there is no theoretical basis for it whatsoever.

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The Neutrosophic Logic View to Schrödinger's Cat Paradox

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This article discusses Neutrosophic Logic interpretation of the Schrödinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic Logic could take into consideration, whereas other methods including Fuzzy Logic could not. For a balanced discussion, other interpretations have also been discussed.

1 Schrödinger equation

As already known, Schrödinger equation is the most used equation to describe non-relativistic quantum systems. Its relativistic version was developed by Klein-Gordon and Dirac, but Schrödinger equation has wide applicability in particular because it resembles classical wave dynamics. For introduction to non-relativistic quantum mechanics, see [1].

Schrödinger equation begins with definition of total energy $E = \vec{p}^{\,2}/2m$. Then, by using a substitution

$$E = i\hbar\frac{\partial}{\partial t}\,, \qquad P = \frac{\hbar}{i}\,\nabla, \tag{1}$$

one gets [2]

$$\left[i\hbar\frac{\partial}{\partial t} + \hbar\frac{\bar{\nabla}^2}{2m} - U(x)\right]\psi = 0 \tag{2}$$

or

$$\frac{i\partial}{\partial t}\,\psi = H\psi\,. \tag{3}$$

While this equation seems quite clear to represent quantum dynamics, the physical meaning of the wavefunction itself is not so clear. Soon thereafter Born came up with hypothesis that the square of the wavefunction has the meaning of chance to find the electron in the region defined by dx (Copenhagen School). While so far his idea was quickly adopted as "standard interpretation", his original "guiding field" interpretation has been dropped after criticism by Heisenberg over its physical meaning [3]. Nonetheless, a definition of "Copenhagen interpretation" is that it gives the wavefunction a role in the actions of something else, namely of certain macroscopic objects, called "measurement apparatus", therefore it could be related to phenomenological formalism [3].

Nonetheless, we should also note here that there are other approaches different from Born hypothesis, including:

- The square of the wavefunction represents a measure of the density of matter in region defined by dx (Determinism school [3, 4, 5]). Schrödinger apparently preferred this argument, albeit his attempt to demonstrate this idea has proven to be unfruitful;
- The square of wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6];
- The wavefunction in Schrödinger equation represents tendency to make structures;
- The wavemechanics can also be described in terms of topological Aharonov effect, which then it could be related to the notion of topological quantization [7, 8]. Aharonov himself apparently argues in favour of "realistic" meaning of Schrödinger wave equation, whose interpretation perhaps could also be related to Kron's work [9].

So forth we will discuss solution of this paradox.

2 Solution to Schrödinger's cat paradox

2.1 Standard interpretation

It is known that Quantum Mechanics could be regarded more as a "mathematical theory" rather than a physical theory [1, p. 2]. It is wave mechanics allowing a corpuscular duality. Already here one could find problematic difficulties: i. e. while the quantity of wavefunction itself could be computed, the physical meaning of wavefunction itself remains *indefinable* [1]. Furthermore, this notion of wavefunction corresponds to another fundamental indefinable in Euclidean geometry: the point [1, p. 2]. It is always a baffling question for decades, whether the electron could be regarded as wave, a point, or we should introduce a *non-zero* finite entity [4]. Attempts have been made to describe wave equation in such non-zero entity but the question of the physical meaning of wavefunction itself remains mystery.

The standard Copenhagen interpretation advertised by Bohr and colleagues (see DeBroglie, Einstein, Schrödinger who advocated "realistic" interpretation) asserts that it is practically *impossible* to know what really happens in quantum scale. The quantum measurement itself only represents reading in *measurement apparatus*, and therefore it is difficult to separate the object to be measured and the measurement apparatus itself. Bohr's phenomenological viewpoint perhaps could be regarded as pragmatic approach, starting with the request not to attribute a deep meaning to the wave function but immediately go over to statistical likelihood [10]. Consequently, how the process of "wave collapse" could happen remains mystery.

Heisenberg himself once emphasized this viewpoint when asked directly the question: Is there a fundamental level of reality? He replied as follows:

> "This is just the point: I do not know what the words fundamental reality mean. They are taken from our daily life situation where they have a good meaning, but when we use such terms we are usually extrapolating from our daily lives into an area very remote from it, where we cannot expect the words to have a meaning. This is perhaps one of the fundamental difficulties of philosophy: that our thinking hangs in the language. Anyway, we are forced to use the words so far as we can; we try to extend their use to the utmost, and then we get into situations in which they have no meaning" [11].

A modern version of this interpretation suggests that at the time of measurement, the wave collapses instantaneously into certain localized object corresponding to the action of measurement. In other words, the measurement processes define how the wave should define itself. At this point, the wave ceases to become coherent, and the process is known as "decoherence". Decoherence may be thought of as a way of making real for an observer in the large scale world only one possible history of the universe which has a likelihood that it will occur. Each possible history must in addition obey the laws of logic of this large-scale world. The existence of the phenomenon of decoherence is now supported by laboratory experiments [12]. It is worthnoting here, that there are also other versions of decoherence hypothesis, for instance by Tegmark [13] and Vitiello [14].

In the meantime, the "standard" Copenhagen interpretation emphasizes the role of observer where the "decoherence viewpoint" may not. The problem becomes more adverse because the axioms of standard statistical theory themselves are not fixed forever [15, 16]. And here is perhaps the source of numerous debates concerning the interpretation and philosophical questions implied by Quantum Mechanics. From this viewpoint, Neutrosophic Logic offers a new viewpoint to problems where indeterminacy exists. We will discuss this subsequently. For a sense of balance, we also discuss a number of alternative interpretations. Nonetheless this article will not discuss all existing interpretations of the quantum wavefunction in the literature.

2.2 Schrödinger's cat paradox

To make the viewpoint on this paradox a bit clearer, let us reformulate the paradox in its original form.

According to Uncertainty Principle, any measurement of a system must disturb the system under investigation, with a resulting lack of precision in the measurement. Soon after reading Einstein-Podolsky-Rosen's paper discussing incompleteness of Quantum Mechanics, Schrödinger in 1935 came up with a series of papers in which he used the "cat paradox" to give an illustration of the problem of viewing these particles in a "thought experiment" [15, 17]:

> "One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The wave-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared into equal parts."

In principle, Schrödinger's thought experiment asks whether the cat is dead or alive after an hour. The most logical solution would be to wait an hour, open the box, and see if the cat is still alive. However once you open the box to determine the state of the cat you have viewed and hence disturbed the system and introduced a level of uncertainty into the results. The answer, in quantum mechanical terms, is that before you open the box the cat is in a state of being half-dead and half-alive.

Of course, at this point one could ask whether it is possible to find out the state of the cat without having to disturb its wavefunction via action of "observation".

If the meaning of word "observation" here is defined by *to open the box and see the cat*, and then it seems that we could argue whether it is possible to propose another equally possible experiment where we introduce a pair of twin cats, instead of only one. A cat is put in the box while another cat is located in a separate distance, let say 1 meter from the box. If the state of the cat inside the box altered because of poison reaction, it is likely that we could also observe its effect to its twin, perhaps something like "sixth sense" test (perhaps via monitoring frequency of the twin cat's brain).

This plausible experiment could be viewed as an alternative "thought experiment" of well-known Bell-Aspect-type experiment. One could also consider an entangled pair of photons instead of twin cats to conduct this "modified" cat paradox. Of course, for this case then one would get a bit complicated problem because now he/she should consider two probable state: the decaying atom and the photon pair.

We could also say that using this alternative configuration, we know exact information about the Cat outside, while indeterminate information about the Cat inside. However, because both Cats are entangled (twin) we are sure of all the properties of the Cat inside "knows" the state of the Cat outside the box, via a kind of "spooky action at distance" reason (in Einstein's own word)*.

Therefore, for experimental purpose, perhaps it would be useful to simplify the problem by using "modified" Aspect-type experiment [16]. Here it is proposed to consider a decaying atom of Cesium which emits two correlated photons, whose polarization is then measured by Alice (A) on the left and by Bob (B) on the right (see Fig. 1). To include the probable state as in the original cat paradox, we will use a switch instead of Alice A. If a photon comes to this switch, then it will turn on a coffee-maker machine, therefore the observer will get a cup of coffee†. Another switch and coffee-maker set also replace Bob position (see Fig. 2). Then we encapsulate the whole system of decaying atom, switch, and coffee-maker at A, while keeping the system at B side open. Now we can be sure, that by the time the decaying atom of Cesium emits photon to B side and triggers the switch at this side which then turns on the coffee-maker, it is "likely" that we could also observe the same cup of coffee at A side, even if we do not open the box.

We use term "likely" here because now we encounter a "quasi-deterministic" state where there is also small chance that the photon is shifted different from -0.0116, which is indeed what the Aspect, Dalibard and Roger experiment demonstrated in 1982 using a system of two correlated photons [16]. At this "shifted" phase, it could be that the switch will not turn on the coffee-maker at all, so when an observer opens the box at A side he will not get a cup of coffee.

If this hypothetical experiment could be verified in real world, then it would result in some wonderful implications, like prediction of ensembles of multi-particles system, — or a colony of cats.

Another version of this cat paradox is known as GHZ paradox: "The Greenberger-Horne-Zeilinger paradox exhibits some of the most surprising aspects of multiparticle entanglement" [18]. But we limit our discussion here on the original cat paradox.

2.3 Hidden-variable hypothesis

It would be incomplete to discuss quantum paradoxes, in particular Schrödinger's cat paradox, without mentioning hidden-variable hypothesis. There are various versions of this argument, but it could be summarised as an assertion that there is "something else" which should be included in the Quantum Mechanical equations in order to explain thoroughly all quantum phenomena. Sometimes this assertion can be formulated in question form [19]: Can Quantum Mechanics be considered complete? Interestingly, however, the meaning of "complete" itself remains quite abstract (fuzzy).

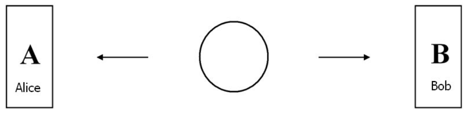

Figure 1: Aspect-type experiment

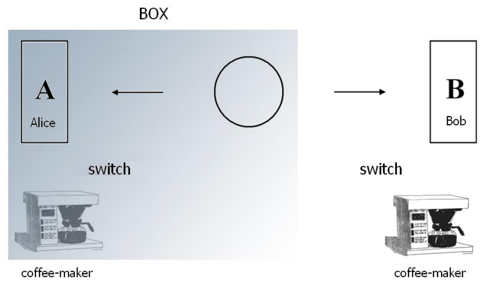

Figure 2: Aspect-type experiment in box

An interpretation of this cat paradox suggests that the problem arises because we mix up the macroscopic systems (observer's wavefunction and apparatus' wavefunction) from microscopic system to be observed. In order to clarify this, it is proposed that "…the measurement apparatus should be described by a classical model in our approach, and the physical system eventually by a quantum model" [20].

2.4 Hydrodynamic viewpoint and diffusion interpretation

In attempt to clarify the meaning of wave collapse and decoherence phenomenon, one could consider the process from (dissipative) hydrodynamic viewpoint [21]. Historically, the hydrodynamic/diffusion viewpoint of Quantum Mechanics has been considered by some physicists since the early years of wave mechanics. Already in 1933, Fuerth showed that Schrödinger equation could be written as a diffusion equation with an imaginary diffusion coefficient [1]

$$D_{qm} = \frac{i\hbar}{2m}\,. \tag{4}$$

But the notion of imaginary diffusion is quite difficult to comprehend. Alternatively, one could consider a classical Markov process of diffusion type to consider wave mechanics equation. Consider a continuity equation

$$\frac{\partial \rho}{\partial t} = -\nabla\left(\rho v\right), \tag{5}$$

where $v = v_0 = D\nabla\ln\rho$ (see [1]), which is a Fokker-Planck equation. Then the expectation value for the energy of particle can be written as [1]

*The authors are grateful to Dmitri Rabounski for his valuable comments discussing a case of entangled twin Cats.

†The "coffee-maker" analogue came to mind after a quote: "A mathematician is a device for turning coffee into theorems" — Alfréd Rényi, a Hungarian mathematician, 1921–1970. (As quoted by Christopher J. Mark.)

$$<E> = \int \left(\frac{mv^2}{2} + \frac{D^2 m}{2} D \ln \rho^2 + eV \right) \rho d^3 x \,. \quad (6)$$

Alternatively, it could be shown that there is exact mapping between Schrödinger equation and viscous dissipative Navier-Stokes equations [6], where the square of the wave-function of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6]. This Navier-Stokes interpretation differs appreciably from more standard Euler-Madelung fluid interpretation of Schrödinger equation [1], because in Euler method the fluid is described only in its inviscid limit.

2.5 How neutrosophy could offer solution to Schrödinger's paradox

In this regard, Neutrosophic Logic as recently discussed by one of these authors [22, 23, 24] could offer an interesting application in the context of Schrödinger's cat paradox. It could explain how the "mixed" state could be. It could be shown, that Neutrosophic probability is useful to those events, which involve some degree of indeterminacy (unknown) and more criteria of evaluation — as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events [25]. This new viewpoint for quantum phenomena is required because it is known that Quantum Mechanics is governed by uncertainty, but the meaning of "uncertainty" itself remains uncertain [16].

For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a problem of "alternative worlds theory well represented by the neutrosophic set theory.

In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function ψ, which describes the superposition of possible states, may be simulated by a neutrosophic function, i. e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Now let's return to our cat paradox [25]. Let's consider a Neutrosophic set of a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^{-}0$ and 1^{+}. For example: x (0.5, 0.2, 0.3) belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is undecidable); or y (0, 0, 1) belongs to A (which normally means y is not for sure in A); or z (0, 1, 0) belongs to A (which means one does know absolutely nothing about z's affiliation with A). More general, x $\{$(0.2–0.3), (0.40–0.45)$\cup$[0.50–0.51], (0.2, 0.24, 0.28)$\}$ belongs to the set A, which mean:

- Owning a likelihood in between 20–30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- Owning a probability of 20% or 24% or 28% x is not in A;
- The indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included);
- The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $n\_\sup = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [25], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i. e. we don't know if it is dead or alive), then herein the Neutrosophic Logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic Logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We could expect that someday this proposition based on Neusotrophic Logic could be transformed into a useful guide for experimental verification of quantum paradox [15, 10].

Above results will be expanded into details in our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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Schrödinger Equation and the Quantization of Celestial Systems

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In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems. While this hypothesis has been described by some authors, including Nottale, here we argue that such a macroquantization was formed by topological superfluid vortice. We also provide derivation of Schrödinger equation from Gross-Pitaevskii-Ginzburg equation, which supports this superfluid dynamics interpretation.

1 Introduction

In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems, based on logarithmic nature of Schrödinger equation, and also its exact mapping to Navier-Stokes equations [1].

While this notion of macro-quantization is not widely accepted yet, as we will see the logarithmic nature of Schrödinger equation could be viewed as a support of its applicability to larger systems. After all, the use of Schrödinger equation has proved itself to help in finding new objects known as extrasolar planets [2, 3]. And we could be sure that new extrasolar planets are to be found in the near future. As an alternative, we will also discuss an outline for how to derive Schrödinger equation from simplification of Ginzburg-Landau equation. It is known that Ginzburg-Landau equation exhibits fractal character, which implies that quantization could happen at *any scale*, supporting topological interpretation of quantized vortices [4].

First, let us rewrite Schrödinger equation in its common form [5]

$$\left[i\,\frac{\partial}{\partial t} + \frac{\bar{\nabla}^2}{2m} - U(x) \right] \psi = 0 \tag{1}$$

or

$$i\,\frac{\partial \psi}{\partial t} = H\,\psi\,. \tag{2}$$

Now, it is worth noting here that Englman and Yahalom [5] argues that this equation exhibits logarithmic character

$$\ln \psi(x,t) = \ln\big(|\psi(x,t)|\big) + i \arg\big(\psi(x,t)\big)\,. \tag{3}$$

Schrödinger already knew this expression in 1926, which then he used it to propose his equation called “eigentliche Wellengleichung” [5]. Therefore equation (1) can be rewritten as follows

$$2m\,\frac{\partial\big(\ln|\psi|\big)}{\partial t} + 2\bar{\nabla}\ln|\psi|\,\bar{\nabla}\arg\big[\psi\big] + \bar{\nabla}\bar{\nabla}\arg\big[\psi\big] = 0\,. \tag{4}$$

Interestingly, Nottale’s scale-relativistic method [2, 3] was also based on generalization of Schrödinger equation to describe quantization of celestial systems. It is known that Nottale-Schumacher’s method [6] could predict new exoplanets in good agreement with observed data. Nottale’s scale-relativistic method is essentially based on the use of first-order scale-differentiation method defined as follows [2]

$$\frac{\partial V}{\partial(\ln \delta t)} = \beta(V) = a + b\,V + \dots\,. \tag{5}$$

Now it seems clear that the natural-logarithmic derivation, which is essential in Nottale’s scale-relativity approach, also has been described properly in Schrödinger’s original equation [5]. In other words, its logarithmic form ensures applicability of Schrödinger equation to describe macro-quantization of celestial systems. [7, 8]

2 Quantization of celestial systems and topological quantized vortices

In order to emphasize this assertion of the possibility to describe quantization of celestial systems, let us quote Fischer’s description [4] of relativistic momentum from superfluid dynamics. Fischer [4] argues that the circulation is in the relativistic dense superfluid, defined as the integral of the momentum

$$\gamma_s = \oint p_\mu\, dx^\mu = 2\pi N_v\,\hbar\,, \tag{6}$$

and is quantized into multiples of Planck’s quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of γ_s. And then Fischer [4] concludes that the Maxwell equations of ordinary electromagnetism can be written in the form of conservation equations of relativistic perfect fluid hydrodynamics [9]. Furthermore, the topological character of equation (6) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [25]. For the plausible linkage between superfluid dynamics and cosmological phenomena, see [16–24].

It is worth noting here, because vortices could be defined as elementary objects in the form of stable topological excitations [4], then equation (6) could be interpreted as Bohr-Sommerfeld-type quantization from *topological quantized vortices*. Fischer [4] also remarks that equation (6) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization for celestial systems is known in literature [7, 8], which here in the context of Fischer's arguments it has special meaning, i. e. it suggests that *quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale* [4]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [16–24].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider the problem of quantization of celestial orbits in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [26, 27]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity [28]; it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s\,dl = 2\pi n\hbar/m$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [28]

$$\oint_\Gamma p\,dx = 2\pi n\,\hbar \tag{6a}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [28]

$$\int_0^T v^2 d\tau = \omega^2 T = 2\pi\omega\,, \tag{7}$$

where $T = 2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion [28]

$$\frac{GMm}{r^2} = \frac{mv^2}{r}\,. \tag{8}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum, a new constant g was introduced [28]

$$mvr = \frac{ng}{2\pi}\,. \tag{9}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [28]

$$r = \frac{n^2 g^2}{4\pi^2 GMm^2}\,, \tag{10}$$

which can be rewritten in the known form of gravitational Bohr-type radius [2, 7, 8]

$$r = \frac{n^2 GM}{v_0^2}\,, \tag{11}$$

where r, n, G, M, v_0 represents orbit radii, quantum number ($n = 1, 2, 3, \ldots$), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (11), we denote [28]

$$v_0 = \frac{2\pi}{g}\,GMm\,. \tag{12}$$

The value of m is an adjustable parameter (similar to g) [7, 8]. In accordance with Nottale, we assert that the specific velocity v_0 is 144 km/sec for planetary systems. By noting that m is meant to be mass of celestial body in question, then we could find g parameter (see also [28] and references cited therein).

Using this equation (11), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and use M in terms of reduced mass $\mu = \frac{(M_1 + M_2)}{M_1 M_2}$. From this viewpoint the result is shown in Table 1 below [28].

For comparison purpose, we also include some recent observation by Brown-Trujillo team from Caltech [29–32]. It is known that Brown et al. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52 AU), 2005FY9 (at 52 AU), 2003VB12 (at 76 AU, dubbed as Sedna). And recently Brown-Trujillo team reported a new planetoid finding, called 2003UB31 (97 AU). This is not to include their previous finding, Quaoar (42 AU), which has orbit distance more or less near Pluto (39.5 AU), therefore this object is excluded from our discussion. It is interesting to remark here that all of those new "planetoids" are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 1). While this prediction is not so precise compared to the observed data, one could argue that the 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until $n = 9$ of Jovian planets (outer solar system), it seems that there are sufficient reasons to suppose that more planetoids in the Oort Cloud will be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we include in Table 1 new expected orbits based

Object	No.	Titius	Nottale	CSV	Observ.	Δ, %
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	−6.95
Mars	6	16	15.4	15.4	15.24	−1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	−10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	−7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	−3.72
2003EL61	7			557.7	520	−7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit) [28].

on the same quantization procedure we outlined before. For Jovian planets corresponding to quantum number $n = 10$ and $n = 11$, our method suggests that it is likely to find new orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new planetoids around these predicted orbits.

As an interesting alternative method supporting this proposition of quantization from superfluid-quantized vortices (6), it is worth noting here that Kiehn has argued in favor of re-interpreting the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [1]. From this viewpoint, Kiehn suggests that there is *exact mapping* from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [1]. Interestingly, de Andrade and Sivaram [33] also suggest that there exists formal analogy between Schrödinger equation and the Navier-Stokes viscous dissipation equation:

$$\frac{\partial V}{\partial t} = \nu \, \nabla^2 V \,, \tag{13}$$

where ν is the kinematic viscosity. Their argument was based on propagation torsion model for quantized vortices [23]. While Kiehn's argument was intended for ordinary fluid, nonetheless the neat linkage between Navier-Stokes equation and superfluid turbulence is known in literature [34, 24].

At this point, it seems worth noting that some criticism arises concerning the use of quantization method for describing the motion of celestial systems. These criticism proponents usually argue that quantization method (wave mechanics) is oversimplifying the problem, and therefore cannot explain other phenomena, for instance planetary migration etc. While we recognize that there are phenomena which do not correspond to quantum mechanical process, at least we can argue further as follows:

1. Using quantization method like Nottale-Schumacher did, one can expect to predict new exoplanets (extra-solar planets) with remarkable result [2, 3];
2. The "conventional" theories explaining planetary migration normally use fluid theory involving *diffusion* process;
3. Alternatively, it has been shown by Gibson et al. [35] that these migration phenomena could be described via Navier-Stokes approach;
4. As we have shown above, Kiehn's argument was based on *exact-mapping* between Schrödinger equation and Navier-Stokes equations [1];
5. Based on Kiehn's vorticity interpretation one these authors published prediction of some new planets in 2004 [28]; which seems to be in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt;
6. To conclude: while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction;
7. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8 AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud);
8. There are of course other theories which have been developed to explain planetoids and exoplanets [36]. Therefore quantization method could be seen as merely a "plausible" theory between others.

All in all, what we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But at least it can be used to predict something quantitatively, i. e. measurable (exoplanets, and new planetoids in the outer solar system etc.).

In the meantime, it seems also interesting here to consider a plausible generalization of Schrödinger equation in particular in the context of viscous dissipation method [1]. First, we could write Schrödinger equation for a charged particle interacting with an external electromagnetic field [1] in the form of Ulrych's unified wave equation [14]

$$\left[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu\psi\right] = \\ = \left[-i2m\,\frac{\partial}{\partial t} + 2mU(x)\right]\psi\,. \tag{14}$$

In the presence of electromagnetic potential, one could include another term into the LHS of equation (14)

$$\left[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu + eA_0\right]\psi = \\ = 2m\left[-i\,\frac{\partial}{\partial t} + U(x)\right]\psi\,. \tag{15}$$

This equation has the physical meaning of Schrödinger equation for a charged particle interacting with an external electromagnetic field, which takes into consideration Aharonov effect [37]. Topological phase shift becomes its immediate implication, as already considered by Kiehn [1].

As described above, one could also derived equation (11) from scale-relativistic Schrödinger equation [2, 3]. It should be noted here, however, that Nottale's method [2, 3] differs appreciably from the viscous dissipative Navier-Stokes approach of Kiehn [1], because Nottale only considers his equation in the Euler-Newton limit [3]. Nonetheless, it shall be noted here that in his recent papers (2004 and up), Nottale has managed to show that his scale relativistic approach has linkage with Navier-Stokes equations.

3 Schrödinger equation derived from Ginzburg-Landau equation

Alternatively, in the context of the aforementioned superfluid dynamics interpretation [4], one could also derive Schrödinger equation from simplification of Ginzburg-Landau equation. This method will be discussed subsequently. It is known that Ginzburg-Landau equation can be used to explain various aspects of superfluid dynamics [16, 17]. For alternative approach to describe superfluid dynamics from Schrödinger-type equation, see [38, 39].

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m^* can be described as [40]

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \kappa\,|\psi|^2\,\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{16}$$

For some conditions, it is possible to replace the potential energy term in equation (16) with Hulthen potential. This substitution yields

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + V_{\text{Hulthen}}\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{17}$$

where

$$V_{\text{Hulthen}} = -Ze^2\,\frac{\delta\,e^{-\delta r}}{1 - e^{-\delta r}}\,. \tag{18}$$

This equation (18) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (18) approximates the effective Coulomb potential, in particular for large radius

$$V^{\text{eff}}_{\text{Coulomb}} = -\frac{e^2}{r} + \frac{\ell(\ell+1)\,\hbar^2}{2mr^2}\,. \tag{19}$$

By inserting (19), equation (17) could be rewritten as

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \left[-\frac{e^2}{r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2}\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{20}$$

For large radii, second term in the square bracket of LHS of equation (20) reduces to zero [41],

$$\frac{\ell(\ell+1)\,\hbar^2}{2mr^2} \to 0\,, \tag{21}$$

so we can write equation (20) as

$$\left[-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2 + U(x)\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{22}$$

where Coulomb potential can be written as

$$U(x) = -\frac{e^2}{r}\,. \tag{22a}$$

This equation (22) is nothing but Schrödinger equation (1), except for the mass term now we get mass of Cooper pairs. In other words, we conclude that it is possible to re-derive Schrödinger equation from simplification of (Gross-Pitaevskii) Ginzburg-Landau equation for superfluid dynamics [40], in the limit of small screening parameter, δ. Calculation shows that introducing this Hulthen effect (18) into equation (17) will yield essentially similar result to (1), in particular for small screening parameter. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (20) is *essentially* the same with the result derived from equation (1). Now, to derive gravitational Bohr-type radius equation (11) from Schrödinger equation, one could use Nottale's scale-relativistic method [2, 3].

4 Concluding remarks

What we would emphasize here is that this derivation of Schrödinger equation from (Gross-Pitaevskii) Ginzburg-Landau equation is in good agreement with our previous conjecture that equation (6) implies macroquantization corresponding to superfluid-quantized vortices. This conclusion is the main result of this paper. Furthermore, because Ginzburg-Landau equation represents superfluid dynamics at low-temperature [40], the fact that we can derive quantization of celestial systems from this equation seems to support the idea of Bose-Einstein condensate cosmology [42, 43]. Nonetheless, this hypothesis of Bose-Einstein condensate cosmology deserves discussion in another paper.

Above results are part of our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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Non-Euclidean Geometry and Gravitation

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A great deal of misunderstandings and mathematical errors are involved in the currently accepted theory of the gravitational field generated by an isotropic spherical mass. The purpose of the present paper is to provide a short account of the rigorous mathematical theory and exhibit a new formulation of the problem. The solution of the corresponding equations of gravitation points out several new and unusual features of the stationary gravitational field which are related to the non-Euclidean structure of the space. Moreover it precludes the black hole from being a mathematical and physical notion.

1 Introduction

If the structure of the spacetime is actually non-Euclidean as is postulated by general relativity, then several non-Euclidean features will manifest themselves in the neighbourhoods of the sources of the gravitational field. So, a spherical distribution of matter will appear as a non-Euclidean ball and the concentric with it spheres will possess the structure of non-Euclidean spheres. Specifically, if this distribution of matter is isotropic, such a sphere will be characterised completely by its radius, say ρ, and its curvature radius which is a function of ρ, say $g(\rho)$, defining the area $4\pi(g(\rho))^2$ of the sphere as well as the length of circumference $2\pi g(\rho)$ of the corresponding great circles. It is then expected that the function $g(\rho)$ will play a significant part in the conception of the metric tensor related to the gravitational field of the spherical mass. Of course, in formulating the problem, we must distinguish clearly the radius ρ, which is introduced as a given length, from the curvature radius $g(\rho)$, the determination of which depends on the equations of gravitation. However the classical approach to the problem suppresses this distinction and assumes that the radius af the sphere is the unknown function $g(\rho)$. This glaring mistake underlies the pseudo-theorem of Birkhoff as well as the classical solutions, which have distorted the theory of the gravitational field.

Another glaring mistake of the classical approach to the problem is related to the topological space which underlies the definition of the metric tensor. The spatial aspect of the problem suggests to identify the centre of the spherical mass with the origin of the vector space $\mathbb{R}^3$ which is moreover considered with the product topology of three real lines. Regarding the time t , several assumptions suggest to consider it (or rather ct) as a variable describing the real line $\mathbb{R}$. It follows that the topological space pertaining to the considered situation is the space $\mathbb{R}\times\mathbb{R}^3$ equipped with the product topology of four real lines. This simple and clear algebraic and topological situation has been altered from the beginnings of general relativity by the introduction of the so-called polar coordinates of $\mathbb{R}^3$ which destroy the topological structure of $\mathbb{R}^3$ and replace it by the manifold with boundary $[0,+\infty[\times S^2$. The use of polar coordinates is allowed in the theory of integration, because the open set $]0,+\infty[\times]0,2\pi[\times]0,\pi[$, described by the point (r,ϕ,θ), is transformed diffeomorphically onto the open set

$$\mathbb{R}^3-\{(x_1,x_2,x_3)\in\mathbb{R}^3;\ \ x_1\geqslant 0,\ \ x_2=0\}$$

and moreover the half-plane

$$\{(x_1,x_2,x_3)\in\mathbb{R}^3;\ \ x_1\geqslant 0,\ \ x_2=0\}$$

is of zero measure in $\mathbb{R}^3$. But in general relativity this half-plane cannot be omitted. Then by choosing two systems of geographic coordinates covering all of S^2, we define a C^∞ mapping of $[0,+\infty[\times S^2$ onto $\mathbb{R}^3$ transgressing the fundamental principle according to which only diffeomorphisms are allowed. In fact, this mapping is not even one-to-one: All of $\{0\}\times S^2$ is transformed into the origin of $\mathbb{R}^3$. This situation gives rise to inconsistent assertions. So, although the origin of $\mathbb{R}^3$ disappears in polar coordinates, the meaningless term "the origin $r=0$" is commonly used. Of course, the value $r=0$ does not define a point but the boundary $\{0\}\times S^2$ which is an abstract two-dimensional sphere without physical meaning. In accordance with the idea that the value $r=0$ defines the origin, the relativists introduce transformations of the form $r=h(\overline{r})$, $\overline{r}\geqslant 0$, in order to "change the origin". This extravagant idea goes back to Droste, who claims that by setting $r=\overline{r}+2\mu$, $\mu=\frac{km}{c^2}$, we define a "new radial coordinate $\overline{r}$" such that the sphere $r=2\mu$ reduces to the "new origin $\overline{r}=0$". Rosen [2] claims also that the transformation $r=\overline{r}+2\mu$ allows to consider a mass point placed at the origin $\overline{r}=0$! The same extravagant ideas are introduced in the definition of the so-called harmonic coordinates by Lanczos (1922) who begins by the introduction of the transformation $r=\overline{r}+\mu$ in order to define the "new radial coordinate $\overline{r}$".

The introduction of the manifold with boundary $[0,+\infty[\times S^2$ instead of $\mathbb{R}^3$, hence also the introduction of $\mathbb{R}\times[0,+\infty[\times S^2$ instead of $\mathbb{R}\times\mathbb{R}^3$, gives also rise to misunderstandings and mistakes regarding the space metrics and the spacetime metrics as well.

Given a C^∞ Riemannian metric on $\mathbb{R}^3$, its transform in polar coordinates is a C^∞ quadratic form on $[0,+\infty[\times S^2$, positive definite on $]0, +\infty[\times S^2$ and null on $\{0\}\times S^2$. (This is, in particular, true for the so-called metric of $\mathbb{R}^3$ in polar coordinates, namely $ds^2 = dr^2 + r^2 d\omega^2$ with $d\omega^2 = \sin^2\theta d\phi^2 + d\theta^2$ in the domain of validity of (ϕ, θ).) But the converse is not true. A C^∞ form on $[0, +\infty[\times S^2$ satisfying the above conditions is associated in general with a form on $\mathbb{R}^3$ presenting discontinuities at the origin of $\mathbb{R}^3$. So the C^∞ form $2dr^2 + r^2 d\omega^2$, conceived on $[0, +\infty[\times S^2$, results from a uniquely defined form on $\mathbb{R}^3$, namely

$$dx^2 + \frac{(xdx)^2}{\|x\|^2},$$

(here $dx^2 = dx_1^2 + dx_2^2 + dx_3^2$, $xdx = x_1dx_1 + x_2dx_2 + x_3dx_3$) which is discontinuous at $x = (0, 0, 0)$.

Now, given a C^∞ spacetime metric on $\mathbb{R}\times\mathbb{R}^3$, its transform in polar coordinates is a C^∞ form degenerating on the boundary $\mathbb{R}\times\{0\}\times S^2$. But the converse is not true. A C^∞ spacetime form on $\mathbb{R}\times[0, +\infty[\times S^2$ degenerating on the boundary $\mathbb{R}\times\{0\}\times S^2$ results in general from a spacetime form on $\mathbb{R}\times\mathbb{R}^3$ presenting discontinuities. For instance, the so-called Bondi metric

$$ds^2 = e^{2A}dt^2 + 2e^{A+B}dtdr - r^2 d\omega^2$$

where $A = A(t, r)$, $B = B(t, r)$, conceals singularities, because it results from a uniquely defined form on $\mathbb{R}\times\mathbb{R}^3$, namely

$$ds^2 = e^{2A}dt^2 + 2e^{A+B}\frac{(xdx)}{\|x\|}dt - dx^2 + \frac{(xdx)^2}{\|x\|}$$

which is discontinuous at $x = (0, 0, 0)$. It follows that the current practice of formulating problems with respect to $\mathbb{R}\times[0, +\infty[\times S^2$, instead of $\mathbb{R}\times\mathbb{R}^3$, gives rise to misleading conclusions. The problems must be always conceived with respect to $\mathbb{R}\times\mathbb{R}^3$.

2 SΘ(4)-invariant and Θ(4)-invariant tensor fields on $\mathbb{R}\times\mathbb{R}^3$.

The metric tensor is conceived naturally as a tensor field invariant by the action of the rotation group $SO(3)$. However, although $SO(3)$ acts naturally on $\mathbb{R}^3$, it does not the same on $\mathbb{R}\times\mathbb{R}^3$, and this is why we are led to introduce the group SΘ(4) consisting of the matrices

$$\begin{pmatrix} 1 & O_H \\ O_V & A \end{pmatrix}$$

with $O_H = (0, 0, 0)$, $O_V = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $A\in SO(3)$. We introduce also the group Θ(4) consisting of the matrices of the same form for which $A\in O(3)$. Obviously SΘ(4) is a subgroup of Θ(4).

With these notations, the metric tensor related to the isotropic distribution of matter is conceived as a SΘ(4)-invariant tensor field on $\mathbb{R}\times\mathbb{R}^3$. SΘ(4)-invariant tensor fields appear in several problems of relativity, so that it is convenient to study them in detail. Their rigorous theory appears in a previous paper [7] together with the theory of the *pure SΘ(4)-invariant tensor fields* which are not used in the present paper.

It is easily seen that a function $h(x_0, x_1, x_2, x_3)$ is SΘ(4)-invariant (or Θ(4)-invariant) if and only if it is of the form $f(x_0, \|x\|)$. Of course we confine ourselves to the case where $f(x_0, \|x\|)$ is C^∞ with respect to the coordinates x_0, x_1, x_2, x_3 on $\mathbb{R}\times\mathbb{R}^3$.

Proposition 2.1 *$f(x_0, \|x\|)$ is C^∞ on $\mathbb{R}\times\mathbb{R}^3$ if and only if the function $f(x_0, u)$ with $(x_0, u)\in\mathbb{R}\times[0, +\infty[$ is C^∞ on $\mathbb{R}\times[0, +\infty[$ and such that its derivatives of odd order with respect to u at $u = 0$ vanish.*

The functions satisfying these conditions constitute an algebra which will be denoted by Γ_0. As a corollary, we see that $f(x_0, \|x\|)$ belongs to Γ_0 if and only if the function $h(x_0, u)$ defined by setting

$$h(x_0, u) = h(x_0, -u) = f(x_0, u), \quad u\geqslant 0$$

is C^∞ on $\mathbb{R}\times\mathbb{R}$. It follows in particular that, if the function $f(x_0, \|x\|)$ belongs to Γ_0 and is strictly positive, then the functions $\frac{1}{f(x_0, \|x\|)}$ and $\sqrt{f(x_0, \|x\|)}$ belong also to Γ_0. Now, if $T(x_0, x)$, $x = (x_1, x_2, x_3)$, is an SΘ(4)-invariant (or Θ(4)-invariant) tensor field on $\mathbb{R}\times\mathbb{R}^3$, then, for every function $f\in\Gamma_0$, the tensor field $f(x_0, \|x\|)\, T(x_0, x)$ is also SΘ(4)-invariant (or Θ(4)-invariant). Consequently the set of SΘ(4)-invariant (or Θ(4)-invariant) tensor fields constitutes a Γ_0-module. In particular, we are interesting in the sub-module consisting of the covariant tensor fields of degree 2. The proof of the following proposition is given in the paper [7].

Proposition 2.2 *Let $T(x_0, x)$ be an SΘ(4)-invariant C^∞ covariant symmetric tensor field of degree 2 on $\mathbb{R}\times\mathbb{R}^3$. Then there exist four functions $q_{00}\in\Gamma_0$, $q_{01}\in\Gamma_0$, $q_{11}\in\Gamma_0$, $q_{22}\in\Gamma_0$ such that*

$$\begin{aligned} T(x_0, x) &= q_{00}(x_0, \|x\|)(dx_0\otimes dx_0) + \\ &+ q_{01}(x_0, \|x\|)\big(dx_0\otimes F(x) + F(x)\otimes dx_0\big) + \\ &+ q_{11}(x_0, \|x\|)\,E(x) + q_{22}(x_0, \|x\|)\big(F(x)\otimes F(x)\big), \end{aligned}$$

where $E(x) = \sum_1^3(dx_i\otimes dx_i)$ and $F(x) = \sum_1^3 x_i dx_i$. Moreover $T(x_0, x)$ is Θ(4)-invariant.

So, the components $g_{\alpha\beta}$ of $T(x_0, x)$ are defined by means of the four functions q_{00}, q_{01}, q_{11}, q_{22} as follows

$$g_{00} = q_{00}, \qquad g_{0i} = g_{i0} = x_i q_{01},$$
$$g_{ii} = q_{11} + x_i^2 q_{22}, \qquad g_{ij} = x_i x_j q_{22},$$

where $i, j = 1, 2, 3$; $i\neq j$. Suppose now that the tensor field $T(x_0, x)$ is a metric tensor, namely a symmetric tensor field of signature $(+1, -1, -1, -1)$. Then we write it usually as a quadratic form

$$ds^2 = q_{00}dx_0^2 + 2q_{01}(xdx)\,dx_0 + q_{11}dx^2 + q_{22}(xdx)^2.$$

Since $x_0 = t$ is the time coordinate, we have $q_{00} = q_{00}(x_0, \|x\|) > 0$ for all $(x_0, x) \in \mathbb{R}\times\mathbb{R}^3$, so the function $f = f(x_0, \|x\|) = \sqrt{q_{00}(x_0, \|x\|)}$ is strictly positive and C^∞ on $\mathbb{R}\times\mathbb{R}^3$. Consequently the function $f_1 = \frac{q_{01}}{f}$ is also C^∞ on $\mathbb{R}\times\mathbb{R}^3$, namely a function belonging to Γ_0, and we can write the metric into the form

$$ds^2 = \big(f dt + f_1 (x dx)\big)^2 + q_{11} dx^2 + \big(q_{22} - f_1^2\big)(x dx)^2$$

which makes explicit the corresponding spatial (positive definite) metric $-q_{11}dx^2 - \big(q_{22} - f_1^2\big)(x dx)^2$ with $-q_{11} > 0$ and $-q_{11} - \big(q_{22} - f_1^2\big)\|x\|^2 > 0$ on $\mathbb{R}\times\mathbb{R}^3$. So we can introduce the strictly positive C^∞ functions

$$\ell_1 = \ell_1(t, \|x\|) = \sqrt{-q_{11}(t, \|x\|)}$$

and

$$\ell = \ell(t, \|x\|) = \sqrt{\ell_1^2 - \|x\|^2 \big(q_{22} - f_1^2\big)}$$

which possess a clear geometrical meaning:

> ℓ_1 serves to define the curvature radius $g(t,\rho) = g(t, \|x\|) = \|x\|\,\ell_1(t, \|x\|) = \rho\,\ell_1(t,\rho)$, $(\rho = \|x\|)$, of the non-Euclidean spheres centered at the origin of $\mathbb{R}^3$, whereas ℓ defines the element of length on the spatial radial geodesics.

Consequently it is very convenient to put the metric into a form exhibiting explicitly ℓ_1 and ℓ. This is obtained by remarking that the C^∞ function $q_{22} - f_1^2$ can be written as

$$\frac{\ell_1^2 - \ell^2}{\rho^2}\,.$$

Of course the last expression is C^∞ everywhere on account of the condition $\ell_1(t,0) = \ell(t,0)$ and the fact that $\ell_1 \in \Gamma_0$, $\ell \in \Gamma_0$. It follows that

$$ds^2 = \big(f dt + f_1 (x dx)\big)^2 - \ell_1^2 dx^2 - \frac{\ell^2 - \ell_1^2}{\rho^2}(x dx)^2 \tag{2.1}$$

or

$$ds^2 = f^2 dt^2 + 2 f f_1 (x dx)\, dt - \ell_1^2 dx^2 + \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right)(x dx)^2 \tag{2.2}$$

with the components

$$g_{00} = f^2, \qquad g_{0i} = g_{i0} = x_i f f_1\,,$$

$$g_{ii} = -\ell_1^2 + x_i^2\left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right),$$

$$g_{ij} = x_i x_j\left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right), \qquad i,j = 1,2,3; \quad i \neq j\,.$$

There are two significant functions which do not appear in (2. 1) and are not C^∞ on $\mathbb{R}\times\mathbb{R}^3$:

1. First the already considered curvature radius $g(t,\rho) = \rho\,\ell_1(t,\rho)$ of the non-Euclidean spheres centered at the origin;
2. Secondly the function $h(t,\rho) = \rho f_1(t,\rho)$ which appears in the equations defining the radial motions of photons outside the matter, namely the equations

$$(f dt + f_1\,\rho\, d\rho)^2 = \ell^2 d\rho^2 \quad\text{or}\quad f dt + \rho f_1 d\rho = \pm \ell\, d\rho$$

which imply necessarily $|h| \leqslant \ell$ in order that both the ingoing and outgoing motions be possible [4]. In any case the condition $|h| \leqslant \ell$ must also be valid inside the matter in order that the nature of the variable t as time coordinate be preserved. Moreover h vanishes for $\rho = 0$.

Of course g and h are C^∞ with respect to $(t,\rho) \in \mathbb{R}\times[0, +\infty\,[$, but since $\rho = \|x\|$ is not differentiable at the origin, they are not differentiable on the subspace $\mathbb{R}\times\{(0,0,0)\}$ of $\mathbb{R}\times\mathbb{R}^3$. However, on account of their geometrical and physical significance, we introduce them in the computations remembering that, for any global solution on $\mathbb{R}\times\mathbb{R}^3$, the functions $\ell_1 = \frac{g}{\rho}$ and $f_1 = \frac{h}{\rho}$ appearing in (2.1) must be elements of the algebra Γ_0.

3 The Ricci tensor and the equations of gravitation

In order to obtain the equations of gravitation related to (2.1), we have first to introduce the Christoffel symbols and then compute the components of the Ricci tensor. At first sight the computations seem to be extremely complicated, but the $\Theta(4)$-invariance of the metric allows to obtain a great deal of simplification in accordance with the following proposition, the proof of which is given in the paper [8].

Proposition 3.1 **(a)** *The Christoffel symbols of the first kind as well as those of the second kind related to (2.2) are the components of a* $\Theta(4)$*-invariant tensor field*; **(b)** *The curvature tensor, the Ricci tensor, and the scalar curvature related to (2.2) are* $\Theta(4)$*-invariant*; **(c)** *If an energy-momentum tensor satisfies the equations of gravitation related to (2.2), it is* $\Theta(4)$*-invariant*.

Corollary 3.1. *The Christoffel symbols of the second kind related to (2.2) depend on ten* C^∞ *functions* $B_\alpha = B_\alpha(t,\rho)$, $(\alpha = 0, 1, 2, \ldots, 9)$, *as follows*:

$$\Gamma^0_{00} = B_0\,, \qquad \Gamma^0_{0i} = \Gamma^0_{i0} = B_1 x_i\,, \qquad \Gamma^i_{00} = B_2 x_i\,,$$
$$\Gamma^0_{ii} = B_3 + B_4 x_i^2, \qquad \Gamma^0_{ij} = \Gamma^0_{ji} = B_4 x_i x_j\,,$$
$$\Gamma^i_{i0} = \Gamma^i_{0i} = B_5 + B_6 x_i^2, \qquad \Gamma^i_{j0} = \Gamma^i_{0j} = B_6 x_i x_j\,,$$
$$\Gamma^i_{ii} = B_7 x_i^3 + (B_8 + 2B_9)\,x_i\,,$$
$$\Gamma^i_{jj} = B_7 x_i x_i^2 + B_8 x_i\,, \quad \Gamma^j_{ij} = \Gamma^j_{ji} = B_7 x_i x_j^2 + B_9 x_i\,,$$
$$\Gamma^i_{jk} = B_7 x_i x_j x_k\,, \qquad i,j,k = 1,2,3; \quad j \neq k \neq i\,.$$

Regarding the Ricci tensor $R_{\alpha\beta}$, since it is symmetric and $\Theta(4)$-invariant, its components are defined, according to proposition 2.2, by four functions $Q_{00} = Q_{00}(t,\rho)$, $Q_{01} = Q_{01}(t,\rho)$, $Q_{11} = Q_{11}(t,\rho)$, $Q_{22} = Q_{22}(t,\rho)$ as follows:

$$R_{00} = Q_{00}\,, \quad R_{0i} = R_{i0} = Q_{01} x_i\,, \quad R_{ii} = Q_{11} + x_i^2 Q_{22}\,,$$
$$R_{ij} = x_i x_j Q_{22}\,, \qquad i,j = 1,2,3; \quad i \neq j\,.$$

In the same way, an energy-momentum tensor $W_{\alpha\beta}$ satisfying the equations of gravitation related to (2.2) is defined by four functions of (t, ρ), say E_{00}, E_{01}, E_{11}, E_{22}:

$$W_{00} = E_{00}\,, \qquad W_{0i} = x_i E_{01}\,, \qquad W_{ii} = E_{11} + x_i^2 E_{22}\,,$$
$$W_{ij} = x_i x_j E_{22}\,, \qquad i,j = 1,2,3; \;\; i \neq j\,.$$

Moreover, since the scalar curvature $R = Q$ is $\Theta(4)$-invariant, it is a function of (t,ρ): $R = Q = Q(t,\rho)$.

It follows that the equations of gravitation (with cosmological constant -3λ)

$$R_{\alpha\beta} - \left(\frac{Q}{2} + 3\lambda\right) g_{\alpha\beta} + \frac{8\pi k}{c^4} W_{\alpha\beta} = 0$$

can be written from the outset as a system of four equations depending uniquely on (t,ρ):

$$Q_{00} - \left(\frac{Q}{2} + 3\lambda\right) f^2 + \frac{8\pi k}{c^4} E_{00} = 0\,,$$
$$Q_{01} - \left(\frac{Q}{2} + 3\lambda\right) f f_1 + \frac{8\pi k}{c^4} E_{01} = 0\,,$$
$$Q_{11} + \left(\frac{Q}{2} + 3\lambda\right) \ell_1^2 + \frac{8\pi k}{c^4} E_{11} = 0\,,$$
$$Q_{22} - \left(\frac{Q}{2} + 3\lambda\right) \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right) + \frac{8\pi k}{c^4} E_{22} = 0\,.$$

Note that it is often convenient to replace the last equation by the equation

$$Q_{11} + \rho^2 Q_{22} - \left(\frac{Q}{2} + 3\lambda\right)\left(\rho^2 f_1^2 - \ell^2\right) + \frac{8\pi k}{c^4}\left(E_{11} + \rho^2 E_{22}\right) = 0\,.$$

In order to apply these equations to special situations, it is necessary to give the explicit expressions of Q_{00}, Q_{01}, Q_{11}, Q_{22} by means of the functions $B_\alpha, (\alpha = 0,1,2,...,9)$, appearing in the Christoffel symbols. We recall the results of computation

$$Q_{00} = \frac{\partial}{\partial t}\left(3B_5 + \rho^2 B_6\right) - \rho\,\frac{\partial B_2}{\partial \rho} -$$
$$- B_2\left(3 + 4\rho^2 B_9 - \rho^2 B_1 + \rho^2 B_8 + \rho^2 B_7\right) -$$
$$- 3B_0 B_5 + 3B_5^2 + \rho^2 B_6\left(-B_0 + 2B_5 + \rho^2 B_6\right),$$

$$Q_{01} = \frac{\partial}{\partial t}\left(\rho^2 B_7 + B_8 + 4B_9\right) - \frac{1}{\rho}\frac{\partial B_5}{\partial \rho} - \rho\,\frac{\partial B_6}{\partial \rho} +$$
$$+ B_2\left(B_3 + \rho^2 B_4\right) - 2B_6\left(2 + \rho^2 B_9\right) - B_1\left(3B_5 + \rho^2 B_6\right),$$

$$Q_{11} = -\frac{\partial B_3}{\partial t} - \rho\frac{\partial B_8}{\partial \rho} - \left(B_0 + B_5 + \rho^2 B_6\right) B_3 +$$
$$+ \left(1 - \rho^2 B_8\right)\left(B_1 + \rho^2 B_7 + B_8 + 2B_9\right) - 3B_8\,,$$

$$Q_{22} = -\frac{\partial B_4}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(B_1 + B_8 + 2B_9\right) + B_1^2 + B_8^2 -$$
$$- 2B_9^2 - 2B_1 B_9 + 2B_3 B_6 + \left(-B_0 - B_5 + \rho^2 B_6\right) B_4 +$$
$$+ \left(-3 + \rho^2\left(-B_1 + B_8 - 2B_9\right)\right) B_7\,.$$

4 Stationary vacuum solutions

The radial motion of the isotropic spherical distribution of matter generates a non-stationary (dynamical) gravitational field extending beyond the boundary in the exterior space. This field is defined by non-stationary $\Theta(4)$-invariant vacuum solutions of the equations of gravitation and exhibits essential and unusual features related to the propagation of gravitation. Several problems related to it are not yet clarified. But, in any case, in order to establish and understand the dynamical solutions, a previous knowledge of the stationary solutions is necessary. This is why, in the sequel we confine ourselves to the simple problems related to the stationary vacuum solutions. So we suppose that we have a stationary metric

$$ds^2 = \left(f dt + f_1\,(x dx)\right)^2 - \ell_1^2 dx^2 - \frac{\ell^2 - \ell_1^2}{\rho^2}\,(x dx)^2\,, \quad (4.1)$$

where $f = f(\rho)$, $f_1 = f_1(\rho)$, $\ell_1 = \ell_1(\rho)$, $\ell = \ell(\rho)$.

Of course, we have also to take into account the significant functions

$$h = h(\rho) = \rho f_1(\rho)\,, \qquad g = g(\rho) = \rho\,\ell_1(\rho)\,,$$

which are not differentiable at the origin $(0,0,0)$. Every half-line issuing from the origin, $x_1 = \alpha_1\rho$, $x_2 = \alpha_2\rho$, $x_3 = \alpha_3\rho$ (where $0 \leqslant \rho < +\infty$ and $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$) is a geodesic of the spatial metric $\ell_1^2 dx^2 + \frac{\ell^2 - \ell_1^2}{\rho^2}\,(x dx)^2$ so that the geodesic distance δ of the origin from the point $x = (x_1, x_2, x_3)$ is defined by the integral

$$\delta = \int_0^\rho \ell(u) du\,, \qquad \rho = \|x\|.$$

As already noticed, the function $\ell(\rho)$, where $0 \leqslant \rho < +\infty$, is strictly positive, but it cannot be arbitrarily given. Suppose, for instance, that

$$\ell(\rho) = \frac{\epsilon}{\rho^2}\,, \qquad \epsilon = \text{const} > 0$$

on $[1, +\infty[$. Then the geodesic distance $\delta = \int_0^1 \ell(u) du + \int_1^\rho \frac{\epsilon}{u^2} du = \int_0^1 \ell(u) du + \epsilon - \frac{\epsilon}{\rho}$ tends to the finite value $\int_0^1 \ell(u) du + \epsilon$ as $\rho \to \infty$, which cannot be physically accepted. Consequently the positive function $\ell(\rho)$ is allowable only if the integral $\int_0^\rho \ell(u) du$ tends to $+\infty$ as $\rho \to +\infty$.

This being said, it is easy to see that the functions $B_\alpha = B_\alpha(\rho)$, $(\alpha = 0, 1, \ldots, 9)$, occurring in the Christoffel symbols resulting from the stationary metric (4.1) are defined by the following formulae:

$$B_0 = -\frac{h f'}{\ell^2}\,, \qquad B_1 = \frac{f'}{\rho f} - \frac{h^2 f'}{\rho f \ell^2}\,,$$
$$B_2 = \frac{f f'}{\rho\,\ell^2}\,, \qquad B_3 = \frac{h g g'}{\rho^2 f \ell^2}\,,$$
$$B_4 = \frac{h f'}{\rho^2 f^2} - \frac{h^3 f'}{\rho^2 f^2 \ell^2} + \frac{h'}{\rho^2 f} - \frac{h \ell'}{\rho^2 f \ell} - \frac{h g g'}{\rho^4 f \ell^2}\,,$$

$$B_5 = 0\,, \qquad B_6 = \frac{hf'}{\rho^2\ell^2}\,,$$
$$B_7 = \frac{h^2 f'}{\rho^3 f\ell^2} + \frac{\ell'}{\rho^3\ell} + \frac{gg'}{\rho^5\ell^2} - \frac{2g'}{\rho^3 g} + \frac{1}{\rho^4}\,,$$
$$B_8 = \frac{1}{\rho^2} - \frac{gg'}{\rho^3\ell^2}\,, \qquad B_9 = -\frac{1}{\rho^2} + \frac{g'}{\rho g}\,.$$

Then inserting these expressions in the formulae brought out at the end of the previous section, we find the functions

$$Q_{00} = f\left(-\frac{f''}{\ell^2} + \frac{f'\ell'}{\ell^3} - \frac{2f'g'}{\ell^2 g}\right), \qquad g = \rho\ell_1\,,$$
$$Q_{01} = \frac{h}{\rho f}Q_{00}\,, \qquad h = \rho f_1\,,$$
$$Q_{11} = \frac{1}{\rho^2}\left(-1 + \frac{g'^2}{\ell^2} + \frac{gg''}{\ell^2} - \frac{\ell' gg'}{\ell^3} + \frac{f'gg'}{f\ell^2}\right),$$
$$Q_{11} + \rho^2 Q_{22} = \frac{f''}{f} + \frac{2g''}{g} - \frac{f'\ell'}{f\ell} - \frac{2\ell' g'}{\ell g} + \frac{h^2}{f^2}Q_{00}\,,$$

which are everywhere valid, namely outside as well as inside the matter, Specifically, by using them, we can establish the gravitational equations outside the matter with electromagnetic field and cosmological constant. However, in the present short account, our purpose is to put forward the most significant elementary facts, and this is why we confine ourselves to the pure gravitational field outside the matter without cosmological constant. Then $Q = R = 0$, $\lambda = 0$, so that $Q_{00} = 0$, $Q_{01} = 0$, $Q_{11} = 0$, $Q_{11} + \rho^2 Q_{22} = 0$. Since $Q_{00} = 0$ implies $Q_{01} = 0$, we have finally the following three equations

$$-f'' + \frac{f'\ell'}{\ell} - \frac{2f'g'}{g} = 0\,, \tag{4.2}$$
$$-1 + \frac{g'^2}{\ell^2} + \frac{gg''}{\ell^2} - \frac{\ell' gg'}{\ell^3} + \frac{f'gg'}{f\ell^2} = 0\,, \tag{4.3}$$
$$f'' + \frac{2fg''}{g} - \frac{f'\ell'}{\ell} - \frac{2f\ell' g'}{\ell g} = 0\,, \tag{4.4}$$

By adding (4.2) to (4.4) we obtain

$$\frac{f'g'}{f} = g'' - \frac{\ell' g'}{\ell} \tag{4.5}$$

and inserting this expression of $\frac{f'g'}{f}$ into (4.3), we find the equation

$$-1 + \frac{g'^2}{\ell^2} + \frac{2gg''}{\ell^2} - \frac{2\ell' gg'}{\ell^3} = 0$$

which implies $\frac{d}{d\rho}\big(-g + \frac{gg'^2}{\ell^2}\big) = 0$ so that

$$-g + \frac{gg'^2}{\ell^2} = -2A = \text{const.} \tag{4.6}$$

On the other hand (4.5) can be written as $(f\ell)' g' = (f\ell)\, g''$ whence $\frac{d}{d\rho}\big(\frac{g'}{f\ell}\big) = 0$ and

$$f\ell = cg', \quad (\text{where } c = \text{const})\,. \tag{4.7}$$

The equations (4.6) and (4.7) define the general stationary solution outside the matter. The function h does not appear in them, but it is not empty of physical meaning as is usually believed. It occurs in the problem as a function satisfying the condition $|h| \leqslant \ell$. The different allowable choices of h correspond to different significations of the time coordinate.

Proposition 4.1. *If $A = 0$, the solution defined by (4.6) and (4.7) is a pseudo-Euclidean metric (or, better, a family of pseudo-Euclidean metrics).*

Proof. On account of $A = 0$, (4.6) implies $g' = \ell$ and next (4.7) gives $f = c$. Referring to (4.1) and setting $\int_0^\rho v f(v)dv = \alpha(\rho)$, we have

$$d\alpha(\rho) = \rho f_1(\rho)d\rho = f_1(\rho)x dx$$

and

$$f(\rho)dt + f_1(\rho)x dx = d\big(ct + \alpha(\rho)\big)\,,$$

which suggests the transformation $\tau = t + \frac{\alpha(\rho)}{c}$. On the other hand, since $\ell = g' = (\rho\ell_1)' = \rho\ell_1' + \ell_1$, we have

$$\ell_1^2 dx^2 + \frac{\ell^2 - \ell_1^2}{\rho^2}(x dx)^2 = \ell_1^2 dx^2 + 2\ell_1\ell_1'\frac{(x dx)^2}{\|x\|} + \ell'^2_1 (x dx)^2 =$$
$$= \left(\ell_1 dx_1 + x_1\ell_1'\frac{x dx}{\rho}\right)^2 + \left(\ell_1 dx_2 + x_2\ell_1'\frac{x dx}{\rho}\right)^2 +$$
$$+ \left(\ell_1 dx_3 + x_3\ell_1'\frac{x dx}{\rho}\right)^2 = \big(d(\ell_1 x_1)\big)^2 + \big(d(\ell_1 x_2)\big)^2 + \big(d(\ell_1 x_3)\big)^2$$

so that by setting $y_1 = \ell_1 x_1$, $y_2 = \ell_1 x_2$, $y_3 = \ell_1 x_3$, we obtain the metric in the standard pseudo-Euclidean form $ds^2 = c^2 d\tau^2 - (dy_1^2 + dy_2^2 + dy_3^2)$. In the sequel we give up this trivial case and assume $A \neq 0$.

5 Punctual sources of the gravitational field do not exist

(4.6) is a first order differential equation with respect to the unknown function $g = g(\rho)$, so that its general solution depends on an arbitrary constant. But (4.6) contains already the constant A and moreover the function $\ell = \ell(\rho)$ which is not given. Consequently the general solution of (4.6) contains two constants. Moreover, it seems that it depends on the function $\ell(\rho)$, namely that to every allowable function $\ell(\rho)$ there corresponds a solution of (4.6) depending on two constants. However, we can prove that the function $\ell(\rho)$ is not actually involved in the general solution of (4.6).

Since the geodesic distance $\delta = \int_0^\rho \ell(u)du = \beta(\rho)$ is a strictly increasing function of ρ tending to $+\infty$ as $\rho \to +\infty$, the inverse function $\rho = \gamma(\delta)$ is also a strictly increasing function of δ tending to $+\infty$ as $\delta \to +\infty$. Consequently $g(\rho)$ can be considered as a function of δ:

$$G(\delta) = g\big(\gamma(\delta)\big).$$

It follows that the determination of $G(\delta)$ as a function of the geodesic distance δ, which possesses an intrinsic meaning with respect to the stationary metric, allows its definition with respect to any other radial coordinate depending diffeomorphically on δ.

Now, since $\delta = \beta\left(\gamma(\delta)\right)$, we have $1 = \frac{d\beta}{d\rho}\frac{d\rho}{d\delta} = \ell(\rho)\gamma'(\delta)$ and $G' = G'(\delta) = g'(\rho)\gamma'(\delta) = \frac{g'(\rho)}{\ell(\rho)}$, so that the equation (4.6) takes the form $-G + GG'^2 = -2A$ or

$$GG'^2 = G - 2A \tag{5.1}$$

which does not contain the function ℓ.

Regarding (4.7), it is obviously replaced by the equation

$$F = cG'$$

with $F = F(\delta) = f\left(\gamma(\delta)\right)$. The functions F and G are related to a stationary metric which results from the stationary metric (4.1) by the introduction of the new space coordinates:

$$y_i = \frac{\delta}{\rho}x_i = \frac{\beta(\rho)}{\rho}x_i\,, \tag{5.2}$$

where $i = 1, 2, 3$; $\|y\| = \delta$; $\|x\| = \rho$. This transformation is C^∞ everywhere, even at the origin, because the function $B(\rho) = \frac{\beta(\rho)}{\rho}$ (where $B(0) = \ell(0)$) belongs to the algebra Γ_0. In fact, since $\beta'(\rho) = \ell(\rho)$, we have $\beta(\rho) = \rho\int_0^1 \beta'(\rho u)du = \rho\int_0^1 \ell(\rho u)du$ and

$$B(\rho) = \int_0^1 \ell(\rho u)du\,,$$

consequently $B^{(2m+1)}(\rho) = \int_0^1 \ell^{(2m+1)}(\rho u)\,u^{2m+1}du$ and since $\ell \in \Gamma_0$ implies $\ell^{(2m+1)}(0) = 0$, we obtain

$$B^{(2m+1)}(0) = 0\,, \qquad (m = 0, 1, 2, 3, \dots)$$

and, from proposition 2.1, it follows that $B \in \Gamma_0$.

The inverse of (5.2) is defined by the equations

$$x_i = \Delta(\delta)\,y_i\,, \qquad i = 1, 2, 3\,, \tag{5.3}$$

where $\Delta(\delta) = \frac{\rho}{\beta(\rho)} = \frac{\gamma(\delta)}{\delta}$. Since $\gamma(\delta) = \delta\int_0^1 \gamma'(\delta u)du = \delta\int_0^1 \frac{du}{\ell(\gamma(\delta u))}$, it can be shown by induction that the function $\Delta(\delta) = \frac{\gamma(\delta)}{\delta} = \int_0^1 \frac{du}{\ell(\gamma(\delta u))}$ is an element of the algebra Γ_0, so that (5.3) is universally valid. A simple computation gives

$$xdx = \sum_1^3 x_i\,dx_i = \frac{\gamma\gamma'}{\delta}\,(ydy)\,,$$

$$dx^2 = \sum_1^3 dx_i^2 = \left(\frac{\gamma'^2}{\delta^2} - \frac{\gamma^2}{\delta^4}\right)(ydy)^2 + \frac{\gamma^2}{\delta^2}dy^2$$

so that, by setting $F(\delta) = f\left(\gamma(\delta)\right)$, $F_1(\delta) = f_1\left(\gamma(\delta)\right)\frac{\gamma(\delta)\gamma'(\delta)}{\delta}$, $L_1(\delta) = \ell_1\left(\gamma(\delta)\right)\frac{\gamma(\delta)}{\delta}$, $L(\delta) = \ell\left(\gamma(\delta)\right)\gamma'(\delta) = 1$, we obtain the transformed metric

$$ds^2 = \left(Fdt + F_1(ydy)\right)^2 - \left(L_1^2dy^2 + \frac{1 - L_1^2}{\delta^2}(ydy)^2\right) \tag{5.4}$$

which is related to the geodesic distance $\delta = \|y\|$ and the functions F and G. Instead of $h(\rho)$, we have now the function $H = H(\delta) = \delta F_1(\delta)$, and moreover the invariant curvature radius of the spheres $\delta = \text{const.}$ is given by the function

$$G = G(\delta) = \delta L_1(\delta)\,.$$

Before solving the equation (5.1), we can anticipate that the values of the solution $G(\delta)$ do not cover the whole half-line $[0, +\infty[$ or, possibly, the whole open half-line $]0, +\infty[$, because by taking a sequence of positive values $\delta_n \to 0$, we have $G(\delta_n) \to 0$ and then the equation (5.1) implies $A = 0$ contrary to our assumption $A \neq 0$. (This conclusion follows also from (4.6), because $g(0) = 0$ implies $A = 0$.) So, we are led to anticipate that the values of the solution $G(\delta)$ cover a half-line $[\alpha, +\infty[$ with $\alpha > 0$. This important property, which implies that the source of the field cannot be reduced to a point, will be verified by the explicit expression of the solution.

Now, since $G(\delta) \geqslant \alpha > 0$ and $G - 2A \geqslant 0$ according to (5.1), the function $G(\delta)$ is obtained by the equation

$$\frac{dG}{d\delta} = \sqrt{1 - \frac{2A}{G}}$$

and since $\sqrt{1 - \frac{2A}{G}} > 0$, $G(\delta)$ is a strictly increasing function of δ. Moreover $G(\delta)$ can not remain bounded because $\frac{dG}{d\delta} \to 1$ as $G \to +\infty$.

Technically, we have first to obtain the inverse function $\delta = P(G)$ by integrating the equation

$$\frac{d\delta}{dG} = \frac{1}{\sqrt{1 - \frac{2A}{G}}}$$

which implies also that $\delta = P(G)$ is a strictly increasing and not bounded function of G. Now, we introduce an auxiliary fixed positive length ξ which will not appear in the final result, but it is needed in order to carry out correctly the computations. In fact, since G, A, $G - 2A$ represent also lengths, the ratios $\frac{G}{\xi}$, $\frac{G-2A}{\xi}$ are dimensionless, so that we can introduce the logarithm

$$\ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G - 2A}{\xi}}\right)$$

and since $\frac{d}{dG}\left(\sqrt{G\,(G - 2A)} + 2A\ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G-2A}{\xi}}\right)\right) = \frac{1}{\sqrt{1 - \frac{2A}{G}}}$ the preceding equation gives $\delta = P(G)$,

$$\delta = B + \sqrt{G(G - 2A)} + 2A\ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G - 2A}{\xi}}\right) \tag{5.5}$$

where $B = \text{const.}$ It follows that

$$\frac{\delta}{G(\delta)} = \frac{P(G)}{G} = \frac{B}{G} + \sqrt{1 - \frac{2A}{G}} + \frac{2A}{G}\ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G - 2A}{\xi}}\right)$$

and since we have $\frac{2A}{G}\ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G-2A}{\xi}}\right) = \frac{2A}{G}\ln\sqrt{\frac{G}{\xi}} + \frac{2A}{G}\ln\left(1 + \sqrt{1 - \frac{2A}{G}}\right) \to 0$ as $G \to +\infty$ we have

$$\frac{\delta}{G(\delta)} = 1 + \epsilon(\delta), \qquad \frac{G(\delta)}{\delta} = \frac{1}{1+\epsilon(\delta)}$$

with $\epsilon(\delta) \to 0$ as $\delta \to +\infty$. This property allows to determine the constant A by using the so-called Newtonian approximation of the metric (5.4) for the great values of the distance δ. Classically this approximation is referred to the static metric, namely to the case where $F_1 = 0$. We have already seen that $|\delta F_1(\delta)| \leqslant 1$, but this condition does not imply that $\delta F_1(\delta)$ possesses a limit as $\delta \to +\infty$. So we accept the condition $F_1(\delta) = 0$ for the derivation of the Newtonian approximation, without forgetting that we have to do with a specific choice of F_1 used for convenience in the case of a special problem.

This being said, the Newtonian approximation is obtained by setting $\epsilon(\delta) = 0$ and $F_1 = 0$. Then since $F = cG' = c\sqrt{1 - \frac{2A}{G}} = c\sqrt{1 - \frac{2A}{\delta} - \frac{2A\epsilon(\delta)}{\delta}}$, $1 - L_1^2 = 1 - \left(\frac{1}{1+\epsilon(\delta)}\right)^2$, and $\frac{\|y\|}{\delta} = 1$, we get the form

$$ds^2 = c^2\left(1 - \frac{2A}{\delta}\right)dt^2 - dy^2$$

which, by means of a known argument, leads to identify $\frac{c^2 A}{\delta}$ with $\frac{km}{\delta}$, whence $A = \frac{km}{c^2} = \mu$.

Since $G - 2A \geqslant 0$, we have $G(\delta) \geqslant 2\mu$, so that, as anticipated, $G(\delta)$ possesses the strictly positive greatest lower bound 2μ, which, as we see, is independent of the second constant B appearing in the solution (5.5). It follows that the strictly increasing function $G(\delta)$ appears as an implicit function defined by the equation

$$\delta = B + \sqrt{G(G - 2\mu)} + 2\mu \ln\left(\sqrt{\frac{G}{\xi}} + \sqrt{\frac{G - 2\mu}{\xi}}\right).$$

The greatest lower bound 2μ is obtained for $\delta = B + 2\mu \ln\sqrt{\frac{2\mu}{\xi}}$ and this is why it is convenient to introduce, instead of B, the constant $\delta_0 = B + 2\mu\ln\sqrt{\frac{2\mu}{\xi}}$, which allows to write $\delta = \delta_0 + \sqrt{G(G - 2\mu)} + 2\mu\ln\left(\sqrt{\frac{G}{2\mu}} + \sqrt{\frac{G}{2\mu} - 1}\right)$ or

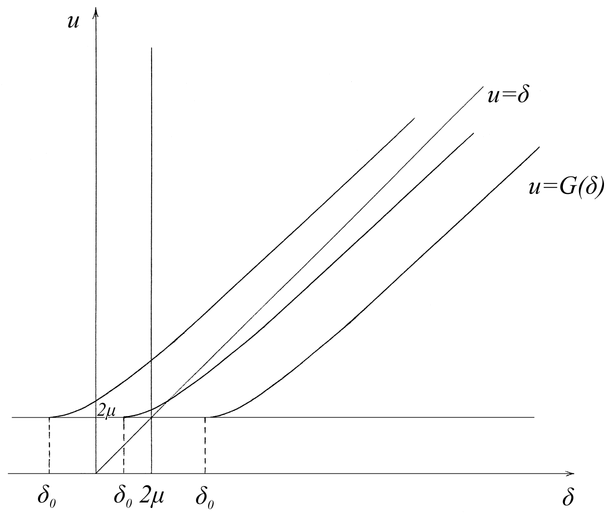
$$\delta = \delta_0 + \int_{2\mu}^{G} \frac{du}{\sqrt{1 - \frac{2\mu}{u}}}, \qquad G = G(\delta) \geqslant 2\mu$$

which does not contain the auxiliary length ξ. The solution is completed by the determination of the function

$$F = cG' = c\sqrt{1 - \frac{2\mu}{G(\delta)}}.$$

As far as $H(\delta) = \delta F_1(\delta)$ is concerned, we repeat that it is introduced simply as a C^∞ function vanishing for $\delta = 0$ and satisfying the condition $|H(\delta)| \leqslant 1$.

What about the new constant δ_0? From the mathematical point of view, negative values of δ_0 are not excluded. So, we distinguish the following cases (see Figure):



(a) $\delta_0 < 0$. Then the values of $G(\delta)$ for $\delta_0 \leqslant \delta < 0$ are meaningless physically, because $G(\delta)$ is conceived on $[0, +\infty[$. But the value $\delta = 0$ is also excluded because
$$\int_{2\mu}^{G(0)} \frac{du}{\sqrt{1 - \frac{2\mu}{u}}} = -\delta_0 > 0$$
implies $G(0) > 2\mu$ contrary to the geometrical condition $G(0) = 0$. Consequently there exists a constant $\delta_1 > 0$ (the radius of the considered distribution of matter) such that only the restriction of $G(\delta)$ to $[\delta_1, +\infty[$ is taken into account.

(b) $\delta_0 = 0$. Then
$$\int_{2\mu}^{G(0)} \frac{du}{\sqrt{1 - \frac{2\mu}{u}}} = 0$$
so that $G(0) = 2\mu$ contrary to the geometrical condition $G(0) = 0$. Consequently the solution is valid, as previously, on a half-line $[\delta_1, +\infty[$ with $\delta_1 > 0$.

(c) $\delta_0 > 0$. Then $G(\delta_0) = 2\mu$, $F(\delta_0) = 0$, so that the metric degenerates for $\delta = \delta_0$. A degenerate metric does not possess physical meaning. Consequently, there exists a constant $\delta_1 > \delta_0$ (the radius of the sphere bounding the matter) such that the solution is physically valid only on the half-line $[\delta_1, +\infty[$.

Whatever the case may be, the vacuum solution is not defined for $\delta < \delta_1$. In other words, the ball $\|y\| \leqslant \delta_1$ is occupied by matter, so that the source of the field cannot be reduced to a point. The constant δ_0 is related to a boundary condition, namely the value of the curvature radius of the sphere bounding the matter. In fact, if δ_1 is the radius of this sphere, and the value $G(\delta_1)$ is known, then the value δ_0 is easily obtained:

$$\delta_0 = \delta_1 - \sqrt{G(\delta_1)\left(G(\delta_1) - 2\mu\right)} - 2\mu\ln\left(\sqrt{\frac{G(\delta_1)}{2\mu}} + \sqrt{\frac{G(\delta_1)}{2\mu} - 1}\right).$$

However, it is difficult, even impossible, to obtain $G(\delta_1)$ by direct measurements. So the value δ_0 is to be found indirectly by taking into account the phenomena induced by δ_0. This problem will be treated in another paper.

The most impressive characteristic of the solution is perhaps the non-Euclidean structure of the space and specifically the strong non-Euclidean properties in the neighbourhood of the origin. If the theory is applicable to the elementary particles, then strong deviations from the Euclidean geometry are to be expected in the world of microphysics. Together with the new geometrical ideas, the solution brings about an improvement of the law of gravitation in accordance with Poincaré's prediction: "Il est difficile de ne pas supposer que la loi véritable contient des termes complémentaires qui deviendraient sensibles aux petites distances" [1]. In fact, the Newton potential

$$-\frac{km}{\delta}$$

is an approximation of the more accurate expression

$$-\frac{km}{G(\delta)}$$

which depends on the curvature radius $G(\delta)$. There is a significant discrepancy between the two formulae. Although, as shown earlier, the ratio $\frac{G(\delta)}{\delta}$ converges to 1, the difference

$$\delta - G(\delta) = P(G) - G = \delta_0 + \\ + 2\mu \ln\left(\sqrt{\frac{G}{2\mu}} + \sqrt{\frac{G}{2\mu} - 1}\right) - \frac{2\mu}{1 + \sqrt{1 - \frac{2\mu}{G}}}$$

tends to $+\infty$ as $\delta \to +\infty$. Moreover $G(\delta)$ depends not only on the radius δ, but also on the constant δ_0. Of course, the distinction between Newton's theory and Einstein's theory does not reduce to the distinction between δ and $G(\delta)$. Einstein's theory provides a new entity, namely a spacetime metric.

A last question regards the "boundary conditions at infinity". Classically it is required that the metric admit as limit form the standard pseudo-Euclidean metric as $\delta \to +\infty$. Since, as already remarked, $\delta F_1(\delta)$ does not possess a limit as $\delta \to +\infty$, this requirement presupposes that $F_1 = 0$, namely that we are dealing with a static metric. Then the metric can be written as

$$ds^2 = c^2\left(1 - \frac{2\mu}{G(\delta)}\right)dt^2 - \\ - \left(\left(\frac{G(\delta)}{\delta}\right)^2 dy^2 + \frac{1}{\delta^2}\left(1 - \left(\frac{G(\delta)}{\delta}\right)^2\right)(y\,dy)^2\right)$$

and since $G(\delta) \to +\infty$, $\frac{G(\delta)}{\delta} \to 1$, $\frac{\|y\|}{\delta} = 1$, we find, in fact, "at infinity" the standard pseudo-Euclidean form

$$ds^2 = c^2 dt^2 - dy^2.$$

Note that, if we introduce the so-called polar coordinates, this conclusion fails. In fact, then we have the form

$$ds^2 = c^2\left(1 - \frac{2\mu}{G(\delta)}\right)dt^2 - \left(d\delta^2 + \big(G(\delta)\big)^2\left(\sin^2\theta\, d\phi^2 + d\theta^2\right)\right)$$

which does not possess a limit form as $\delta \to +\infty$.

6 Black holes do not exist

The pseudo-theory of black holes appeared as a consequence of misunderstandings and mathematical errors brought out in detail in the papers [3, 5, 6]. We emphasize that the so-called "horizon" does not represent an observable value of the curvature radius $G(\delta)$. According to the established rigorous solution, 2μ is the greatest lower bound of the vacuum solution $G(\delta)$ and is defined for a certain value δ_0 of the new constant. If $\delta_0 \leqslant 0$ there exists no real sphere with the curvature radius 2μ, and the physically valid part of the solution is defined for $\delta \geqslant \delta_1$, where δ_1 is a strictly positive value such that $G(\delta_1) > 2\mu$. On the other hand, if $\delta_0 > 0$, the degeneracy of the metric for $\delta = \delta_0$ implies that the corresponding sphere lies inside the matter, so that the vacuum solution is valid for $\delta \geqslant \delta_1$ where $\delta_1 > \delta_0$ and $G(\delta_1) > 2\mu$. Whatever the case may be, the notion of black hole is inconceivable.

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Gravitational Perturbations as a Possible Cause for Instability in the Measurements of Positron Annihilation

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The annihilation of positrons is measured in a wide range of studies in the field of physical chemistry [1, 2]. One of the problems in these studies is the instability of the results of measurements [3–5]. As shown in our research, instability may result from the change of nonregistering gravitational effects related to alteration of the tidal forces upon the change of moon phases and the seasonal changes of the distance between the Earth and the Sun.

1 Materials and methods

A sample of ^{22}Na (5 mCu) was used as a source of positrons. The yield of positronium (I_2) and the parameters of its annihilation at the passage through organic liquids were measured by two techniques: either angular (parapositronium) or temporal (orthopositronium) correlations of annihilation quanta were registered. The yield of positronium was measured with a setup of "fast-slow coincidences". The setup was assembled according to a typical scheme, had the time resolution of 0.5 ns and was connected to a multichannel amplitude recorder [1, 2].

2 Results

In the experiments, I_2-parameter and spectra of triplet positronium were measured in the toluol samples purified from oxygen by the method of vacuum freezing-out and the samples under oxygen (0.6 atm). The measurements were conducted daily over a period of 3 months (November, 1981 — February, 1982).

Fig. 1 shows that in the oxygen-depleted samples, regular fluctuations in the positronium yield are observed, which correlate with the changes of the moon phase. The yield is maximal in the times close to the new moon and minimal in the times close to the full moon.

In the presence of oxygen, Fig. 2, no reliable effects were revealed. It can be explained by a specific influence of oxygen on the processes of formation and annihilation of positronium [1, 2]. However, these experiments indicate stability of the setup itself.

In addition to periodical fluctuations, one can see a trend in the series of measurements: the mean level of I_2 grows from November to February. This trend may be due to the seasonal change of the distance between the Earth and the Sun.

In more large scale the seasonal changes of positronium yield apparent from Fig. 3, which presents average results

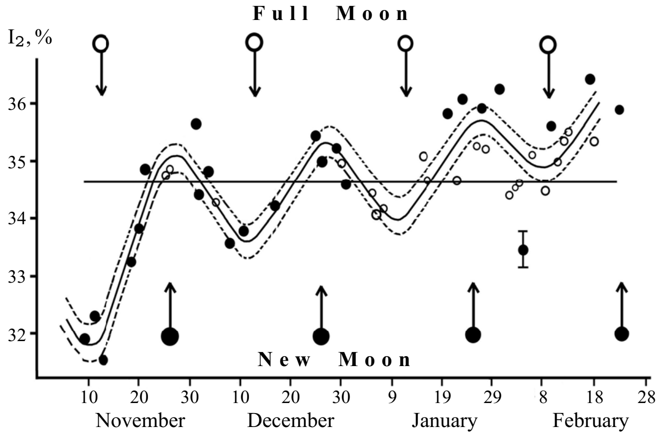

Fig. 1: Yield of positronium correlate with the changes of the Moon phases.

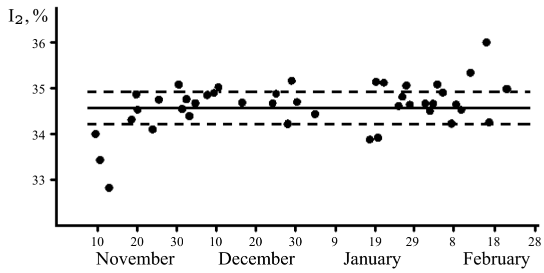

Fig. 2: Yield of positronium in the presence of oxygen.

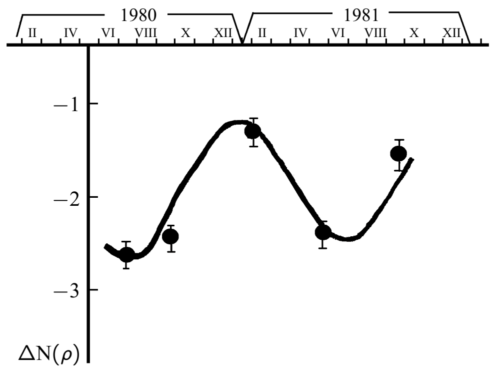

Fig. 3: Seasonal changes of positronium yield in 1980–1981.

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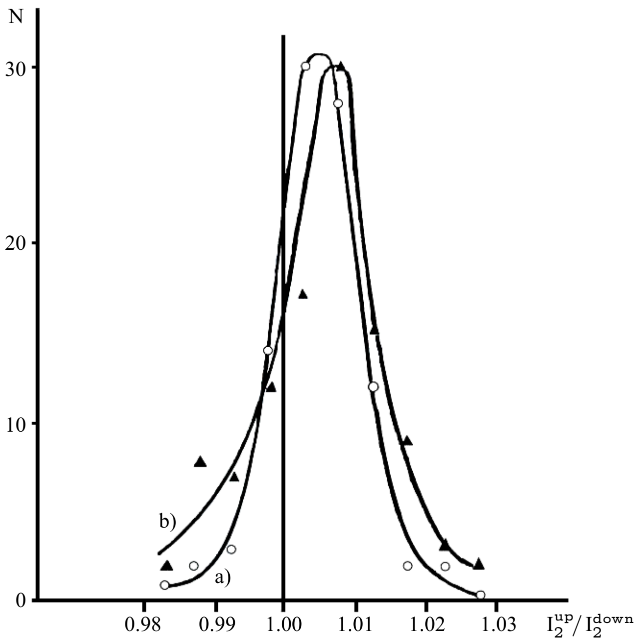

Fig. 4: Distribution of measuring results of positronium yield at the case of the experimental setup relocating up (triangles) and down (circles), with the height difference 1.5 m.

of large number of experiments provided in 1980–1981. It's possible to see that minimal yield is observable for summer solstice (June — July) and maximal yield for winter solstice (December — February).

The reliability of the conclusion that the yield of positronium depends on the tidal changes in gravity force was checked in the experiments of 1984–1985, in which the setup used for measuring I_2 was relocating up-and-down, with the height difference of 1.5 m. The measurements (1500 in total) were alternated (up/down) every 20 min. Finally, an I_2^{up}/I_2^{down} ratio was calculated. Fig. 4 shows a smoothed distribution of the results obtained. The mean of this ratio is 1.00447 for all the measurements. The root-mean-square error equals to $\pm\,0.00064$. Thus, lifting the setup 1.5 m higher results in a reliable increase in the positronium yield (the difference amounts to 7σ).

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Photon Physics of Revised Electromagnetics

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Conventional theory, as based on Maxwell's equations and associated quantum electrodynamical concepts in the vacuum, includes the condition of zero electric field divergence. In applications to models of the individual photon and to dense light beams such a theory exhibits several discrepancies from experimental evidence. These include the absence of angular momentum (spin), and the lack of spatially limited geometry in the directions transverse to that of the propagation. The present revised theory includes on the other hand a nonzero electric field divergence, and this changes the field equations substantially. It results in an extended quantum electrodynamical approach, leading to nonzero spin and spatially limited geometry for photon models and light beams. The photon models thereby behave as an entirety, having both particle and wave properties and possessing wave-packet solutions which are reconcilable with the photoelectric effect, and with the dot-shaped marks and interference patterns on a screen by individual photons in a two-slit experiment.

1 Introduction

Conventional electromagnetic theory based on Maxwell's equations and quantum mechanics has been very successful in its applications to numerous problems in physics, and has sometimes manifested itself in an exceptionally good agreement with experiments. Nevertheless there exist areas within which these joint theories do not provide fully adequate descriptions of physical reality. As stated by Feynman [1], there are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics. Thus the classical theory of electromagnetism is in its conventional form an unsatisfactory theory of its own.

Maxwell's equations have served as a guiding line and basis for conventional quantum electrodynamics (QED) in which there is a vacuum state with a vanishing electric field divergence [2]. The quantized equations become equivalent to the classical field equations in which all field quantities are replaced by their expectations values [3]. According to Schiff [2] and Heitler [3], the Poynting vector further forms the basis for the quantized field momentum. Consequently, QED will also become subject to the shortcomings of a conventional field theory.

When applying such a theory to photon physics, it will lead to irrelevant results in a number of important cases. This occurs with the experimentally confirmed existence of angular momentum of individual photons and of light beams with a spatially limited cross-section, with the behaviour of individual photons as needle radiation in the photoelectric effect and in two-slit experiments, and with the particle-wave duality of the photon.

As a consequence of the revealed limitations, modified theories leading beyond Maxwell's equations have been elaborated by several authors. Among these there is an approach described in this paper [4–9]. It is based on a vacuum state that can give rise to local space charges and currents in addition to the displacement current. This changes the field equations in a substantial way, thus resulting in an extended quantum electrodynamical ("EQED") approach.

In the applications to photon physics the nonzero electric field divergence may appear as small, but it still comes out to have an essential effect on the end result. In other applications of the present theory, such as on an electron model [6, 7] not being treated here, the electric field divergence terms appear as large contributions already in the basic field equations.

2 Basis of present theory

The vacuum is not merely an empty space. There is a nonzero level of its ground state, the zero-point-energy, which derives from the quantum states of the harmonic oscillator [2]. An experimentally verified example of the related electromagnetic vacuum fluctuations is the Casimir effect [10]. Electron-positron pair formation due to an energetic photon also indicates that local positive and negative electric charges can be created out of an electrically neutral vacuum state. The basic physical concept of the present theory is therefore the appearance of a local charge density in such a state. In its turn, this becomes associated with a nonzero electric field divergence. The inclusion of the latter can as well be taken as a starting point of a corresponding field theory.

2.1 Lorentz invariant field equations

In presence of electric space charges and related current densities, a general form of the Proca-type equation

$$\Box A_\mu \equiv \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu\,, \quad \mu = 1,2,3,4 \quad (1)$$

can be taken as a four-dimensional starting point of the present field equations, given in SI units. Here $A_\mu = (\mathbf{A}, i\phi/c)$, where $\mathbf{A}$ and ϕ are the magnetic vector potential and the electrostatic potential in three-space, and

$$J_\mu = (\mathbf{j},\, \mathrm{i}c\bar{\rho}) \tag{2}$$

is the four-current density. The right-hand member of equation (1) and the form (2) are now given a new interpretation, where $\bar{\rho}$ is the nonzero electric charge density in the vacuum, and $\mathbf{j}$ stands for an associated three-space current density. Maxwell's equations are recovered when the current density (2) disappears, and equation (1) reduces to the d'Alembert equation. The present charge and current densities should not become less conceivable than the conventional concepts of a nonzero curl of the magnetic field and an associated displacement current. All these concepts can be regarded as intrinsic properties of the electromagnetic field.

Physical experience further supports that also the present revised and extended field equations should remain Lorentz invariant. This implies that the current (2) has to transform as a four-vector, and its square then becomes invariant to a transform from one inertial frame K to another such frame K$'$. Thus

$$j^2 - c^2\bar{\rho}^2 = j'^2 - c^2\bar{\rho}'^2 = \text{const}\,. \tag{3}$$

In addition, the current density $\mathbf{j}$ should exist only when there is also a charge density $\bar{\rho}$, and this implies that the constant in equation (3) vanishes. Since $\mathbf{j}$ and $\bar{\rho}$ must behave as space and time parts of J_μ, the disappearance of this constant merely becomes analogous to the choice of origin for the space and time coordinates. Consequently the final form of he current density (2) becomes

$$\mathbf{j} = \bar{\rho}\,(\mathbf{C},\, \mathrm{i}c) \qquad \mathbf{C}^2 = c^2. \tag{4}$$

It is obvious that $\bar{\rho}$ as well as the velocity vector $\mathbf{C}$ vary from one inertial frame to another and do not become Lorentz invariant, whereas this is the case of J_μ.

It can be shown [6, 7] that there is a connection between the current density (4) and the electron theory by Dirac. A different form of the current density in equation (1) has further been introduced by de Broglie and Vigier [11] and applied by Evans and Vigier [12]. It explicitly includes a small nonzero photon rest mass.

The three-dimensional representation of the extended equations *in the vacuum* now becomes

$$\text{curl}\,\mathbf{B}/\mu_0 = \varepsilon_0\,(\text{div}\,\mathbf{E})\,\mathbf{C} + \varepsilon_0\partial\mathbf{E}/\partial t \tag{5}$$

$$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t \tag{6}$$

$$\text{div}\,\mathbf{E} = \bar{\rho}/\varepsilon_0 \tag{7}$$

for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. Here the first term of the right-hand member of equation (5) and equation (7) are the new parts introduced. Thus, there is a change from a homogeneous to an inhomogeneous system of equations, a new degree of freedom is introduced by the nonzero electric field divergence, and the latter produces an asymmetry in the appearance of the electric and magnetic fields.

The presence in equations (5) and (7) of the dielectric constant ε_0 and the magnetic permeability μ_0 of the conventional vacuum may require further explanation. In the present approach the vacuum is considered not to include electrically polarized or magnetized atoms or molecules. In this respect equation (7) is the same as in the theory of plasmas which contain freely moving charged particles in a background of empty vacuum space.

A nonzero magnetic field divergence is not adopted in this theory, because this is so far a possible but not experimentally supported supposition which is here left as an open question.

Using vector identities, the basic equations (5)–(7) yield the local momentum equation

$$\text{div}^2\mathbf{S} = \bar{\rho}\,(\mathbf{E} + \mathbf{C}\times\mathbf{B}) + \frac{\partial}{\partial t}\,\mathbf{g} \tag{8}$$

and the local energy equation

$$-\text{div}\,\mathbf{S} = \bar{\rho}\,\mathbf{E}\cdot\mathbf{C} + \frac{\partial}{\partial t}\,w_f\,. \tag{9}$$

Here $\frac{1}{c^2}\,\mathbf{S}$ is the electromagnetic stress tensor,

$$\mathbf{g} = \varepsilon_0\,\mathbf{E}\times\mathbf{B} = \frac{1}{c^2}\,\mathbf{S} \tag{10}$$

can be interpreted as an electromagnetic momentum density with $\mathbf{S}$ denoting the Poynting vector, and

$$w_f = \frac{1}{2}\,\varepsilon_0\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right) \tag{11}$$

representing the electromagnetic field energy density. The angular momentum density becomes

$$\mathbf{s} = \mathbf{r}\times\mathbf{S}/c^2 \tag{12}$$

where $\mathbf{r}$ is the radius vector from the origin.

Combination of equations (5) and (6) results in an extended wave equation for the electric field

$$\left(\frac{\partial^2}{\partial t^2} - c^2\,\nabla^2\right)\mathbf{E} + \left(c^2\,\nabla + \mathbf{C}\,\frac{\partial}{\partial t}\right)(\text{div}\,\mathbf{E}) = 0\,. \tag{13}$$

A divergence operation on equation (5) results in

$$\left(\frac{\partial}{\partial t} + \mathbf{C}\cdot\nabla\right)(\text{div}\,\mathbf{E}) = 0\,, \tag{14}$$

provided that div $\mathbf{C} = 0$.

2.2 Quantization of the revised theory

In the conventional QED formalism Maxwell's equations with a vanishing electric field divergence have been used as a basis, also including the Poynting vector in the representation of the quantized field momentum [2, 3]. The quantized equations then become equivalent to the classical ones in which the field quantities are replaced by their expectation values.

A similar situation also has to apply to the present revised equations. The resulting solutions are expected not to be too far from the truth, by representing the most probable trajectories. A rigorous extended quantum electrodynamical (EQED) formulation would imply that the field equations are quantized already from the outset. However, to simplify the analysis, we will instead solve the extended equations as they stand, and impose relevant quantum conditions afterward. For the considered photon models these conditions are given by the energy $h\nu$ related to the frequency ν, and by the angular momentum $h/2\pi$ of the individual photon with the property of a boson particle.

3 Axisymmetric model of the individual photon

When elaborating a model of the individual photon as a propagating boson, a wave or wave-packet with preserved and limited geometrical shape and undamped motion in a defined direction of space has to be taken as a starting point. This leads to cylindrical geometry and waves. A cylindrical frame (r, φ, z) becomes appropriate, with its z-axis in the direction of propagation. We further introduce a velocity vector of helical geometry

$$\mathbf{C} = c\,(0, \cos\alpha, \sin\alpha) \tag{15}$$

where the angle α is constant and $0 < \cos\alpha \ll 1$ for reasons to be clarified later. As will be shown, the component C_z is related to the wave propagation in the axial direction, and the component C_φ to the angular momentum and an associated small nonzero rest mass. Here we choose the positive values of $\sin\alpha$ and $\cos\alpha$, but have in general both signs representing the two directions of propagation and the two spin directions.

The wave equation (13) now leads to

$$\left(D_1 - \frac{1}{r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_r - \frac{2}{r^2}\frac{\partial}{\partial\varphi} E_\varphi = \frac{\partial}{\partial r}(\mathrm{div}\,\mathbf{E}) \tag{16}$$

$$\begin{aligned}\left(D_1 - \frac{1}{r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_\varphi - \frac{2}{r^2}\frac{\partial}{\partial\varphi} E_r &= \\ = \left[\frac{1}{r}\frac{\partial}{\partial\varphi} + \frac{1}{c}(\cos\alpha)\frac{\partial}{\partial t}\right](\mathrm{div}\,\mathbf{E})&\end{aligned} \tag{17}$$

$$\left(D_1 + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_z = \left[\frac{\partial}{\partial z} + \frac{1}{c}(\sin\alpha)\frac{\partial}{\partial t}\right](\mathrm{div}\,\mathbf{E}) \tag{18}$$

where

$$D_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,. \tag{19}$$

Equation (14) further becomes

$$\left(\frac{\partial}{\partial t} + c\cos\alpha\,\frac{1}{r}\frac{\partial}{\partial\varphi} + c\sin\alpha\,\frac{\partial}{\partial z}\right)(\mathrm{div}\,\mathbf{E}) = 0\,. \tag{20}$$

In this section restriction is made to purely axisymmetric normal modes for which $\partial/\partial\varphi = 0$, and where every quantity Q has the form $Q = \hat{Q}(r)\exp\left[\mathrm{i}(-\omega t + kz)\right]$ with a given angular frequency ω and wave number k.

3.1 Shortcomings of a conventional model

Turning first to a model based on Maxwell's equations, the system (16)–(18) with a vanishing electric field divergence results in the electric field components

$$\begin{aligned}\hat{E}_r &= k_{1r}\, r + k_{2r}/r \\ \hat{E}_\varphi &= k_{1\varphi}\, r + k_{2\varphi}/r \\ \hat{E}_z &= k_{1z}\ln r + k_{2z}\end{aligned} \tag{21}$$

and similar expressions for the magnetic field. The solutions for E_r and E_φ were first deduced by Thomson [13] who called attention to their divergent character, thus becoming unsuitable for a limited model.

However, an even more serious shortcoming arises from the requirement that the divergences of the fields have to vanish. Thus the second order equations (16)–(18) and their solutions (21) have to be checked with respect to the first order equations of an *identically* vanishing field divergence. This implies that

$$2k_{r_1} + \mathrm{i}k\,(k_{1z}\ln r + k_{2z}) \equiv 0 \tag{22}$$

for all k and r. Consequently E_z and k_{1r} have to vanish, only E_φ and k_{2r} remain nonzero, and similar results apply to the magnetic field. This implies that the wave becomes purely transverse, that the Poynting vector (10) has a component in the direction of propagation only, and that there is no spin in the axial direction.

3.2 Axisymmetric models in revised theory

For an axisymmetric normal mode, equation (20) of the revised theory yields the dispersion relation

$$\omega/k = c\sin\alpha = v\,, \qquad k^2 - \omega^2/c^2 = k^2\cos^2\alpha \tag{23}$$

where v stands for the phase velocity which becomes equal to the group velocity $\partial\omega/\partial k$. The parameter $\cos\alpha$ must be small here, such as not to get in conflict with experiments of the Michelson-Morley type. For $\cos\alpha \leqslant 10^{-4}$ the difference between v and c would thus become less than a change in the eight decimal of c. Equations (16), (17) and (23) further combine to

$$-k^2\cos^2\alpha\, E_r = \mathrm{i}k\,\frac{\partial E_z}{\partial r} \tag{24}$$

$$\begin{aligned}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} - k^2\cos^2\alpha\right) E_\varphi &= \\ = -(\mathrm{tg}\,\alpha)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k^2\cos^2\alpha\right) E_z\,.&\end{aligned} \tag{25}$$

A *generating function*

$$G_0 \cdot G = E_z + (\cot\alpha)E_\varphi\,, \quad G = R(\rho)\,\mathrm{e}^{\mathrm{i}(-\omega t + kz)} \quad (26)$$

can now be found which has the amplitude G_0, a normalized dimensionless part G, the normalized coordinate $\rho = r/r_0$, and the characteristic radius r_0 of the configuration represented by the radial function R. The function (26) generates the solutions

$$E_r = -\mathrm{i}G_0\left(\theta\cos^2\alpha\right)^{-1}\frac{\partial}{\partial\rho}\Big[\left(1-\rho^2 D\right)G\Big] \quad (27)$$

$$E_\varphi = G_0\,(\mathrm{tg}\,\alpha)\,\rho^2 DG \quad (28)$$

$$E_z = G_0\left(1-\rho^2 D\right)G \quad (29)$$

where

$$D = D_\rho - \theta^2\cos^2\alpha\,, \qquad D_\rho = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} \quad (30)$$

and $\theta = kr_0$. The solutions (27)–(29) are reconfirmed by insertion into equations (16)–(18). The magnetic field components are derived from the induction law (6). The helical-like structure of the obtained solution, with its axial field components, is similar but not identical to that deduced by Evans and Vigier [12].

From the normal modes a wave packet solution is now formed which has finite extensions, and a narrow line width as required by experimental observation. We are free to rewrite the amplitude factor (26) as $G_0 = g_0\cos^2\alpha$. The wave packet has the amplitude

$$A_k = \left(\frac{k}{k_0^2}\right)\mathrm{e}^{-z_0^2(k-k_0)^2} \quad (31)$$

in the interval dk being centered around the main wave number k_0. Here $2z_0$ represents the axial length of the packet. With $z = \bar{z} - vt$ and the notation

$$\bar{E}_0 = E_0(\bar{z}) = \left(\frac{g_0}{k_0 r_0}\right)\left(\frac{\sqrt{\pi}}{k_0 z_0}\right)\exp\left[-\left(\frac{\bar{z}}{2z_0}\right)^2 + \mathrm{i}k_0\bar{z}\right] \quad (32)$$

the spectral averages of the field components are

$$\bar{E}_r = -\mathrm{i}E_0\left[R_5 + (\theta_0')^2 R_1\right] \quad (33)$$

$$\bar{E}_\varphi = E_0\theta_0(\sin\alpha)(\cos\alpha)\left[R_3 - (\theta_0')^2 R_1\right] \quad (34)$$

$$\bar{E}_z = E_0\theta_0(\cos^2\alpha)\left[R_4 + (\theta_0')^2 R_1\right] \quad (35)$$

where $\theta_0 = k_0 r_0$, $\theta_0' = \theta_0\cos\alpha$ and

$$R_1 = \rho^2 R, \quad R_3 = \rho^2 D_\rho R, \quad R_4 = \left(1-\rho^2 D_\rho\right)R, \quad (36)$$

$$R_5 = \frac{d}{d\rho}\Big[\left(1-\rho^2 D_\rho\right)R\Big], \quad R_7 = \left(\frac{d}{d\rho} + \frac{1}{\rho}\right)\left(\rho^2 R\right). \quad (37)$$

The packet components $\left(\bar{E}_\varphi, \bar{E}_z, \bar{B}_r\right)$ are in phase with the generating function (26). The components $\left(\bar{E}_r, \bar{B}_\varphi, \bar{B}_z\right)$ are ninety degrees out of phase with it. We now choose the real part of the function (26), i.e. $G = R(\rho)\cos k\bar{z}$, which is symmetric with respect to the axial center $\bar{z} = 0$ of the moving wave packet. Then the real part of the form (32) is adopted, from which $\left(\bar{E}_\varphi, \bar{E}_z, \bar{B}_r\right)$ become symmetric and $\left(\bar{E}_r, \bar{B}_\varphi, \bar{B}_z\right)$ antisymmetric. Under these conditions the integrated electric charge and magnetic moment vanish.

The electromagnetic field energy density (11) defines an equivalent total mass

$$m = \frac{1}{c^2}\int w_f\,dV \cong \frac{2\pi\varepsilon_0}{c^2}\int_{-\infty}^{+\infty} r\,E_r^2\,dr d\bar{z} \quad (38)$$

to lowest order. Integration and quantization yields

$$m = a_0 W_m \cong \frac{h\nu_0}{c^2}\,, \qquad W_m = \int \rho\,R_5^2\,d\rho\,, \quad (39)$$

where

$$a_0 = \varepsilon_0\,\pi^{5/2}\sqrt{2}\,z_0\left(g_0/ck_0^2 z_0\right)^2 \equiv 2a_0^* g_0^2 \quad (40)$$

and $\nu_0 = c/\lambda_0$. Here the slightly reduced phase and group velocity (23) can be associated with a very small nonzero photon rest mass $m_0 = m\cos\alpha$.

Turning finally to the momentum balance, all integrated volume forces in equation (8) vanish on account of the symmetry properties, and expression (12) gives

$$s = \int s_z\,dV = -2\pi\varepsilon_0\int_{-\infty}^{+\infty}\int r^2\bar{E}_r\bar{B}_z\,drd\bar{z}\,. \quad (41)$$

It reduces to the quantum condition

$$s = a_0 r_0 c(\cos\alpha)W_s = \frac{h}{2\pi}\,, \quad W_s = -\int\rho^2 R_5 R_7\,d\rho\,. \quad (42)$$

So far the radial function R has not been specified. We first consider the case where it is finite at the axis $\rho = 0$ and tends to zero at large ρ, as in the form

$$R(\rho) = \rho^\gamma\mathrm{e}^{-\rho} \quad (43)$$

where $\gamma > 0$. In principle, the factor in front of the exponential would have to be replaced by a power series of ρ, but since we will proceed to the limit of large γ, only one dominating term becomes sufficient. The exponential factor in (43) is further included to secure the convergence of any moment with R. The function (43) has a sharply defined maximum at the radius $\hat{r} = \gamma r_0$. Combination of relations (39) and (42) finally results in an effective photon diameter

$$2\hat{r} = \frac{\lambda_0}{\pi\cos\alpha} \quad (44)$$

being independent of γ and the exponential factor in equation (43).

We next consider the alternative of a radial function R which *diverges* at the axis, i.e.

$$R(\rho) = \rho^{-\gamma}\mathrm{e}^{-\rho}. \quad (45)$$

Here $\hat{r} = r_0$ can be taken as an effective radius. To obtain *finite* integrated values of the total mass m and spin s, small lower limits ρ_m and ρ_s are now introduced in the integrals of W_m and W_s. We further introduce

$$r_0 = c_r \cdot \varepsilon\,, \qquad g_0 = c_g \cdot \varepsilon^{\beta} \tag{46}$$

such as to make the characteristic radius r_0 and the factor g_0 shrink to small but nonzero values as the lower limits ρ_m and ρ_s approach zero. In equations (46), ε is an independent smallness parameter, $0 < \varepsilon \ll 1$, and c_r, c_g and β stand for positive constants. Equations (40), (39) and (44) combine to

$$m = a_0^* \gamma^5 c_g^2 \left(\varepsilon^{2\beta}/\rho_m^{2\gamma}\right) \cong h/\lambda_0 c\,, \tag{47}$$

$$s = a_0^* \gamma^5 c_g^2 c_r c (\cos\alpha) \left(\varepsilon^{2\beta+1}/\rho_m^{2\gamma-1}\right) = h/2\pi\,. \tag{48}$$

To obtain finite m and s it is then necessary that

$$\rho_m = \varepsilon^{\beta/\gamma}, \qquad \rho_s = \varepsilon^{(2\beta+1)(2\gamma-1)}. \tag{49}$$

We are here free to choose $\beta = \gamma \gg 1$ by which $\rho_s \cong \rho_m = \varepsilon$. This leads to a similar set of geometrical configurations which have a shape being independent of ρ_m, ρ_s and ε in the range of small ε. From equations (47) and (48) the effective photon diameter finally becomes

$$2\hat{r} = \frac{\varepsilon \lambda_0}{\pi \cos\alpha} \tag{50}$$

which is independent of γ, β and the exponential factor.

3.3 Summary on the photon model

- Conventional theory results in a model of the individual photon which has no spin, and is not reconcilable with a limited geometrical shape.
- The present axisymmetric wave packet model is radially polarized, does not consist of purely transverse elementary waves as in conventional theory, has a nonzero spin and an associated very small rest mass, and a helical-like field structure.
- The spatial dimensions of the present model are limited. The solutions are reconcilable with the concepts of needle radiation proposed by Einstein. This provides an explanation of the photoelectric effect in which a photon knocks out an electron from an atom, and of the dot-shaped marks on a screen in two-slit experiments on individual photons as reported by Tsuchiya et al. [14]. As an example with $\cos\alpha = 10^{-4}$ and $\lambda_0 = 3\times 10^{-7}$m, equation (44) yields a diameter $2\hat{r} = 10^{-3}$m, and equation (50) results in $2\hat{r} \leqslant 10^{-7}$m when $\varepsilon \leqslant \cos\alpha$ for needle-like radiation.
- The present individual photon model is relevant in respect to particle-wave dualism. A subdivision into a particle and an associated pilot wave is not necessary, because the rest mass merely constitutes an integrating part of the total field energy. The wave packet behaves as an entirety, having particle and wave properties at the same time. There is a particle like behaviour such as by needle radiation and a nonzero rest mass, and a wave-like behaviour in terms of interference between cylindrical waves. The rest mass may make possible "photon oscillations" between different modes [8], such as those of the results (44) and (50).

4 Screw-shaped light

In a review by Battersby [15] new results have been reported on twisted light in which the energy travels along a cork-screw-shaped path. These discoveries are expected to become important in communication and microbiology.

In this section, equations (16)–(18) will be applied to screw-shaped waves with the factor

$$\exp\left[\mathrm{i}(-\omega t + \bar{m}\varphi + kz)\right] = \exp\left(\mathrm{i}\theta_m\right) \tag{51}$$

and $\bar{m}$ as a positive or negative integer. Since the analysis is similar to that of Section 3.2, we shall leave out its details.

4.1 Shortcomings of the conventional analysis

With Maxwell's equations the system (16)–(18) reduces to

$$\left[D_\rho - \frac{\left(1+\bar{m}^2\right)}{\rho^2}\right](E_r, \mathrm{i}E_\varphi) - \frac{2\bar{m}^2}{\rho^2}\left(\mathrm{i}E_\varphi\,, E_r\right) = 0\,, \tag{52}$$

$$\left[D_\rho - \frac{\bar{m}^2}{\rho^2}\right]E_z = 0\,. \tag{53}$$

For nonzero values of $\bar{m}$, equations (52) combine to

$$\hat{E}_r = c_{1r}\,\rho^{1\pm\bar{m}} + c_{2r}\,\rho^{-(1\pm\bar{m})} = \pm \mathrm{i}\hat{E}_\varphi \tag{54}$$

when $1 \pm \bar{m} \neq 0$ and

$$\hat{E}_r = c_{1r0} + c_{2r0}\ln\rho = \pm \mathrm{i}\hat{E}_\varphi \tag{55}$$

when $1 \pm \bar{m} = 0$. Further equation (53) gives

$$\hat{E}_z = c_{1z}\,\rho^{\bar{m}} + c_{2z}\,\rho^{-\bar{m}}. \tag{56}$$

As in Section 3.1 these results become divergent.

An even more serious shortcoming is again due to an identically vanishing electric and magnetic field divergence. This makes the axial components E_z and B_z disappear, thus resulting in a vanishing spin.

4.2 Twisted modes in revised theory

For nonzero values of $\bar{m}$, the second term in equation (20) introduces complications. This problem is approached by limiting the analysis to sufficient small $\cos\alpha$, and the dispersion relation to be approximated by relations (23). From equation (18) can be seen that E_z is of the order of $\cos^2\alpha$ as compared to E_r and E_φ when $\bar{m} \neq 0$. Equation (16) then takes the approximate form

$$E_r \cong -\left(\frac{r}{\bar{m}}\right)\left[1-k^2\left(\cos^2\alpha\right)\left(\frac{r}{\bar{m}}\right)^2\right]\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)(\mathrm{i}E_\varphi)\,. \quad (57)$$

When inserting this relation into equation (17), the latter is identically satisfied up to the order $\cos^2\alpha$. The component $\mathrm{i}E_\varphi$ can be used as a generating function

$$\mathrm{i}E_\varphi = G_0 G \qquad G = R(\rho)\,\mathrm{e}^{\mathrm{i}\theta_m}. \quad (58)$$

The analysis proceeds in forming a wave packet of narrow line width, as given in detail elsewhere [9]. The radial forms (43) and (45) lead to effective diameters for which a factor $|\bar{m}|^{3/2}$ has to be added in the denominators of expressions (44) and (50). These diameters also apply to radially polarized dense light beams, because the mass and angular momentum are both proportional to the same number of photons.

5 Boundary conditions and spin of light beams

A light beam of low photon density can merely be regarded as a stream of non-interacting photons. At high photon densities a unidirectional beam of limited cross-section becomes more complex. The observed angular momentum of such a linearly or elliptically polarized beam has been proposed to be due to transverse spatial derivatives at its boundary [3, 16]. The angular momentum which would have existed for the individual photons in the beam core have been imagined to be substituted by the momentum generated in the boundary region. However, the detailed explanation is so far not clear.

In this section a dense light beam is considered where the individual photons in the beam core overlap each other, such as to form a plane classical electromagnetic (EM) wave as conceived in earlier considerations [7, 8]. Outside the beam there is a vacuum region. The main purpose is to analyze the boundary conditions and the angular momentum of this system.

5.1 Definitions of beam conditions

A beam is considered having an arbitrary cross-section of large size as compared to its characteristic wave lengths. The analysis of a general case with elliptically polarized modes of various wave lengths can be subdivided into a study on each of the included elementary and linearly polarized modes of a specific wave length. A further simplification is provided by the narrow boundary region where the boundary conditions can be applied separately to every small local segment. The analysis is then limited to one linearly polarized core wave. In its turn, this wave can be subdivided into two waves of the same frequency, but having electric field vectors which are perpendicular and parallel to the local segment.

The following analysis starts with an investigation in terms of Maxwell's equations. It then proceeds by the revised theory, first on a flat-shaped configuration with main electric field vectors being either perpendicular or parallel to the boundary. Finally a simplified study is undertaken on a beam of circular cross-section.

5.2 Shortcomings of the conventional analysis

We consider a beam which propagates in the z-direction of a frame (x, y, z) and where every field quantity Q has the form $\hat{Q}(x,y)\exp\left[\mathrm{i}\,(-\omega t + kz)\right]$. The conventional limit of the field equation (13) then reduces to

$$\left[k_0^2 - \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right](\mathbf{E},\mathbf{B}) = 0 \quad (59)$$

where $k_0^2 = k^2 - \left(\frac{\omega}{c}\right)^2$ can have any value. A separable form $X(x)\cdot Y(y)$ of each component then leads to

$$k_0^2 = k_{0x}^2 + k_{0y}^2\,, \quad X''/X = k_{0x}^2\,, \quad Y''/Y = k_{0y}^2\,, \quad (60)$$

where k_{0x}^2 and k_{0y}^2 can have any sign and value. The solution for the electric field becomes

$$\begin{aligned}\bar{E}_\nu = {} & \left[a_{\nu 1}\exp(k_{0x}x) + a_{\nu 2}\exp(-k_{0x}x)\right]\cdot \\ & \cdot\left[b_{\nu 1}\exp(k_{0y}y) + b_{\nu 2}\exp(-k_{0y}y)\right]\end{aligned} \quad (61)$$

with $\nu = x, y, z$ and an analogous form for the magnetic field. The divergences have to vanish identically. With the solution (61), this leads to a purely transverse wave with zero spin as shown by equation (12). Further one should either have $E_x = E_y = 0$ and $B_x = B_y = 0$, or $k_{0x} = k_{0y} = k_0 = 0$ and $\omega^2 = k^2c^2$. There are no transverse derivatives in an exact solution.

The alternative has also be taken into account where k_0 is zero already from the beginning. Then

$$\bar{E}_\nu = (c_{\nu 1}x + c_{\nu 2})\,(d_{\nu 1}y + d_{\nu 2})\,. \quad (62)$$

With these solutions inserted into the condition of vanishing field divergence

$$\bar{E}_x = c_0\,x + c_1\,y + c_2\,,\ \bar{E}_y = c_3\,x - c_0\,y + c_4\,,\ \bar{E}_z = 0\,. \quad (63)$$

All the obtained solutions thus have a vanishing spin, and are not reconcilable with a beam of spatially limited cross-section.

5.3 Revised equations of flat-shaped geometry

We now proceed to a revised analysis of flat-shaped beam geometry. With z still in the direction of propagation and x along the normal of the boundaries of a slab-like beam, all field quantities become independent of y. The velocity vector is given by a form similar to (15), having a small component C_y along the boundary and a large component C_z in the direction of propagation.

Now equation (14) yields the same dispersion relation as (23), and the three component equations reduce to

$$E_x = -\frac{\mathrm{i}}{k\cos^2\alpha}\frac{\partial E_z}{\partial x}\,, \tag{64}$$

$$\left(k^2\cos^2\alpha - \frac{\partial^2}{\partial x^2}\right)\left(E_y + \frac{\sin\alpha}{\cos\alpha}E_z\right) = 0\,. \tag{65}$$

Consequently E_z can be considered as a generating function of E_x and E_y. One solution of equation (65) is found where E_y has the same spatial profile as E_z and

$$E_y = -\frac{\sin\alpha}{\cos\alpha}\,E_z\,. \tag{66}$$

5.4 Two special cases of flat-shaped geometry

A flat-shaped (slab-like) beam is now considered which has a core region $-a < x < a$ and two narrow boundary regions, $-(a+b) < x < -a$ and $a < x < a+b$, with thickness $d = b - a$. With the frame chosen in Section 5.3, we first consider the case where E_x is the main electric component. Within the core a homogeneous linearly polarized EM wave is assumed to exist, having the constant components E_{0x} and B_{0y}. In the boundary region an axial field component E_z is chosen which increases linearly with x, from zero at $x = a$, and in such a way that E_x of equation (64) becomes matched to E_{0x} at $x = a$. In the same region the field E_z further passes a maximum, and then drops to zero at the vacuum interface $x = a + b$. The resulting field E_x is reversed in the boundary layer, having a maximum strength of the order of E_{0x}. With $E_{0x} = \mathrm{O}(1)$ in respect to the smallness parameter $\cos\alpha$, equations (64) and (66) show that $E_z = \mathrm{O}(\cos^2\alpha)$ and $E_y = \mathrm{O}(\cos\alpha)$. Here B_y is of zero order and matches B_{0y} at the edge of the core. The components of the Poynting vector are $S_x = 0$ and

$$S_y \cong c\,(\cos\alpha)\,\varepsilon_0\,E_x^2\,, \qquad S_z \cong c\,(\sin\alpha)\,\varepsilon_0\,E_x^2\,. \tag{67}$$

Thus there is a primary flow of momentum S_z in the direction of propagation, and a secondary flow S_y along the boundary, but no flow across it. The field energy density finally becomes $w_f \cong \varepsilon_0\,E_x^2$.

Turning to the second case where E_y is the main electric component and is parallel to the boundary, there is an EM core wave with the components E_{0y} and B_{0x}. In a small range of x near $x = a$ the axial field E_z is assumed to be constant, and $E_x = 0$. Relation (66) then makes it possible to match E_y to E_{0y} at $x = a$. Moreover, the field E_z is chosen to decrease towards zero when approaching the outer boundary $x = a + b$. According to equation (64) the field E_x increases from zero at $x = a$ to a maximum, and then drops towards zero when approaching the outer boundary at $x = a + b$. Combination of equations (64) and (66) yields

$$|E_x/E_y| = \lambda/2\pi L_{cy}\cos\alpha \tag{68}$$

where $\lambda = 2\pi/k$ and L_{cy} stands for the characteristic length of the derivative of E_y. As an example with $\lambda/L_{cy} = 10^{-4}$ and $\cos\alpha = 10^{-4}$, equation (68) gives a ratio of about 0.16. The Poynting vector components become $S_x = 0$ and

$$S_y = c\,(\cos\alpha)\,\varepsilon_0\,E_y^2\left[1 + \sin^2\alpha\,(E_x/E_y)^2\right]/\sin^2\alpha\,, \tag{69}$$

$$S_z = c\,\varepsilon_0\,E_y^2\left[1 + \sin^2\alpha\,(E_x/E_y)^2\right]/\sin^2\alpha\,. \tag{70}$$

The energy density is $w_f \cong \varepsilon_0 E_y^2$ as long as $E_x^2 \ll E_y^2$.

5.5 Simplified analysis on the spin of a beam

A simplified analysis is performed on a beam of circular cross-section. The frame is redefined for a linearly polarized EM core wave $\mathbf{E}_0 = (E_0, 0, 0)$ and $\mathbf{B}_0 = (0, B_0, 0)$. With the angle θ between the y-direction and the radial direction, the electric components are

$$E_{0\perp} = E_0\sin\theta\,, \qquad E_{0\parallel} = E_0\cos\theta \tag{71}$$

in the perpendicular and parallel directions of the boundary. The solutions of Section 5.4 are now matched to these core components at the inner surface of the boundary layer. The energy density is $w_f = \varepsilon_0\,\mathbf{E}^2$ where $\mathbf{E}^2 = \mathbf{E}_0^2$ at the edge of the beam core.

With the numerical example of Section 5.4 as a reference where $E_x^2 \ll E_y^2$, the Poynting vector components in the transverse direction now add up to

$$S_t = c(\cos\alpha)\,\varepsilon_0\,\mathbf{E}^2\,. \tag{72}$$

The energy density of the beam core can be written as

$$\varepsilon E_0^2 = n_p hc/\lambda \tag{73}$$

where n_p is the number of equivalent photons per unit volume. With the spin $h/2\pi$ of each photon, the core contains a total angular momentum per unit length

$$s_c = r_0^2 n_p h/2 = \varepsilon_0 E_0^2 \lambda r_0^2/2c \tag{74}$$

with r_0 standing for the core radius. From equations (12) and (72) the angular momentum generated per axial length in the boundary layer becomes on the other hand

$$s_b = 2\pi(\cos\alpha)\,\varepsilon\,E_0^2\,f_E\,r_0^2\,d/c \tag{75}$$

where d is the thickness of the boundary layer and $f_E < 1$ is a profile factor of $\mathbf{E}^2$ across the layer. Thus

$$\frac{s_b}{s_c} = \frac{4\pi\,(\cos\alpha) f_E\,d}{\lambda}\,. \tag{76}$$

Here $s_b = s_c$ when the equivalent angular momentum of the core is compensated by that generated in the boundary layer. As an example, for $\lambda = 3{\times}10^{-7}$m, $f_E = 0.2$ and $d = 10^{-3}$m this becomes possible when $\cos\alpha = 10^{-4}$.

5.6 Summary of the analysis on a dense light beam

- Conventional theory leads to a vanishing spin, and is not reconcilable with a beam of limited extensions in its transverse directions. A limited cross-section can only appear in an approximate solution when the characteristic lengths of the transverse derivatives are much larger than the included wavelengths.
- The present revised theory leads to a Poynting vector with a primary component in the direction of propagation, and a secondary component in the transverse directions which generates a spin.
- The angular momentum represented by the spin of the photons in the beam core is substituted by a real spin generated in the boundary layer.
- Even large transverse spatial derivatives and a corresponding limited beam cross-section can exist according to the revised theory.

6 Conclusions

Conventional theory which is based on Maxwell's equations and the associated quantum electrodynamical concepts in the vacuum state includes the condition of zero electric field divergence. When being applied to the physics of the individual photon and of dense light beams, such a theory exhibits a number of discrepancies from experimental evidence. These shortcomings include the absence of spin and of spatially limited geometry in the directions which are transverse to that of the propagation.

The present revised theory on the vacuum state is based on a nonzero electric field divergence which introduces an additional degree of freedom into the field equations, thereby changing the latter and their solutions substantially as compared to the conventional ones. The resulting extended quantum electrodynamics (EQED) makes it possible for both individual photons and for dense light beams to possess a nonzero spin, and to have a spatially limited geometry in the transverse directions. Moreover the individual photon models behave as an entirety in having both particle and wave properties. There are wave-packet solutions with the character of needle radiation which become reconcilable with the photoelectric effect, and with the dot-shaped marks and interference patterns due to individual photons in two-slit experiments.

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On Area Coordinates and Quantum Mechanics in Yang's Noncommutative Spacetime with a Lower and Upper Scale

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We explore Yang's Noncommutative space-time algebra (involving *two* length scales) within the context of QM defined in Noncommutative spacetimes and the holographic area-coordinates algebra in Clifford spaces. Casimir invariant wave equations corresponding to Noncommutative coordinates and momenta in d-dimensions can be recast in terms of *ordinary* QM wave equations in $d+2$-dimensions. It is conjectured that QM over Noncommutative spacetimes (Noncommutative QM) may be described by ordinary QM in *higher* dimensions. Novel Moyal-Yang-Fedosov-Kontsevich star products deformations of the Noncommutative Poisson Brackets are employed to construct star product deformations of scalar field theories. Finally, generalizations of the Dirac-Konstant and Klein-Gordon-like equations relevant to the physics of D-branes and Matrix Models are presented.

1 Introduction

Yang's noncommutative space time algebra [1] is a generalization of the Snyder algebra [2] (where now both coordinates and momenta are not commuting) that has received more attention recently, see for example [3] and references therein. In particular, Noncommutative p-brane actions, for even $p+1$ $=2n$-dimensional world-volumes, were written explicitly [15] in terms of the *novel* Moyal-Yang (Fedosov-Kontsevich) star product deformations [11, 12] of the Noncommutative Nambu Poisson Brackets (NCNPB) that are associated with the *noncommuting* world-volume coordinates q^A, p^A for $A =$ $= 1, 2, 3, \ldots n$. The latter noncommuting coordinates obey the noncommutative Yang algebra with an ultraviolet L_P (Planck) scale and infrared (R) scale cutoff. It was shown why the novel p-brane actions in the "classical" limit $\hbar_{\text{eff}} =$ $= \hbar L_P/R \to 0$ still acquire nontrivial noncommutative *corrections* that differ from ordinary p-brane actions. Super p-branes actions in the light-cone gauge are also amenable to Moyal-Yang star product deformations as well due to the fact that p-branes moving in flat spacetime backgrounds, in the light-cone gauge, can be recast as gauge theories of volume-preserving diffeomorphisms. The most general construction of noncommutative super p-branes actions based on non (anti) commuting superspaces and quantum group methods remains an open problem.

The purpose of this work is to explore further the consequences of Yang's Noncommutative spacetime algebra within the context of QM in Noncommutative spacetimes and the holographic area-coordinates algebra in Clifford spaces [14]. In section 2 we study the interplay among Yang's Noncommutative spacetime algebra and the former area-coordinates algebra in Clifford spaces. In section 3 we show how Casimir invariant wave equations corresponding to Noncommutative coordinates and momenta in D-dimensions, can be recast in terms of ordinary QM wave equations in $D+2$-dimensions. In particular, we shall present explicit solutions of the D'Alambertian operator in the *bulk* of AdS spaces and explain its correspondence with the Casimir invariant wave equations associated with the Yang's Noncommutative spacetime algebra at the projective *boundary* of the conformally compactified AdS spacetime. We conjecture that QM over Noncommutative spacetimes (Noncommutative QM) may be described by ordinary QM in *higher* dimensions.

In section 4 we recur to the *novel* Moyal-Yang (Fedosov-Kontsevich) star products [11, 12] deformations of the Noncommutative Poisson Brackets to construct Moyal-Yang star product deformations of scalar field theories. The role of star products in the construction of p-branes actions from the large N limit of $SU(N)$ Yang-Mills can be found in [6] and in the Self-Dual Gravity/$SU(\infty)$ Self Dual Yang-Mills relation in [7, 8, 9, 10]. Finally, in the conclusion 5, we present the generalizations of the Dirac-Konstant equations (and their "square" Klein-Gordon type equations) that are relevant to the incorporation of fermions and the physics of D-branes and Matrix Models.

2 Noncommutative Yang's spacetime algebra in terms of area-coordinates in Clifford spaces

The main result of this section is that there is a *subalgebra* of the C-space operator-valued coordinates [13] which is *isomorphic* to the Noncommutative Yang's spacetime algebra [1, 3]. This, in conjunction to the discrete spectrum of angular momentum, leads to the discrete area quantization in multiples of Planck areas. Namely, the $4D$ Yang's Noncommutative space-time algebra [3] (written in terms of $8D$ phase-space coordinates) is isomorphic to the 15-dimensional *subalgebra* of the C-space operator-valued coordinates associated with the *holographic areas* of C-space. This connection between Yang's algebra and the $6D$ Clifford algebra is possible because the $8D$ phase-space coordinates x^μ, p^μ (associated to a $4D$ spacetime) have a one-to-one correspondence to the $\hat{X}^{\mu 5}$; $\hat{X}^{\mu 6}$ holographic area-coordinates of the C-space (corresponding to the $6D$ Clifford algebra). Furhermore, Tanaka [3] has shown that the Yang's algebra [1] (with 15 generators) is related to the $4D$ conformal algebra (15 generators) which in turn is isomorphic to a subalgebra of the $4D$ Clifford algebra because it is known that the 15 generators of the $4D$ conformal algebra $SO(4,2)$ can be explicitly realized in terms of the $4D$ Clifford algebra as shown in [13].

The correspondence between the holographic area coordinates $X^{AB} \leftrightarrow \lambda^2 \Sigma^{AB}$ and the angular momentum variables when $A, B = 1, 2, 3, \ldots 6$ yields an isomorphism between the holographic area coordinates algebra in Clifford spaces [14] and the noncommutative Yang's spacetime algebra in $D = 4$. The scale λ is the ultraviolet lower Planck scale. We begin by writing the exchange algebra between the position and momentum coordinates encapsulated by the commutator

$$\begin{gathered} \left[\hat{X}^{\mu 6}, \hat{X}^{56}\right] = -i\lambda^2 \eta^{66} \hat{X}^{\mu 5} \;\leftrightarrow \\ \left[\frac{\lambda^2 R}{\hbar}\, \hat{p}^\mu,\; \lambda^2 \Sigma^{56}\right] = -i\lambda^2 \eta^{66} \lambda \hat{x}^\mu \end{gathered} \tag{2.1}$$

from which we can deduce that

$$\left[\hat{p}^\mu,\; \Sigma^{56}\right] = -i\eta^{66} \frac{\hbar}{\lambda R}\, \hat{x}^\mu, \tag{2.2}$$

hence, after using the definition $\mathcal{N} = (\lambda/R)\,\Sigma^{56}$, where R is the infrared upper scale, one has the exchange algebra commutator of p^μ and $\mathcal{N}$ of the Yang's spacetime algebra given by

$$[\hat{p}^\mu, \mathcal{N}] = -i\eta^{66} \frac{\hbar}{R^2}\, \hat{x}^\mu. \tag{2.3}$$

From the commutator

$$\begin{gathered} \left[\hat{X}^{\mu 5}, \hat{X}^{56}\right] = -\left[\hat{X}^{\mu 5}, \hat{X}^{65}\right] = i\eta^{55} \lambda^2 \hat{X}^{\mu 6} \;\leftrightarrow \\ \left[\lambda \hat{x}^\mu, \lambda^2 \Sigma^{56}\right] = i\eta^{55} \lambda^2 \lambda^2 \frac{R}{\hbar}\, \hat{p}^\mu \end{gathered} \tag{2.4}$$

we can deduce that

$$\left[\hat{x}^\mu, \Sigma^{56}\right] = i\eta^{55} \frac{\lambda R}{\hbar}\, \hat{p}^\mu \tag{2.5}$$

and after using the definition $\mathcal{N} = (\lambda/R)\,\Sigma^{56}$ one has the exchange algebra commutator of x^μ and $\mathcal{N}$ of the Yang's spacetime algebra

$$[\hat{x}^\mu, \mathcal{N}] = i\eta^{55} \frac{\lambda^2}{\hbar}\, \hat{p}^\mu. \tag{2.6}$$

The other relevant holographic area-coordinates commutators in C-space are

$$\left[\hat{X}^{\mu 5}, \hat{X}^{\nu 5}\right] = -i\eta^{55} \lambda^2 \hat{X}^{\mu\nu} \leftrightarrow [\hat{x}^\mu, \hat{x}^\nu] = -i\eta^{55} \lambda^2 \Sigma^{\mu\nu} \tag{2.7}$$

that yield the noncommuting coordinates algebra after having used the representation of the C-space operator holographic area-coordinates

$$i\hat{X}^{\mu\nu} \leftrightarrow i\lambda^2 \frac{1}{\hbar} \mathcal{M}^{\mu\nu} = i\lambda^2 \Sigma^{\mu\nu}, \quad i\hat{X}^{56} \leftrightarrow i\lambda^2 \Sigma^{56}, \tag{2.8}$$

where we appropriately introduced the Planck scale λ as one should to match units. From the correspondence

$$\hat{p}^\mu = \frac{\hbar}{R}\, \Sigma^{\mu 6} \leftrightarrow \frac{\hbar}{R} \frac{1}{\lambda^2}\, \hat{X}^{\mu 6} \tag{2.9}$$

one can obtain nonvanishing momentum commutator

$$\left[\hat{X}^{\mu 6}, \hat{X}^{\nu 6}\right] = -i\eta^{66} \lambda^2 \hat{X}^{\mu\nu} \leftrightarrow [\hat{p}^\mu, \hat{p}^\nu] = -i\eta^{66} \frac{\hbar^2}{R^2} \Sigma^{\mu\nu}. \tag{2.10}$$

The signatures for AdS_5 space are $\eta^{55} = +1$; $\eta^{66} = -1$ and for the *Euclideanized* AdS_5 space are $\eta^{55} = +1$ and $\eta^{66} = +1$. Yang's space-time algebra corresponds to the latter case. Finally, the *modified* Heisenberg algebra can be read from the following C-space commutators

$$\begin{gathered} \left[\hat{X}^{\mu 5}, \hat{X}^{\nu 6}\right] = i\eta^{\mu\nu} \lambda^2 \hat{X}^{56} \;\leftrightarrow \\ [\hat{x}^\mu, \hat{p}^\mu] = i\hbar \eta^{\mu\nu} \frac{\lambda}{R}\, \Sigma^{56} = i\hbar \eta^{\mu\nu} \mathcal{N}. \end{gathered} \tag{2.11}$$

Eqs-(2.1–2.11) are the defining relations of Yang's Noncommutative $4D$ spacetime algebra [1] involving the $8D$ phase-space variables. These commutators obey the Jacobi identities. There are other commutation relations like $[\mathcal{M}^{\mu\nu}, x^\rho], \ldots$ that we did not write down. These are just the well known rotations (boosts) of the coordinates and momenta.

When $\lambda \to 0$ and $R \to \infty$ one recovers the ordinary *commutative* spacetime algebra. The Snyder algebra [2] is recovered by setting $R \to \infty$ while leaving λ intact. To recover the ordinary Weyl-Heisenberg algebra is more subtle. Tanaka [3] has shown the the *spectrum* of the operator $\mathcal{N} = (\lambda/R)\,\Sigma^{56}$ is discrete given by $n(\lambda/R)$. This is not suprising since the angular momentum generator $\mathcal{M}^{56}$ associated with the *Euclideanized* AdS_5 space is a rotation in the now compact $x^5 - x^6$ directions. This is not the case in AdS_5 space since $\eta^{66} = -1$ and this timelike direction is no longer compact. Rotations involving timelike directions are equivalent to noncompact boosts with a continuous spectrum.

In order to recover the standard Weyl-Heisenberg algebra from Yang's Noncommutative spacetime algebra, and the standard uncertainty relations $\Delta x \Delta p \geqslant \hbar$ with the ordinary $\hbar$ term, rather than the $n\hbar$ term, one needs to take the limit $n \to \infty$ limit in such a way that the net combination of $n\frac{\lambda}{R} \to 1$. This can be attained when one takes the *double* scaling limit of the quantities as follows

$$\begin{gathered} \lambda \to 0, \quad R \to \infty\,, \quad \lambda R \to L^2, \\ \lim_{n\to\infty} n\,\frac{\lambda}{R} = n\,\frac{\lambda^2}{\lambda R} = \frac{n\lambda^2}{L^2} \to 1. \end{gathered} \tag{2.12}$$

From eq-(2.12) one learns then that

$$n\lambda^2 = \lambda R = L^2. \tag{2.13}$$

The spectrum n corresponds to the quantization of the angular momentum operator in the $x^5 - x^6$ direction (after embedding the $5D$ hyperboloid of throat size R onto $6D$). Tanaka [3] has shown why there is a *discrete spectra* for the *spatial* coordinates and *spatial* momenta in Yang's spacetime algebra that yields a *minimum* length λ (ultraviolet cutoff in energy) and a minimum momentum $p=\hbar/R$ (maximal length R, infrared cutoff). The energy and temporal coordinates had a continous spectrum.

The physical interpretation of the double-scaling limit of eq-(2.12) is that the the area $L^2=\lambda R$ becomes now *quantized* in units of the Planck area λ^2 as $L^2=n\lambda^2$. Thus the quantization of the area (via the double scaling limit) $L^2=\lambda R=n\lambda^2$ is a result of the *discrete* angular momentum spectrum in the x^5-x^6 directions of the Yang's Noncommutative spacetime algebra when it is realized by (angular momentum) differential operators acting on the *Euclideanized* AdS_5 space (two branches of a $5D$ hyperboloid embedded in $6D$). A general interplay between quantum of areas and quantum of angular momentum, for arbitrary values of spin, in terms of the square root of the Casimir $\mathbf{A}\sim\lambda^2\sqrt{j\,(j+1)}$, has been obtained a while ago in Loop Quantum Gravity by using spin-networks techniques and highly technical area-operator regularization procedures [4].

The advantage of this work is that we have arrived at similar (not identical) area-quantization conclusions in terms of minimal Planck areas and a discrete angular momentum spectrum n via the double scaling limit based on Clifford algebraic methods (C-space holographic area-coordinates). This is not surprising since the norm-squared of the holographic Area operator has a correspondence with the quadratic Casimir $\Sigma_{AB}\Sigma^{AB}$ of the conformal algebra $SO(4,2)$ ($SO(5,1)$ in the Euclideanized AdS_5 case). This quadratic Casimir must not be confused with the $SU(2)$ Casimir J^2 with eigenvalues $j\,(j+1)$. Hence, the correspondence given by eqs-(2.3–2.8) gives $\mathbf{A}^2\leftrightarrow\lambda^4\Sigma_{AB}\Sigma^{AB}$.

In [5] we have shown why AdS_4 gravity with a topological term; i. e. an Einstein-Hilbert action with a cosmological constant plus Gauss-Bonnet terms can be obtained from the vacuum state of a **BF**-Chern-Simons-Higgs theory *without* introducing by *hand* the zero torsion condition imposed in the McDowell-Mansouri-Chamsedine-West construction. One of the most salient features of [5] was that a *geometric mean* relationship was found among the cosmological constant Λ_c, the Planck area λ^2 and the AdS_4 throat size squared R^2 given by $(\Lambda_c)^{-1}=(\lambda)^2(R^2)$. Upon setting the throat size to be of the order of the Hubble scale R_H and $\lambda=L_P$ (Planck scale), one recovers the observed value of the cosmological constant $L_P^{-2}R_H^{-2}=L_P^{-4}(L_P/R_H)^2\sim10^{-120}M_P^4$. A similar geometric mean relation is also obeyed by the condition $\lambda R= =L^2(=n\lambda^2)$ in the double scaling limit of Yang's algebra which suggests to identify the cosmological constant as $\Lambda_c= =L^{-4}$. This geometric mean condition remains to be investigated further. In particular, we presented the preliminary steps how to construct a Noncommutative Gravity via the Vasiliev-Moyal star products deformations of the $SO(4,2)$ algebra used in the study of higher conformal massless spin theories in AdS spaces by taking the inverse-throat size $1/R$ as a deformation parameter of the $SO(4,2)$ algebra. A Moyal deformation of ordinary Gravity via $SU(\infty)$ gauge theories was advanced in [7].

3 Noncommutative QM in Yang's spacetime from ordinary QM in higher dimensions

In order to write wave equations in non-commuting spacetimes we start with a Hamiltonian written in *dimensionless* variables involving the terms of the relativistic oscillator (let us say oscillations of the center of mass) and the rigid rotor/top terms (rotations about the center of mass)

$$H=\left(\frac{p_\mu}{\hbar/R}\right)^2+\left(\frac{x_\mu}{L_P}\right)^2+\left(\Sigma^{\mu\nu}\right)^2 \tag{3.1}$$

with the fundamental difference that the coordinates x^μ and momenta p^μ obey the non-commutative Yang's space time algebra. For this reason one *cannot* naively replace p^μ any longer by the differential operator $-i\hbar\partial/\partial x^\mu$ nor write the $\Sigma^{\mu\nu}$ generators as $\frac{1}{\hbar}(x^\mu\partial_{x_\nu}-x^\nu\partial_{x_\mu})$. The correct coordinate realization of Yang's noncommutative spacetime algebra requires, for example, embedding the 4-dim space into 6-dim and expressing the coordinates and momenta operators as follows

$$\begin{aligned}
&\frac{p_\mu}{(\hbar/R)}\;\leftrightarrow\;\Sigma^{\mu6}=i\,\frac{1}{\hbar}\left(X^\mu\partial_{X_6}-X^6\partial_{X_\mu}\right),\\
&\frac{x_\mu}{L_P}\;\leftrightarrow\;\Sigma^{\mu5}=i\,\frac{1}{\hbar}\left(X^\mu\partial_{X_5}-X^5\partial_{X_\mu}\right),\\
&\Sigma^{\mu\nu}\;\leftrightarrow\;i\,\frac{1}{\hbar}\left(X^\mu\partial_{X_\nu}-X^\nu\partial_{X_\mu}\right),\\
&\mathcal{N}=\Sigma^{56}\;\leftrightarrow\;i\,\frac{1}{\hbar}\left(X^5\partial_{X_6}-X^6\partial_{X_5}\right).
\end{aligned} \tag{3.2}$$

This allows to express H in terms of the standard angular momentum operators in 6-dim. The $X^A=X^\mu,X^5,X^6$ coordinates ($\mu=1,2,3,4$) and $P^A=P^\mu,P^5,P^6$ momentum variables obey the standard commutation relations of ordinary QM in 6-dim, namely — $\left[X^A,X^B\right]=0$, $\left[P^A,P^B\right]=0$, $\left[X^A,P^B\right]=i\hbar\eta^{AB}$, so that the momentum admits the standard realization as $P^A=-i\hbar\partial/\partial X_A$.

Therefore, concluding, the Hamiltonian H in eq-(3.1) associated with the non-commuting coordinates x^μ and momenta p^μ in $d-1$-dimensions can be written in terms of the standard angular momentum operators in $(d-1)+2=d+1$-dim as $H=\mathcal{C}_2-\mathcal{N}^2$, where $\mathcal{C}_2$ agrees precisely with the quadratic Casimir operator of the $SO(d-1,2)$ algebra in the spin $s=0$ case,

$$\mathcal{C}_2=\Sigma_{AB}\Sigma^{AB}=(X_A\partial_B-X_B\partial_A)(X^A\partial^B-X^B\partial^A). \tag{3.4}$$

One remarkable feature is that $\mathcal{C}_2$ also agrees with the d'Alambertian operator for the Anti de Sitter Space AdS_d of *unit radius* (throat size) $(D_\mu D^\mu)_{AdS_d}$ as shown by [18].

The proof requires to show that the d'Alambertian operator for the $d+1$-dim embedding space (expressed in terms of the X^A coordinates) is related to the d'Alambertian operator in AdS_d space of *unit* radius expressed in terms of the $z^1, z^2, \ldots, z^d$ *bulk intrinsic* coordinates as

$$(D_\mu D^\mu)_{R^{d+1}} = -\frac{\partial^2}{\partial\rho^2} - \frac{d}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}(D_\mu D^\mu)_{AdS} \Rightarrow$$
$$C_2 = \rho^2 (D_\mu D^\mu)_{R^{d+1}} + \left[(d-1) + \rho\frac{\partial}{\partial\rho}\right]\rho\frac{\partial}{\partial\rho} = (D_\mu D^\mu)_{AdS_d}. \tag{3.5}$$

This result is just the hyperbolic-space generalization of the standard decomposition of the Laplace operator in spherical coordinates in terms of the radial derivatives plus a term containing the square of the orbital angular momentum operator L^2/r^2. In the case of nontrivial spin, the Casimir $C_2 = \Sigma_{AB}\Sigma^{AB} + S_{AB}S^{AB}$ has additional terms stemming from the spin operator.

The quantity $\Phi(z^1, z^2, \ldots, z^d)|_{\text{boundary}}$ restricted to the $d-1$-dim projective boundary of the conformally compactified AdS_d space (of unit throat size, whose topology is $S^{d-2} \times S^1$) is the sought-after solution to the Casimir invariant wave equation associated with the non-commutative x^μ coordinates and momenta p^μ of the Yang's algebra ($\mu = 1, 2, \ldots, d-1$). Pertaining to the boundary of the conformally compactified AdS_d space, there are two radii R_1, R_2 associated with S^{d-2} and S^1, respectively, and which must not be confused with the two scales R, L_P appearing in eq-(3.1). One can choose the units such that the present value of the Hubble scale (taking the Hubble scale as the infrared cutoff) is $R=1$. In these units the Planck scale L_P will be of the order of $L_P \sim 10^{-60}$. In essence, there has been a trade-off of two scales L_P, R with the two radii R_1, R_2.

Once can parametrize the coordinates of $AdS_d = AdS_{p+2}$ by writing there, according to [17], $X_0 = R\cosh(\rho)\cos(\tau)$, $X_{p+1} = R\cosh(\rho)\sin(\tau)$, $X_i = R\sinh(\rho)\,\Omega_i$.

The metric of $AdS_d = AdS_{p+2}$ space in these coordinates is $ds^2 = R^2\left[-(\cosh^2\rho)d\tau^2 + d\rho^2 + (\sinh^2\rho)d\Omega^2\right]$, where $0 \leqslant \rho$ and $0 \leqslant \tau < 2\pi$ are the global coordinates. The topology of this hyperboloid is $S^1 \times R^{p+1}$. To study the causal structure of AdS it is convenient to unwrap the circle S^1 (closed-timelike coordinate τ) to obtain the universal covering of the hyperboloid without closed-timelike curves and take $-\infty \leqslant \tau \leqslant +\infty$. Upon introducing the new coordinate $0 \leqslant \theta < \frac{\pi}{2}$ related to ρ by $\tan(\theta) = \sinh(\rho)$, the metric is

$$ds^2 = \frac{R^2}{\cos^2\theta}\left[-d\tau^2 + d\theta^2 + (\sinh^2\rho)\,d\Omega^2\right]. \tag{3.6}$$

It is a conformally-rescaled version of the metric of the Einstein static universe. Namely, $AdS_d = AdS_{p+2}$ can be conformally mapped into one-half of the Einstein static universe, since the coordinate θ takes values $0 \leqslant \theta < \frac{\pi}{2}$ rather than $0 \leqslant \theta < \pi$. The boundary of the conformally compactified AdS_{p+2} space has the topology of $S^p \times S^1$ (identical to the conformal compactification of the $p+1$-dim Minkowski space). Therefore, the equator at $\theta = \frac{\pi}{2}$ is a *boundary* of the space with the topology of S^p. Ω_p is the solid angle coordinates corresponding to S^p and τ is the coordinate which parametrizes S^1. For a detailed discussion of AdS spaces and the AdS/CFT duality see [17].

The d'Alambertian in AdS_d space (of radius R, later we shall set $R=1$) is

$$D_\mu D^\mu = \frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu) =$$
$$= \frac{\cos^2\theta}{R^2}\left[-\partial_\tau^2 + \frac{1}{(R\tan\theta)^p}\partial_\theta\left((R\tan\theta)^p\partial_\theta\right)\right] + \frac{\mathcal{L}^2}{R^2\tan^2\theta} \tag{3.7}$$

where $\mathcal{L}^2$ is the Laplacian operator in the p-dim sphere S^p whose eigenvalues are $l\,(l+p-1)$.

The scalar field can be decomposed as follows $\Phi = e^{\omega R\tau} Y_l(\Omega_p) G(\theta)$; the wave equation $(D_\mu D^\mu - m^2)\Phi = 0$ leads to the equation $\Big[\cos^2\theta\left(\omega^2 + \partial_\theta^2 + \frac{p}{\tan\theta\cos^2\theta}\,\partial_\theta\right) + \frac{l(l+p-1)}{\tan^2\theta} - m^2R^2\Big]G(\theta) = 0$, whose solution is

$$G(\theta) = (\sin\theta)^l\,(\cos\theta)^{\lambda_\pm}\,{}_2F_1(a, b, c; \sin\theta)\,. \tag{3.8}$$

The hypergeometric function is defined as

$${}_2F_1(a, b, c, z) = \sum \frac{(a)_k(b)_k}{(c)_k k!}\,z^n, \tag{3.9}$$

where $|z| < 1$, $(\lambda)_0 = 1$, $(\lambda)_k = \frac{\Gamma(\lambda+k)}{\Gamma(\lambda)} = \lambda(\lambda+1)(\lambda+2)\ldots(\lambda+k-1)$, $k = 1, 2, \ldots$, while $a = \frac{1}{2}(l + \lambda_\pm - \omega R)$, $b = \frac{1}{2}(l + \lambda_\pm + \omega R)$, $c = l + \frac{1}{2}(p+1) > 0$, $\lambda_\pm = \frac{1}{2}(p+1) \pm \frac{1}{2}\sqrt{(p+1)^2 + 4(mR)^2}$.

The analytical continuation of the hypergeometric function for $|z| \geqslant 1$ is

$${}_2F_1(a, b, c, z) =$$
$$= \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt \tag{3.10}$$

with $Real\,(c) > 0$ and $Real\,(b) > 0$. The boundary value when $\theta = \frac{\pi}{2}$ gives

$$\lim_{z\to 1^-} F(a, b, c; z) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\,. \tag{3.11}$$

Let us study the behaviour of the solution $G(\theta)$ in the massless case: $m = 0$, $\lambda_- = 0$, $\lambda_+ = p+1$.

Solutions with $\lambda_+ = p+1$ yield a trivial value of $G(\theta) = 0$ at the boundary $\theta = \frac{\pi}{2}$ since $\cos\left(\frac{\pi}{2}\right)^{p+1} = 0$. Solutions with $\lambda_- = 0$ lead to $\cos(\theta)^{\lambda_-} = \cos(\theta)^0 = 1$ *prior* to taking the limit $\theta = \frac{\pi}{2}$. The expression $\cos\left(\frac{\pi}{2}\right)^{\lambda_-} = 0^0$ is ill defined. Upon using l'Hospital rule it yields 0. Thus, the limit $\theta = \frac{\pi}{2}$ must be taken afterwards the limit $\lambda_- = 0$:

$$\lim_{\theta\to\frac{\pi}{2}}\left[\cos(\theta)^{\lambda_-}\right] = \lim_{\theta\to\frac{\pi}{2}}\left[\cos(\theta)^0\right] = \lim_{\theta\to\frac{\pi}{2}}[1] = 1. \tag{3.12}$$

In this fashion the value of $G(\theta)$ is well defined and nonzero at the boundary when $\lambda_- = 0$ and leads to the value of the wavefunction at the boundary of the conformally compactified AdS_d (for $d = p + 2$ with radius R)

$$\Phi_{\text{bound}} = e^{i\omega\tau} Y_l(\Omega_p) \frac{\Gamma\left(l + \frac{p+1}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\omega R + \frac{l+p+1}{2}\right)\Gamma\left(-\omega R + \frac{l+p+1}{2}\right)} . \quad (3.13a)$$

upon setting the radius of AdS_d space to *unity* it gives

$$\Phi_{\text{bound}} = e^{i\omega\tau} Y_l(\Omega_p) \frac{\Gamma\left(l + \frac{p+1}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\omega + \frac{l+p+1}{2}\right)\Gamma\left(-\omega + \frac{l+p+1}{2}\right)} . \quad (3.13b)$$

Hence, Φ_{bound} in eq-(3.13b) is the solution to the Casimir invariant wave equation in the *massless* $m = 0$ case

$$C_2\Phi = \left[\left(\frac{p_\mu}{\hbar/R}\right)^2 + \left(\frac{x_\mu}{L_P}\right)^2 + \left(\Sigma^{\mu\nu}\right)^2 + \mathcal{N}^2\right]\Phi = 0 \quad (3.14)$$

and (when $R = 1$)

$$\left[\left(\frac{p_\mu}{\hbar/R}\right)^2 + \left(\frac{x_\mu}{L_P}\right)^2 + \left(\Sigma^{\mu\nu}\right)^2\right]\Phi = \left[C_2 - \mathcal{N}^2\right]\Phi = -\omega^2\Phi \quad (3.15)$$

since $\mathcal{N} = \Sigma^{56}$ is the rotation generator along the S^1 component of AdS space. It acts as $\partial/\partial\tau$ only on the $e^{i\omega R\tau}$ piece of Φ. Concluding: $\Phi(z^1, z^2, \ldots, z^d)|_{\text{boundary}}$, restricted to the $d-1$-dim projective boundary of the conformally compactified AdS_d space (of *unit* radius and topology $S^{d-2} \times S^1$) given by eq-(3.12), is the sought-after solution to the wave equations (3.13, 3.14) associated with the non-commutative x^μ coordinates and momenta p^μ of the Yang's algebra and where the indices μ range over the dimensions of the *boundary* $\mu = 1, 2, \ldots, d-1$. This suggests that QM over Yang's Noncommutative Spacetimes could be well defined in terms of ordinary QM in *higher* dimensions! This idea deserves further investigations. For example, it was argued by [16] that the *quantized* Nonabelian gauge theory in d dimensions can be obtained as the infrared limit of the corresponding *classical* gauge theory in $d + 1$-dim.

4 Star products and noncommutative QM

The ordinary Moyal star-product of two functions in phase space $f(x,p)$, $g(x,p)$ is

$$\begin{aligned}(f * g)(x,p) = \sum_s \frac{\hbar^s}{s!} \sum_{t=0}^{s} (-1)^t C(s,t) \times \\ \times \left(\partial_x^{s-t}\partial_p^t f(x,p)\right)\left(\partial_x^t \partial_p^{s-t} g(x,p)\right)\end{aligned} \quad (4.1)$$

where $C(s,t)$ is the binomial coefficient $s!/t!(s-t)!$. In the $\hbar \to 0$ limit the star product $f * g$ reduces to the ordinary pointwise product fg of functions. The Moyal product of two functions of the $2n$-dim phase space coordinates (q_i, p_i) with $i = 1, 2 \ldots n$ is

$$\begin{aligned}(f * g)(x,p) = \sum_i \sum_s \frac{\hbar^s}{s!} \sum_{t=0}^{s} (-1)^t C(s,t) \times \\ \times \left(\partial_{x_i}^{s-t}\partial_{p_i}^t f(x,p)\right)\left(\partial_{x_i}^t \partial_{p_i}^{s-t} g(x,p)\right).\end{aligned} \quad (4.2)$$

The noncommutative, associative Moyal bracket is

$$\{f, g\}_{\text{MB}} = \frac{1}{i\hbar}\left(f * g - g * f\right). \quad (4.3)$$

The task now is to construct *novel* Moyal-Yang star products based on the noncommutative spacetime Yang's algebra. A novel star product deformations of (super) p-brane actions based on the noncommutative spacetime Yang's algebra where the deformation parameter is $\hbar_{\text{eff}} = \hbar L_P/R$, for nonzero values of $\hbar$, was obtained in [15] The modified (noncommutative) Poisson bracket is now given by

$$\begin{aligned}&\{\mathcal{F}(q^m, p^m), \mathcal{G}(q^m, p^m)\}_\Omega = \\ &= (\partial_{q^m}\mathcal{F})\{q^m, q^n\}(\partial_{q^n}\mathcal{G}) + (\partial_{p^m}\mathcal{F})\{p^m, p^n\}(\partial_{p^n}\mathcal{G}) + \\ &+ (\partial_{q^m}\mathcal{F})\{q^m, p^n\}(\partial_{p^n}\mathcal{G}) + (\partial_{p^m}\mathcal{F})\{p^m, q^n\}(\partial_{q^n}\mathcal{G}),\end{aligned} \quad (4.4)$$

where the entries $\{q^m, q^n\} \neq 0$, $\{p^m, p^n\} \neq 0$, and also $\{p^m, q^n\} = -\{q^n, p^m\}$ can be read from the commutators described in section 2 by simply defining the deformation parameter $\hbar_{\text{eff}} \equiv \hbar(L_P/R)$. One can generalize Yang's original 4-dim algebra to noncommutative $2n$-dim world-volumes and/or spacetimes by working with the $2n+2$-dim angular-momentum algebra $SO(d,2) = SO(p+1,2) = SO(2n,2)$.

The Noncommutative Poisson brackets $\Omega(q^m, q^n) = \{q^m, q^n\}_{\text{NCPB}}$, $\Omega(p^m, p^n) = \{p^m, p^n\}_{\text{NCPB}}$, $\Omega(q^m, p^n) = -\Omega(p^n, q^m) = \{q^m, p^n\}_{\text{NCPB}}$

$$\Omega(q^m, q^n) = \lim_{\hbar_{\text{eff}} \to 0} \frac{1}{i\hbar_{\text{eff}}}\left[q^m, q^n\right] = -\frac{L^2}{\hbar}\Sigma^{mn}, \quad (4.5a)$$

$$\Omega(p^m, p^n) = \lim_{\hbar_{\text{eff}} \to 0} \frac{1}{i\hbar_{\text{eff}}}\left[p^m, p^n\right] = -\frac{\hbar}{L^2}\Sigma^{mn}, \quad (4.5b)$$

$$\Omega(q^m, p^n) = \lim_{\hbar_{\text{eff}} \to 0} \frac{1}{i\hbar_{\text{eff}}}\left[q^m, p^n\right] = -\eta^{mn}, \quad (4.5c)$$

defined by above expressions, where Σ^{mn} is the "classical" $\hbar_{\text{eff}} = (\hbar L_P/R) \to 0$ limit ($R \to \infty$, $L_P \to 0$, $RL_P = L^2$, $\hbar \neq 0$) of the quantity $\Sigma^{mn} = \frac{1}{\hbar}(X^m P^n - X^n P^m)$, after embedding the $d-1$-dimensional spacetime (boundary of AdS_d) into an ordinary $(d-1)+2$-dimensional one. In the $R \to \infty$, ... limit, the AdS_d space (the hyperboloid) degenerates into a *flat* Minkowski spacetime and the coordinates q^m, p^n, in that infrared limit, *coincide* with the coordinates X^m, P^n. Concluding, in the "classical" limit ($R \to \infty, \ldots$, flat limit) one has

$$\Sigma^{mn} \equiv \frac{1}{\hbar}\left(X^m P^n - X^n P^m\right) \to \frac{1}{\hbar}\left(q^m p^n - q^n p^m\right) \quad (4.5d)$$

and then one recovers in that limit the ordinary definition of the angular momentum in terms of commuting coordinates q's and commuting momenta p's.

Denoting the coordinates (q^m, p^m) by Z^m and when the Poisson structure Ω^{mn} is given in terms of *constant* numerical coefficients, the Moyal star product is defined in terms of the deformation parameter $\hbar_{\text{eff}} = \hbar L_P/R$ as

$$(\mathcal{F} * \mathcal{G})(z) \equiv \\ \equiv \exp\left[(i\hbar_{\text{eff}})\,\Omega^{mn}\,\partial_m^{(z_1)}\partial_n^{(z_2)}\right]\mathcal{F}(z_1)\mathcal{G}(z_2)|_{z_1=z_2=z} \tag{4.6}$$

where the derivatives $\partial_m^{(z_1)}$ act only on the $\mathcal{F}(z_1)$ term and $\partial_n^{(z_2)}$ act only on the $\mathcal{G}(z_2)$ term. In our case the generalized Poisson structure Ω^{mn} is given in terms of *variable* coefficients, it is a function of the coordinates, then $\partial\Omega^{mn} \neq 0$, since the Yang's algebra is basically an angular momentum algebra, therefore the suitable Moyal-Yang star product given by Kontsevich [11] will contain the appropriate *corrections* $\partial\Omega^{mn}$ to the ordinary Moyal star product.

Denoting by $\partial_m = \partial/\partial z^m = (\partial/\partial q^m; \partial/\partial p^m)$ the Moyal-Yang-Kontsevich star product, let us say, of the Hamiltonian $H(q,p)$ with the density distribution in phase space $\rho\,(q,p)$ (not necessarily positive definite), $H(q,p) * \rho\,(q,p)$ is

$$\begin{aligned} &H\rho + i\hbar_{\text{eff}}\Omega^{mn}(\partial_m H \partial_n \rho) + \\ &+ \frac{(i\hbar_{\text{eff}})^2}{2}\,\Omega^{m_1n_1}\Omega^{m_2n_2}(\partial^2_{m_1m_2}H)(\partial^2_{n_1n_2}\rho) + \\ &+ \frac{(i\hbar_{\text{eff}})^2}{3}\left[\Omega^{m_1n_1}(\partial_{n_1}\Omega^{m_2n_2}) \times \right. \\ &\left. \times (\partial_{m_1}\partial_{n_2}H\partial_{n_2}\rho - \partial_{m_2}H\partial_{m_1}\partial_{n_2}\rho)\right] + \mathrm{O}(\hbar^3_{\text{eff}})\,, \end{aligned} \tag{4.7}$$

where the explicit components of Ω^{mn} are given by eqs-(4.5a–4.5d). The Kontsevich star product is associative up to second order [11] $(f * g) * h = f * (g * h) + \mathrm{O}(\hbar^3_{\text{eff}})$.

The most general expression of the Kontsevich star product in Poisson manifold is quite elaborate and shall not be given here. Star products in *curved* phase spaces have been constructed by Fedosov [12]. Despite these technical subtlelties it did not affect the final expressions for the "classical" Noncommutative p-brane actions as shown in [15] when one takes the $\hbar_{\text{eff}} \to 0$ "classical" limit. In that limit there are still *nontrivial noncommutative corrections* to the ordinary p-brane actions.

In the Weyl-Wigner-Gronewold-Moyal quantization scheme in phase spaces one writes

$$H(x,p) * \rho\,(x,p) = \rho\,(x,p) * H(x,p) = E\rho\,(x,p)\,, \tag{4.8}$$

where the Wigner density function in phase space associated with the Hilbert space state $|\Psi>$ is

$$\rho\,(x,p,\hbar) = \frac{1}{2\pi}\int dy\;\Psi^*\Big(x - \frac{\hbar y}{2}\Big)\Psi\Big(x + \frac{\hbar y}{2}\Big)e^{\frac{ipy}{\hbar}} \tag{4.9}$$

plus their higher dimensional generalizations. It remains to be studied if this Weyl-Wigner-Gronewold-Moyal quantization scheme is appropriate to study QM over Noncommutative Yang's spacetimes when we use the above Moyal-Yang-Kontsevich star products. A recent study of the Yang's Noncommutative algebra and *discrete* Hilbert (Buniy-Hsu-Zee) spaces was undertaken by Tanaka [3].

Let us write down the Moyal-Yang-Konstevich star deformations of the field theory Lagrangian corresponding to the scalar field $\Phi = \Phi(X^{AB})$ which depends on the holographic-area coordinates X^{AB} [13]. The reason one should *not* try to construct the star product of $\Phi(x^m) * \Phi(x^n)$ based on the Moyal-Yang-Kontsevich product, is because the latter star product given by eq-(4.7) will introduce explicit *momentum* terms in the r.h.s of $\Phi(x^m) * \Phi(x^m)$, stemming from the expression $\Sigma^{mn} = x^m p^n - x^n p^m$ of eq-(4.5d), and thus it invalidates writing $\phi = \phi(x)$ in the first place. If the Σ^{mn} were *numerical constants*, like Θ^{mn}, then one could write the $\Phi(x^m) * \Phi(x^m)$ in a straightforward fashion as it is done in the literature.

The reason behind choosing $\Phi = \Phi(X^{AB})$ is more clear after one invokes the area-coordinates and angular momentum correspondence discussed in detail in section 2. It allows to properly define the star products. A typical Lagrangian is

$$\begin{aligned} \mathcal{L} = &-\Phi * \partial^2_{X^{AB}}\Phi\big(X^{AB}\big) + \frac{m^2}{2}\,\Phi\big(X^{AB}\big) * \Phi\big(X^{AB}\big) + \\ &+ \frac{g^n}{n}\,\Phi\big(X^{AB}\big) * \Phi\big(X^{AB}\big) * \cdots *_n \Phi\big(X^{AB}\big) \end{aligned} \tag{4.10}$$

and leads to the equations of motion

$$\begin{aligned} &-\big(\partial/\partial X^{AB}\big)\big(\partial/\partial X^{AB}\big)\Phi\big(X^{AB}\big) + m^2\Phi\big(X^{AB}\big) + \\ &+ g^n\,\Phi\big(X^{AB}\big) * \Phi\big(X^{AB}\big) * \cdots *_{n-1}\Phi\big(X^{AB}\big) = 0 \end{aligned} \tag{4.11}$$

when the multi-symplectic Ω^{ABCD} form is coordinate-independent, the star product is

$$\begin{aligned} &(\Phi * \Phi)(Z^{AB}) \equiv \exp\Big[\big(i\lambda\Omega^{ABCD}\partial_{X^{AB}}\partial_{Y^{AB}}\big)\Big] \times \\ &\times \Phi(X^{AB})\,\Phi\big(Y^{AB}\big)\big|_{X=Y=Z} = \\ &= \exp\Big[\big(\Sigma^{ABCD}\partial_{X^{AB}}\partial_{Y^{AB}}\big)\Big]\Phi\big(X^{AB}\big)\Phi\big(Y^{AB}\big)\big|_{X=Y=Z} \end{aligned} \tag{4.12}$$

where Σ^{ABCD} is derived from the structure constants of the holographic area-coordinate algebra in C-spaces [14] as: $\big[X^{AB}, X^{CD}\big] = \Sigma^{ABCD} \equiv iL_P^2\big(\eta^{AD}X^{BC} - \eta^{AC}X^{BD} + \eta^{BC}X^{AD} - \eta^{BD}X^{AC}\big)$. There are nontrivial derivative terms acting on Σ^{ABCD} in the definition of the star product $(\Phi * \Phi)(Z^{MN})$ as we have seen in the definition of the Kontsevich star product $H(x,p) * \rho\,(x,p)$ in eq-(4.7). The expansion parameter in the star product is the Planck scale squared $\lambda = L_P^2$. The star product has the same functional form as (4-7) with the only difference that now we are taking derivatives w.r.t the area-coordinates X^{AB} instead of derivatives w.r.t the variables x, p, hence to order $\mathrm{O}(L_P^4)$, the star product is

$$\begin{aligned} &\Phi * \Phi = \Phi^2 + \Sigma^{ABCD}(\partial_{AB}\,\Phi\,\partial_{CD}\,\Phi) + \\ &+ \frac{1}{2}\,\Sigma^{A_1B_1C_1D_1}\Sigma^{A_2B_2C_2D_2}(\partial^2_{A_1B_1A_2B_2}\Phi)(\partial^2_{C_1D_1C_2D_2}\Phi) + \\ &+ \frac{1}{3}\left[\Sigma^{A_1B_1C_1D_1}(\partial_{C_1D_1}\Sigma^{A_2B_2C_2D_2}) \times \right. \\ &\left. \times (\partial_{A_1B_1}\partial_{A_2B_2}\Phi\,\partial_{C_2D_2}\Phi - B_1 \leftrightarrow B_2)\right]. \end{aligned} \tag{4.13}$$

Notice that the powers of iL_P^2 are encoded in the definition of Σ^{ABCD}. The star product is noncommutative but is also nonassociative at the order $O(L_P^6)$ and beyond. The Jacobi identities would be anomalous at that order and beyond. The derivatives acting on Σ^{ABCD} are

$$\begin{aligned}&(\partial_{C_1D_1}\Sigma^{A_2B_2C_2D_2}) = \\ &= i\,L_P^2\left(\eta^{A_2D_2}\,\delta^{B_2C_2}_{C_1D_1} - \eta^{A_2C_2}\,\delta^{B_2D_2}_{C_1D_1}\right) + \\ &+ i\,L_P^2\left(\eta^{B_2C_2}\,\delta^{A_2D_2}_{C_1D_1} - \eta^{B_2D_2}\,\delta^{A_2C_2}_{C_1D_1}\right).\end{aligned} \tag{4.14}$$

where $\delta^{AB}_{CD} = \delta^A_C\delta^B_D - \delta^A_D\delta^B_C$ and the higher derivatives like $\partial^2_{A_1B_1C_1D_1}\Sigma^{A_2B_2C_2D_2}$ will be zero.

5 On the generalized Dirac-Konstant equation in Clifford spaces

To conclude this work we will discuss the wave equations relevant to fermions. The "square" of the Dirac-Konstant equation $(\gamma^{[\mu\nu]}\Sigma_{\mu\nu})\Psi = \lambda\Psi$ yields

$$\begin{aligned}&(\gamma^{[\mu\nu]}\gamma^{[\rho\tau]}\Sigma_{\mu\nu}\Sigma_{\rho\tau})\Psi = \lambda^2\Psi \;\Rightarrow \\ &\Big[\gamma^{[\mu\nu\rho\tau]} + (\eta^{\mu\rho}\gamma^{[\nu\tau]} - \eta^{\mu\tau}\gamma^{[\nu\rho]} + \ldots) + \\ &+ (\eta^{\mu\rho}\eta^{\nu\tau}\mathbf{1} - \eta^{\mu\tau}\eta^{\nu\rho}\mathbf{1})\Big]\,\Sigma_{\mu\nu}\Sigma_{\rho\tau}\Psi = \lambda^2\Psi\end{aligned} \tag{5.2}$$

where we omitted numerical factors. The generalized Dirac equation in Clifford spaces is given by [13]

$$\begin{aligned}-i\Bigg(&\frac{\partial}{\partial\sigma} + \gamma^\mu\frac{\partial}{\partial x^\mu} + \gamma^{[\mu\nu]}\frac{\partial}{\partial x^{\mu\nu}} + \ldots \\ &+ \gamma^{[\mu_1\mu_2\ldots\mu_d]}\frac{\partial}{\partial x^{\mu_1\mu_2\ldots\mu_d}}\Bigg)\Psi = \lambda\Psi\,,\end{aligned} \tag{5.3}$$

where $\sigma, x^\mu, x^{\mu\nu}, \ldots$ are the generalized coordinates associated with the Clifford *polyvector* in C-space

$$X = \sigma\mathbf{1} + \gamma^\mu x_\mu + \gamma^{\mu_1\mu_2}x_{\mu_1\mu_2} + \ldots\gamma^{\mu_1\mu_2\ldots\mu_d}x_{\mu_1\mu_2\ldots\mu_d} \tag{5.4}$$

after the length scale expansion parameter is set to unity. The generalized Dirac-Konstant equations in Clifford-spaces are obtained after introducing the generalized angular momentum operators [14]

$$\begin{aligned}&\Sigma^{[[\mu_1\mu_2\ldots\mu_n][\nu_1\nu_2\ldots\nu_n]]} = X^{[[\mu_1\mu_2\ldots\mu_n]}P^{[\nu_1\nu_2\ldots\nu_n]]} = \\ &= X^{[\mu_1\mu_2\ldots\mu_n]}\frac{i\partial}{\partial X_{[\nu_1\nu_2\ldots\nu_n]}} - X^{[\nu_1\nu_2\ldots\nu_n]}\frac{i\partial}{\partial X_{[\mu_1\mu_2\ldots\mu_n]}}\end{aligned} \tag{5.5}$$

by writing

$$\sum_n \gamma^{[[\mu_1\mu_2\ldots\mu_n]}\gamma^{[\nu_1\nu_2\ldots\nu_n]]}\Sigma_{[[\mu_1\mu_2\ldots\mu_n][\nu_1\nu_2\ldots\nu_n]]}\Psi = \lambda\Psi \tag{5.6}$$

and where we sum over all polyvector-valued indices (antisymmetric tensors of arbitrary rank). Upon squaring eq-(5.4), one obtains the Clifford space extensions of the $D0$-brane field equations found in [3] which are of the form

$$\begin{aligned}&\left[X^{AB}\frac{\partial}{\partial X_{CD}} - X^{CD}\frac{\partial}{\partial X_{AB}}\right]\times \\ &\qquad\times\left[X_{AB}\frac{\partial}{\partial X^{CD}} - X_{CD}\frac{\partial}{\partial X^{AB}}\right]\Psi = 0\,,\end{aligned} \tag{5.6}$$

where $A, B = 1, 2, \ldots, 6$. It is warranted to study all these equations in future work and their relation to the physics of D-branes and Matrix Models [3]. Yang's Noncommutative algebra should be extended to superspaces, meaning non-anti-commuting Grassmanian coordinates and noncommuting bosonic coordinates.

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Open Letter by the Editor-in-Chief: Declaration of Academic Freedom (Scientific Human Rights)
*The Spanish Translation**

Declaración de Libertad Académica

(Derechos científicos del Ser Humano)

Artículo 1: Preámbulo

El comienzo del siglo XXI refleja, más que en cualquier otra época de la historia de la Humanidad, el profundo significado del papel de la Ciencia y la Tecnología en los asuntos humanos.

La naturaleza poderosamente influyente de la Ciencia y Tecnología modernas ha conducido a la percepción general de que los descubrimientos más importantes pueden realizarse principalmente o solamente mediante grandes grupos de investigación gubernamentales o corporativos con acceso a un instrumental enormemente caro y una gran cantidad de personal de apoyo.

La impresión general es, sin embargo, mítica, y oculta la naturaleza real de cómo se realizan los descubrimientos científicos. Los caros y enormes proyectos tecnológicos, independientemente de su complejidad, no son sino el resultado de la aplicación de profundas ideas científicas de pequeños grupos de investigadores incansables o científicos solitarios, quienes con frecuencia trabajan aislados. Un científico que trabaja solo es ahora y en el futuro, y como sucedió en el pasado, capaz de hacer un descubrimiento que pueda influir sustancialmente en el destino de la Humanidad y cambiar la faz del planeta entero en el cual somos unos habitantes insignificantes.

Los descubrimientos revolucionarios son realizados generalmente por individuos que trabajan en posiciones subordinadas en agencias gubernamentales, instituciones de enseñanza e investigación, o en empresas comerciales. Por lo tanto, el investigador está también con frecuencia ligado o limitado por los directores de instituciones y corporaciones, quienes trabajando en una dirección diferente, buscan controlar y aplicar el descubrimiento y la investigación científica para beneficio personal o de una organización, o incluso para su gloria personal.

La memoria histórica de los descubrimientos científicos está repleta de ejemplos de supresión y ridiculización por los poderes establecidos, que aún así se revelaron y reivindicaron en años posteriores por la marcha inexorable de la necesidad práctica y la ilustración intelectual. Así mismo, el registro histórico es modificado y mancillado por el plagio y la deliberada mala interpretación, perpetrados por gente sin escrúpulos, motivados por la envidia y avaricia. Y así es también hoy día.

El objetivo de esta Declaración es mantener y fomentar la doctrina fundamental acerca de que la investigación científica debe estar libre de influencias represivas latentes o abiertas de dirigentes burocráticos, políticos, religiosos o monetarios, y que la creación científica es un derecho humano no inferior a otros derechos y manifestar fervientes esperanzas de que logren expresarse en los tratados internacionales y en la ley internacional.

Todos los científicos que la apoyen, deberán ser fieles a los principios de esta Declaración, como signo de solidaridad con la comunidad científica internacional en cuestión, y para conceder el Derecho de los ciudadanos del mundo a la creación científica de acuerdo a sus aptitudes individuales y disposición, para el avance de la ciencia y, con su extrema capacidad como ciudadanos decentes en un mundo indecente, para el beneficio de la Humanidad. La Ciencia y la Tecnología han sido demasiado tiempo siervos de la opresión.

Artículo 2: Quién es un científico

Un científico es cualquier persona que hace Ciencia. Cualquier persona que colabora con un científico en el desarrollo y propuesta de ideas y datos en la investigación o aplicación es también un científico. La posesión de una cualificación formal no es un prerrequisito para que una persona sea un científico.

Artículo 3: Dónde se produce la Ciencia

La investigación científica se puede desarrollar en cualquier lugar, por ejemplo, en un lugar de trabajo, durante un curso formal de educación, durante un programa académico patrocinado, en grupos, o como individuos que llevan a cabo una investigación independiente en casa.

Artículo 4: Libertad de elección del tema de investigación

Muchos científicos que trabajan en aras de obtener un grado de investigador avanzado o en otros programas de instituciones académicas tales como universidades y centros de estudios avanzados, están limitados para trabajar en un tema de investigación de su propia elección por académicos se-

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nior y/o funcionarios administrativos, no a causa de la falta de instrumentos de apoyo sino a causa de que la jerarquía académica y/o otros funcionarios simplemente no aprueban la línea de pensamiento debido a su potencial conflicto con el dogma preestablecido, teorías favoritas en boga, o la financiación de otros proyectos que pueden ser desacreditados por la investigación propuesta. La autoridad de la mayoría ortodoxa es invocada bastante a menudo para obstaculizar un proyecto de investigación para que la autoridad y los presupuestos no se vean cuestionados. Esta práctica común es una obstrucción deliberada al libre pensamiento científico, es extremadamente anticientífica, y es criminal. No puede ser tolerada.

Un científico que trabaja para cualquier institución académica, autoridad o agencia, tiene que ser completamente libre para elegir un tema de investigación, limitado solamente por los recursos materiales disponibles y las aptitudes intelectuales capaces de ser ofrecidas por la institución académica, agencia o autoridad. Si un científico lleva a cabo una investigación como un miembro de un grupo de colaboración, los directores de investigación y líderes del equipo deberán estar limitados a labores consultivas en relación a la escogencia de un tema relevante de investigación por un científico del grupo.

Artículo 5: Libertad de elección de métodos de investigación

Ocurre con frecuencia que se ejerce presión sobre un científico por parte del personal administrativo o académicos senior en relación a un proyecto de investigación realizado en un medio académico, para forzar al científico a adoptar métodos de investigación diferentes a aquellos que el científico hubiera elegido, sin más razón que la preferencia personal, sesgo, política institucional, mandatos editoriales, o la autoridad colectiva. Esta práctica, que está bastante extendida, es una negación deliberada de la libertad de pensamiento y no debe ser permitida.

Un científico no comercial o académico tiene el derecho de desarrollar un tema de investigación en cualquier forma razonable y por cualquier medio razonable que él considere más efectivo. La decisión final acerca de cómo será realizada debe ser tomada por el científico mismo.

Si un científico no comercial o académico trabaja como un miembro de un equipo no comercial o académico de científicos, los líderes del proyecto y directores de investigación deberán tener solamente derechos consultivos y no deberán en modo alguno influenciar, entorpecer o limitar los métodos o tema de investigación del científico en el grupo.

Artículo 6: Libertad de colaboración y participación en la investigación

Hay un elemento significativo de rivalidad institucional en la práctica de la Ciencia moderna, unida a elementos de envidia personal y la preservación de la reputación y crédito personal a toda costa, independiente de las realidades científicas. Esto ha conducido a menudo a los científicos a abstenerse de invitar a colegas competentes localizados en instituciones rivales u otros sin afiliación académica. Esta práctica es también una obstrucción deliberada del progreso científico.

Si un científico no comercial o académico necesita la colaboración de otra persona y esa otra persona está de acuerdo en ofrecérsela, el científico tiene libertad de invitar a esa persona para prestarle ésa y cualquier otra ayuda, en el caso en que tal ayuda esté en un presupuesto de investigación asociado. Si el auxilio es independiente de las consideraciones del presupuesto, el científico es libre de escoger a la persona a su discreción, libre de toda interferencia por cualquier otra persona quien quiera que sea.

Artículo 7: Libertad de desacuerdo en la discusión científica

Debido a los celos furtivos y al interés adquirido, la Ciencia moderna aborrece la discusión abierta y premeditadamente proscribe a aquellos científicos que cuestionan los puntos de vista ortodoxos. Muy a menudo, científicos de excepcional capacidad, que señalan las deficiencias en las teorías actuales o la interpretación de los datos, son calificados de chiflados, de forma que sus ideas puedan ser convenientemente ignoradas. Ellos son pública y privadamente denostados y sistemáticamente barridos de las convenciones científicas, seminarios y coloquios para que sus ideas no puedan encontrar audiencia. La falsificación deliberada de datos y la interpretación errónea de la teoría son ahora instrumentos frecuentes de personas sin escrúpulos en la supresión de los hechos, tanto técnicos como históricos. Se han formado comités internacionales de científicos malvados y estos comités albergan y dirigen convenciones internacionales a las que solamente sus acólitos pueden presentar artículos, independientemente de la calidad de los mismos. Estas comisiones obtienen grandes sumas de dinero público para financiar sus proyectos patrocinados, por medio del engaño y la mentira. Cualquier objeción a las bases científicas de sus propuestas es silenciada por cualquier medio a su disposición, de forma que el dinero pueda continuar fluyendo a las cuentas de sus proyectos, y sean mantenidos en sus empleos bien remunerados. Científicos opuestos a esta praxis han sido despedidos por orden suya; otros han sido impedidos de ocupar posiciones académicas por una red de cómplices corruptos. En otras situaciones, algunos han sido expulsados de su candidatura a programas de educación superior tales como la tesis doctoral, por expresar ideas que minan una teoría de moda, independientemente del tiempo que una teoría ortodoxa pueda tener. El hecho fundamental de que ninguna teoría científica es definitiva e inviolable, y que es entonces susceptible de discutirse y reexaminarse, es ignorado completamente. De esta forma, también ignoran el hecho de que un fenómeno puede tener varias explicaciones plausibles, y maliciosamente desacreditan cualquier explicación que vaya en contra de la ortodoxia, recurriendo sin vacilación al uso de argumentos no científicos para justificar sus opiniones sesgadas.

Todos los científicos deben ser libres de discutir su investigación y la de los demás sin temor de ser ridiculizados sin fundamento en público o en privado, o ser acusados, desacreditados o impugnados de cualquier otra forma mediante alegatos insustanciales. Ningún científico deberá ser puesto en posición de arriesgar su sustento o reputación por expresar una opinión científica. La libertad de expresión científica debe ser lo principal. El uso de la autoridad para refutar un argumento científico no es científico y no se usará para amordazar, suprimir, intimidar, condenar al ostracismo, o ejercer cualquier forma de coacción o supresión contra un científico. La supresión deliberada de hechos o argumentos científicos bien por omisión o bien por acción, y la manipulación deliberada de datos para apoyar un argumento o para desacreditar una idea contraria es un fraude a la Ciencia, es un verdadero crimen científico. Los principios de la evidencia deberán guiar toda discusión científica, sean estos de naturaleza teórica o experimental, o bien una combinación de ambos.

Artículo 8: Libertad de publicar resultados científicos

Una lamentable censura de artículos científicos ha llegado ahora a ser la práctica estándar de los comités editoriales de las principales revistas y archivos electrónicos, y su séquito de alegados revisores expertos. Los revisores son, en su mayoría, protegidos por el anonimato de forma que un autor no pueda verificar su alegada calidad de experto. Los artículos son ahora rutinariamente rechazados si el autor no está de acuerdo o contradice una teoría preferida y la ortodoxia establecida. Muchos artículos se rechazan ahora automáticamente por virtud de la aparición en la lista de autores de un científico particular que no ha encontrado favores con los editores, los revisores u otros censores expertos, sin cualquier consideración acerca del contenido del artículo. Hay una lista negra de científicos disidentes y esta lista se comunica entre las directivas editoriales participantes. Todas estas prácticas amenazan con crecer el sesgo y constituyen una supresión del libre pensamiento, y deben ser condenadas por la comunidad científica internacional.

Todos los científicos deberán tener el derecho de presentar sus resultados científicos, enteros o en parte, en conferencias científicas relevantes, y a publicar los mismos en revistas científicas impresas, archivos electrónicos, y cualquier otro medio. A ningún científico deberá rechazársele sus artículos o informes cuando se les envíe para publicación a las revistas científicas, archivos electrónicos o cualquier otro medio, simplemente porque su trabajo cuestione la opinión actual de la mayoría, entre en conflicto con las ideas de una dirección editorial, mine las bases de otros proyectos actuales o planificados por otros científicos, esté en conflicto con cualquier dogma político o credo religioso, o la opinión personal de otro, y ningún científico será colocado en listas negras o en cualquier otra forma censurado o impedido de publicar por cualquier otra persona sea quien sea. Ningún científico bloqueará, modificará o de otra forma interferirá con la publicación del trabajo de un científico bajo la promesa de recibir cualquier contrapartida o cualquier otro soborno.

Artículo 9: Coautoría de artículos científicos

Es un secreto a voces en los círculos científicos que muchos coautores de artículos de investigación tienen de hecho poco o nada que ver con la investigación referida en su interior. Muchos supervisores de estudiantes graduados, por ejemplo, no son contrarios a poner sus nombres en los artículos escritos por aquellos que sólo en forma nominal trabajan bajo su supervisión. En muchos casos, la persona que en realidad escribe el artículo tiene un intelecto superior al supervisor nominal. En otros casos, de nuevo por propósitos de notoriedad, reputación, dinero, prestigio, y similares, personas que no participan son incluidas en un artículo como coautores. Los autores reales de tales artículos pueden solamente objetar, al riesgo de ser posteriormente penalizados de alguna manera, o incluso expulsados de la candidatura a sus estudios superiores de investigación o del equipo de investigación, según el caso. Muchos han sido de hecho expulsados bajo tales circunstancias. Esta práctica espantosa no debe tolerarse. Sólo aquellos individuos responsables de una investigación deberían ser acreditados como autores.

Ningún científico invitará a otro a ser incluido y ningún científico deberá permitir que su nombre sea incluido como coautor de un artículo científico si no contribuyeron significativamente a la investigación presentada en el artículo. Ningún científico deberá permitir que él o ella mismos sean coaccionados por cualquier representante de una institución académica, corporación, agencia gubernamental, o cualquier otra persona, para incluir su nombre como coautor de la concerniente investigación a la que ellos no contribuyeron significativamente a cambio de contrapartidas u otros sobornos. Ninguna persona deberá inducir o intentar inducir a un científico en cualquier forma para permitir que el nombre del científico sea incluido como coautor de un artículo científico a cuyos temas no contribuyeron de forma significativa.

Artículo 10: Independencia de afiliación

Muchos científicos se contratan a corto plazo. Con el fin del contrato, termina también la afiliación académica. Es a menudo política de las directivas editoriales el no permitir la publicación por parte de personas que no posean afiliación académica o comercial. En ausencia de afiliación, muchos recursos no están disponibles para el científico, y se reducen las posibilidades de presentar charlas y artículos en las conferencias. Esto es una práctica viciosa que debe detenerse. La Ciencia no reconoce afiliaciones de ningún tipo.

Ningún científico será rechazado para presentar artículos en conferencias, coloquios o seminarios, de la publicación en cualquier medio, del acceso a las bibliotecas académicas o publicaciones científicas, de la asistencia a encuentros científicos, o de dar conferencias, por carecer de una afiliación con una institución académica, instituto científico, gobierno o laboratorio comercial, o cualquier otra organización.

Artículo 11: Acceso abierto a la información científica

La mayoría de los libros especializados sobre asuntos científicos y muchas revistas científicas rinden poca o ninguna ganancia de forma que los editores comerciales no están dispuestos a publicarlos sin una contribución de dinero por parte de instituciones académicas, agencias gubernamentales, fundaciones filantrópicas y similares. Bajo estas circunstancias, los editores comerciales deberían permitir el acceso libre a las versiones electrónicas de las publicaciones, y esforzarse por mantener el coste de los materiales impresos a un mínimo.

Todos los científicos se esforzarán en asegurar que sus artículos de investigación estén disponibles para la comunidad científica internacional libre de coste, o de forma alternativa, si no puede evitarse, al mínimo coste. Todos los científicos deberían tomar medidas activas para hacer sus libros técnicos accesibles al precio más bajo posible para que la información científica esté disponible a la más amplia audiencia científica internacional.

Artículo 12: Responsabilidad ética de los científicos

La Historia atestigua que los descubrimientos científicos se usan para ambos extremos, el bien y el mal, para el beneficio de la Humanidad y la destrucción de otros. Ya que el progreso de la Ciencia y la Tecnología no puede parar, deberían establecerse medios para evitar la aplicación maléfica de las mismas. Sólo un gobierno democráticamente elegido, libre de sesgos raciales, religiosos o de cualquier otro tipo, puede salvaguardar la civilización. Sólo comités, tribunales y gobiernos democráticamente elegidos pueden proteger el derecho de la libre creación científica. En la época actual, varios estados no democráticos y regímenes totalitarios conducen una investigación activa en Física Nuclear, Química, Virología, Ingeniería Genética, etcétera, para producir armas químicas, nucleares y biológicas. Ningún científico debería colaborar voluntariamente con estados no democráticos o regímenes totalitarios. Cualquier científico coaccionado a trabajar en el desarrollo de armas para tales estados debería encontrar formas y medios para ralentizar o incluso detener el progreso de programas de investigación y de reducir la producción científica para que la civilización y la democracia puedan finalmente prevalecer.

Todos los científicos adquieren una responsabilidad moral por sus creaciones y descubrimientos científicos. Ningún científico se unirá voluntariamente al diseño o construcción de armas de cualquier tipo para estados no democráticos o regímenes totalitarios o permitirá que sus aptitudes científicas y conocimientos sean aplicados al desarrollo de nada nocivo para la Humanidad. Un científico vivirá bajo el dictado de que todo gobierno no democrático y la violación de los Derechos Humanos son criminales.

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